SE 267A HW2

Problem 1

In[∘]:=
```
(* Define the original analog signal and calculate its properties. *)
Print[Style["Angular frequency Ωi (rad/s) for each harmonic signal:", Bold, FontFamily → "Times", FontSize → 14]]
Omega = {2000 π, 6000 π, 12 000 π, 15 000 π, 20 000 π}

Print[Style["Fundamental frequency fpi (Hz) for each harmonic signal:", Bold, FontFamily → "Times", FontSize → 14]]
Frequency = Omega / (2 π)

Print[Style["Fundamental period Tpi (s) for each harmonic signal:", Bold, FontFamily → "Times", FontSize → 14]]
Period = 1 / Frequency

Print[Style["Amplitude Ai for each harmonic signal:", Bold, FontFamily → "Times", FontSize → 14]]
Amplitude = {1, 5, 10, 20, 10}

Print[Style["Phase difference Φi (rad) for each harmonic signal:", Bold, FontFamily → "Times", FontSize → 14]]
Phi = {0, -π / 2, 0, 0, -π / 2}

f1 = Amplitude[[1]] * Cos[Omega[[1]] * t + Phi[[1]]];
f2 = Amplitude[[2]] * Cos[Omega[[2]] * t + Phi[[2]]];
f3 = Amplitude[[3]] * Cos[Omega[[3]] * t + Phi[[3]]];
f4 = Amplitude[[4]] * Cos[Omega[[4]] * t + Phi[[4]]];
f5 = Amplitude[[5]] * Cos[Omega[[5]] * t + Phi[[5]]];

Print[Style["Original analog signal: ", Bold, FontFamily → "Times", FontSize → 14],
 Style["f(t)", Italic, Bold, FontFamily → "Times", FontSize → 14]]
f = f1 + f2 + f3 + f4 + f5;
f /. Sin[var_] :→ HoldForm[Cos[var - π / 2]]
```

```
(* Calculate the fundamental time period of the original analog signal. *)
Print[Style["The fundamental time period Tp (second) of the original analog signal:",
  Bold, FontFamily → "Times", FontSize → 14]]
Tp = LCM[Period[[1]], Period[[2]], Period[[3]], Period[[4]], Period[[5]]]

(* Calculate the intergration for Dirichlet conditions of convergence. *)
Print[Style["Integration result for Dirichlet conditions of convergence:", Bold, FontFamily → "Times", FontSize → 14]]
NIntegrate[Abs[f], {t, -Tp / 2, Tp / 2}]
```

Angular frequency Ω_i (rad/s) for each harmonic signal:

Out[∘]=

$\{2000\pi, 6000\pi, 12\,000\pi, 15\,000\pi, 20\,000\pi\}$

Fundamental frequency f_{pi} (Hz) for each harmonic signal:

Out[∘]=

$\{1000, 3000, 6000, 7500, 10\,000\}$

Fundamental period T_{pi} (s) for each harmonic signal:

Out[∘]=

$\left\{\frac{1}{1000}, \frac{1}{3000}, \frac{1}{6000}, \frac{1}{7500}, \frac{1}{10\,000}\right\}$

Amplitude A_i for each harmonic signal:

Out[∘]=

$\{1, 5, 10, 20, 10\}$

Phase difference Φ_i (rad) for each harmonic signal:

Out[∘]=

$\left\{0, -\frac{\pi}{2}, 0, 0, -\frac{\pi}{2}\right\}$

Original analog signal: $f(t)$

Out[∘]=

$\text{Cos}[2000\pi t] + 10\,\text{Cos}[12\,000\pi t] + 20\,\text{Cos}[15\,000\pi t] + 5\,\text{Cos}\left[6000\pi t - \frac{\pi}{2}\right] + 10\,\text{Cos}\left[20\,000\pi t - \frac{\pi}{2}\right]$

The fundamental time period T_p (second) of the original analog signal:

Out[ ]=

$$\frac{1}{500}$$

Integration result for Dirichlet conditions of convergence:

Out[ ]=

0.029185

The function is absolutely integrable over its fundamental period. Also, this is a continuous function and it contains a finite number of maximum and minimum values over its fundamental period. So Dirichlet conditions of convergence hold for this function. The Fourier coefficients exist and the function can be represented by Fourier series.

Problem 2

In[∘]:=
```
(* Calculate the fundamental frequency of the original analog signal. *)
Print[
 Style["The fundamental frequency f_p (Hz) of the original analog signal:", Bold, FontFamily → "Times", FontSize → 14]]
fp = 1 / Tp

(* Obtain the maximum frequency of the original analog signal. *)
Print[Style["The maximum frequency f_p,max (Hz) of the original analog signal:", Bold, FontFamily → "Times", FontSize → 14]]
MaxFrequency = Max[Frequency]

(* Calculate the maximum index for the exact representation using Fourier series. *)
Print[Style["The maximum index m_max for the exact representation using Fourier series:",
  Bold, FontFamily → "Times", FontSize → 14]]
MaxIndex = MaxFrequency / fp
```

The fundamental frequency f_p (Hz) of the original analog signal:

Out[∘]=

500

The maximum frequency $f_{p,max}$ (Hz) of the original analog signal:

Out[∘]=

10 000

The maximum index m_{max} for the exact representation using Fourier series:

Out[∘]=

20

Problem 3

```
In[◦]:= m = Range[1, MaxIndex]; (* Index for the harmonics in the Fourier serires. *)
fm = m * fp; (* Frequency. *)
OmegaM = 2 π * fm; (* Angular frequency. *)
Fm = Integrate[f * Exp[-I * OmegaM * t], {t, -Tp / 2, Tp / 2}] / Tp; (* Fourier coefficients. *)
FmMagnitude = Abs[Fm]; (* Magnitude of Fourier coefficients. *)
theta = Arg[Fm]; (* Phase of Fourier coefficients. *)
PowerDensity = FmMagnitude^2; (* Power dentsity spectrum. *)

(* List a table for the Fourier coefficients. *)
Title = {"Harmonic index m", "Frequency fm", "Angular frequency Ωm",
   "Magnitude |Fm|", "Phase θm ", "Power density spectrum", "Fourier coefficient Fm"};
Data = Transpose[{m, fm, OmegaM, FmMagnitude, theta, PowerDensity, Fm}];
AppendData = Prepend[Data, Title];
StyledData = Map[Style[#, FontFamily → "Times", FontSize → 8, FontWeight → Bold] &, AppendData, {2}];
Grid[StyledData, Frame → All, ItemSize → {8, 1}, Alignment → Center]
```

Out[ ]=

Harmonic index m	Frequency f_m	Angular frequency Ω_m	Magnitude $\lvert F_m \rvert$	Phase θ_m	Power density spectrum	Fourier coefficient F_m
1	500	1000π	0	0	0	0
2	1000	2000π	$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{2}$
3	1500	3000π	0	0	0	0
4	2000	4000π	0	0	0	0
5	2500	5000π	0	0	0	0
6	3000	6000π	$\frac{5}{2}$	$-\frac{\pi}{2}$	$\frac{25}{4}$	$-\frac{5i}{2}$
7	3500	7000π	0	0	0	0
8	4000	8000π	0	0	0	0
9	4500	9000π	0	0	0	0
10	5000	$10\,000\pi$	0	0	0	0
11	5500	$11\,000\pi$	0	0	0	0
12	6000	$12\,000\pi$	5	0	25	5
13	6500	$13\,000\pi$	0	0	0	0
14	7000	$14\,000\pi$	0	0	0	0
15	7500	$15\,000\pi$	10	0	100	10
16	8000	$16\,000\pi$	0	0	0	0
17	8500	$17\,000\pi$	0	0	0	0
18	9000	$18\,000\pi$	0	0	0	0
19	9500	$19\,000\pi$	0	0	0	0
20	10 000	$20\,000\pi$	5	$-\frac{\pi}{2}$	25	$-5i$

Problem 4

In[ ]:=
```
m = Range[-30, 30]; (* Index for the harmonics in the Fourier serires. *)
fm = m * fp; (* Frequency. *)
OmegaM = 2 π * fm; (* Angular frequency. *)
Fm = Integrate[f * Exp[-I * OmegaM * t], {t, -Tp / 2, Tp / 2}] / Tp; (* Fourier coefficients. *)
FmMagnitude = Abs[Fm]; (* Magnitude of Fourier coefficients. *)
theta = Arg[Fm]; (* Phase of Fourier coefficients. *)
PowerDensity = FmMagnitude^2; (* Power dentsity spectrum. *)

(* List a table for the Fourier coefficients. *)
Title = {"Harmonic index m", "Frequency f_m", "Angular frequency Ω_m",
   "Magnitude |F_m|", "Phase θ_m ", "Power density spectrum", "Fourier coefficient F_m"};
Data = Transpose[{m, fm, OmegaM, FmMagnitude, theta, PowerDensity, Fm}];
AppendData = Prepend[Data, Title];
StyledData = Map[Style[#, FontFamily → "Times", FontSize → 8, FontWeight → Bold] &, AppendData, {2}];
Grid[StyledData, Frame → All, ItemSize → {8, 1}, Alignment → Center]
```

Out[ ]=

Harmonic index m	Frequency f_m	Angular frequency Ω_m	Magnitude $\lvert F_m \rvert$	Phase θ_m	Power density spectrum	Fourier coefficient F_m
−30	−15 000	$-30\,000\,\pi$	0	0	0	0
−29	−14 500	$-29\,000\,\pi$	0	0	0	0
−28	−14 000	$-28\,000\,\pi$	0	0	0	0
−27	−13 500	$-27\,000\,\pi$	0	0	0	0
−26	−13 000	$-26\,000\,\pi$	0	0	0	0
−25	−12 500	$-25\,000\,\pi$	0	0	0	0
−24	−12 000	$-24\,000\,\pi$	0	0	0	0
−23	−11 500	$-23\,000\,\pi$	0	0	0	0
−22	−11 000	$-22\,000\,\pi$	0	0	0	0
−21	−10 500	$-21\,000\,\pi$	0	0	0	0
−20	−10 000	$-20\,000\,\pi$	5	$\frac{\pi}{2}$	25	$5\,i$
−19	−9500	$-19\,000\,\pi$	0	0	0	0
−18	−9000	$-18\,000\,\pi$	0	0	0	0
−17	−8500	$-17\,000\,\pi$	0	0	0	0

