9.1. A 16 in. square column is reinforced with four No. 11 (No. 36) bars, one in each corner, with cover distances 3 in. to the steel center in each direction. Material strengths are $f_c' = 5000$ psi and $f_y = 60{,}000$ psi. Construct the interaction diagram relating axial strength P_n and flexural strength M_n. Bending will be about an axis parallel to one face. Calculate the coordinates for P_o, P_b, and at least three other representative points on the curve.

9.2. Starting with the column in Problem 9.1, perform enough additional calculations to determine the effects of increasing f_c' from 5000 to 8000 psi on column capacity at both high and low axial loads. Assuming that a compressive strength of 8000 psi is appropriate for the lower stories of a high-rise structure, would you recommend using concrete with $f_c' = 8000$ psi for the columns supporting all stories within the building? Use your analysis to support your answer.

```
% RC Columns P-M Diagram.
% Solution Code for SE151A_WI25_HW8.
% Author: Ruipu Ji.

% Initialization and default plot settings. -------------------------------
clc; clear; close all;

set(0, 'DefaultTextInterpreter', 'latex');
set(0, 'DefaultLegendInterpreter', 'latex');
set(0, 'DefaultAxesTickLabelInterpreter', 'latex');

set(0, 'DefaultAxesFontSize', 15);
set(0, 'DefaultTextFontSize', 15);

% Enter input information. ------------------------------------------------
% Section dimension.
b = 16; % Section width (unit = inch).
h = 16; % Section height (unit = inch).

% Material properties.
fc_1 = 5; % Concrete compressive strength for Problem 1 (unit = ksi).
fc_2 = 8; % Concrete compressive strength for Problem 2 (unit = ksi).
beta_1 = 0.85 - 0.05*(fc_1-4)/1;
beta_2 = 0.85 - 0.05*(fc_2-4)/1;

fy = 60; % Steel reinforcement yield strength (unit = ksi).
Es = 29000; % Elastic modulus of steel reinforcement (unit = ksi).
eps_cu = -0.003; % Strain of the most compressive fiber at the peak capacity.

% Reinforcement layout.
d = [3 3 13 13]; % Distance between the center of each reinforcement and the
top of the section (unit = inch).
As = [1.56 1.56 1.56 1.56]; % Area of each reinforcement (unit = inch^2).

% Calculate P-M interaction. ----------------------------------------------
% Initialize an 101x9 array for result output.
% (100 rows for P-M interaction and the last row for pure axial load P0).
Results = zeros(101, 9);

for i = 1:100
    % Calculate the neutral axis distance c.
    c = 0.01*i*h;

    % Calculate strain of each reinforcement based on strain profile.
    % Sign convention: Compression is negative (-); Tension is positive (+).
    eps_s = eps_cu*(c-d)/c;

    % Calculate stress of each reinforcement based on stress-strain
relationship.
    % Sign convention: Compression is negative (-); Tension is positive (+).
    fs = Es*eps_s;
    fs = max(min(fs, fy), -fy); % Limit stress values between -fy and fy.
