SE 267A HW1

Problem 1

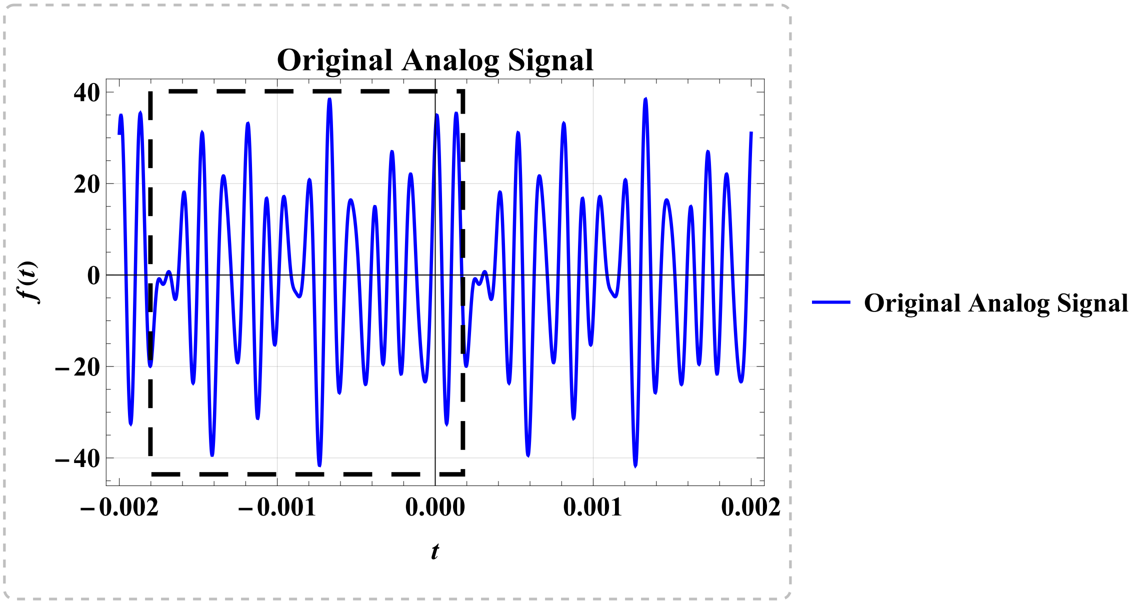
In[133]:=

```
(* Define each harmonic signal. *)
f1[t_] = Cos[(2000π)t];
f2[t_] = 5Sin[(6000π)t];
f3[t_] = 10Cos[(12000π)t];
f4[t_] = 20Cos[(15000π)t];
f5[t_] = 10Sin[(20000π)t];

(* Sum up all the harmonics to get the original analog signal. *)
Print[Style["Original analog signal: ", Bold, FontFamily→"Times", FontSize→14],
    Style["f(t)", Italic, Bold, FontFamily→"Times", FontSize→14]]
f[t_] = f1[t]+f2[t]+f3[t]+f4[t]+f5[t]

(* Plot the original analog signal. *)
Plot[{f[t]}, {t, -0.002, 0.002}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.005]},
    PlotLegends→{Style["Original Analog Signal"]}, FrameLabel→{Style["t", Italic], Style["f(t)",
    PlotLabel→"Original Analog Signal", GridLines→Automatic, LabelStyle→{RGBColor[0,0,0], Bold,
    FontSize→14, FontFamily→"Times"}, Frame→True]

(* Plot each harmonic signal separately. *)
Plot[{f1[t], f2[t], f3[t], f4[t], f5[t]}, {t, -0.002, 0.002}, ImageSize→{400, 300},
    PlotStyle→{Blue, Red, Green, Orange, Black}, PlotLegends→{Style[Subscript["f", 1][t],
    Italic], Style[Subscript["f", 2][t], Italic], Style[Subscript["f", 3][t], Italic],
    Style[Subscript["f", 4][t], Italic], Style[Subscript["f", 5][t], Italic]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]}, PlotLabel→"Harmonic Signals",
    GridLines→Automatic, LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"},
    Frame→True]
```

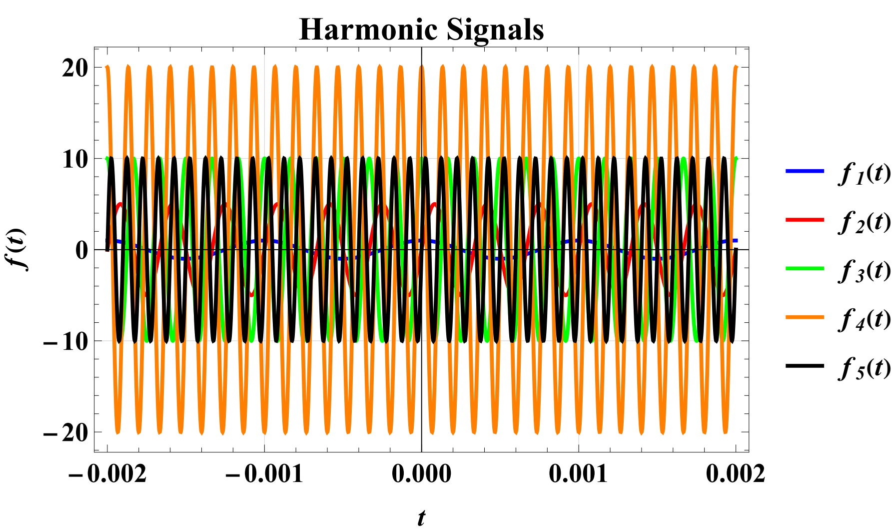
Original analog signal: ***f*(*t*)**

Out[139]=

$\text{Cos}[2000\,\pi\,t] + 10\,\text{Cos}[12\,000\,\pi\,t] + 20\,\text{Cos}[15\,000\,\pi\,t] + 5\,\text{Sin}[6000\,\pi\,t] + 10\,\text{Sin}[20\,000\,\pi\,t]$



Out[ ]=



There are 5 distinct frequencies composing the signal.

By visual inspection, the fundamental period of the signal is T_p = 0.002 s.
The full cycle of the signal is marked with black dash lines in the plot.

Problem 2

In[10]:=
```
(* Define each harmonic signal in the cosine function form. *)
Index = {1, 2, 3, 4, 5};

Print[Style["Angular frequency Ωi (rad/s) for each harmonic signal:", Bold, FontFamily→"Times", F
```

```
Omega = {2000π, 6000π, 12000π, 15000π, 20000π}

Print[Style["Fundamental frequency fpi (Hz) for each harmonic signal:", Bold, FontFamily→"Times",
Frequency = Omega/(2π)

Print[Style["Fundamental period Tpi (s) for each harmonic signal:", Bold, FontFamily→"Times", Font
Period = 1/Frequency

Print[Style["Amplitude Ai for each harmonic signal:", Bold, FontFamily→"Times", FontSize→14]]
Amplitude = {1, 5, 10, 20, 10}

Print[Style["Phase difference Φi (rad) for each harmonic signal:", Bold, FontFamily→"Times", Font
Phi = {0, -π/2, 0, 0, -π/2}

f1[t_] = Amplitude⟦1⟧ * Cos[Omega⟦1⟧*t + Phi⟦1⟧];
f2[t_] = Amplitude⟦2⟧ * Cos[Omega⟦2⟧*t + Phi⟦2⟧];
f3[t_] = Amplitude⟦3⟧ * Cos[Omega⟦3⟧*t + Phi⟦3⟧];
f4[t_] = Amplitude⟦4⟧ * Cos[Omega⟦4⟧*t + Phi⟦4⟧];
f5[t_] = Amplitude⟦5⟧ * Cos[Omega⟦5⟧*t + Phi⟦5⟧];

(* List a table for the characteristics of harmonics. *)
Title = {"Harmonic index i", "Angular frequency Ωi (rad/s)", "Fundamental frequency fpi (Hz)",
    "Fundamental period Tpi (s)", "Amplitude Ai", "Phase difference Φi (rad)"};
Data = Transpose[{Index, Omega, Frequency, Period, Amplitude, Phi}];
AppendData = Prepend[Data, Title];
StyledData = Map[Style[#, FontFamily→"Times", FontSize→10, FontWeight→Bold] &, AppendData, {2}];
Grid[StyledData, Frame→All, ItemSize→{8, 2}, Alignment→Center]

(* Sum up all the harmonics to get the original analog signal. *)
f[t_] = f1[t]+f2[t]+f3[t]+f4[t]+f5[t];

(* Print all the harmonics and the original analog signal. *)
Print[Style["Harmonic signal: ", Bold, FontFamily→"Times", FontSize→14],
    Style["f1(t)", Italic, Bold, FontFamily→"Times", FontSize→14]]
f1[t]/.Sin[var_]:→HoldForm[Cos[var-π/2]]

Print[Style["Harmonic signal: ", Bold, FontFamily→"Times", FontSize→14],
    Style["f2(t)", Italic, Bold, FontFamily→"Times", FontSize→14]]
f2[t]/.Sin[var_]:→HoldForm[Cos[var-π/2]]

Print[Style["Harmonic signal: ", Bold, FontFamily→"Times", FontSize→14],
    Style["f3(t)", Italic, Bold, FontFamily→"Times", FontSize→14]]
f3[t]/.Sin[var_]:→HoldForm[Cos[var-π/2]]

Print[Style["Harmonic signal: ", Bold, FontFamily→"Times", FontSize→14],
    Style["f4(t)", Italic, Bold, FontFamily→"Times", FontSize→14]]
f4[t]/.Sin[var_]:→HoldForm[Cos[var-π/2]]

Print[Style["Harmonic signal: ", Bold, FontFamily→"Times", FontSize→14],
    Style["f5(t)", Italic, Bold, FontFamily→"Times", FontSize→14]]
f5[t]/.Sin[var_]:→HoldForm[Cos[var-π/2]]
```

```
Print[Style["Original analog signal: ", Bold, FontFamily→"Times", FontSize→14],
    Style["f(t)", Italic, Bold, FontFamily→"Times", FontSize→14]]
f[t]/.Sin[var_]:→HoldForm[Cos[var-π/2]]
```

Angular frequency Ω_i (rad/s) for each harmonic signal:

Out[12]=

$\{2000\pi, 6000\pi, 12\,000\pi, 15\,000\pi, 20\,000\pi\}$

Fundamental frequency f_{pi} (Hz) for each harmonic signal:

Out[14]=

$\{1000, 3000, 6000, 7500, 10\,000\}$

Fundamental period T_{pi} (s) for each harmonic signal:

Out[16]=

$\left\{\frac{1}{1000}, \frac{1}{3000}, \frac{1}{6000}, \frac{1}{7500}, \frac{1}{10\,000}\right\}$

Amplitude A_i for each harmonic signal:

Out[18]=

$\{1, 5, 10, 20, 10\}$

Phase difference Φ_i (rad) for each harmonic signal:

Out[20]=

$\left\{0, -\frac{\pi}{2}, 0, 0, -\frac{\pi}{2}\right\}$

Out[30]=

Harmonic index i	Angular frequency Ω_i (rad/s)	Fundamental frequency f_{pi} (Hz)	Fundamental period T_{pi} (s)	Amplitude A_i	Phase difference Φ_i (rad)
1	2000π	1000	$\frac{1}{1000}$	1	0
2	6000π	3000	$\frac{1}{3000}$	5	$-\frac{\pi}{2}$
3	$12\,000\pi$	6000	$\frac{1}{6000}$	10	0
4	$15\,000\pi$	7500	$\frac{1}{7500}$	20	0
5	$20\,000\pi$	10 000	$\frac{1}{10\,000}$	10	$-\frac{\pi}{2}$

Harmonic signal: $f_1(t)$

Out[33]=

$\mathrm{Cos}[2000\pi t]$

Harmonic signal: $f_2(t)$

Out[35]=

$5\,\mathrm{Cos}\left[6000\pi t - \frac{\pi}{2}\right]$

Harmonic signal: $f_3(t)$

Out[37]=

$10\,\mathrm{Cos}[12\,000\pi t]$

Harmonic signal: $f_4(t)$

Out[39]=

$20\,\text{Cos}[15\,000\,\pi\,t]$

Harmonic signal: $f_5(t)$

Out[41]=

$$10\,\text{Cos}\left[20\,000\,\pi\,t - \frac{\pi}{2}\right]$$

Original analog signal: $f(t)$

Out[43]=

$$\text{Cos}[2000\,\pi\,t] + 10\,\text{Cos}[12\,000\,\pi\,t] + 20\,\text{Cos}[15\,000\,\pi\,t] + 5\,\text{Cos}\left[6000\,\pi\,t - \frac{\pi}{2}\right] + 10\,\text{Cos}\left[20\,000\,\pi\,t - \frac{\pi}{2}\right]$$

Problem 3

In[44]:=

```
(* Calculate the fundamental time period of the original analog signal. *)
Print[Style["The fundamental time period Tp (second) of the original analog signal:", Bold,
    FontFamily→"Times", FontSize→14]]
Tp = LCM[Period[[1]], Period[[2]], Period[[3]], Period[[4]], Period[[5]]]
```

The fundamental time period T_p (second) of the original analog signal:

Out[45]=

$$\frac{1}{500}$$

The fundamental time period of signal is $f(t)$ = 0.002 s, which can match the result from visual inspection.

Problem 4

In[46]:=

```
(* Calculate the fundamental frequency and angular frequency. *)
Print[Style["The fundamental frequency fp (Hz) of the original analog signal:", Bold,
    FontFamily→"Times", FontSize→14]]
Analogfp = 1/Tp

Print[Style["The angular frequency Ω (rad/s) of the original analog signal:", Bold,
    FontFamily→"Times", FontSize→14]]
AnalogOmega = 2π*Analogfp
```

The fundamental frequency f_p (Hz) of the original analog signal:

Out[47]=

500

The angular frequency Ω (rad/s) of the original analog signal:

Out[49]=

$1000\,\pi$

Problem 5

In[50]:=
```
(* Obtain the maximum frequency of the original analog signal. *)
Print[Style["The maximum frequency fp,max (Hz) of the original analog signal:", Bold,
    FontFamily→"Times", FontSize→14]]
MaxFrequency = Max[Frequency]

(* Calculate the Nyquist frequency of the original analog signal. *)
Print[Style["The Nyquist frequency fN (Hz) of the original analog signal:", Bold,
    FontFamily→"Times", FontSize→14]]
NyquistFrequency = 2*MaxFrequency
```

The maximum frequency $f_{p,max}$ (Hz) of the original analog signal:

Out[51]=

10 000

The Nyquist frequency f_N (Hz) of the original analog signal:

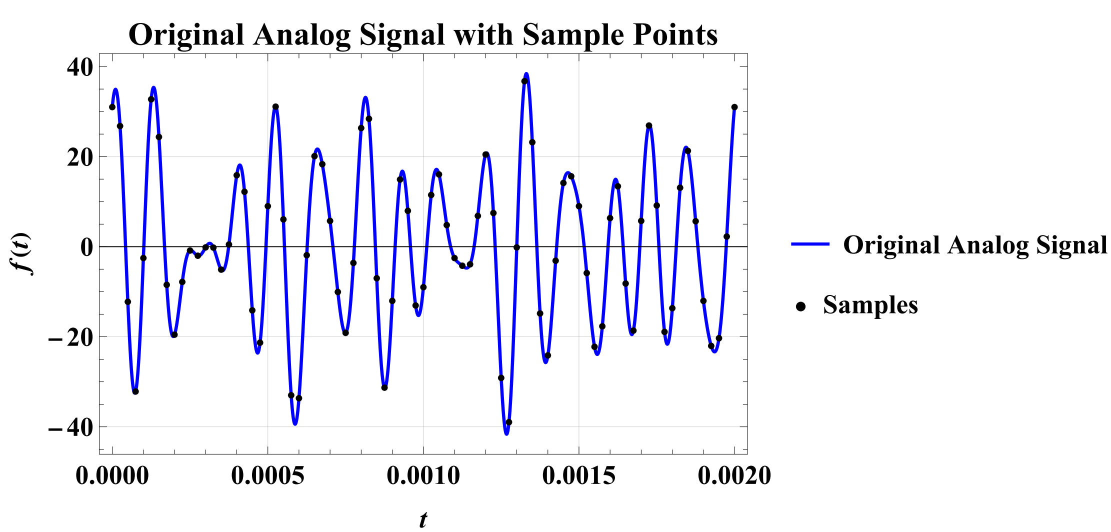
Out[53]=

20 000

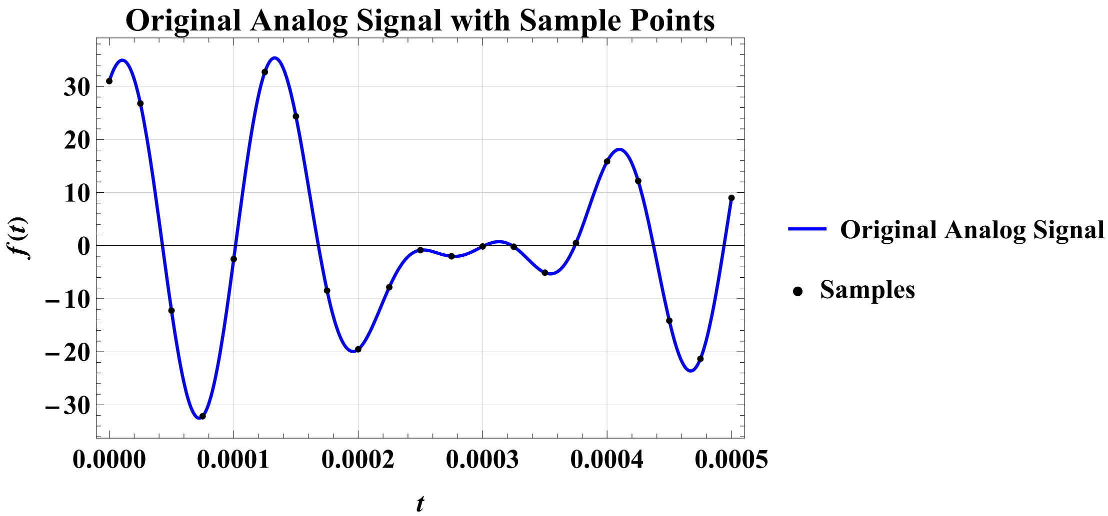
Problem 6

In[54]:=
```
(* Given the sampling frequency equal to two times the Nyquist frequency. *)
SamplingFrequency = 2 * NyquistFrequency;
SamplingTime = Range[0, 0.002, 1/SamplingFrequency];

(* Sample the signal. *)
fSeq = Table[N[f[SamplingTime[[k]]]], {k, 1, Length[SamplingTime]}];
fSeqList = Transpose[{SamplingTime, fSeq}];

(* Plot the original signal and the sampled signal. *)
Show[Plot[f[t], {t, 0, 0.002}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.005]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]},
    PlotLabel→"Original Analog Signal with Sample Points", GridLines→Automatic,
    LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Frame→True],
ListPlot[fSeqList, PlotStyle→{Directive[PointSize[0.01], Black]}, PlotLegends→{Style["Samples",
    Bold, FontSize→14, FontFamily→"Times"]}]]

Show[Plot[f[t], {t, 0, 0.0005}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.005]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]},
    PlotLabel→"Original Analog Signal with Sample Points", GridLines→Automatic,
    LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Frame→True],
ListPlot[fSeqList, PlotStyle→{Directive[PointSize[0.01], Black]}, PlotLegends→{Style["Samples",
    Bold, FontSize→14, FontFamily→"Times"]}]]
```

Out[58]=



Out[59]=



Problem 7

In[60]:=

```
(* Calculate ωi for each harmonic signal. *)
Print[Style["ωi (rad/sample) for each harmonic signal:", Bold,
    FontFamily→"Times", FontSize→14]]
w = Omega/SamplingFrequency

(* Calculate the analytical form of the discrete-time signal after sampling. *)
fSeqAnalytical[n_] = Total[Amplitude * Cos[w*n + Phi]];

Print[Style["Discrete-time signal: ", Bold, FontFamily→"Times", FontSize→14],
    Style["f(n)", Italic, Bold, FontFamily→"Times", FontSize→14]]
fSeqAnalytical[n]/.Sin[var_]:>HoldForm[Cos[var-π/2]]
```

ω_i (rad/sample) for each harmonic signal:

Out[61]=

$$\left\{\frac{\pi}{20}, \frac{3\pi}{20}, \frac{3\pi}{10}, \frac{3\pi}{8}, \frac{\pi}{2}\right\}$$

Discrete−time signal: *f*(*n*)

Out[64]=

$$\mathrm{Cos}\left[\frac{n\pi}{20}\right] + 10\,\mathrm{Cos}\left[\frac{3n\pi}{10}\right] + 20\,\mathrm{Cos}\left[\frac{3n\pi}{8}\right] + 5\,\mathrm{Cos}\left[\frac{3n\pi}{20} - \frac{\pi}{2}\right] + 10\,\mathrm{Cos}\left[\frac{n\pi}{2} - \frac{\pi}{2}\right]$$

Problem 8

See the handwritten pages at the end.