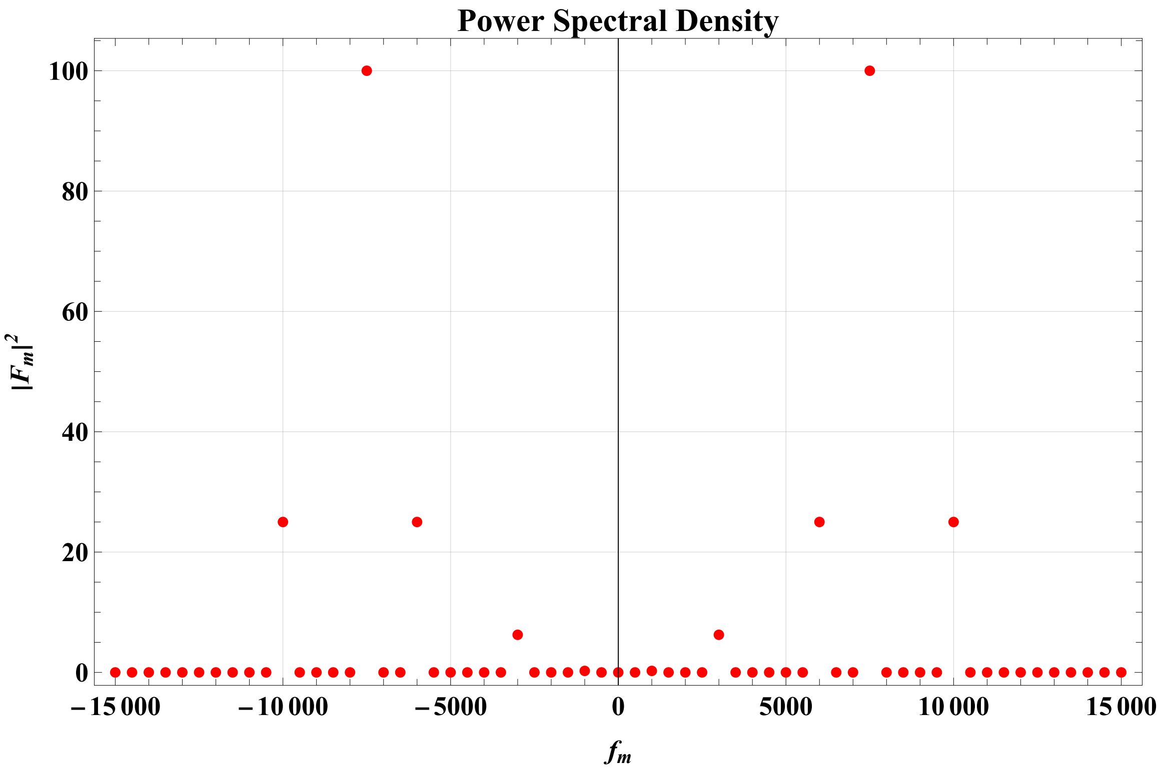
−16	−8000	-16000π	0	0	0	0
−15	−7500	-15000π	10	0	100	10
−14	−7000	-14000π	0	0	0	0
−13	−6500	-13000π	0	0	0	0
−12	−6000	-12000π	5	0	25	5
−11	−5500	-11000π	0	0	0	0
−10	−5000	-10000π	0	0	0	0
−9	−4500	-9000π	0	0	0	0
−8	−4000	-8000π	0	0	0	0
−7	−3500	-7000π	0	0	0	0
−6	−3000	-6000π	$\frac{5}{2}$	$\frac{\pi}{2}$	$\frac{25}{4}$	$\frac{5i}{2}$
−5	−2500	-5000π	0	0	0	0
−4	−2000	-4000π	0	0	0	0
−3	−1500	-3000π	0	0	0	0
−2	−1000	-2000π	$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{2}$
−1	−500	-1000π	0	0	0	0
0	0	0	0	0	0	0
1	500	1000π	0	0	0	0
2	1000	2000π	$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{2}$
3	1500	3000π	0	0	0	0
4	2000	4000π	0	0	0	0
5	2500	5000π	0	0	0	0
6	3000	6000π	$\frac{5}{2}$	$-\frac{\pi}{2}$	$\frac{25}{4}$	$-\frac{5i}{2}$
7	3500	7000π	0	0	0	0
8	4000	8000π	0	0	0	0
9	4500	9000π	0	0	0	0
10	5000	10000π	0	0	0	0
11	5500	11000π	0	0	0	0
12	6000	12000π	5	0	25	5
13	6500	13000π	0	0	0	0
14	7000	14000π	0	0	0	0
15	7500	15000π	10	0	100	10
16	8000	16000π	0	0	0	0
17	8500	17000π	0	0	0	0
18	9000	18000π	0	0	0	0

19	9500	19 000 π	0	0	0	0
20	10 000	20 000 π	5	$-\frac{\pi}{2}$	25	−5 i
21	10 500	21 000 π	0	0	0	0
22	11 000	22 000 π	0	0	0	0
23	11 500	23 000 π	0	0	0	0
24	12 000	24 000 π	0	0	0	0
25	12 500	25 000 π	0	0	0	0
26	13 000	26 000 π	0	0	0	0
27	13 500	27 000 π	0	0	0	0
28	14 000	28 000 π	0	0	0	0
29	14 500	29 000 π	0	0	0	0
30	15 000	30 000 π	0	0	0	0

Problem 5

In[ ]:=
```
(* Plot the power density spectrum. *)
PowerSpectral = Transpose[{fm, PowerDensity}];
Show[ListPlot[PowerSpectral, ImageSize → {600, 400}, PlotStyle → {Directive[PointSize[0.01], Red]},
  FrameLabel → {Style["fm", Italic], Style[Superscript["|Fm|", 2], Italic]}, PlotLabel → "Power Spectral Density",
  GridLines → Automatic, LabelStyle → {RGBColor[0, 0, 0], Bold, FontSize → 14, FontFamily → "Times"} , Frame → True]]
```

Out[ ]=


Power Spectral Density
$|F_m|^2$
100
80
60
40
20
0
−15 000
−10 000
−5000
0
5000
10 000
15 000
f_m

In[∘]:=
```
(* Check for Parseval's theorem. *)
Print[Style["Calculation of total average power using f(t):", Bold, FontFamily → "Times", FontSize → 14]]
Pf1 = NIntegrate[Abs[f]^2, {t, -Tp / 2, Tp / 2}] / Tp
Print[Style["Calculation of total average power using Fm:", Bold, FontFamily → "Times", FontSize → 14]]
Pf2 = Total[FmMagnitude^2]
```

Calculation of total average power using f(t):

Out[∘]=

313.

Calculation of total average power using F_m:

Out[∘]=

313

The Parseval's equality holds for this signal. It represents that the total average power in the periodic signal equals to the sum of the average powers in all the harmonics constructing the original signal.

Problem 6

In[∘]:=
```
(* Calculate the reconstructed signal. *)
Print[Style["The reconstructed signal:", Bold, FontFamily → "Times", FontSize → 14]]
ReconstructedSignal = Total[Fm * Exp[I * OmegaM * t]]
ExpToTrig[ReconstructedSignal] /. Sin[var_] :→ HoldForm[Cos[var - π / 2]]

(* Plot the original analog signal and the reconstructed signal. *)
Show[Plot[f, {t, 0, 0.0005}, ImageSize → {600, 400}, PlotStyle → {Blue, Thickness[0.01]},
  PlotLegends → {Style["Original Analog Signal", Bold, FontSize → 14, FontFamily → "Times"]},
  FrameLabel → {Style["t", Italic], Style["f(t)", Italic]},
  PlotLabel → "Original analog signal and its Fourier series approximation", GridLines → Automatic,
  LabelStyle → {RGBColor[0, 0, 0], Bold, FontSize → 14, FontFamily → "Times"}, Frame → True],
 Plot[ReconstructedSignal, {t, 0, 0.0005}, PlotStyle → {Red, Thickness[0.005]},
  PlotLegends → {Style["Fourier Series Approximation", Bold, FontSize → 14, FontFamily → "Times"]}]]
```

The reconstructed signal:

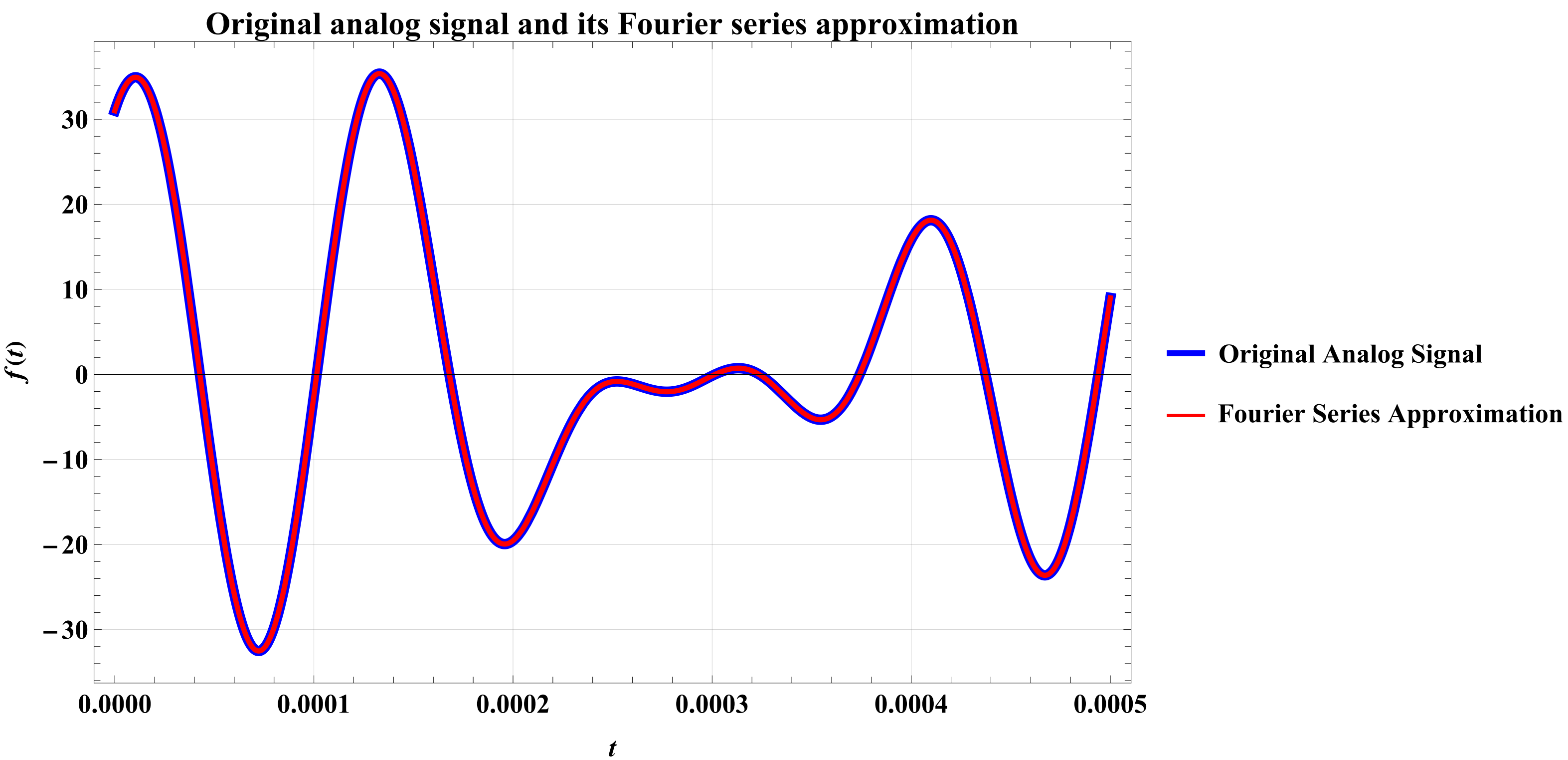
Out[∘]=

$$\frac{1}{2} e^{-2000 i \pi t}+\frac{1}{2} e^{2000 i \pi t}+\frac{5}{2} i e^{-6000 i \pi t}-\frac{5}{2} i e^{6000 i \pi t}+5 e^{-12000 i \pi t}+5 e^{12000 i \pi t}+10 e^{-15000 i \pi t}+10 e^{15000 i \pi t}+5 i e^{-20000 i \pi t}-5 i e^{20000 i \pi t}$$

Out[∘]=

$$\operatorname{Cos}[2000 \pi t]+10 \operatorname{Cos}[12000 \pi t]+20 \operatorname{Cos}[15000 \pi t]+5 \operatorname{Cos}\left[6000 \pi t-\frac{\pi}{2}\right]+10 \operatorname{Cos}\left[20000 \pi t-\frac{\pi}{2}\right]$$

Out[∘]=



Problem 7

In[25]:=
```
(*  Clear all the previous variables. *)
ClearAll["Global`*"];

(* Define the aperiodic function. *)
Print[Style["The aperiodic function:", Bold, FontFamily → "Times", FontSize → 14]]
f = Piecewise[{{0, t < 0},
   {Cos[2000 π t] + 5 Sin[6000 π t] + 10 Cos[12 000 π t] + 20 Cos[15 000 π t] + 10 Sin[20 000 π t], 0 ≤ t ≤ 0.002}, {0, t > 0.002}}]

(* Calculate the Fourier coefficient. *)
Print[Style["The Fourier coefficient:", Bold, FontFamily → "Times", FontSize → 14]]
F = Integrate[f * Exp[-I * 2 π * k * t], {t, 0, 0.002}]
F2 = NIntegrate[f * Exp[-I * 2 π * k * t], {t, 0, 0.002}];

(* Calculate engergy and phase spectrum. *)
EnergySpectralDensity = Abs[F2]^2;
PhaseSpectralDensity = Arg[F2];

(* Plot the power density spectrum. *)
Show[Plot[EnergySpectralDensity, {k, -20 000, 20 000}, ImageSize → {600, 400}, PlotStyle → {Blue, Thickness[0.003]},
  PlotRange → All, PlotLegends → {Style["Energy Spectral Density", Bold, FontSize → 14, FontFamily → "Times"]},
  FrameLabel → {Style["f", Italic], Style[Superscript["|Ff|", 2], Italic]}, PlotLabel → "Energy Spectral Density",
  GridLines → Automatic, LabelStyle → {RGBColor[0, 0, 0], Bold, FontSize → 14, FontFamily → "Times"}, Frame → True]]

(* Plot the phase spectrum. *)
Show[Plot[PhaseSpectralDensity, {k, -20 000, 20 000}, ImageSize → {600, 400}, PlotStyle → {Blue, Thickness[0.003]},
  PlotRange → All, PlotLegends → {Style["Phase Spectral Density", Bold, FontSize → 14, FontFamily → "Times"]},
  FrameLabel → {Style["f", Italic], Style[θ, Italic]}, PlotLabel → "Phase Spectral Density",
  GridLines → Automatic, LabelStyle → {RGBColor[0, 0, 0], Bold, FontSize → 14, FontFamily → "Times"}, Frame → True]]
```

