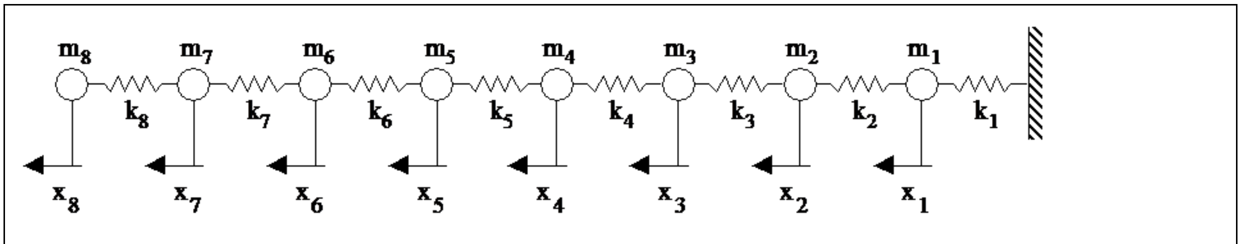
SE 165/265 Homework 4
Assigned April 24th, 2024; Due May 1st, 2024 (11:59 pm Pacific Time)

Assignment Goal: Become familiar with model updating approach to damage detection

1. Extract damage sensitive features based on modal parameters and use those features to update models that will locate and quantify the damage.
2. Reinforce material present in Lectures 7 and 8.

<u>**Task 1:**</u> We will continue to work with the 8 DOF lumped mass system from HW 3. You may use your solutions from the past assignment.

A. Form the mass matrix, m, for 8 degree of freedom (DOF) system shown below. Display your matrix on the command window.



$m_1=m_2\ldots=m_8=0.4194$kg

$k_1=k_2\ldots=k_8=56700$N/m

Note that we will specify the mass matrix such that there is no inertial coupling (off-diagonal terms). Therefore, the mass matrix will only have diagonal elements and these element are identical to the respective lumped mass (i.e. m(1,1)=m(2,2)….=m(8,8)=0.4194 Kg, all off-diagonal terms = 0).

Remember, for an 8 DOF systems the mass matrix will be of dimension 8 rows x 8 columns.

B. Form the stiffness matrix, $[K^u]$, for the 8 DOF system. This will be the stiffness matrix of the undamaged structure. **Display your matrix on the command window.**

C. Now, consider a 10% reduction in the stiffness of spring k_6, between masses 5 and 6. Form the damaged stiffness matrix, $[K^d]$. Display your matrix on the command window.

D. For the DAMAGED structure only, solve for the system eigenvalues (resonant frequencies) and eigenvectors (mode shapes).

- Solve for eigenvalue $[\lambda^d]$and eigenvectors $[\phi^d]$using MATLAB command **eig** such that:

$$[\phi^d,\lambda^d]=eig(-K^d,M)$$

Notice that this time, we will use $-K^d$ inside the eig function. This is to simplify the sorting process for the next steps from Task E onwards.

$[\phi^d]$ will be an 8x8 matrix with each column corresponding to the eigenvectors or modes shapes.
$[\lambda^d]$ will be an 8x8 matrix whose diagonal values are the eigenvalues. Note that these values correspond to the square of the natural frequency, ω, in radians/s, $\lambda_i = \omega_i^2$

- Note the eigenvectors and eigenvalues may have small imaginary components due to finite precision computing. We are only interested in the real portions of the components so use the "*real*" command to eliminate the imaginary parts of $[\phi^d]$ and $[\lambda^d]$.

- Knowing that the diagonal values of $[\lambda^d]$ correspond to the eigenvalues, save them into a new vector $\{\lambda^d\}_{diag}$. Store the matrix $[\lambda^d]$ and the vector $\{\lambda^d\}_{diag}$. We will need both later on.

E. Modal force Error [E]

The motivation for this analysis:

On HW 3, the mode shapes associated with the 8-DOF system were analyzed to investigate the use of mode shapes as damage-sensitive features in a more quantifiable manner. The last plots generated showed the changes in the first and eighth mode shapes, respectively, for the 8-DOF system when a 10% spring stiffness reduction was sequentially introduced at each spring.