Problem 9

In[65]:=

```
(* Calculate the frequency, amplitude and phase difference values. *)
MirrorFrequency = SamplingFrequency - Frequency;
MirrorAmplitude = Amplitude;
MirrorPhi = -Phi;
MirrorOmega = 2π * MirrorFrequency;
MirrorW = MirrorOmega/SamplingFrequency;

CopyFrequency = SamplingFrequency + Frequency;
CopyAmplitude = Amplitude;
CopyPhi = Phi;
CopyOmega = 2π * CopyFrequency;
CopyW = CopyOmega/SamplingFrequency;

(* Discrete-time signal for the 1st mirror alias. *)
fSeqMirror[n_] = Total[MirrorAmplitude * Cos[MirrorW*n + MirrorPhi]];
Print[Style["Discrete-time signal for the 1st mirror alias: ", Bold, FontFamily→"Times", FontSize
TraditionalForm[fSeqMirror[n]/.Sin[var_]:→HoldForm[Cos[var-π/2]]]

(* Discrete-time signal for the 1st copy alias. *)
fSeqCopy[n_] = Total[CopyAmplitude * Cos[CopyW*n + CopyPhi]];
Print[Style["Discrete-time signal for the 1st copy alias: ", Bold, FontFamily→"Times", FontSize→1
TraditionalForm[fSeqCopy[n]/.Sin[var_]:→HoldForm[Cos[var-π/2]]]

(* Analog signal for the 1st mirror alias. *)
fMirror[t_] = Total[MirrorAmplitude * Cos[MirrorOmega*t + MirrorPhi]];
Print[Style["Analog signal for the 1st mirror alias: ", Bold, FontFamily→"Times", FontSize→14]]
TraditionalForm[fMirror[t]/.Sin[var_]:→HoldForm[Cos[var-π/2]]]

(* Analog signal for the 1st copy alias. *)
fCopy[t_] = Total[CopyAmplitude * Cos[CopyOmega*t + CopyPhi]];
Print[Style["Analog signal for the 1st copy alias: ", Bold, FontFamily→"Times", FontSize→14]]
TraditionalForm[fCopy[t]/.Sin[var_]:→HoldForm[Cos[var-π/2]]]
```

Discrete−time signal for the 1st mirror alias:

Out[77]//TraditionalForm=

$$20\cos\left(\frac{13\pi n}{8}\right)+10\cos\left(\frac{17\pi n}{10}\right)+\cos\left(\frac{39\pi n}{20}\right)-10\cos\left(\frac{3n\pi}{2}-\frac{\pi}{2}\right)-5\cos\left(\frac{37n\pi}{20}-\frac{\pi}{2}\right)$$

Discrete−time signal for the 1st copy alias:

Out[80]//TraditionalForm=

$$\cos\left(\frac{41\pi n}{20}\right)+10\cos\left(\frac{23\pi n}{10}\right)+20\cos\left(\frac{19\pi n}{8}\right)+5\cos\left(\frac{43n\pi}{20}-\frac{\pi}{2}\right)+10\cos\left(\frac{5n\pi}{2}-\frac{\pi}{2}\right)$$

Analog signal for the 1st mirror alias:

Out[83]//TraditionalForm=

$$20\cos(65\,000\,\pi\,t)+10\cos(68\,000\,\pi\,t)+\cos(78\,000\,\pi\,t)-10\cos\left(60\,000\,\pi\,t-\frac{\pi}{2}\right)-5\cos\left(74\,000\,\pi\,t-\frac{\pi}{2}\right)$$

Analog signal for the 1st copy alias:

Out[86]//TraditionalForm=

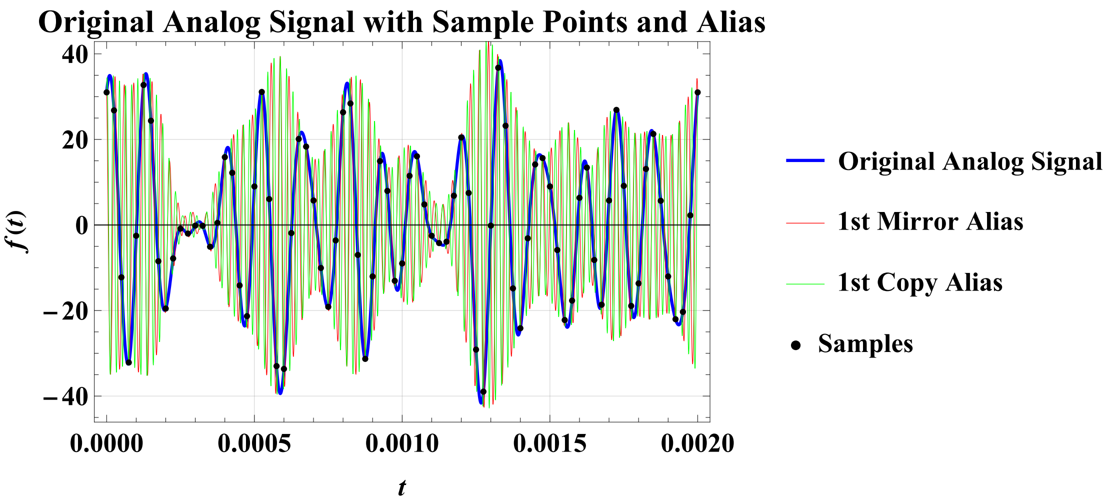
$$\cos(82\,000\,\pi\,t)+10\cos(92\,000\,\pi\,t)+20\cos(95\,000\,\pi\,t)+5\cos\left(86\,000\,\pi\,t-\frac{\pi}{2}\right)+10\cos\left(100\,000\,\pi\,t-\frac{\pi}{2}\right)$$

Problem 10

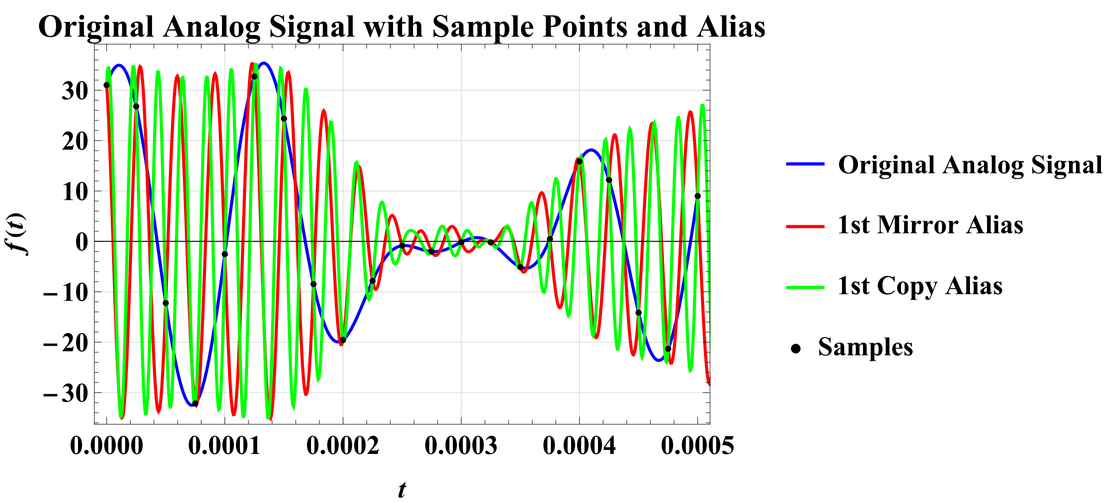
In[87]:=

```
(* Plot the original signal, the sampled signal and the alias. *)
Show[Plot[f[t], {t, 0, 0.002}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.005]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]},
    PlotLabel→"Original Analog Signal with Sample Points and Alias", GridLines→Automatic,
    LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Frame→True],
Plot[fMirror[t], {t, 0, 0.002}, PlotStyle→{Red, Thickness[0.001]},
    PlotLegends→{Style["1st Mirror Alias", Bold, FontSize→14, FontFamily→"Times"]}],
Plot[fCopy[t], {t, 0, 0.002}, PlotStyle→{Green, Thickness[0.001]},
    PlotLegends→{Style["1st Copy Alias", Bold, FontSize→14, FontFamily→"Times"]}],
ListPlot[fSeqList, PlotStyle→{Directive[PointSize[0.01], Black]}, PlotLegends→{Style["Samples",
    Bold, FontSize→14, FontFamily→"Times"]}]]

Show[Plot[f[t], {t, 0, 0.0005}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.005]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]},
    PlotLabel→"Original Analog Signal with Sample Points and Alias", GridLines→Automatic,
    LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Frame→True],
Plot[fMirror[t], {t, 0, 0.002}, PlotStyle→{Red, Thickness[0.005]},
    PlotLegends→{Style["1st Mirror Alias", Bold, FontSize→14, FontFamily→"Times"]}],
Plot[fCopy[t], {t, 0, 0.002}, PlotStyle→{Green, Thickness[0.005]},
    PlotLegends→{Style["1st Copy Alias", Bold, FontSize→14, FontFamily→"Times"]}],
ListPlot[fSeqList, PlotStyle→{Directive[PointSize[0.01], Black]}, PlotLegends→{Style["Samples",
    Bold, FontSize→14, FontFamily→"Times"]}]]
```