The aperiodic function:

Out[•]=

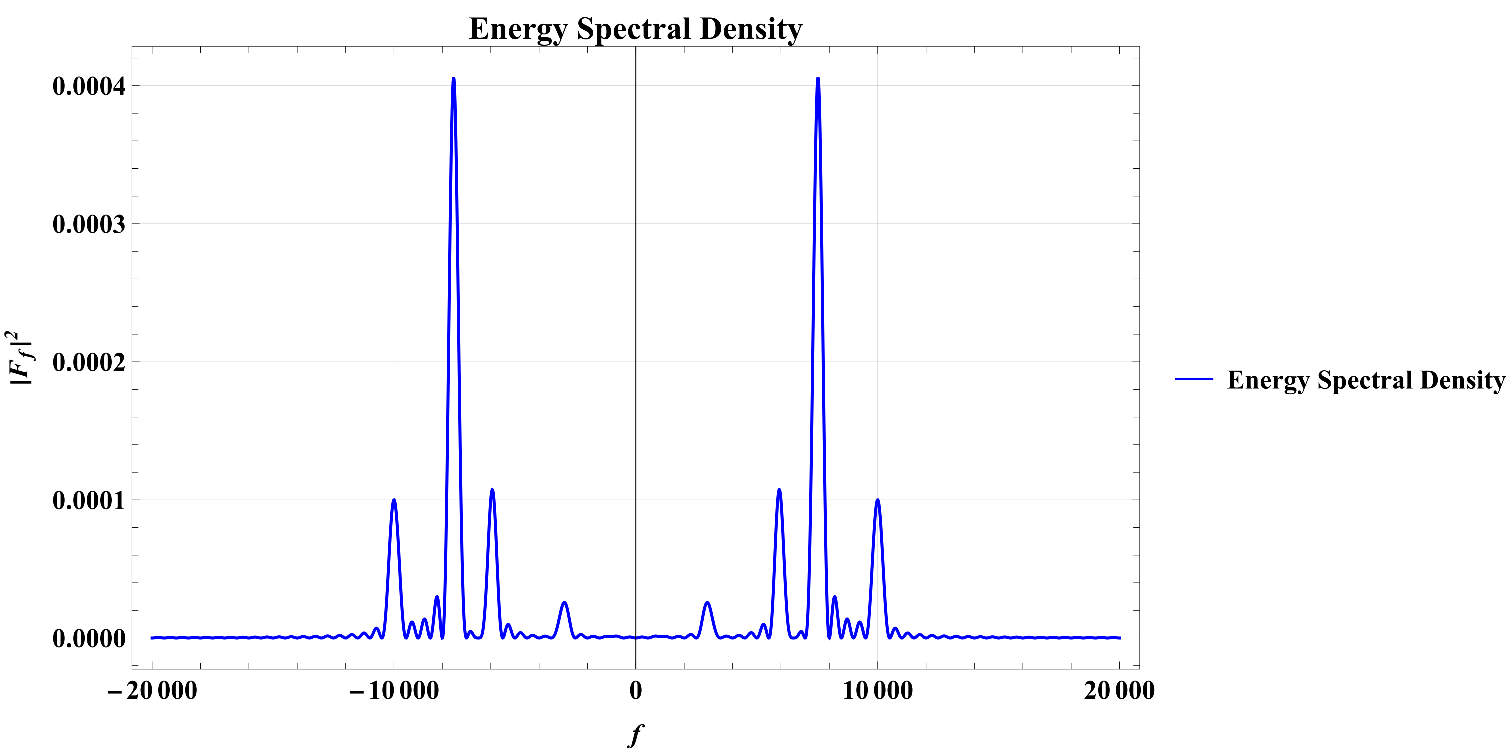
$$\begin{cases} 0 & t < 0 \\ \text{Cos}[2000\,\pi\,t] + 10\,\text{Cos}[12\,000\,\pi\,t] + 20\,\text{Cos}[15\,000\,\pi\,t] + 5\,\text{Sin}[6000\,\pi\,t] + 10\,\text{Sin}[20\,000\,\pi\,t] & 0 \le t \le 0.002 \\ 0 & \text{True} \end{cases}$$

The Fourier coefficient:

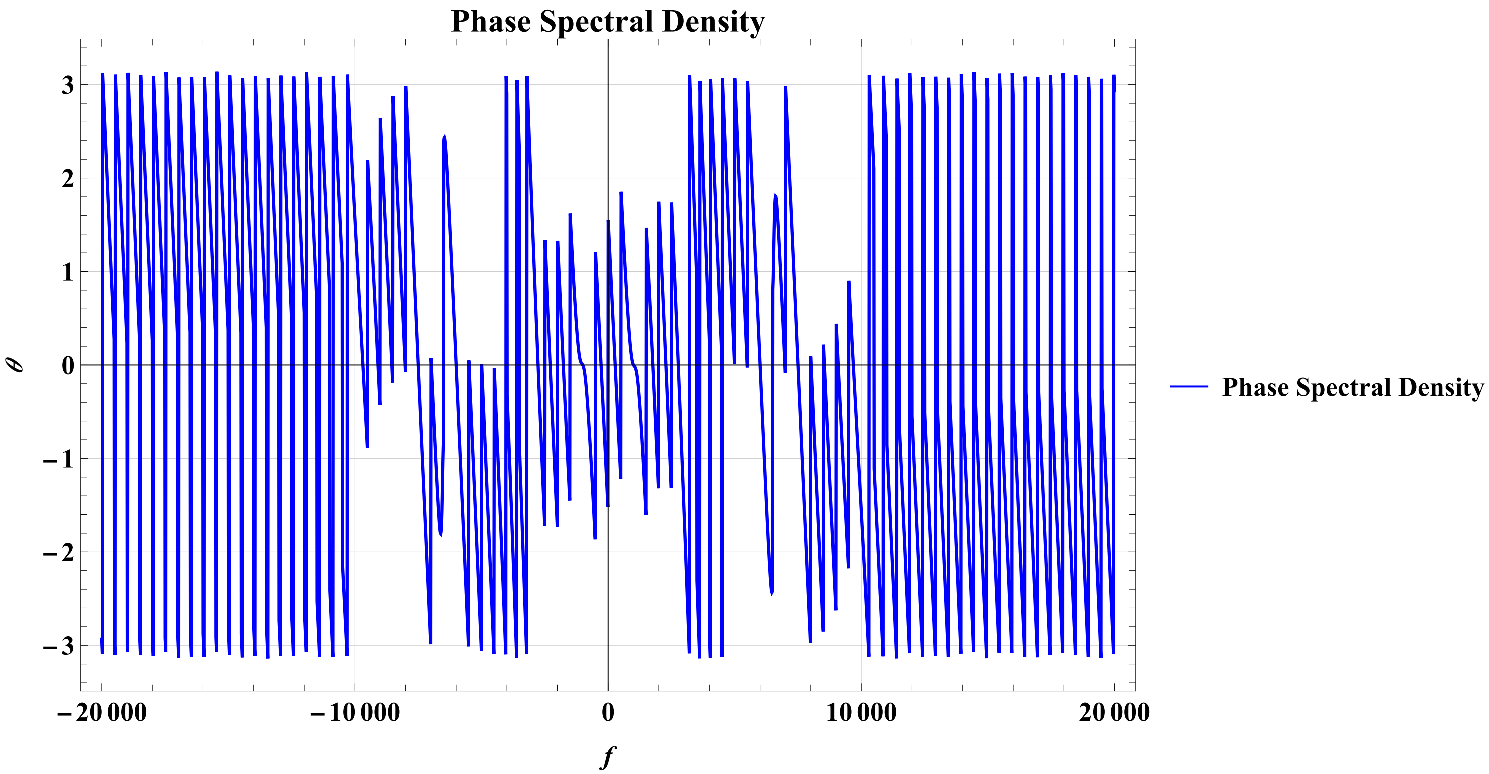
Out[•]=

$$\left(0.159155\, e^{(0.-0.0125664\, i)\, k} \left(-1. + e^{(0.+0.0125664\, i)\, k}\right) \left(-4.86\times10^{33} - \left(0. + 2.97675\times10^{30}\, i\right) k + 5.31428\times10^{27}\, k^2 + \left(0. + 1.62479\times10^{24}\, i\right) k^3 - 4.67284\times10^{20}\, k^4 - \left(0. + 1.84325\times10^{17}\, i\right) k^5 + 1.31238\times10^{13}\, k^6 + \left(0. + 4.78375\times10^{9}\, i\right) k^7 - 115\,000.\, k^8 - (0. + 31.\, i)\, k^9\right)\right) \Big/ \left(\left(-1.\times10^{8} + k^2\right) \left(-5.625\times10^{7} + k^2\right) \left(-3.6\times10^{7} + k^2\right) \left(-9.\times10^{6} + k^2\right) \left(-1.\times10^{6} + k^2\right)\right)$$

Out[•]=



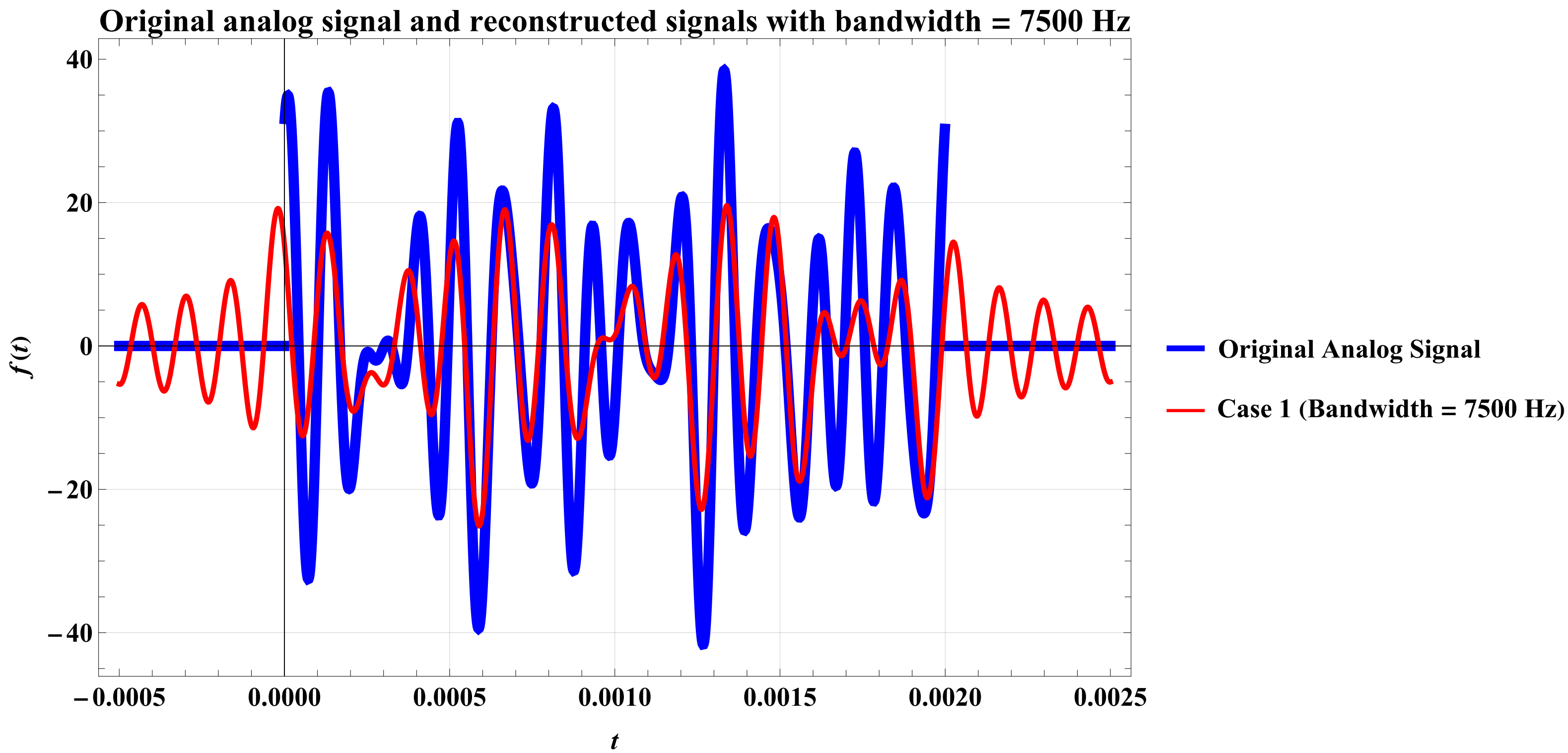
Out[ ]=


Phase Spectral Density
3
2
1
0
-1
-2
-3
θ
-20 000
-10 000
0
10 000
20 000
f
Phase Spectral Density