```

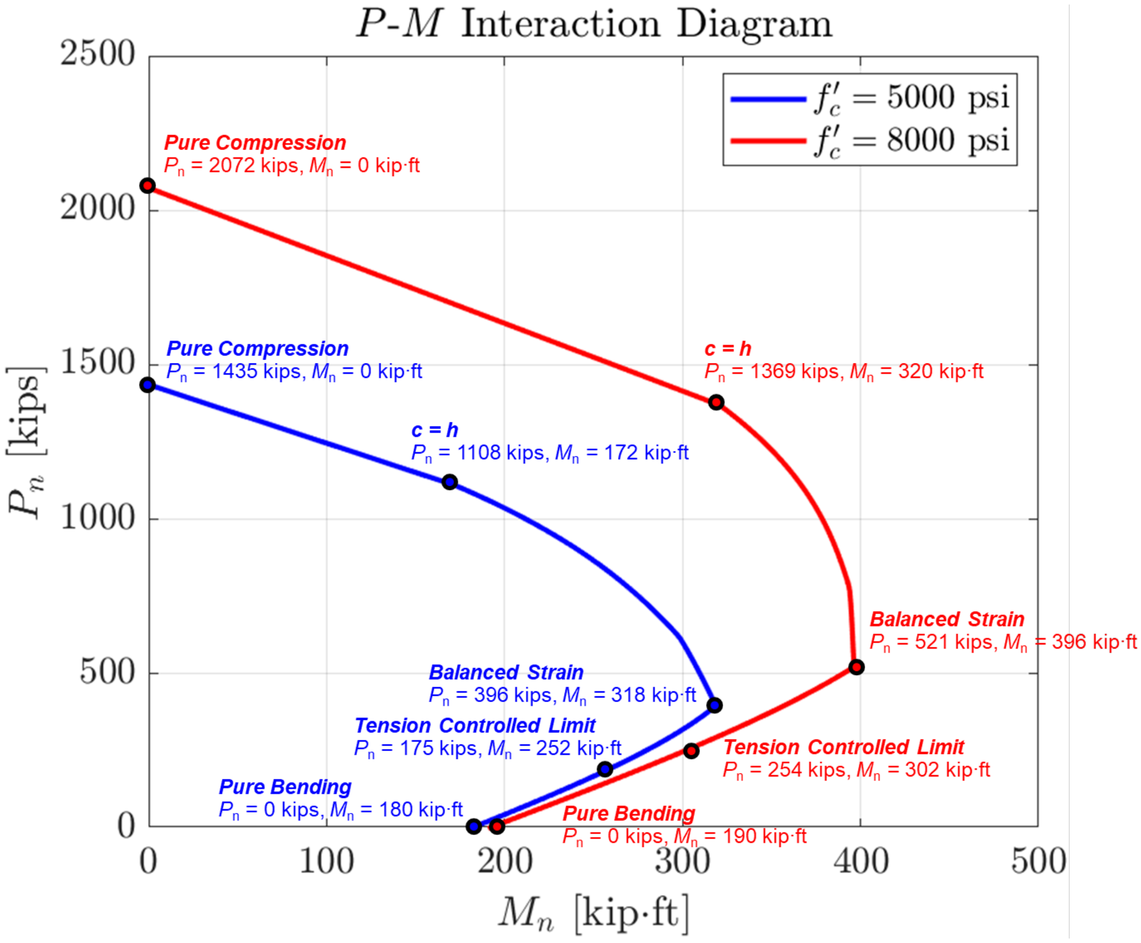
```
    % Calculate the axial load capacity Pn (unit = kips).
    % Sign convention: Compression is positive (+); Tension is negative (-).
    Pn_1 = 0.85*fc_1*b*beta_1*c - sum(As.*fs); % Pn for fc = 5 ksi.
    Pn_2 = 0.85*fc_2*b*beta_2*c - sum(As.*fs); % Pn for fc = 8 ksi.

    % Calculate the moment capacity Mn (unit = kip*ft).
    Mn_1 = (0.85*fc_1*b*beta_1*c*(h/2-beta_1*c/2) - sum(As.*fs.*(h/2-d))) /
12; % Mn for fc = 5 ksi.
    Mn_2 = (0.85*fc_2*b*beta_2*c*(h/2-beta_2*c/2) - sum(As.*fs.*(h/2-d))) /
12; % Mn for fc = 8 ksi.

    % Result output.
    Results(i,:) = [c, eps_s(1), fs(1), eps_s(end), fs(end), Pn_1, Mn_1,
Pn_2, Mn_2];
end

% Caclulate P0 (pure axial load capacity). --------------------------------
Ag = b*h; % Gross cross section area.
Ast = sum(As); % Total area of all the steel reinforcement.

% Calculate P0.
P0_1 = 0.85*fc_1*(Ag-Ast) + Ast*fy; % P0 for fc = 5 ksi.
P0_2 = 0.85*fc_2*(Ag-Ast) + Ast*fy; % P0 for fc = 8 ksi.

% Result output.