Out[87]=


Original Analog Signal with Sample Points and Alias
f(t)
t
Original Analog Signal
1st Mirror Alias
1st Copy Alias
Samples

Out[88]=


Original Analog Signal with Sample Points and Alias
f(t)
t
Original Analog Signal
1st Mirror Alias
1st Copy Alias
Samples

Problem 11

In[164]:=

```
(* Define the set of shape functions. *)
ShapeFunctionSet = N[Table[Sin[2π*(SamplingFrequency/2)*(t-n/SamplingFrequency)] / (2π*(Samplin
    {n, 0, Length[SamplingTime]-1}]];
Print[Style["The set of shape functions: ", Bold, FontFamily→"Times", FontSize→14]]
TraditionalForm[ShapeFunctionSet]

(* Reconstruct the signal using the samples. *)
ReconstructedSignal = Dot[fSeq, ShapeFunctionSet];
Print[Style["The reconstructed signal using samples: ", Bold, FontFamily→"Times", FontSize→14]]
TraditionalForm[ReconstructedSignal]

(* Plot the reconstructed signal with the original signal and its alias. *)
Show[Plot[f[t], {t, 0, 0.002}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.01]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]}, PlotLabel→"Reconstructed Signal",
    GridLines→Automatic, LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Fr
Plot[fMirror[t], {t, 0, 0.002}, PlotStyle→{Red, Thickness[0.001]},
    PlotLegends→{Style["1st Mirror Alias", Bold, FontSize→14, FontFamily→"Times"]}],
Plot[fCopy[t], {t, 0, 0.002}, PlotStyle→{Green, Thickness[0.001]},
    PlotLegends→{Style["1st Copy Alias", Bold, FontSize→14, FontFamily→"Times"]}],
ListPlot[fSeqList, PlotStyle→{Directive[PointSize[0.015], Black]}, PlotLegends→{Style["Samples",
    Bold, FontSize→14, FontFamily→"Times"]}],
Plot[ReconstructedSignal, {t, 0, 0.002}, PlotStyle→{Orange, Thickness[0.005]},
    PlotLegends→{Style["Reconstructed Signal", Bold, FontSize→14, FontFamily→"Times"]}]]

Show[Plot[f[t], {t, 0, 0.0005}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.01]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]}, PlotLabel→"Reconstructed Signal",
    GridLines→Automatic, LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Fr
Plot[fMirror[t], {t, 0, 0.002}, PlotStyle→{Red, Thickness[0.001]},
    PlotLegends→{Style["1st Mirror Alias", Bold, FontSize→14, FontFamily→"Times"]}],
Plot[fCopy[t], {t, 0, 0.002}, PlotStyle→{Green, Thickness[0.001]},
    PlotLegends→{Style["1st Copy Alias", Bold, FontSize→14, FontFamily→"Times"]}],
ListPlot[fSeqList, PlotStyle→{Directive[PointSize[0.015], Black]}, PlotLegends→{Style["Samples",
    Bold, FontSize→14, FontFamily→"Times"]}],
Plot[ReconstructedSignal, {t, 0, 0.002}, PlotStyle→{Orange, Thickness[0.005]},
    PlotLegends→{Style["Reconstructed Signal", Bold, FontSize→14, FontFamily→"Times"]}]]
```

The set of shape functions:

Out[166]//TraditionalForm=

$$\left\{\frac{7.95775\times10^{-6}\sin(125\,664.\,t)}{t}, \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000025))}{t-0.000025}, \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00005))}{t-0.00005},\right.$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000075))}{t-0.000075}, \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0001))}{t-0.0001},$$

$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000125))}{t-0.000125},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00015))}{t-0.00015},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000175))}{t-0.000175},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0002))}{t-0.0002},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000225))}{t-0.000225},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00025))}{t-0.00025},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000275))}{t-0.000275},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0003))}{t-0.0003},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000325))}{t-0.000325},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00035))}{t-0.00035},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000375))}{t-0.000375},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0004))}{t-0.0004},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000425))}{t-0.000425},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00045))}{t-0.00045},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000475))}{t-0.000475},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0005))}{t-0.0005},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000525))}{t-0.000525},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00055))}{t-0.00055},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000575))}{t-0.000575},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0006))}{t-0.0006},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000625))}{t-0.000625},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00065))}{t-0.00065},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000675))}{t-0.000675},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0007))}{t-0.0007},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000725))}{t-0.000725},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00075))}{t-0.00075},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000775))}{t-0.000775},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0008))}{t-0.0008},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000825))}{t-0.000825},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00085))}{t-0.00085},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000875))}{t-0.000875},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0009))}{t-0.0009},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000925))}{t-0.000925},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00095))}{t-0.00095},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000975))}{t-0.000975},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001))}{t-0.001},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001025))}{t-0.001025},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00105))}{t-0.00105},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001075))}{t-0.001075},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0011))}{t-0.0011},$$

$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001125))}{t-0.001125},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00115))}{t-0.00115},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001175))}{t-0.001175},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0012))}{t-0.0012},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001225))}{t-0.001225},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00125))}{t-0.00125},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001275))}{t-0.001275},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0013))}{t-0.0013},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001325))}{t-0.001325},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00135))}{t-0.00135},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001375))}{t-0.001375},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0014))}{t-0.0014},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001425))}{t-0.001425},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00145))}{t-0.00145},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001475))}{t-0.001475},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0015))}{t-0.0015},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001525))}{t-0.001525},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00155))}{t-0.00155},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001575))}{t-0.001575},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0016))}{t-0.0016},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001625))}{t-0.001625},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00165))}{t-0.00165},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001675))}{t-0.001675},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0017))}{t-0.0017},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001725))}{t-0.001725},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00175))}{t-0.00175},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001775))}{t-0.001775},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0018))}{t-0.0018},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001825))}{t-0.001825},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00185))}{t-0.00185},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001875))}{t-0.001875},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0019))}{t-0.0019},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001925))}{t-0.001925},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00195))}{t-0.00195},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001975))}{t-0.001975},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.002))}{t-0.002}\Bigg\}$$

The reconstructed signal using samples:

Out[169]//TraditionalForm=

$$\frac{0.00024669\sin(125\,664.\,(t-0.002))}{t-0.002}+\frac{0.000017899\sin(125\,664.\,(t-0.001975))}{t-0.001975}-$$

$$\frac{0.000161752\sin(125\,664.\,(t-0.00195))}{t-0.00195}-\frac{0.000175354\sin(125\,664.\,(t-0.001925))}{t-0.001925}-$$

$$\frac{0.0000957829\sin(125\,664.\,(t-0.0019))}{t-0.0019}+\frac{0.0000449546\sin(125\,664.\,(t-0.001875))}{t-0.001875}+$$

$$\frac{0.000169301\sin(125\,664.\,(t-0.00185))}{t-0.00185}+\frac{0.000104191\sin(125\,664.\,(t-0.001825))}{t-0.001825}-$$

$$\frac{0.000108718\sin(125\,664.\,(t-0.0018))}{t-0.0018}-\frac{0.000150561\sin(125\,664.\,(t-0.001775))}{t-0.001775}+$$

$$\frac{0.0000727508\sin(125\,664.\,(t-0.00175))}{t-0.00175}+\frac{0.00021405\sin(125\,664.\,(t-0.001725))}{t-0.001725}+$$

$$\frac{0.0000455189\sin(125\,664.\,(t-0.0017))}{t-0.0017}-\frac{0.000148323\sin(125\,664.\,(t-0.001675))}{t-0.001675}-$$

$$\frac{0.0000651329\sin(125\,664.\,(t-0.00165))}{t-0.00165}+\frac{0.000106722\sin(125\,664.\,(t-0.001625))}{t-0.001625}+$$

$$\frac{0.0000504961\sin(125\,664.\,(t-0.0016))}{t-0.0016}-\frac{0.000140743\sin(125\,664.\,(t-0.001575))}{t-0.001575}-$$

$$\frac{0.000176888\sin(125\,664.\,(t-0.00155))}{t-0.00155}-\frac{0.0000466115\sin(125\,664.\,(t-0.001525))}{t-0.001525}+$$

$$\frac{0.0000716197\sin(125\,664.\,(t-0.0015))}{t-0.0015}+\frac{0.000124441\sin(125\,664.\,(t-0.001475))}{t-0.001475}+$$

$$\frac{0.00011257\sin(125\,664.\,(t-0.00145))}{t-0.00145}-\frac{0.0000248027\sin(125\,664.\,(t-0.001425))}{t-0.001425}-$$

$$\frac{0.000192131\sin(125\,664.\,(t-0.0014))}{t-0.0014}-\frac{0.000117976\sin(125\,664.\,(t-0.001375))}{t-0.001375}+$$

$$\frac{0.000184537\sin(125\,664.\,(t-0.00135))}{t-0.00135}+\frac{0.000292463\sin(125\,664.\,(t-0.001325))}{t-0.001325}-$$

$$\frac{1.25552\times10^{-6}\sin(125\,664.\,(t-0.0013))}{t-0.0013}-\frac{0.000310089\sin(125\,664.\,(t-0.001275))}{t-0.001275}-$$

$$\frac{0.000231906\sin(125\,664.\,(t-0.00125))}{t-0.00125}+\frac{0.0000595018\sin(125\,664.\,(t-0.001225))}{t-0.001225}+$$

$$\frac{0.000162818\sin(125\,664.\,(t-0.0012))}{t-0.0012}+\frac{0.0000543996\sin(125\,664.\,(t-0.001175))}{t-0.001175}-$$

$$\frac{0.0000311872\sin(125\,664.\,(t-0.00115))}{t-0.00115}-\frac{0.0000337007\sin(125\,664.\,(t-0.001125))}{t-0.001125}-$$

$$\frac{0.0000201002\sin(125\,664.\,(t-0.0011))}{t-0.0011}+\frac{0.0000381691\sin(125\,664.\,(t-0.001075))}{t-0.001075}+$$