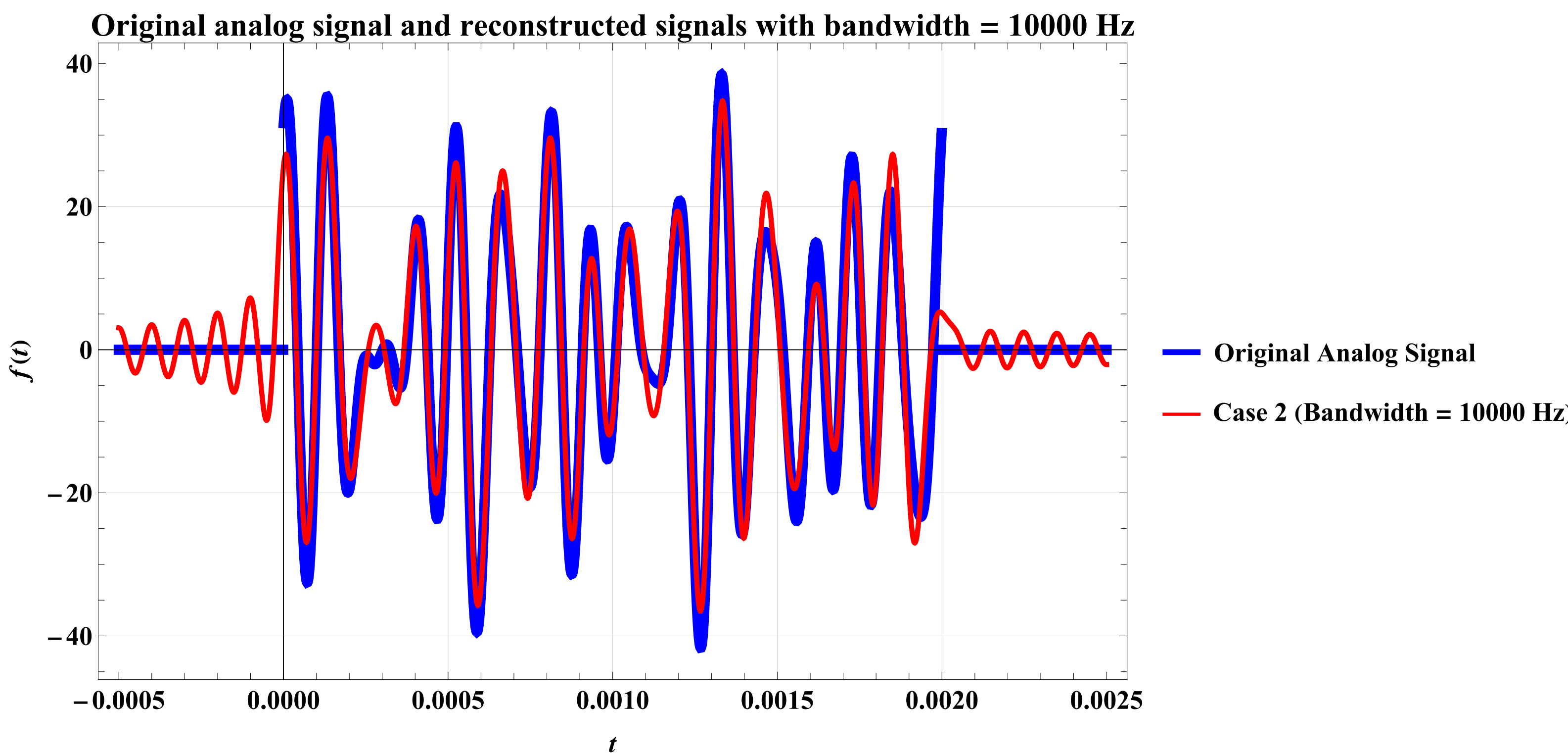
Problem 8

In[*]:=
```
(* Calculate the reconstructed signal. *)
ReconstructedSignal1 = NIntegrate[F * Exp[I * 2 π * k * t], {k, -7500, 7500}];
ReconstructedSignal2 = NIntegrate[F * Exp[I * 2 π * k * t], {k, -10000, 10000}];
ReconstructedSignal3 = NIntegrate[F * Exp[I * 2 π * k * t], {k, -20000, 20000}];

(* Plot the original analog signal and the reconstructed signals. *)
Show[Plot[f, {t, -0.0005, 0.0025}, ImageSize → {600, 400}, PlotStyle → {Blue, Thickness[0.01]},
  PlotLegends → {Style["Original Analog Signal", Bold, FontSize → 14, FontFamily → "Times"]},
  FrameLabel → {Style["t", Italic], Style["f(t)", Italic]},
  PlotLabel → "Original analog signal and reconstructed signals with bandwidth = 7500 Hz", GridLines → Automatic,
  LabelStyle → {RGBColor[0, 0, 0], Bold, FontSize → 14, FontFamily → "Times"}, Frame → True],
 Plot[ReconstructedSignal1, {t, -0.0005, 0.0025}, PlotStyle → {Red, Thickness[0.005]},
  PlotLegends → {Style["Case 1 (Bandwidth = 7500 Hz)", Bold, FontSize → 14, FontFamily → "Times"]}]]

Show[Plot[f, {t, -0.0005, 0.0025}, ImageSize → {600, 400}, PlotStyle → {Blue, Thickness[0.01]},
  PlotLegends → {Style["Original Analog Signal", Bold, FontSize → 14, FontFamily → "Times"]},
  FrameLabel → {Style["t", Italic], Style["f(t)", Italic]},
  PlotLabel → "Original analog signal and reconstructed signals with bandwidth = 10000 Hz", GridLines → Automatic,
  LabelStyle → {RGBColor[0, 0, 0], Bold, FontSize → 14, FontFamily → "Times"}, Frame → True],
 Plot[ReconstructedSignal2, {t, -0.0005, 0.0025}, PlotStyle → {Red, Thickness[0.005]},
  PlotLegends → {Style["Case 2 (Bandwidth = 10000 Hz)", Bold, FontSize → 14, FontFamily → "Times"]}]]

Show[Plot[f, {t, -0.0005, 0.0025}, ImageSize → {600, 400}, PlotStyle → {Blue, Thickness[0.01]},
  PlotLegends → {Style["Original Analog Signal", Bold, FontSize → 14, FontFamily → "Times"]},
  FrameLabel → {Style["t", Italic], Style["f(t)", Italic]},
  PlotLabel → "Original analog signal and reconstructed signals with bandwidth = 20000 Hz", GridLines → Automatic,
  LabelStyle → {RGBColor[0, 0, 0], Bold, FontSize → 14, FontFamily → "Times"}, Frame → True],
 Plot[ReconstructedSignal3, {t, -0.0005, 0.0025}, PlotStyle → {Red, Thickness[0.005]},
  PlotLegends → {Style["Case 3 (Bandwidth = 20000 Hz)", Bold, FontSize → 14, FontFamily → "Times"]}]]
```

Out[ ]=

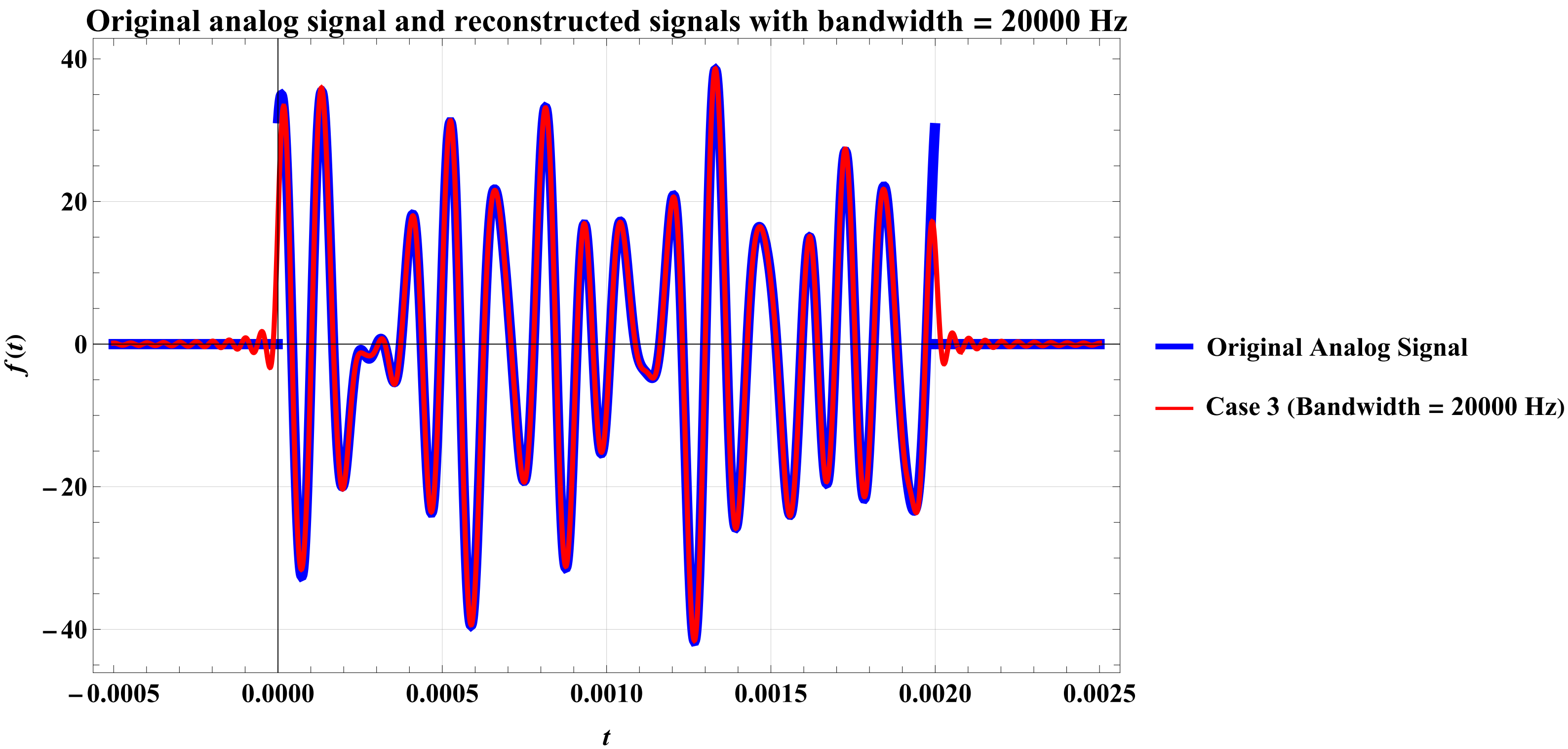

Original analog signal and reconstructed signals with bandwidth = 7500 Hz
40
20
0
−20
−40
f(t)
−0.0005
0.0000
0.0005
0.0010
0.0015
0.0020
0.0025
t
Original Analog Signal
Case 1 (Bandwidth = 7500 Hz)

Out[ ]=



Original analog signal and reconstructed signals with bandwidth = 10000 Hz
40
20
0
-20
-40
f(t)
-0.0005
0.0000
0.0005
0.0010
0.0015
0.0020
0.0025
t
Original Analog Signal
Case 2 (Bandwidth = 10000 Hz)

Out[*]=



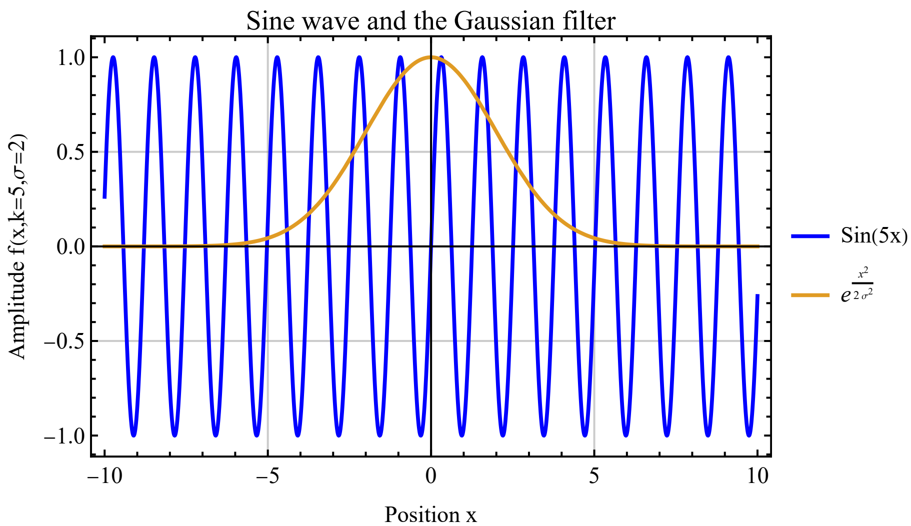
As the bandwidth of frequency increases, more frequency components are included in the reconstruction and the reconstructed signal is closer to the original analog signal. However, because the original analog signal is not continuous, small vibration is observed outside the region between 0 and 0.002 second (Gibbs phenomenon).