Results(end,:) = [0, -fy/Es, -fy, -fy/Es, -fy, P0_1, 0, P0_2, 0];

% Plot P-M interaction diagram. -------------------------------------------
Results = array2table(Results, 'VariableNames', {'c [inch]', 'Epsilon''',
'fs'' [ksi]', 'Epsilon', 'fs [ksi]', 'Pn_1 [kips]', 'Mn_1 [kip*ft]', 'Pn_2
[kips]', 'Mn_2 [kip*ft]'});

figure('Position', [0, 0, 600, 500]);
hold on;

plot(Results{:,7}, Results{:,6}, 'Color', 'b', 'LineWidth', 2);
plot(Results{:,9}, Results{:,8}, 'Color', 'r', 'LineWidth', 2);
grid on;
box on;
xlim([0 500]);
xticks(0:100:500);
ylim([0 2500]);
yticks(0:500:2500);
xlabel('$M_n$ [kip$\cdot$ft]');
ylabel('$P_n$ [kips]');
legend('$f''_c = 5000$ psi', '$f''_c = 8000$ psi', 'Location', 'northeast');
title('$P$-$M$ Interaction Diagram');
```

Summarized Calculation Results:

c [inch]	ε'_s	f'_s [ksi]	ε_s	f_s [ksi]	f'_c = 5000 psi		f'_c = 8000 psi		Note	Controlled Category
					P_n [kips]	M_n [kip*ft]	P_n [kips]	M_n [kip*ft]		
0.16	0.0533	60.00	0.2408	60.00	-365.70	5.76	-363.08	7.49		
0.32	0.0251	60.00	0.1189	60.00	-356.99	11.42	-351.77	14.89		
0.48	0.0158	60.00	0.0783	60.00	-348.29	16.99	-340.45	22.19		
0.64	0.0111	60.00	0.0579	60.00	-339.58	22.47	-329.14	29.39		
0.80	0.0083	60.00	0.0458	60.00	-330.88	27.85	-317.82	36.49		
0.96	0.0064	60.00	0.0376	60.00	-322.18	33.14	-306.51	43.50		
1.12	0.0050	60.00	0.0318	60.00	-313.47	38.34	-295.19	50.40		
1.28	0.0040	60.00	0.0275	60.00	-304.77	43.45	-283.88	57.21		
1.44	0.0033	60.00	0.0241	60.00	-296.06	48.46	-272.56	63.92		
1.60	0.0026	60.00	0.0214	60.00	-287.36	53.38	-261.25	70.53		
1.76	0.0021	60.00	0.0192	60.00	-278.66	58.21	-249.93	77.05		
1.92	0.0017	48.94	0.0173	60.00	-235.44	77.33	-204.10	97.84		
2.08	0.0013	38.48	0.0158	60.00	-194.11	95.56	-160.16	117.75		
2.24	0.0010	29.52	0.0144	60.00	-157.44	111.77	-120.88	135.62		
2.40	0.0008	21.75	0.0133	60.00	-124.50	126.32	-85.33	151.84		
2.56	0.0005	14.95	0.0122	60.00	-94.59	139.52	-52.81	166.70		**Tension Controlled**
2.72	0.0003	8.96	0.0113	60.00	-67.17	151.59	**-22.78**	**180.43**	***M_0* = 190.83 kip*ft**	
2.88	0.0001	3.63	0.0105	60.00	-41.84	162.69	**5.16**	**193.18**	**(*f'_c* = 8000 psi)**	
3.04	0.0000	-1.14	0.0098	60.00	**-18.25**	**172.98**	31.36	205.11	***M_0* = 180.88 kip*ft**	
3.20	-0.0002	-5.44	0.0092	60.00	**3.85**	**182.55**	56.07	216.33	**(*f'_c* = 5000 psi)**	
3.36	-0.0003	-9.32	0.0086	60.00	24.67	191.50	79.50	226.91		
3.52	-0.0004	-12.85	0.0081	60.00	44.39	199.90	101.83	236.93		
3.68	-0.0006	-16.08	0.0076	60.00	63.15	207.80	123.21	246.46		