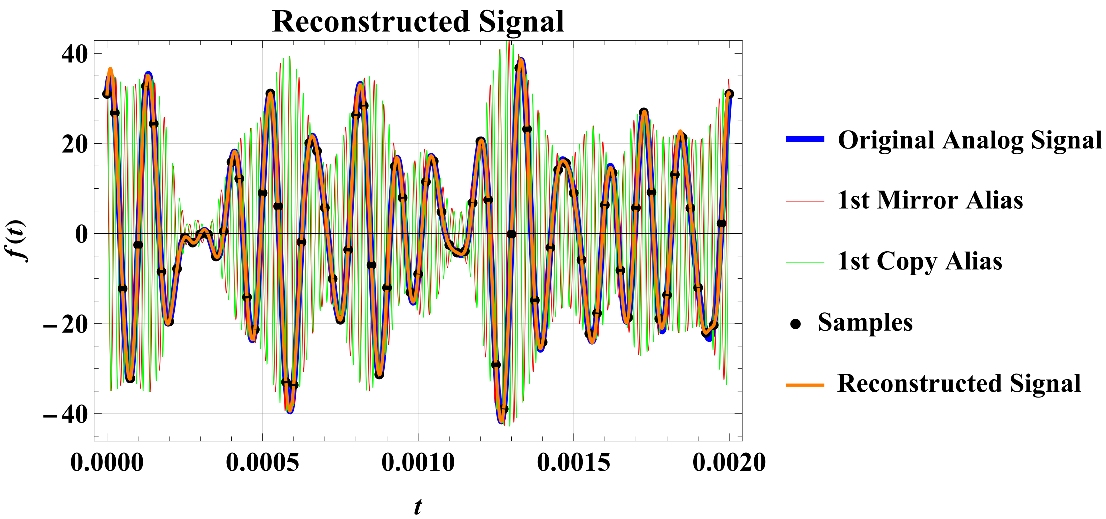
$$\frac{0.000127707\sin(125\,664.\,(t-0.00105))}{t-0.00105}+\frac{0.0000913695\sin(125\,664.\,(t-0.001025))}{t-0.001025}-$$

$$\frac{0.0000716197\sin(125\,664.\,(t-0.001))}{t-0.001}-\frac{0.000103913\sin(125\,664.\,(t-0.000975))}{t-0.000975}+$$

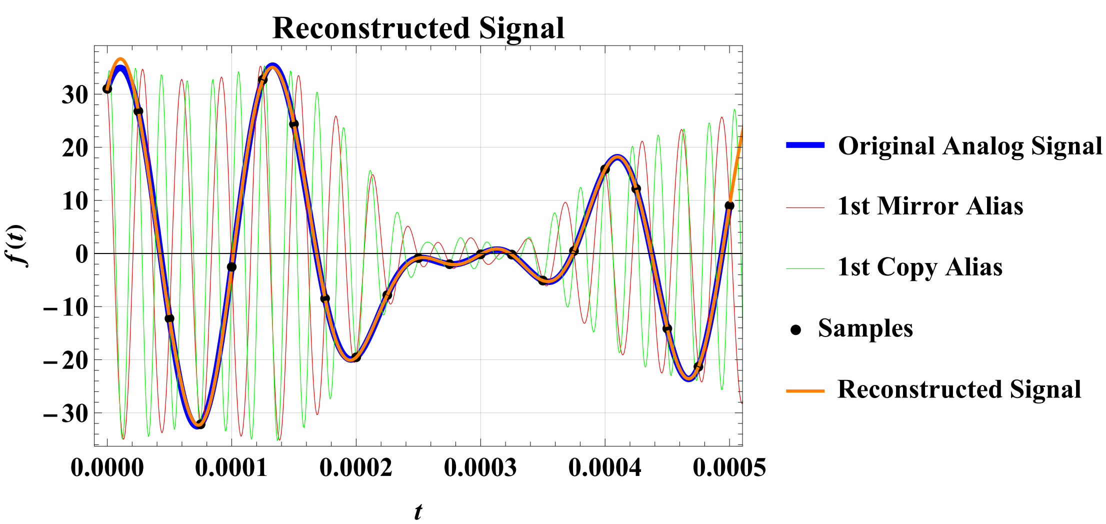
$$\frac{0.0000633273\sin(125\,664.\,(t-0.00095))}{t-0.00095}+\frac{0.000118726\sin(125\,664.\,(t-0.000925))}{t-0.000925}-$$

$$\frac{0.0000957829 \sin(125\,664.\,(t-0.0009))}{t-0.0009} - \frac{0.000249125 \sin(125\,664.\,(t-0.000875))}{t-0.000875} - \frac{0.000055778 \sin(125\,664.\,(t-0.00085))}{t-0.00085} + \frac{0.000226003 \sin(125\,664.\,(t-0.000825))}{t-0.000825} + \frac{0.000209592 \sin(125\,664.\,(t-0.0008))}{t-0.0008} - \frac{0.0000287491 \sin(125\,664.\,(t-0.000775))}{t-0.000775} - \frac{0.000152328 \sin(125\,664.\,(t-0.00075))}{t-0.00075} - \frac{0.0000800298 \sin(125\,664.\,(t-0.000725))}{t-0.000725} + \frac{0.0000455189 \sin(125\,664.\,(t-0.0007))}{t-0.0007} + \frac{0.000145757 \sin(125\,664.\,(t-0.000675))}{t-0.000675} + \frac{0.000159946 \sin(125\,664.\,(t-0.00065))}{t-0.00065} - \frac{0.0000150904 \sin(125\,664.\,(t-0.000625))}{t-0.000625} - \frac{0.000267814 \sin(125\,664.\,(t-0.0006))}{t-0.0006} - \frac{0.000262555 \sin(125\,664.\,(t-0.000575))}{t-0.000575} + \frac{0.0000481907 \sin(125\,664.\,(t-0.00055))}{t-0.00055} + \frac{0.000247468 \sin(125\,664.\,(t-0.000525))}{t-0.000525} + \frac{0.0000716197 \sin(125\,664.\,(t-0.0005))}{t-0.0005} - \frac{0.000169639 \sin(125\,664.\,(t-0.000475))}{t-0.000475} - \frac{0.000112509 \sin(125\,664.\,(t-0.00045))}{t-0.00045} + \frac{0.0000970092 \sin(125\,664.\,(t-0.000425))}{t-0.000425} + \frac{0.000126179 \sin(125\,664.\,(t-0.0004))}{t-0.0004} + \frac{3.8364\times 10^{-6} \sin(125\,664.\,(t-0.000375))}{t-0.000375} - \frac{0.0000405421 \sin(125\,664.\,(t-0.00035))}{t-0.00035} - \frac{1.61692\times 10^{-6} \sin(125\,664.\,(t-0.000325))}{t-0.000325} - \frac{1.25552\times 10^{-6} \sin(125\,664.\,(t-0.0003))}{t-0.0003} - \frac{0.0000160088 \sin(125\,664.\,(t-0.000275))}{t-0.000275} - \frac{6.82667\times 10^{-6} \sin(125\,664.\,(t-0.00025))}{t-0.00025} - \frac{0.0000623101 \sin(125\,664.\,(t-0.000225))}{t-0.000225} - \frac{0.000155492 \sin(125\,664.\,(t-0.0002))}{t-0.0002} - \frac{0.0000674123 \sin(125\,664.\,(t-0.000175))}{t-0.000175} + \frac{0.000193892 \sin(125\,664.\,(t-0.00015))}{t-0.00015} + \frac{0.000260379 \sin(125\,664.\,(t-0.000125))}{t-0.000125} - \frac{0.0000201002 \sin(125\,664.\,(t-0.0001))}{t-0.0001} - \frac{0.000255911 \sin(125\,664.\,(t-0.000075))}{t-0.000075} - \frac{0.0000973723 \sin(125\,664.\,(t-0.00005))}{t-0.00005} + \frac{0.000213181 \sin(125\,664.\,(t-0.000025))}{t-0.000025} + \frac{0.00024669 \sin(125\,664.\,t)}{t}$$