Problem 9

In[2]:=
```
ClearAll["Global`*"];
(*Here I define the signal as a dynamic function*)
f[x_, k_, σ_] := Sin[k * x] * Exp[- ((x - 0) ^2 / (2 σ^2))];
```

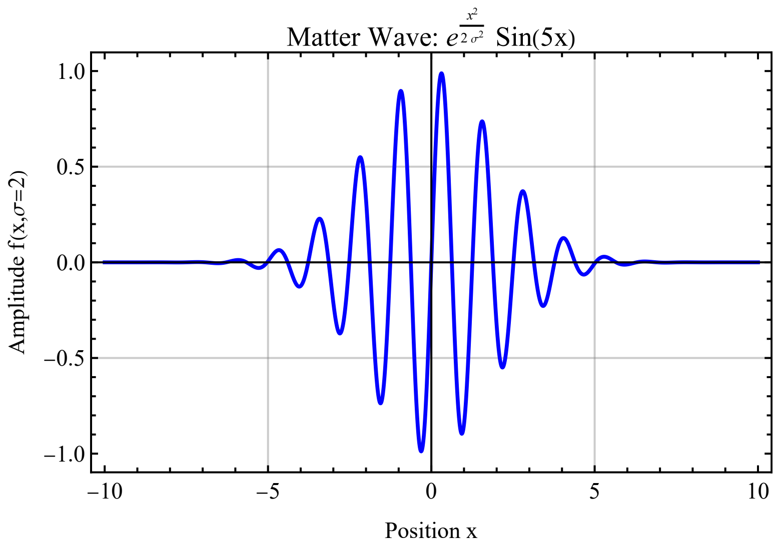
In[4]:=
```
(*Here is how the sine and the Gaussian filter looks like separately*)
Plot[{Sin[5 * x], Exp[- ((x - 0) ^2 / (2 × 2^2))]}, {x, -10, 10}, PlotRange → All,
 PlotStyle → {Blue, Thick}, FrameLabel → {"Position x", "Amplitude f(x,k=5,σ=2)"},
 PlotLabel → "Sine wave and the Gaussian filter", PlotLegends → {"Sin(5x)", "e^(x^2/(2σ^2))"},
 ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
 GridLines → Automatic, Frame → True, PlotRange → All]
```

Out[4]=



In[5]:=
```
(*Here is how the function e^(x²/(2σ²)) Sin(5x) looks like*)
Plot[f[x, 5, 2], {x, -10, 10}, PlotRange → All,
 PlotStyle → {Blue, Thick}, FrameLabel → {"Position x", "Amplitude f(x,σ=2)"},
 PlotLabel → "Matter Wave: e^(x²/(2σ²)) Sin(5x)", ImageSize → 400,
 LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
 GridLines → Automatic, Frame → True, PlotRange → All]
```

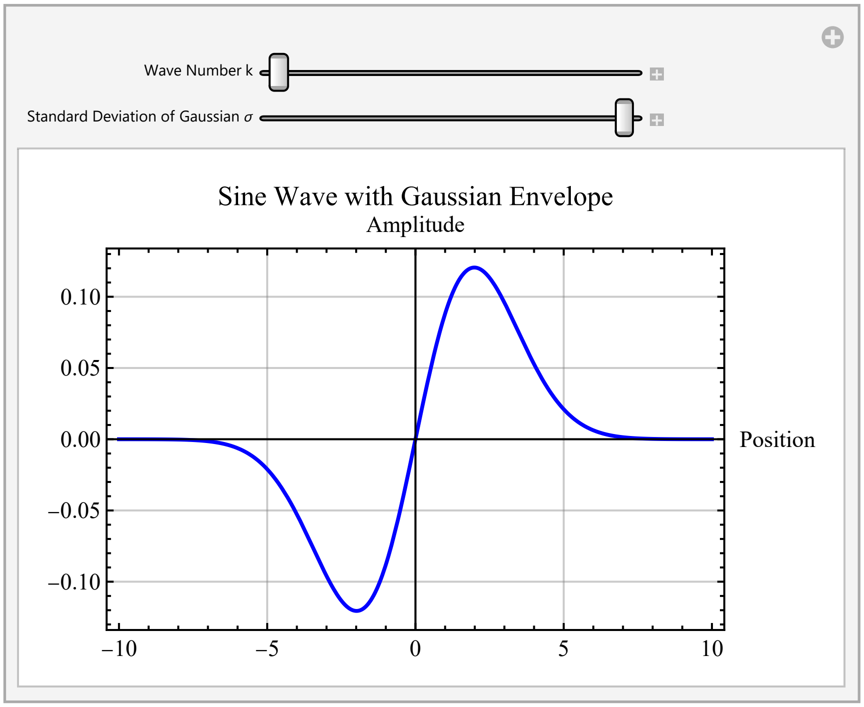
Out[5]=



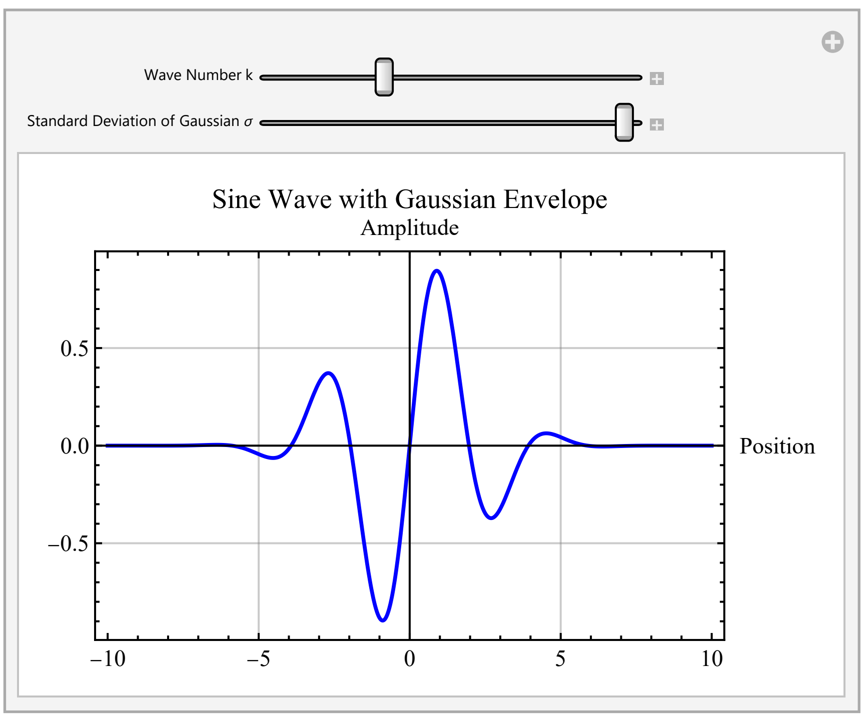
Problem 9 - Part 1 (Effect of the wave number *k*)

In[ ]:=
```
(*PART 9 of ASSINGMENT2:*)
(*I use Manipulate command to visually see how the
 function changes for different values of the wave number k and σ*)
Manipulate[Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue, Thick},
  AxesLabel → {"Position", "Amplitude"}, PlotLabel → "Sine Wave with Gaussian Envelope",
  ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
  GridLines → Automatic, Frame → True, PlotRange → All], {{k, 1, "Wave Number k"},
  0.1, 5, 0.1}, {{σ, 1, "Standard Deviation of Gaussian σ"}, 0.5, 2, 0.1}]
Manipulate[Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue, Thick},
  AxesLabel → {"Position", "Amplitude"}, PlotLabel → "Sine Wave with Gaussian Envelope",
  ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
  GridLines → Automatic, Frame → True, PlotRange → All],
 {{k, 1, "Wave Number k"}, 0.1, 5, 0.1},
 {{σ, 1, "Standard Deviation of Gaussian σ"}, 0.5, 2, 0.1}]
Manipulate[Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue, Thick},
  AxesLabel → {"Position", "Amplitude"}, PlotLabel → "Sine Wave with Gaussian Envelope",
  ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
  GridLines → Automatic, Frame → True, PlotRange → All],
 {{k, 1, "Wave Number k"}, 0.1, 5, 0.1},
 {{σ, 1, "Standard Deviation of Gaussian σ"}, 0.5, 2, 0.1}]
```

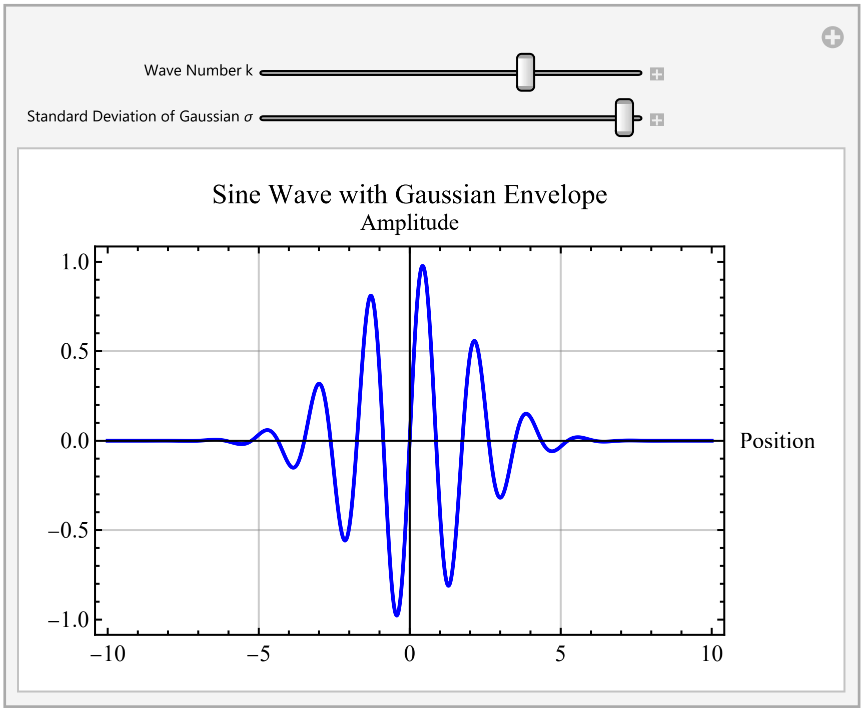
Out[ ]=


Wave Number k
Standard Deviation of Gaussian σ
Sine Wave with Gaussian Envelope
Amplitude
0.10
0.05
0.00
−0.05
−0.10
Position
−10
−5
0
5
10

Out[ ]=


Wave Number k
Standard Deviation of Gaussian σ
Sine Wave with Gaussian Envelope
Amplitude
0.5
0.0
−0.5
Position
−10
−5
0
5
10