3.84	-0.0007	-19.03	0.0072	60.00	81.07	215.27	143.74	255.54		
4.00	-0.0008	-21.75	0.0068	60.00	98.26	222.33	163.54	264.22		
4.16	-0.0008	-24.26	0.0064	60.00	114.79	229.03	182.69	272.52		
4.32	-0.0009	-26.58	0.0060	60.00	130.75	235.39	201.25	280.49		
4.48	-0.0010	-28.74	0.0057	60.00	146.18	241.44	219.30	288.14		
4.64	-0.0011	-30.75	0.0054	60.00	161.16	247.21	236.88	295.50		
4.80	**-0.0011**	**-32.63**	**0.0051**	**60.00**	**175.71**	**252.71**	**254.05**	**302.59**	**$\varepsilon_s = \varepsilon_y + 0.003$**	
4.96	-0.0012	-34.38	0.0049	60.00	189.89	257.96	270.83	309.42		
5.12	-0.0012	-36.02	0.0046	60.00	203.72	262.98	287.28	316.01		
5.28	-0.0013	-37.57	0.0044	60.00	217.24	267.77	303.41	322.38		
5.44	-0.0013	-39.02	0.0042	60.00	230.48	272.36	319.27	328.52		
5.60	-0.0014	-40.39	0.0040	60.00	243.47	276.74	334.86	334.47		
5.76	-0.0014	-41.69	0.0038	60.00	256.21	280.93	350.21	340.21		
5.92	-0.0015	-42.91	0.0036	60.00	268.73	284.93	365.35	345.77		
6.08	-0.0015	-44.07	0.0034	60.00	281.06	288.76	380.28	351.14		
6.24	-0.0016	-45.17	0.0033	60.00	293.20	292.42	395.03	356.34		**Transition**
6.40	-0.0016	-46.22	0.0031	60.00	305.16	295.92	409.61	361.37		
6.56	-0.0016	-47.21	0.0029	60.00	316.97	299.25	424.03	366.24		
6.72	-0.0017	-48.16	0.0028	60.00	328.63	302.43	438.30	370.94		
6.88	-0.0017	-49.06	0.0027	60.00	340.15	305.46	452.43	375.49		
7.04	-0.0017	-49.93	0.0025	60.00	351.55	308.35	466.44	379.89		
7.20	-0.0018	-50.75	0.0024	60.00	362.82	311.09	480.32	384.14		
7.36	-0.0018	-51.54	0.0023	60.00	373.98	313.69	494.10	388.25		
7.52	-0.0018	-52.29	0.0022	60.00	385.04	316.16	507.77	392.21		
7.68	**-0.0018**	**-53.02**	**0.0021**	**60.00**	**396.00**	**318.49**	**521.34**	**396.04**	**$\varepsilon_s = \varepsilon_y$**	**Balanced Strain**

c [inch]	ε'_s	f'_s [ksi]	ε_s	f_s [ksi]	f'_c = 5000 psi		f'_c = 8000 psi		Note	Controlled Category
					P_n [kips]	M_n [kip*ft]	P_n [kips]	M_n [kip*ft]		
7.84	-0.0019	-53.71	0.0020	57.26	415.42	317.13	543.37	396.16		
8.00	-0.0019	-54.38	0.0019	54.38	435.20	315.46	565.76	395.97		
8.16	-0.0019	-55.01	0.0018	51.60	454.55	313.80	587.72	395.79		
8.32	-0.0019	-55.63	0.0017	48.94	473.49	312.15	609.27	395.61		
8.48	-0.0019	-56.22	0.0016	46.37	492.04	310.52	630.43	395.44		
8.64	-0.0020	-56.79	0.0015	43.90	510.23	308.88	651.23	395.27		
8.80	-0.0020	-57.34	0.0014	41.52	528.07	307.24	671.69	395.09		
8.96	-0.0020	-57.87	0.0014	39.23	545.59	305.60	691.82	394.90		
9.12	-0.0020	-58.38	0.0013	37.01	562.80	303.94	711.64	394.68		