Out[170]=



Out[171]=

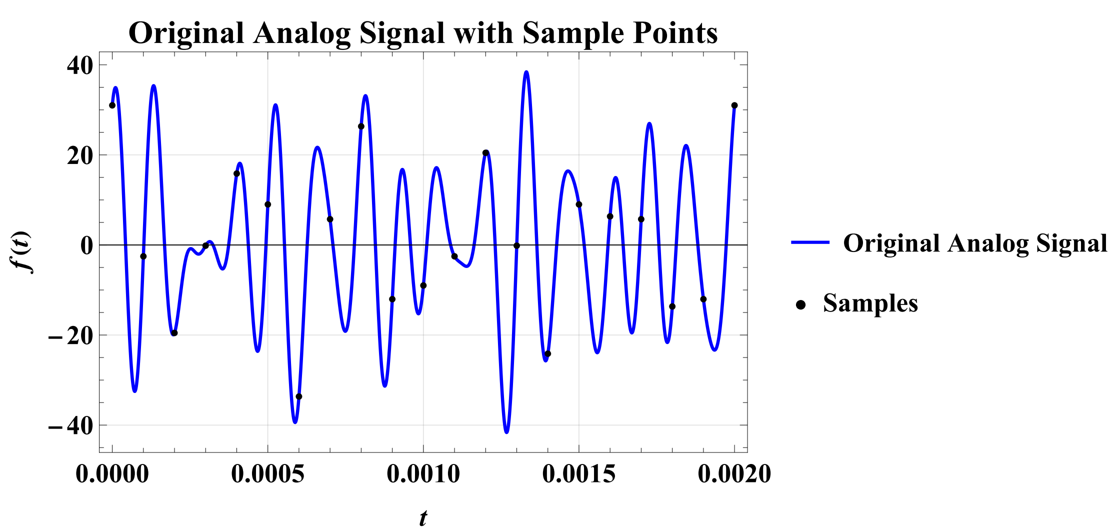


Problem 12

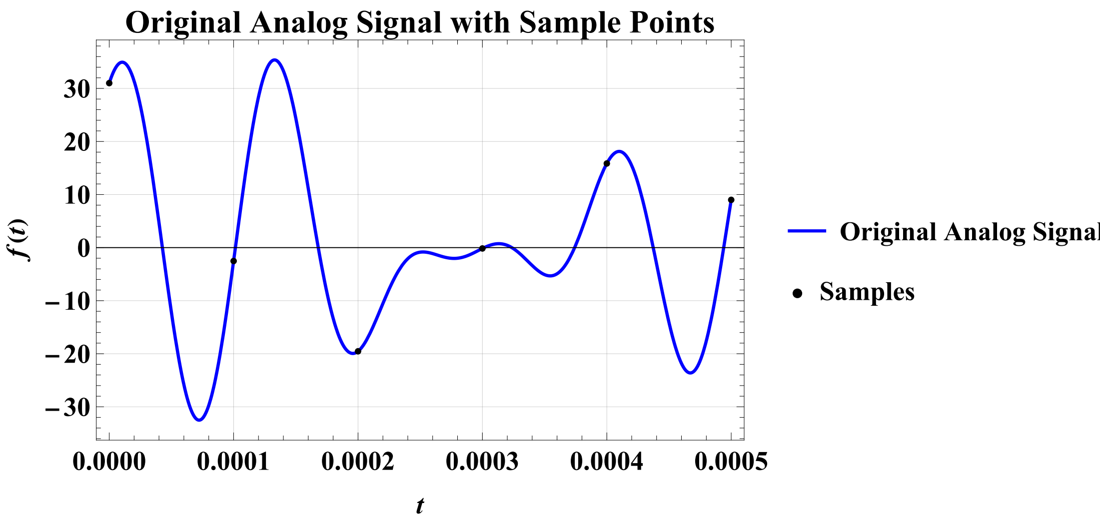
In[188]:=

```
(* Given the sampling frequency is the half of the Nyquist frequency. *)
SamplingFrequency2 = NyquistFrequency/2;
SamplingTime2 = Range[0, 0.002, 1/SamplingFrequency2];

(* Sample the signal. *)
fSeq2 = Table[N[f[SamplingTime2[[i]]]], {i, 1, Length[SamplingTime2]}];
fSeqList2 = Transpose[{SamplingTime2, fSeq2}];

(* Plot the original signal and the sampled signal. *)
Show[Plot[{f[t]}, {t, 0, 0.002}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.005]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]},
    PlotLabel→"Original Analog Signal with Sample Points", GridLines→Automatic,
    LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Frame→True],
ListPlot[fSeqList2, PlotStyle→{Directive[PointSize[0.01], Black]}, PlotLegends→{Style["Samples",
    Bold, FontSize→14, FontFamily→"Times"]}]]

Show[Plot[{f[t]}, {t, 0, 0.0005}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.005]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]},
    PlotLabel→"Original Analog Signal with Sample Points", GridLines→Automatic,
    LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Frame→True],
ListPlot[fSeqList2, PlotStyle→{Directive[PointSize[0.01], Black]}, PlotLegends→{Style["Samples",
    Bold, FontSize→14, FontFamily→"Times"]}]]
```

Out[192]=



Out[193]=


Original Analog Signal with Sample Points
30
20
10
0
−10
−20
−30
f(t)
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005
t
Original Analog Signal
Samples

Problem 13

In[202]:=

```
(* Define the set of shape functions. *)
ShapeFunctionSet2 = N[Table[Sin[2π*(SamplingFrequency2/2)*(t-n/SamplingFrequency2)] / (2π*(Sampl
    {n, 0, Length[SamplingTime2]-1}]];
Print[Style["The set of shape functions: ", Bold, FontFamily→"Times", FontSize→14]]
TraditionalForm[ShapeFunctionSet]

(* Reconstruct the signal using the samples. *)
ReconstructedSignal2 = Dot[fSeq2, ShapeFunctionSet2];
Print[Style["The reconstructed signal using samples: ", Bold, FontFamily→"Times", FontSize→14]]
TraditionalForm[ReconstructedSignal2]

(* Plot the reconstructed signal with the original signal and its alias. *)
Show[Plot[f[t], {t, 0, 0.002}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.01]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]}, PlotLabel→"Reconstructed Signal",
    GridLines→Automatic, LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Fr
Plot[fMirror[t], {t, 0, 0.002}, PlotStyle→{Red, Thickness[0.001]},
    PlotLegends→{Style["1st Mirror Alias", Bold, FontSize→14, FontFamily→"Times"]}],
Plot[fCopy[t], {t, 0, 0.002}, PlotStyle→{Green, Thickness[0.001]},
    PlotLegends→{Style["1st Copy Alias", Bold, FontSize→14, FontFamily→"Times"]}],
ListPlot[fSeqList2, PlotStyle→{Directive[PointSize[0.015], Black]}, PlotLegends→{Style["Samples"
    Bold, FontSize→14, FontFamily→"Times"]}],
Plot[ReconstructedSignal2, {t, 0, 0.002}, PlotStyle→{Orange, Thickness[0.005]},
    PlotLegends→{Style["Reconstructed Signal", Bold, FontSize→14, FontFamily→"Times"]}]]

Show[Plot[f[t], {t, 0, 0.0005}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.01]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]}, PlotLabel→"Reconstructed Signal",
    GridLines→Automatic, LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Fr
Plot[fMirror[t], {t, 0, 0.002}, PlotStyle→{Red, Thickness[0.001]},
    PlotLegends→{Style["1st Mirror Alias", Bold, FontSize→14, FontFamily→"Times"]}],
Plot[fCopy[t], {t, 0, 0.002}, PlotStyle→{Green, Thickness[0.001]},
    PlotLegends→{Style["1st Copy Alias", Bold, FontSize→14, FontFamily→"Times"]}],
ListPlot[fSeqList2, PlotStyle→{Directive[PointSize[0.015], Black]}, PlotLegends→{Style["Samples"
    Bold, FontSize→14, FontFamily→"Times"]}],
Plot[ReconstructedSignal2, {t, 0, 0.002}, PlotStyle→{Orange, Thickness[0.005]},
    PlotLegends→{Style["Reconstructed Signal", Bold, FontSize→14, FontFamily→"Times"]}]]
```

The set of shape functions:

Out[204]//TraditionalForm=

$$\left\{\frac{7.95775\times 10^{-6}\sin(125\,664.\,t)}{t}, \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.000025))}{t-0.000025}, \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.00005))}{t-0.00005},\right.$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.000075))}{t-0.000075}, \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.0001))}{t-0.0001},$$

$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000125))}{t-0.000125},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00015))}{t-0.00015},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000175))}{t-0.000175},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0002))}{t-0.0002},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000225))}{t-0.000225},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00025))}{t-0.00025},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000275))}{t-0.000275},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0003))}{t-0.0003},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000325))}{t-0.000325},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00035))}{t-0.00035},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000375))}{t-0.000375},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0004))}{t-0.0004},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000425))}{t-0.000425},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00045))}{t-0.00045},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000475))}{t-0.000475},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0005))}{t-0.0005},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000525))}{t-0.000525},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00055))}{t-0.00055},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000575))}{t-0.000575},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0006))}{t-0.0006},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000625))}{t-0.000625},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00065))}{t-0.00065},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000675))}{t-0.000675},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0007))}{t-0.0007},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000725))}{t-0.000725},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00075))}{t-0.00075},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000775))}{t-0.000775},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0008))}{t-0.0008},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000825))}{t-0.000825},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00085))}{t-0.00085},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000875))}{t-0.000875},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0009))}{t-0.0009},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000925))}{t-0.000925},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00095))}{t-0.00095},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.000975))}{t-0.000975},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001))}{t-0.001},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001025))}{t-0.001025},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.00105))}{t-0.00105},$$
$$\frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.001075))}{t-0.001075},\ \frac{7.95775\times10^{-6}\sin(125\,664.\,(t-0.0011))}{t-0.0011},$$

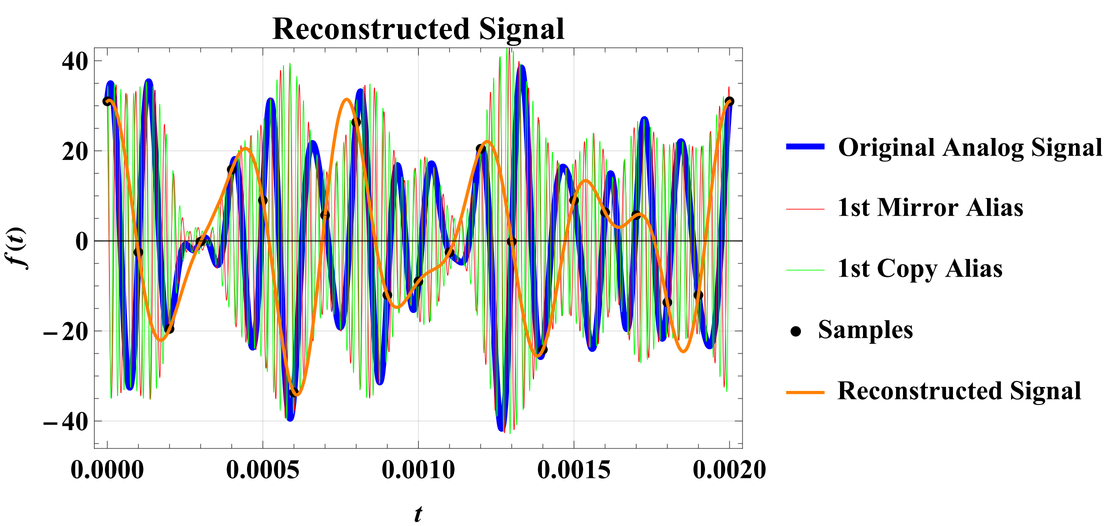
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001125))}{t-0.001125},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.00115))}{t-0.00115},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001175))}{t-0.001175},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.0012))}{t-0.0012},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001225))}{t-0.001225},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.00125))}{t-0.00125},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001275))}{t-0.001275},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.0013))}{t-0.0013},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001325))}{t-0.001325},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.00135))}{t-0.00135},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001375))}{t-0.001375},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.0014))}{t-0.0014},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001425))}{t-0.001425},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.00145))}{t-0.00145},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001475))}{t-0.001475},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.0015))}{t-0.0015},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001525))}{t-0.001525},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.00155))}{t-0.00155},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001575))}{t-0.001575},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.0016))}{t-0.0016},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001625))}{t-0.001625},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.00165))}{t-0.00165},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001675))}{t-0.001675},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.0017))}{t-0.0017},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001725))}{t-0.001725},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.00175))}{t-0.00175},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001775))}{t-0.001775},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.0018))}{t-0.0018},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001825))}{t-0.001825},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.00185))}{t-0.00185},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001875))}{t-0.001875},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.0019))}{t-0.0019},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001925))}{t-0.001925},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.00195))}{t-0.00195},$$
$$\frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.001975))}{t-0.001975},\ \frac{7.95775\times 10^{-6}\sin(125\,664.\,(t-0.002))}{t-0.002}\Big\}$$