Out[ ]=



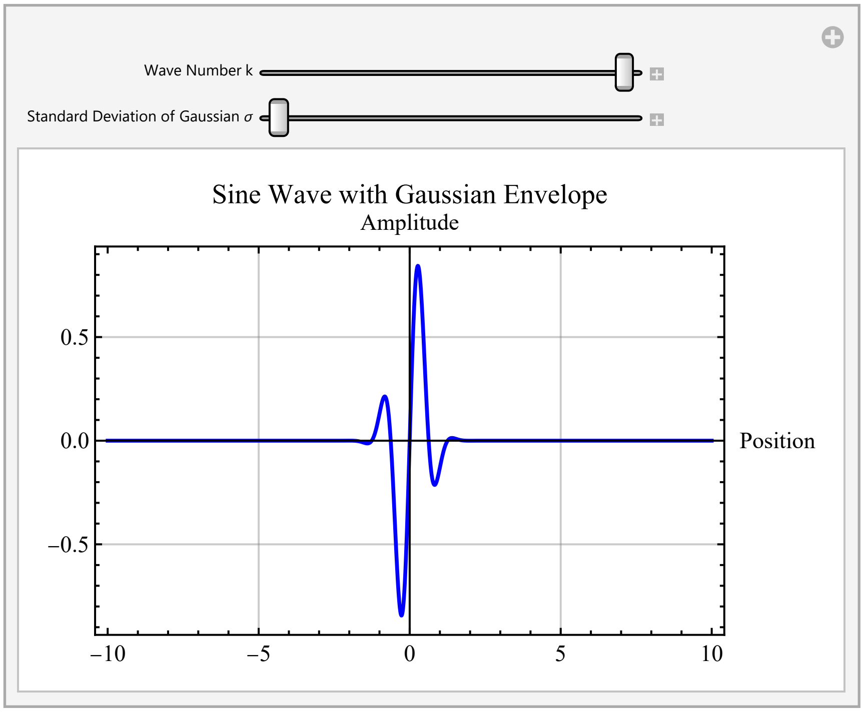
As we can see from the plot, the number of the wave included in the plot increases as the wave number *k* increases. This is because the wave number *k* represents the frequency of the sine wave signal. As the frequency increases, more cycles will be included in a certain time period.

Problem 9 - Part 2 (Effect of the standard deviation σ of Gaussian)

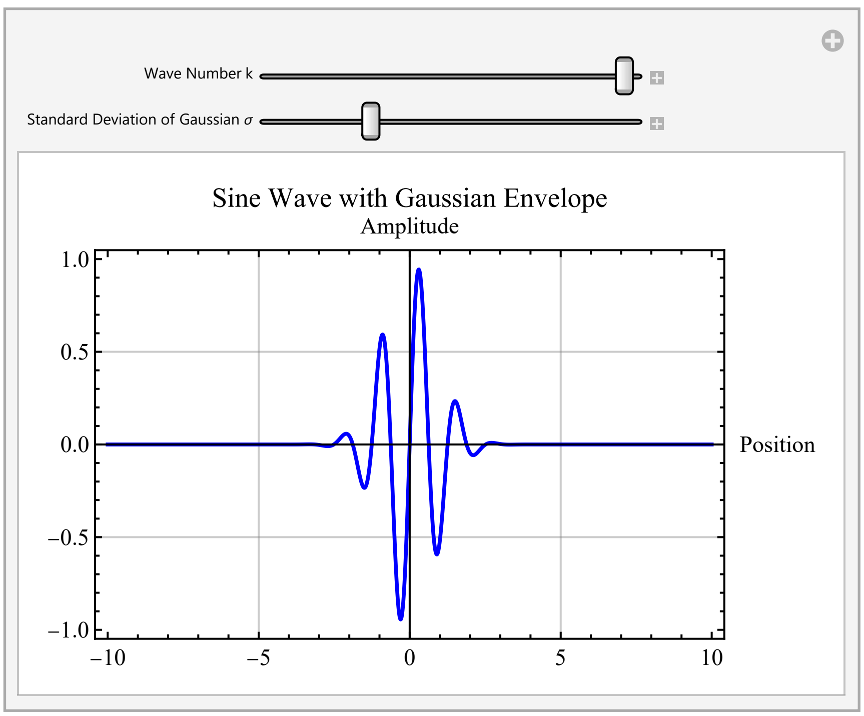
In[ ]:=

```
Manipulate[Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue, Thick},
  AxesLabel → {"Position", "Amplitude"}, PlotLabel → "Sine Wave with Gaussian Envelope",
  ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
  GridLines → Automatic, Frame → True, PlotRange → All],
 {{k, 1, "Wave Number k"}, 0.1, 5, 0.1},
 {{σ, 1, "Standard Deviation of Gaussian σ"}, 0.5, 2, 0.1}]
Manipulate[Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue, Thick},
  AxesLabel → {"Position", "Amplitude"}, PlotLabel → "Sine Wave with Gaussian Envelope",
  ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
  GridLines → Automatic, Frame → True, PlotRange → All],
 {{k, 1, "Wave Number k"}, 0.1, 5, 0.1},
 {{σ, 1, "Standard Deviation of Gaussian σ"}, 0.5, 2, 0.1}]
Manipulate[Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue, Thick},
  AxesLabel → {"Position", "Amplitude"}, PlotLabel → "Sine Wave with Gaussian Envelope",
  ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
  GridLines → Automatic, Frame → True, PlotRange → All],
 {{k, 1, "Wave Number k"}, 0.1, 5, 0.1},
 {{σ, 1, "Standard Deviation of Gaussian σ"}, 0.5, 2, 0.1}]
```

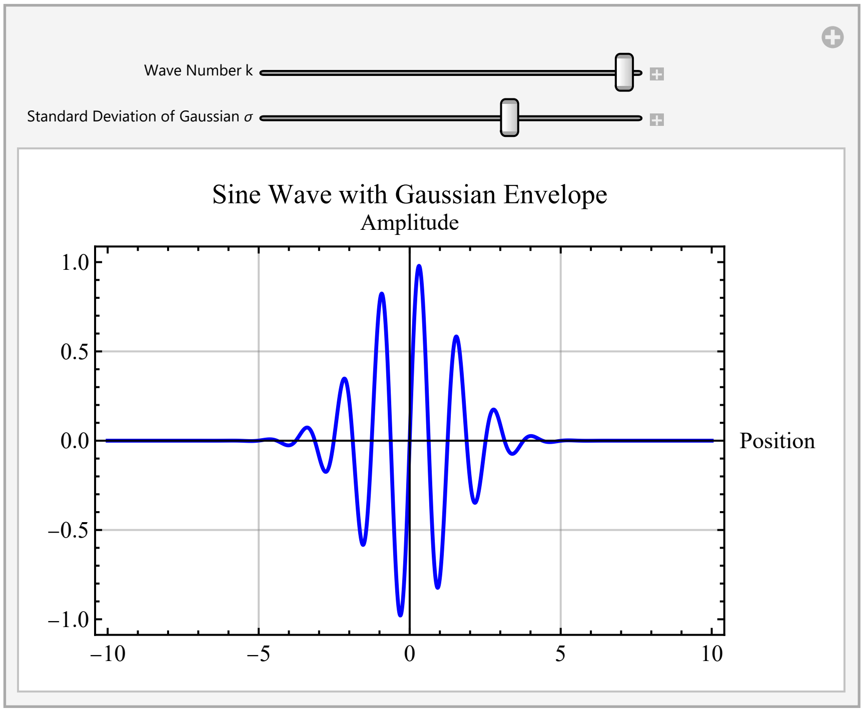
Out[ ]=


Wave Number k
Standard Deviation of Gaussian σ
Sine Wave with Gaussian Envelope
Amplitude
0.5
0.0
−0.5
−10
−5
0
5
10
Position

Out[ ]=


Wave Number k
Standard Deviation of Gaussian σ
Sine Wave with Gaussian Envelope
Amplitude
1.0
0.5
0.0
−0.5
−1.0
−10
−5
0
5
10
Position

Out[ ]=



As we can see from the plot, the range of the sine wave signal with non-zero values increases as the standard deviation σ of Gaussian increases. Since the signal is the standard sine wave multiplied by the Gaussian function, the Gaussian filter can be regarded as the proportion of the signal that is allowed to pass the filter. The increase of the standard deviation σ means the Gaussian is distributed in a larger range. This allows the signal from a larger range of input to pass.

Problem 10

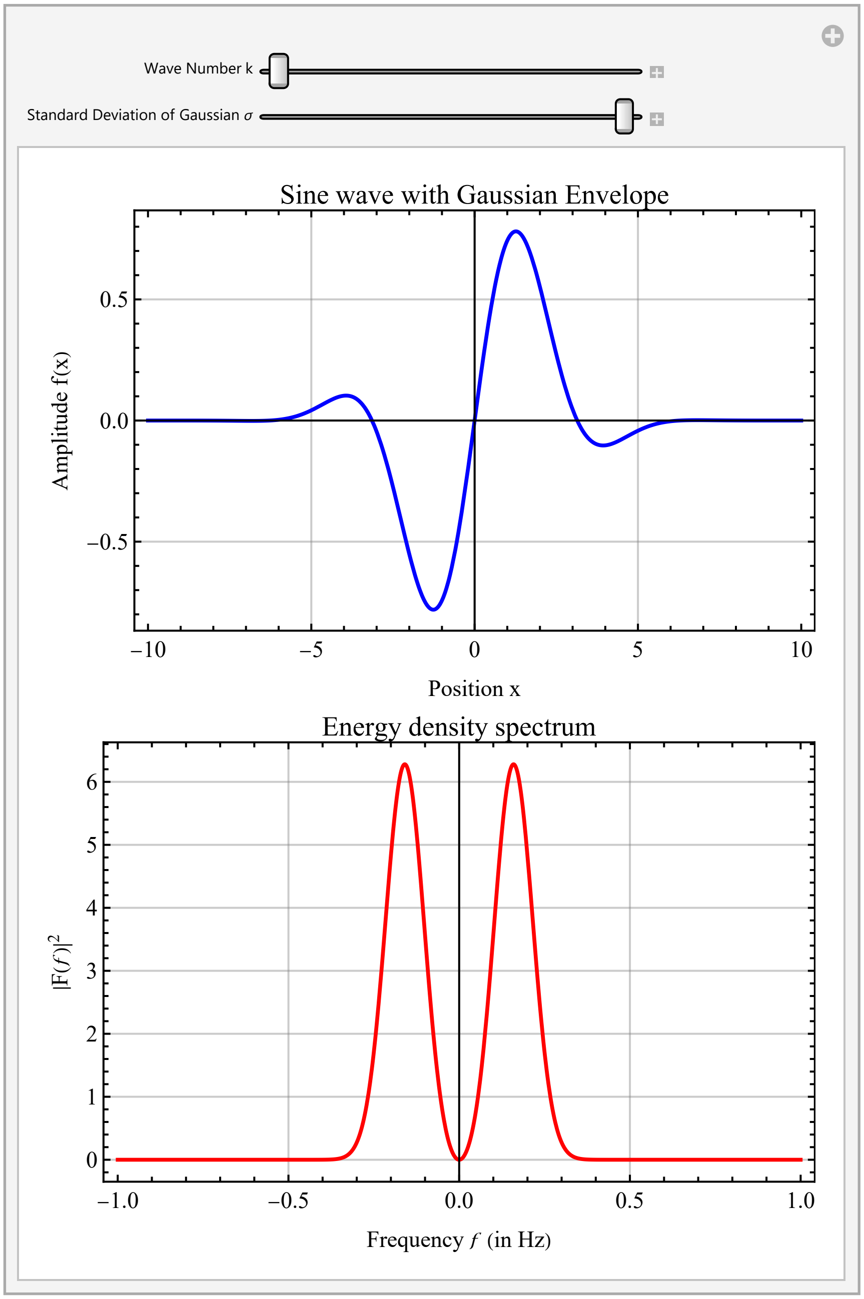
In[6]:=

```
(*Then I obtain the Fourier Coefficients,
its magnitude and Phase as a dynamic function*)
FourierCoeff[freqValue_, k_, σ_] :=
  NIntegrate[f[x, k, σ] Exp[-I 2 Pi freqValue x], {x, -10, 10}];
FourierCoeffMag[freqValue_, k_, σ_] := Abs[FourierCoeff[freqValue, k, σ]];
FourierCoeffPhase[freqValue_, k_, σ_] := Arg[FourierCoeff[freqValue, k, σ]];
```