When plotted on a scale where the entire mode shape was visible, it appeared that there was little change in these mode shapes when the 10% stiffness reduction is introduced at the various locations. However, percentage changes in mode shape amplitudes for mode 1 range from 0.015% to 8.8%, with the largest percentage change occurring at DOF 1 when the 10% reduction in stiffness has been introduced into spring 1.

For mode 8 the largest changes were observed and varied between 0.33% and 30%, with the largest change occurring at DOF 7, and corresponded to the 10% reduction at spring 7. These examples indicate that in certain cases mode shapes can be successfully used as a damage-sensitive feature, but there is a need to quantify these changes better.

The modal force error equation:

$$\left(\left(-\boldsymbol{\omega}_i^{\boldsymbol{d}}\right)^{\mathbf{2}}[\boldsymbol{M}^{\boldsymbol{u}}] + [\boldsymbol{K}^{\boldsymbol{u}}]\right)\{\boldsymbol{\phi}_i^{\boldsymbol{d}}\} = \{\boldsymbol{E}_i\}$$

where $\{E\}_i$ is defined as the *modal force error* (or *residual force*) for the mode i of the damaged structure. Note that for this problem we are assuming damage has only altered the stiffness matrix, so $[\boldsymbol{M}^{\boldsymbol{u}}]$ is the mass matrix you created in task 1.A.

This vector represents the harmonic force excitation that would have to be applied to the undamaged structure at the frequency ω_i^d so that the structure would respond with mode shape$\{\phi_i^d\}$.

MATLAB tasks:

- Calculate the modal force error $\{E_i\}$ for the first three damaged modal vectors. i.e., use the first equation above for i=1, 2, and 3. Recall that the damaged mode shapes refer to the columns of $[\phi^d]$.
- Create a **bar** plot of each of these 3 modal force error vectors. The x axis corresponds to each degree of freedom and the y axis are the error values.

  Hint: Figure 7.51 of the textbook.

Task F. Adding noise to the data.

The results generated on Task E show that the damage is located between the fifth and sixth masses. However, the data used in this assignment were generated from **numerical** simulation and, as such, have no variability that might be present in **measured** data.

In order to investigate the effects of noise, the mode shapes from the damaged system will be perturbed by Gaussian noise. The noise level was taken as 5% of the maximum mode shape amplitude, which is a realistic level. This noise matrix was randomly generated and will be provided to keep the results consistent for the entire class. Download the .mat file from Canvas> Files>Data & Codes> HW4- Noise data.

The resonance frequencies, which typically have less variance than the mode shape estimates, will be left unperturbed.

MATLAB tasks:

- Load the data file 'noise_matrix.mat'.
- Create a new matrix:$[\phi_{noisy}^d] = [\phi^d] + [Noise]$
- Calculate the modal force error vectors for the first 3 damaged modes.
- For each vector, plot them in a bar plot, as in Task E.
- Create another bar plot, plotting the mean of the 3 error vectors obtained with the noisy data.

Task G. Minimum Rank Update

In task F, it was observed that with noise present, the 10% reduction in spring 6 stiffness can still be identified based on modes 1 and 2, but mode 3 gives results that would be more difficult to properly interpret if there was no previous knowledge of the damage location. The problem is that the update with the noisy mode shape vectors has smeared the stiffness change over the whole stiffness matrix.

In an effort to eliminate the problem of smearing the stiffness changes over the entire stiffness matrix, an approach called the minimum tank perturbation theory (MRTP) was developed. The perturbation error equation for the stiffness matrix can be solved using the MRTP equation as:

$$[\Delta K] = [d][B][d]^T$$

where:

$$[B] = ([d]^T[\phi^d])^{-1}$$

And

$$[d] = [M][\phi^d][\lambda^d] \ + [\boldsymbol{K^u}][\phi^d]$$

MATLAB tasks:

- Calculate the perturbation error $[\Delta K]$ for the damaged data (noise free).
- Using **surf**, generate a 3-d colored surface plot: $surf(x, y, \Delta K)$, where x and y are a vector from 1:8 degrees of freedom.
- Type **view(61,24)** to rotate this plot to a better view.
- Add labels to the axis. Hint: you will create a plot similar to Figure 7.53 on the textbook. It might not look exactly the same because of different Matlab versions and figure renderers being used.
- Repeat all these 4 previous steps and **create a new plot** for the **damaged noisy data** that you created on Task F.