The reconstructed signal using samples:

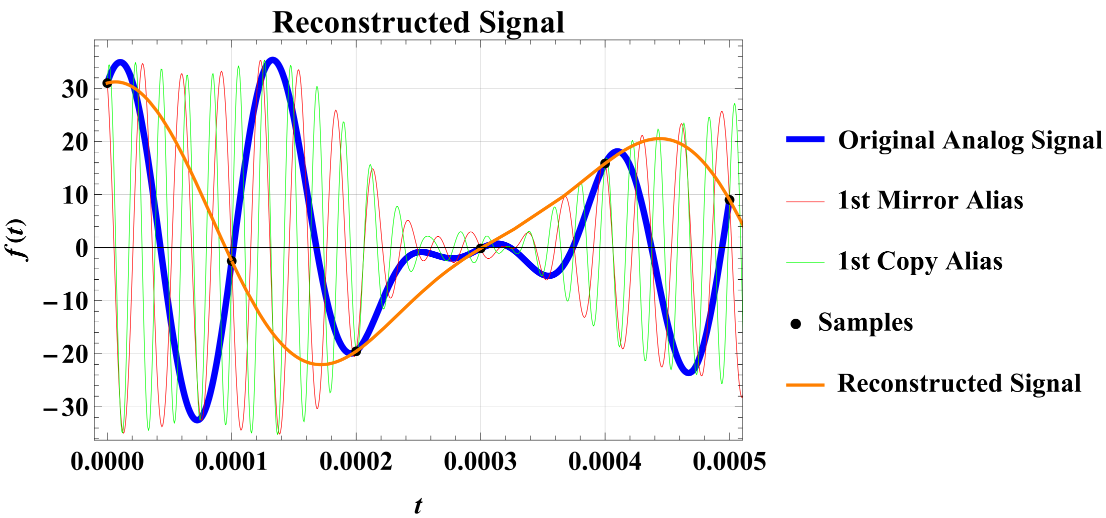
Out[207]//TraditionalForm=

$$\frac{0.000986761 \sin(31\,415.9\,(t-0.002))}{t-0.002} - \frac{0.000383132 \sin(31\,415.9\,(t-0.0019))}{t-0.0019} - \frac{0.000434871 \sin(31\,415.9\,(t-0.0018))}{t-0.0018} + \frac{0.000182076 \sin(31\,415.9\,(t-0.0017))}{t-0.0017} + \frac{0.000201985 \sin(31\,415.9\,(t-0.0016))}{t-0.0016} + \frac{0.000286479 \sin(31\,415.9\,(t-0.0015))}{t-0.0015} - \frac{0.000768524 \sin(31\,415.9\,(t-0.0014))}{t-0.0014} - \frac{5.02208\times 10^{-6} \sin(31\,415.9\,(t-0.0013))}{t-0.0013} + \frac{0.00065127 \sin(31\,415.9\,(t-0.0012))}{t-0.0012} - \frac{0.000080401 \sin(31\,415.9\,(t-0.0011))}{t-0.0011} - \frac{0.000286479 \sin(31\,415.9\,(t-0.001))}{t-0.001} - \frac{0.000383132 \sin(31\,415.9\,(t-0.0009))}{t-0.0009} + \frac{0.000838368 \sin(31\,415.9\,(t-0.0008))}{t-0.0008} + \frac{0.000182076 \sin(31\,415.9\,(t-0.0007))}{t-0.0007} - \frac{0.00107126 \sin(31\,415.9\,(t-0.0006))}{t-0.0006} + \frac{0.000286479 \sin(31\,415.9\,(t-0.0005))}{t-0.0005} + \frac{0.000504715 \sin(31\,415.9\,(t-0.0004))}{t-0.0004} - \frac{5.02208\times 10^{-6} \sin(31\,415.9\,(t-0.0003))}{t-0.0003} - \frac{0.000621969 \sin(31\,415.9\,(t-0.0002))}{t-0.0002} - \frac{0.000080401 \sin(31\,415.9\,(t-0.0001))}{t-0.0001} + \frac{0.000986761 \sin(31\,415.9\,t)}{t}$$

Out[208]=



Out[209]=



The reconstructed signal does not match the original analog signal since the sampling frequency is lower than the Nyquist frequency of the signal. The sampling frequency is 10000 Hz for this case, which means that the maximum frequency that the reconstructed signal can capture is 5000 Hz (half of the sampling frequency). So all the components with frequency higher than 5000 Hz (6000 Hz, 7500 Hz and 10000 Hz) are missed.

Problem 14

In[111]:=

```
(* Quantization by rounding. *)
Print[Style["Samples after quantization by rounding: ", Bold, FontFamily→"Times", FontSize→14]]
fSeqRound = Round[fSeq, 1]
fSeqRoundList = Transpose[{SamplingTime, fSeqRound}];

(* Quantization by truncating. *)
Print[Style["Samples after quantization by truncating: ", Bold, FontFamily→"Times", FontSize→14]]
fSeqTruncate = Floor[fSeq, 1]
fSeqTruncateList = Transpose[{SamplingTime, fSeqTruncate}];

(* Plot the original signal and the rounded samples. *)
Show[Plot[f[t], {t, 0, 0.002}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.005]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]}, PlotLabel→"Quantization by Rounding"
    GridLines→Automatic, LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Fr
ListPlot[fSeqList, PlotStyle→{Directive[PointSize[0.01], Black]}, PlotLegends→{Style["Samples",
    Bold, FontSize→14, FontFamily→"Times"]}],
ListStepPlot[fSeqRoundList, DataRange→{0, Length[fSeqRound]-1}, PlotStyle → {Directive[PointSize
    PlotLegends→{Style["Step Function for Rounded Samples", Bold, FontSize→14, FontFamily→"Times
ListPlot[fSeqRoundList, PlotStyle→{Directive[PointSize[0.01], Green]}, PlotLegends→{Style["Round
    Bold, FontSize→14, FontFamily→"Times"]}]]

(* Plot the original signal and the truncated samples. *)
Show[Plot[f[t], {t, 0, 0.002}, ImageSize→{400, 300}, PlotStyle→{Blue, Thickness[0.005]},
    PlotLegends→{Style["Original Analog Signal", Bold, FontSize→14, FontFamily→"Times"]},
    FrameLabel→{Style["t", Italic], Style["f(t)", Italic]}, PlotLabel→"Quantization by Truncatin
    GridLines→Automatic, LabelStyle→{RGBColor[0,0,0], Bold, FontSize→14, FontFamily→"Times"}, Fr
ListPlot[fSeqList, PlotStyle→{Directive[PointSize[0.01], Black]}, PlotLegends→{Style["Samples",
    Bold, FontSize→14, FontFamily→"Times"]}],
ListStepPlot[fSeqTruncateList, DataRange→{0, Length[fSeqRound]-1}, PlotStyle → {Directive[PointS
    PlotLegends→{Style["Step Function for Truncated Samples", Bold, FontSize→14, FontFamily→"Tim
ListPlot[fSeqTruncateList, PlotStyle→{Directive[PointSize[0.01], Green]}, PlotLegends→{Style["Tr
    Bold, FontSize→14, FontFamily→"Times"]}]]
```

Samples after quantization by rounding:

Out[112]=

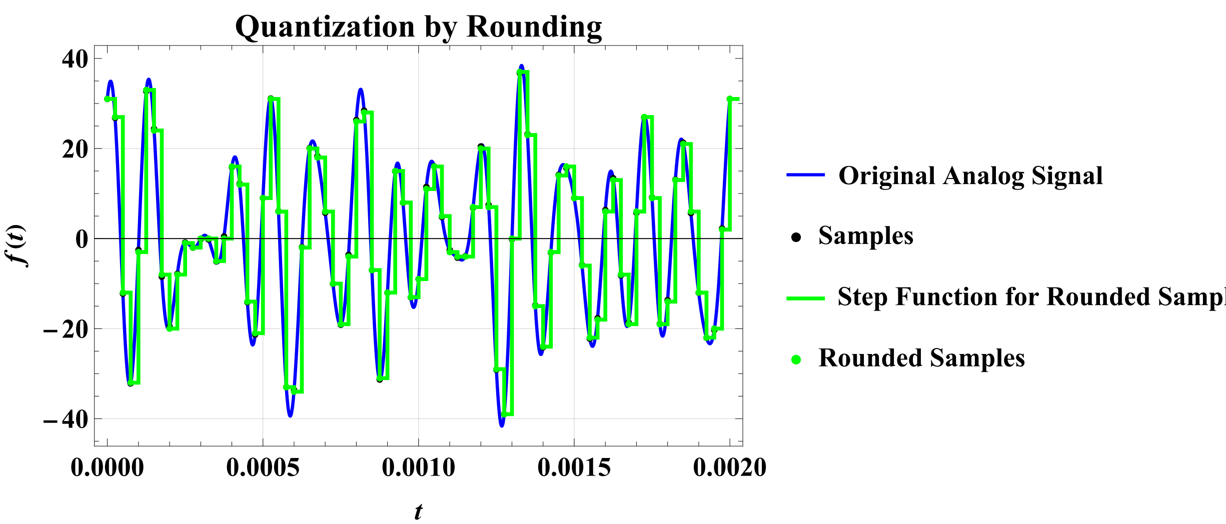
{31, 27, -12, -32, -3, 33, 24, -8, -20, -8, -1, -2, 0, 0, -5, 0, 16, 12, -14, -21,
9, 31, 6, -33, -34, -2, 20, 18, 6, -10, -19, -4, 26, 28, -7, -31, -12, 15, 8, -13,
-9, 11, 16, 5, -3, -4, -4, 7, 20, 7, -29, -39, 0, 37, 23, -15, -24, -3, 14, 16, 9,
-6, -22, -18, 6, 13, -8, -19, 6, 27, 9, -19, -14, 13, 21, 6, -12, -22, -20, 2, 31}