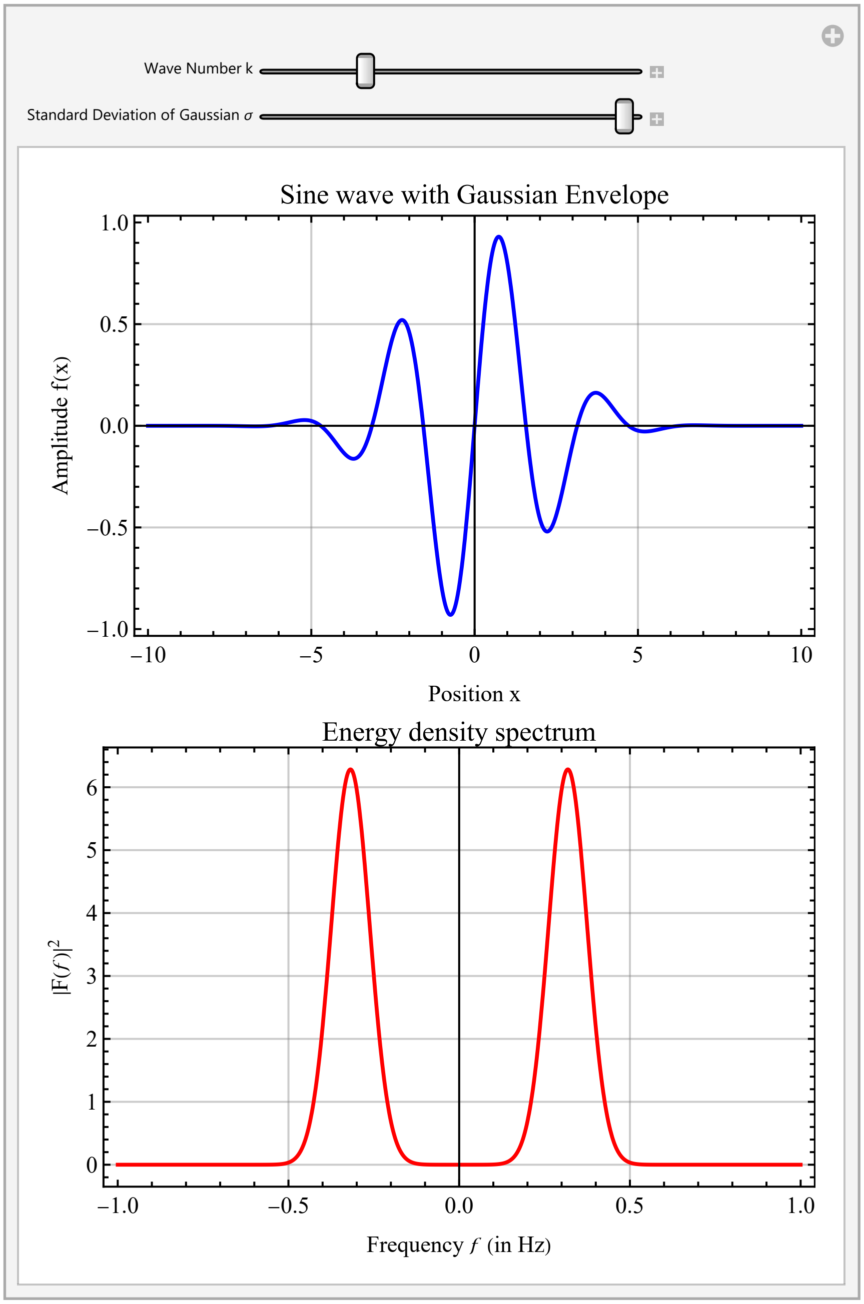
Problem 10 - Part 1 (Effect of the wave number *k*)

In[13]:=
```
(*PART 10 of ASSINGMENT2:*)
(*You can now manipulate the energy density spectrum by changing k and σ *)
Manipulate[Column[{Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All,
    PlotStyle → {Blue}, FrameLabel → {"Position x", "Amplitude f(x)"},
    PlotLabel → "Sine wave with Gaussian Envelope", ImageSize → 400,
    LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All],
   Plot[Abs[FourierCoeffMag[freqValue, k, σ]]^2, {freqValue, -1, 1}, PlotRange → All,
    PlotStyle → {Red}, FrameLabel → {"Frequency f (in Hz)", "|F(f)|^2"},
    PlotLabel → "Energy density spectrum", ImageSize → 400,
    LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All]}],
 {{k, 1, "Wave Number k"}, 1, 5, 1}, {{σ, 1, "Standard Deviation of Gaussian σ"},
  0.5, 2, 0.5}]
Manipulate[
 Column[{Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue}, FrameLabel →
     {"Position x", "Amplitude f(x)"}, PlotLabel → "Sine wave with Gaussian Envelope",
    ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All],
   Plot[Abs[FourierCoeffMag[freqValue, k, σ]]^2, {freqValue, -1, 1}, PlotRange → All,
    PlotStyle → {Red}, FrameLabel → {"Frequency f (in Hz)", "|F(f)|^2"},
    PlotLabel → "Energy density spectrum", ImageSize → 400,
    LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All]}],
 {{k, 1, "Wave Number k"}, 1, 5, 1}, {{σ, 1, "Standard Deviation of Gaussian σ"},
  0.5, 2, 0.5}]
Manipulate[
 Column[{Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue}, FrameLabel →
     {"Position x", "Amplitude f(x)"}, PlotLabel → "Sine wave with Gaussian Envelope",
    ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All],
   Plot[Abs[FourierCoeffMag[freqValue, k, σ]]^2, {freqValue, -1, 1}, PlotRange → All,
    PlotStyle → {Red}, FrameLabel → {"Frequency f (in Hz)", "|F(f)|^2"},
    PlotLabel → "Energy density spectrum", ImageSize → 400,
    LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All]}],
 {{k, 1, "Wave Number k"}, 1, 5, 1}, {{σ, 1, "Standard Deviation of Gaussian σ"},
  0.5, 2, 0.5}]
```

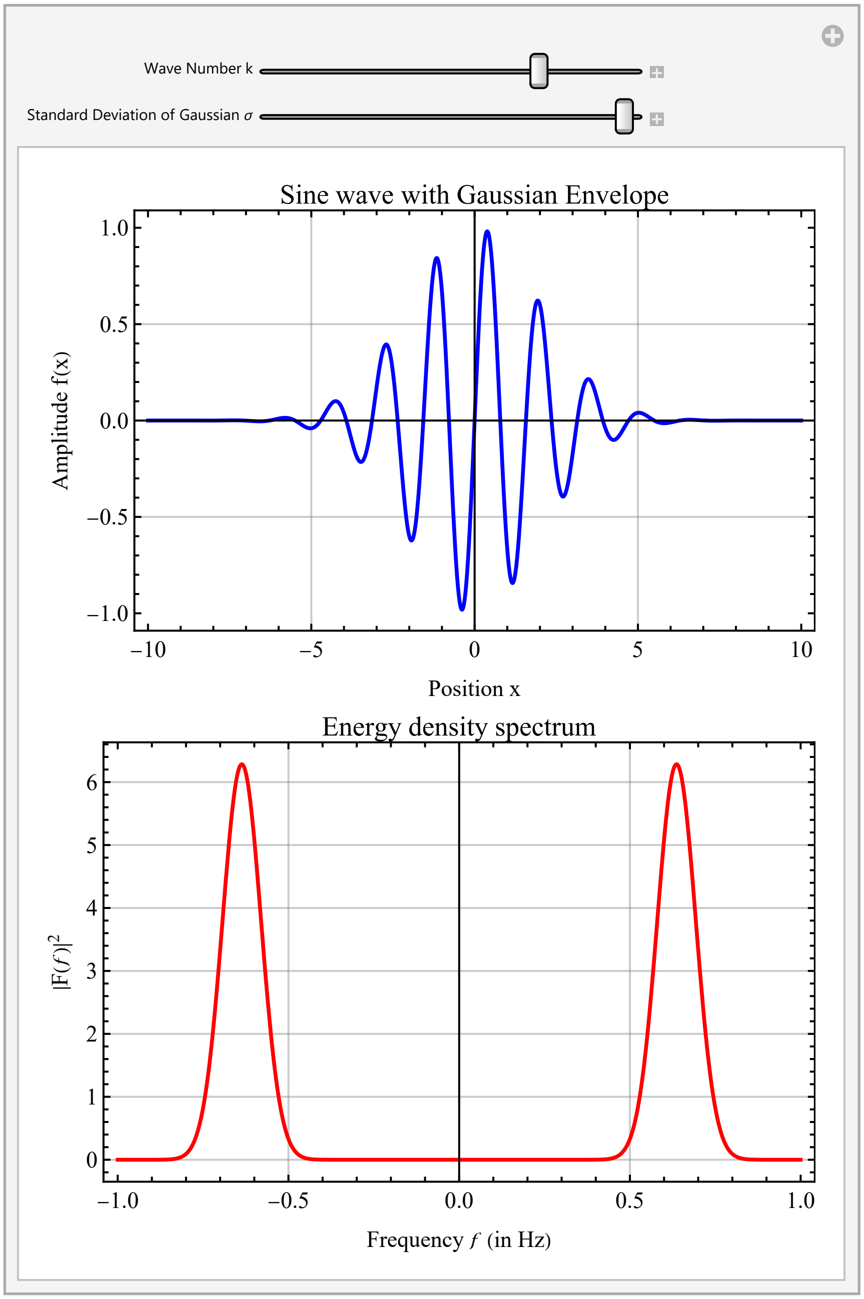
Out[13]=


Wave Number k
Standard Deviation of Gaussian σ
Sine wave with Gaussian Envelope
Amplitude f(x)
0.5
0.0
−0.5
−10
−5
0
5
10
Position x
Energy density spectrum
|F(f)|²
6
5
4
3
2
1
0
−1.0
−0.5
0.0
0.5
1.0
Frequency f (in Hz)

Out[14]=


Wave Number k
Standard Deviation of Gaussian σ
Sine wave with Gaussian Envelope
Amplitude f(x)
Position x
1.0
0.5
0.0
−0.5
−1.0
−10
−5
0
5
10
Energy density spectrum
|F(f)|²
Frequency f (in Hz)
6
5
4
3
2
1
0
−1.0
−0.5
0.0
0.5
1.0