Samples after quantization by truncating:

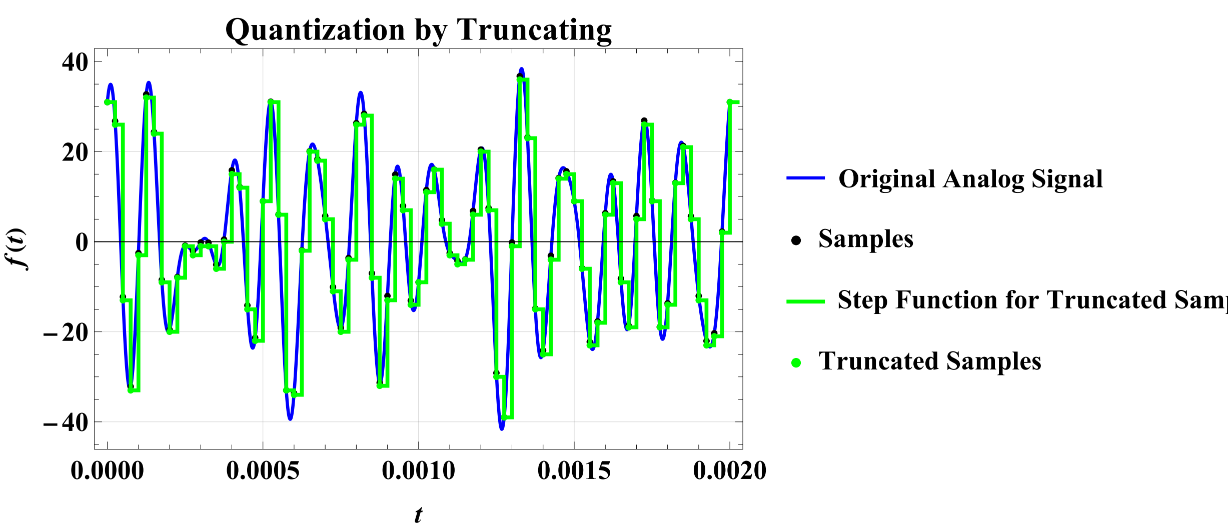
Out[115]=

{31, 26, -13, -33, -3, 32, 24, -9, -20, -8, -1, -3, -1, -1, -6, 0, 15, 12, -15, -22,
9, 31, 6, -33, -34, -2, 20, 18, 5, -11, -20, -4, 26, 28, -8, -32, -13, 14, 7, -14,
-9, 11, 16, 4, -3, -5, -4, 6, 20, 7, -30, -39, -1, 36, 23, -15, -25, -4, 14, 15, 9,
-6, -23, -18, 6, 13, -9, -19, 5, 26, 9, -19, -14, 13, 21, 5, -13, -23, -21, 2, 31}

Out[117]=



Out[118]=



Problem 15

In[252]:=

```
(* Calculate the average sampled sequence power. *)
Print[Style["Average sampled sequence power: ", Bold, FontFamily→"Times", FontSize→14]]
AvgSeqPower = Dot[fSeq, fSeq] / Length[fSeq]

(* Error calculation for rounding. *)
Print[Style["Error sequence for rounded samples: ", Bold, FontFamily→"Times", FontSize→14]]
RoundingError = fSeq - fSeqRound

Print[Style["Quantization error power for rounded samples: ", Bold, FontFamily→"Times", FontSize→
RoundingErrorPower = Dot[RoundingError, RoundingError] / Length[RoundingError]

Print[Style["Signal-to-Noise ratio for rounded samples: ", Bold, FontFamily→"Times", FontSize→14
RoundingSNR = 10log[AvgSeqPower/RoundingErrorPower]

(* Error calculation for truncating. *)
Print[Style["Error sequence for truncated samples: ", Bold, FontFamily→"Times", FontSize→14]]
TruncatingError = fSeq - fSeqTruncate

Print[Style["Quantization error power for truncated samples: ", Bold, FontFamily→"Times", FontSiz
TruncatingErrorPower = Dot[TruncatingError, TruncatingError] / Length[TruncatingError]

Print[Style["Signal-to-Noise ratio for truncated samples: ", Bold, FontFamily→"Times", FontSize→
TruncatingSNR = 10log[AvgSeqPower/TruncatingErrorPower]
```

Average sampled sequence power:

Out[253]=

321.

Error sequence for rounded samples:

Out[255]=

{0., -0.210838, -0.236164, -0.158708, 0.47413, -0.279769, 0.365176, -0.471285, 0.460261,
0.169881, 0.142136, -0.011729, -0.157773, -0.203188, -0.094666, 0.482096, -0.143904,
0.190539, -0.138277, -0.317474, 0., 0.0978023, 0.0558242, 0.00631796, 0.34553,
0.103691, 0.0994353, 0.316338, -0.279921, -0.056845, -0.142136, 0.387283, 0.338113,
0.400397, -0.0092654, -0.306018, -0.0364355, -0.0804097, -0.0420628, -0.0580803,
0., 0.481825, 0.0481072, -0.203526, 0.47413, -0.23495, 0.0809045, -0.163948,
0.460261, 0.477218, -0.142136, 0.0330897, -0.157773, -0.248007, 0.189605, 0.174758,
-0.143904, -0.116799, 0.145994, -0.362293, 0., 0.142621, -0.228447, 0.313655, 0.34553,
0.411028, -0.184836, 0.361156, -0.279921, -0.101664, 0.142136, 0.0799459, 0.338113,
0.0930593, 0.275006, -0.350836, -0.0364355, -0.035591, -0.326334, 0.249257, 0.}

Quantization error power for rounded samples:

Out[257]=

0.0621903

Signal–to–Noise ratio for rounded samples:

Out[259]=

10 log[5161.58]

Error sequence for truncated samples:

Out[261]=

{0., 0.789162, 0.763836, 0.841292, 0.47413, 0.720231, 0.365176, 0.528715, 0.460261, 0.169881, 0.142136, 0.988271, 0.842227, 0.796812, 0.905334, 0.482096, 0.856096, 0.190539, 0.861723, 0.682526, 0., 0.0978023, 0.0558242, 0.00631796, 0.34553, 0.103691, 0.0994353, 0.316338, 0.720079, 0.943155, 0.857864, 0.387283, 0.338113, 0.400397, 0.990735, 0.693982, 0.963564, 0.91959, 0.957937, 0.94192, 0., 0.481825, 0.0481072, 0.796474, 0.47413, 0.76505, 0.0809045, 0.836052, 0.460261, 0.477218, 0.857864, 0.0330897, 0.842227, 0.751993, 0.189605, 0.174758, 0.856096, 0.883201, 0.145994, 0.637707, 0., 0.142621, 0.771553, 0.313655, 0.34553, 0.411028, 0.815164, 0.361156, 0.720079, 0.898336, 0.142136, 0.0799459, 0.338113, 0.0930593, 0.275006, 0.649164, 0.963564, 0.964409, 0.673666, 0.249257, 0.}

Quantization error power for truncated samples:

Out[263]=

0.364601

Signal−to−Noise ratio for truncated samples:

Out[265]=

10 log[880.413]

Problem 8

$f_{p1} = 1000\,Hz$ — $A_1 = 1$ $\phi_1 = 0$
$f_{p2} = 3000\,Hz$ — $A_2 = 5$ $\phi_2 = -\pi/2$
$f_{p3} = 6000\,Hz$ — $A_3 = 10$ $\phi_3 = 0$
$f_{p4} = 7500\,Hz$ — $A_4 = 20$ $\phi_4 = 0$
$f_{p5} = 10000\,Hz$ — $A_5 = 10$ $\phi_5 = -\pi/2$

1st mirror alias

$f_s - f_{p5} = 30000\,Hz$ — $A_5 = 10$ $-\phi_5 = \pi/2$
$f_s - f_{p4} = 32500\,Hz$ — $A_4 = 20$ $-\phi_4 = 0$
$f_s - f_{p3} = 34000\,Hz$ — $A_3 = 10$ $-\phi_3 = 0$
$f_s - f_{p2} = 37000\,Hz$ — $A_2 = 5$ $-\phi_2 = \pi/2$
$f_s - f_{p1} = 39000\,Hz$ — $A_1 = 1$ $-\phi_1 = 0$
$f_s = 40000\,Hz$

1st copy alias

$f_s + f_{p1} = 41000\,Hz$ — $A_1 = 1$ $\phi_1 = 0$
$f_s + f_{p2} = 43000\,Hz$ — $A_2 = 5$ $\phi_2 = -\pi/2$
$f_s + f_{p3} = 46000\,Hz$ — $A_3 = 10$ $\phi_3 = 0$
$f_s + f_{p4} = 47500\,Hz$ — $A_4 = 20$ $\phi_4 = 0$
$f_s + f_{p5} = 50000\,Hz$ — $A_5 = 10$ $\phi_5 = -\pi/2$

2nd mirror alias

$2f_s - f_5 = 70000\,Hz$ — $A_5 = 10$ $-\phi_5 = \pi/2$
$2f_s - f_{p4} = 72500\,Hz$ — $A_4 = 20$ $-\phi_4 = 0$
$2f_s - f_{p3} = 74000\,Hz$ — $A_3 = 10$ $-\phi_3 = 0$
$2f_s - f_{p2} = 77000\,Hz$ — $A_2 = 5$ $-\phi_2 = \pi/2$
$2f_s - f_{p1} = 79000\,Hz$ — $A_1 = 1$ $-\phi_1 = 0$
$2f_s = 80000\,Hz$

2nd copy alias

$2f_s + f_{p1} = 81000\,Hz$ — $A_1 = 1$ $\phi_1 = 0$
$2f_s + f_{p2} = 83000\,Hz$ — $A_2 = 5$ $\phi_2 = -\pi/2$
$2f_s + f_{p3} = 86000\,Hz$ — $A_3 = 10$ $\phi_3 = 0$
$2f_s + f_{p4} = 87500\,Hz$ — $A_4 = 20$ $\phi_4 = 0$
$2f_s + f_{p5} = 90000\,Hz$ — $A_5 = 10$ $\phi_5 = -\pi/2$

Frequency f (Hz)