Out[15]=

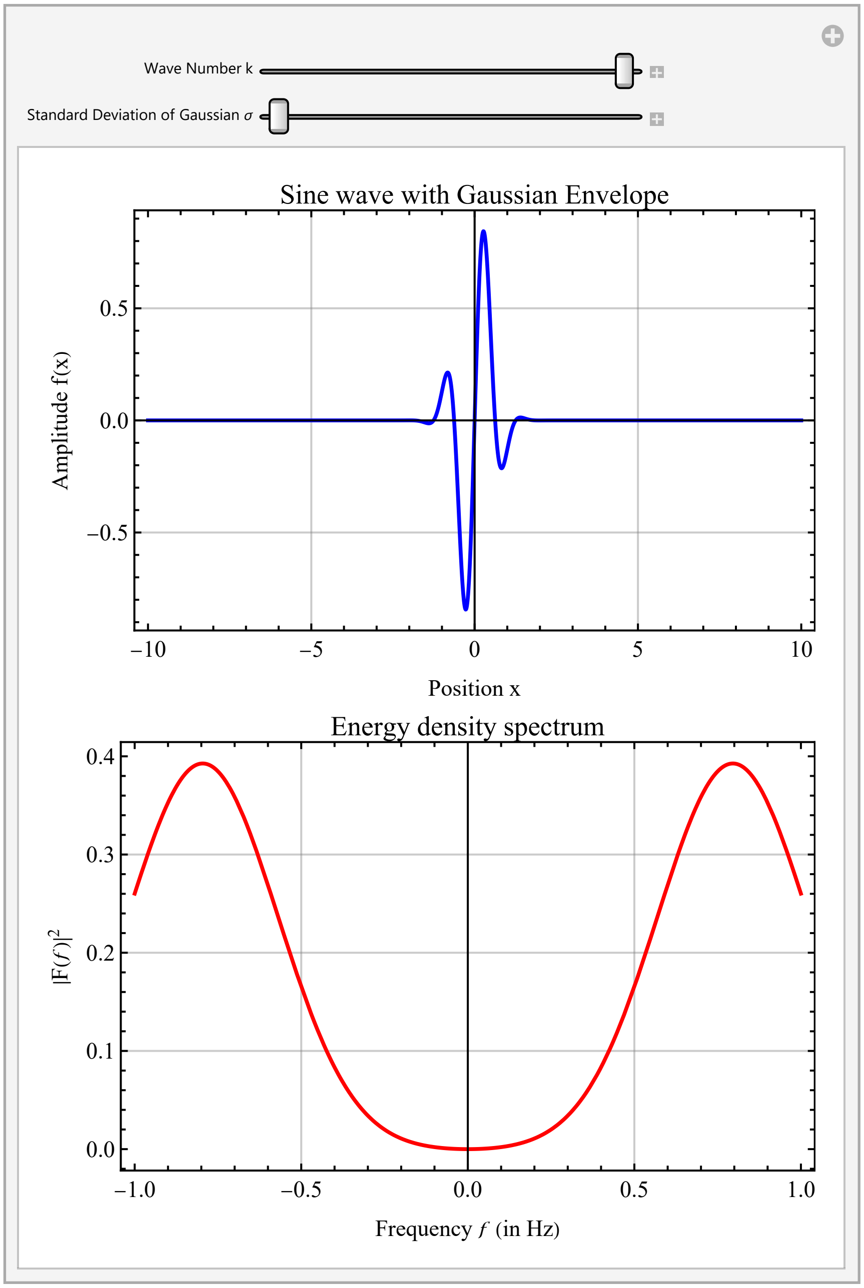


As we can see from the plot, the peak of the energy density spectrum moves to a higher frequency value as the wave number *k* increases. This is because the energy density spectrum shows the contribution of the signal with different frequency and the wave number *k* represents the frequency of the sine wave signal. With a higher frequency of the signal, the peak amplitude moves further away from the origin.

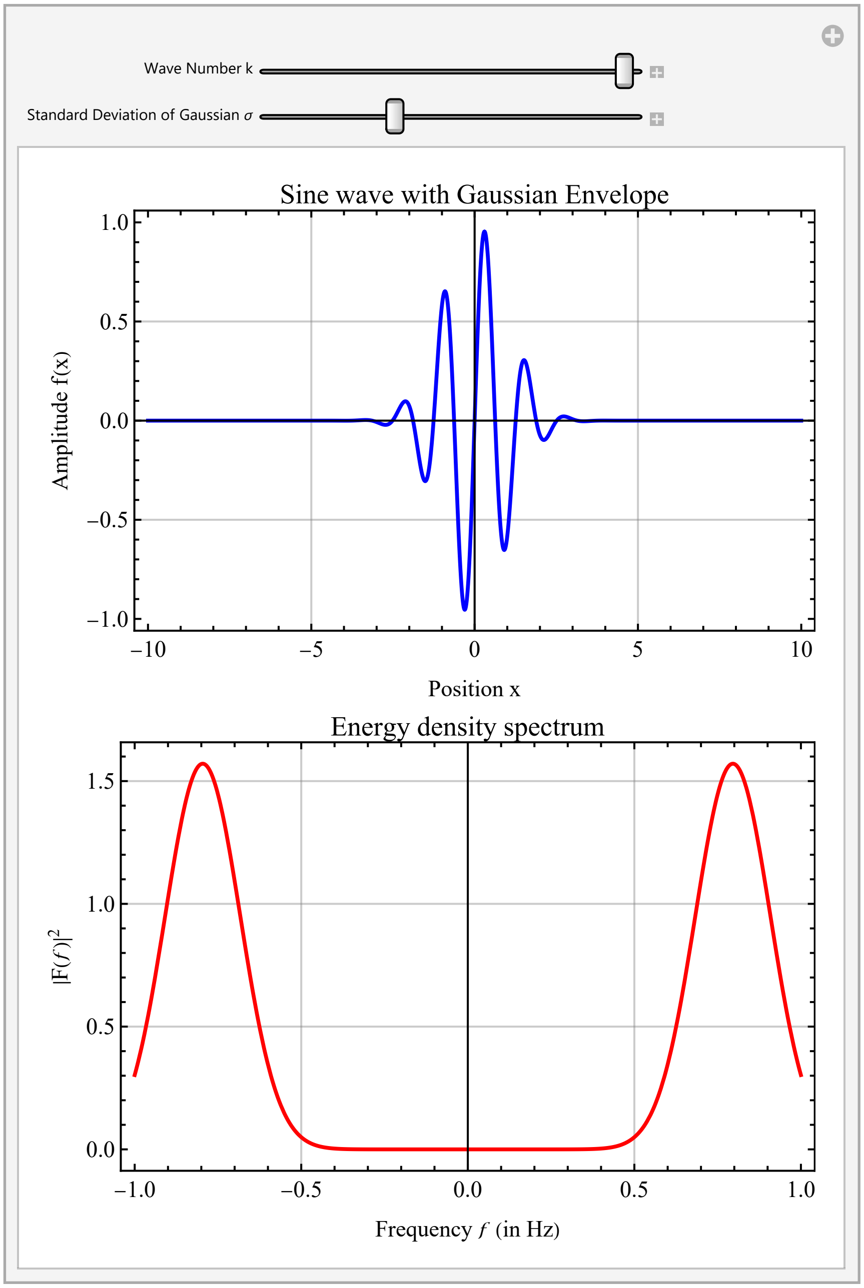
Problem 10 - Part 2 (Effect of the standard deviation σ of Gaussian)

In[16]:=
```
Manipulate[
 Column[{Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue}, FrameLabel →
     {"Position x", "Amplitude f(x)"}, PlotLabel → "Sine wave with Gaussian Envelope",
    ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All],
   Plot[Abs[FourierCoeffMag[freqValue, k, σ]]^2, {freqValue, -1, 1}, PlotRange → All,
    PlotStyle → {Red}, FrameLabel → {"Frequency f (in Hz)", "|F(f)|^2"},
    PlotLabel → "Energy density spectrum", ImageSize → 400,
    LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All]}],
 {{k, 1, "Wave Number k"}, 1, 5, 1}, {{σ, 1, "Standard Deviation of Gaussian σ"},
  0.5, 2, 0.5}]
Manipulate[
 Column[{Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue}, FrameLabel →
     {"Position x", "Amplitude f(x)"}, PlotLabel → "Sine wave with Gaussian Envelope",
    ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All],
   Plot[Abs[FourierCoeffMag[freqValue, k, σ]]^2, {freqValue, -1, 1}, PlotRange → All,
    PlotStyle → {Red}, FrameLabel → {"Frequency f (in Hz)", "|F(f)|^2"},
    PlotLabel → "Energy density spectrum", ImageSize → 400,
    LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All]}],
 {{k, 1, "Wave Number k"}, 1, 5, 1}, {{σ, 1, "Standard Deviation of Gaussian σ"},
  0.5, 2, 0.5}]
Manipulate[
 Column[{Plot[f[x, k, σ], {x, -10, 10}, PlotRange → All, PlotStyle → {Blue}, FrameLabel →
     {"Position x", "Amplitude f(x)"}, PlotLabel → "Sine wave with Gaussian Envelope",
    ImageSize → 400, LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All],
   Plot[Abs[FourierCoeffMag[freqValue, k, σ]]^2, {freqValue, -1, 1}, PlotRange → All,
    PlotStyle → {Red}, FrameLabel → {"Frequency f (in Hz)", "|F(f)|^2"},
    PlotLabel → "Energy density spectrum", ImageSize → 400,
    LabelStyle → {RGBColor[0, 0, 0], FontSize → 12, FontFamily → "Times"},
    GridLines → Automatic, Frame → True, PlotRange → All]}],
 {{k, 1, "Wave Number k"}, 1, 5, 1}, {{σ, 1, "Standard Deviation of Gaussian σ"},
  0.5, 2, 0.5}]
```

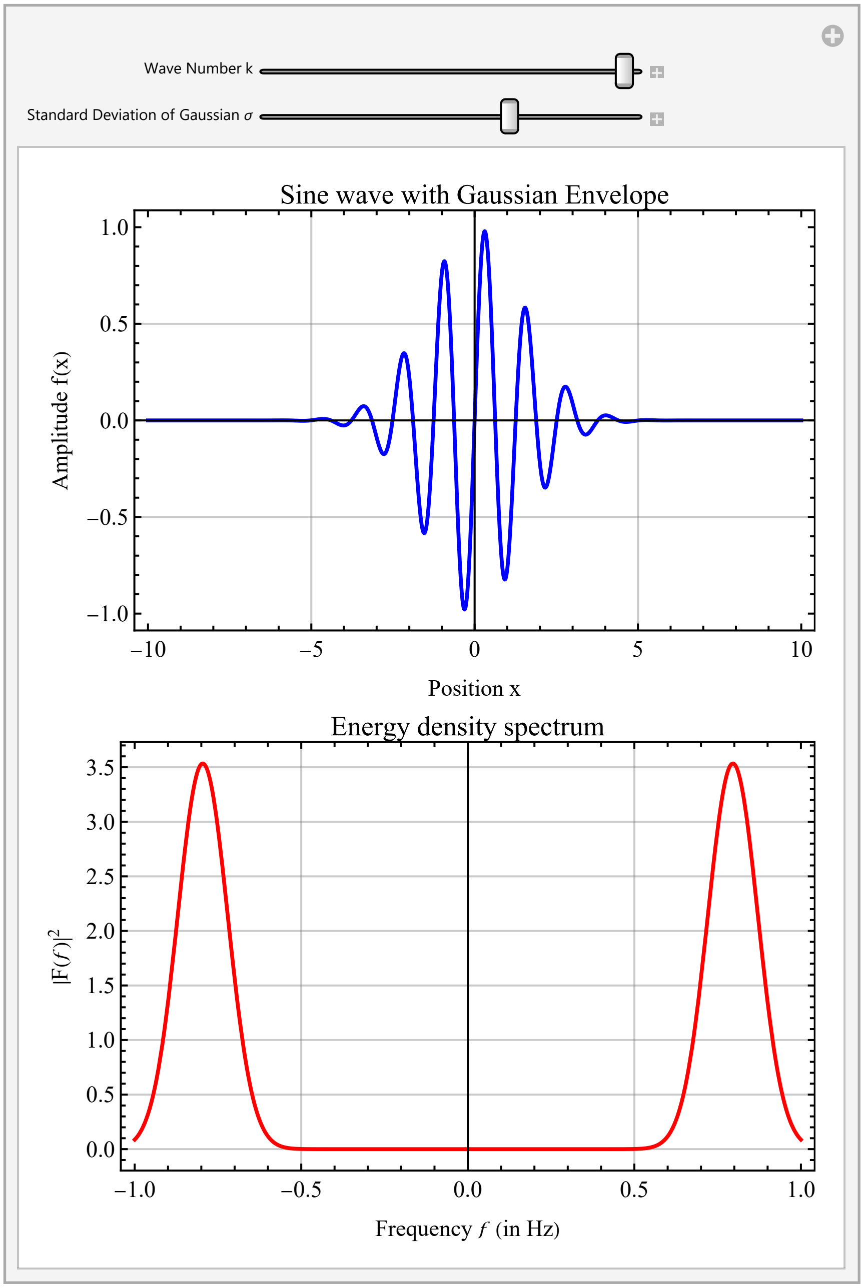
Out[16]=


Wave Number k
Standard Deviation of Gaussian σ
Sine wave with Gaussian Envelope
Amplitude f(x)
Position x
0.5
0.0
−0.5
−10
−5
0
5
10
Energy density spectrum
|F(f)|²
Frequency f (in Hz)
0.4
0.3
0.2
0.1
0.0
−1.0
−0.5
0.0
0.5
1.0

Out[17]=


Wave Number k
Standard Deviation of Gaussian σ
Sine wave with Gaussian Envelope
Amplitude f(x)
Position x
Energy density spectrum
|F(f)|²
Frequency f (in Hz)

Out[18]=



In this case the wave number k is fixed, which means the frequency of the signal remains constant. So the frequency corresponding to the peak amplitude of the energy density spectrum is the same for all the 3 cases. The increase of the standard deviation σ means the Gaussian is distributed in a larger range. This allows the signal from a larger range of input to pass and results in more total energy being included in the signal. So the peak value of the energy density spectrum increases with the increase of the standard deviation σ.