9.28	-0.0020	-58.88	0.0012	34.88	579.71	302.27	731.16	394.45		
9.44	-0.0020	-59.35	0.0011	32.81	596.35	300.57	750.41	394.19		
9.60	-0.0021	-59.81	0.0011	30.81	612.72	298.86	769.39	393.90		
9.76	-0.0021	-60.00	0.0010	28.88	628.03	296.77	787.32	393.25		
9.92	-0.0021	-60.00	0.0009	27.01	642.57	294.44	804.46	392.33		
10.08	-0.0021	-60.00	0.0009	25.20	656.92	292.08	821.43	391.39		
10.24	-0.0021	-60.00	0.0008	23.45	671.09	289.71	838.21	390.43		
10.40	-0.0021	-60.00	0.0008	21.75	685.10	287.32	854.83	389.44		
10.56	-0.0021	-60.00	0.0007	20.10	698.94	284.90	871.28	388.42		
10.72	-0.0022	-60.00	0.0006	18.50	712.64	282.45	887.59	387.36		
10.88	-0.0022	-60.00	0.0006	16.95	726.18	279.97	903.74	386.27		
11.04	-0.0022	-60.00	0.0005	15.45	739.59	277.45	919.76	385.13		
11.20	-0.0022	-60.00	0.0005	13.98	752.86	274.90	935.64	383.96		
11.36	-0.0022	-60.00	0.0004	12.56	766.00	272.31	951.39	382.74		
11.52	-0.0022	-60.00	0.0004	11.18	779.02	269.67	967.02	381.48		
11.68	-0.0022	-60.00	0.0003	9.83	791.92	267.00	982.53	380.16		
11.84	-0.0022	-60.00	0.0003	8.52	804.70	264.27	997.93	378.80		
12.00	-0.0023	-60.00	0.0003	7.25	817.38	261.51	1013.22	377.38		Compression Controlled
12.16	-0.0023	-60.00	0.0002	6.01	829.95	258.69	1028.40	375.90		
12.32	-0.0023	-60.00	0.0002	4.80	842.43	255.82	1043.49	374.38		
12.48	-0.0023	-60.00	0.0001	3.63	854.80	252.89	1058.48	372.79		
12.64	-0.0023	-60.00	0.0001	2.48	867.09	249.92	1073.37	371.14		
12.80	-0.0023	-60.00	0.0000	1.36	879.28	246.88	1088.17	369.44		
12.96	-0.0023	-60.00	0.0000	0.27	891.39	243.79	1102.89	367.67		
13.12	-0.0023	-60.00	0.0000	-0.80	903.41	240.65	1117.53	365.84		
13.28	-0.0023	-60.00	-0.0001	-1.83	915.36	237.44	1132.08	363.94		
13.44	-0.0023	-60.00	-0.0001	-2.85	927.22	234.17	1146.56	361.97		
13.60	-0.0023	-60.00	-0.0001	-3.84	939.02	230.84	1160.97	359.94		
13.76	-0.0023	-60.00	-0.0002	-4.81	950.74	227.45	1175.30	357.85		
13.92	-0.0024	-60.00	-0.0002	-5.75	962.39	223.99	1189.56	355.68		
14.08	-0.0024	-60.00	-0.0002	-6.67	973.97	220.47	1203.76	353.44		
14.24	-0.0024	-60.00	-0.0003	-7.58	985.49	216.89	1217.89	351.13		
14.40	-0.0024	-60.00	-0.0003	-8.46	996.95	213.23	1231.96	348.75		
14.56	-0.0024	-60.00	-0.0003	-9.32	1008.35	209.51	1245.97	346.30		
14.72	-0.0024	-60.00	-0.0004	-10.17	1019.69	205.72	1259.92	343.77		
14.88	-0.0024	-60.00	-0.0004	-10.99	1030.97	201.86	1273.81	341.17		
15.04	-0.0024	-60.00	-0.0004	-11.80	1042.19	197.93	1287.65	338.49		
15.20	-0.0024	-60.00	-0.0004	-12.59	1053.37	193.93	1301.43	335.74		
15.36	-0.0024	-60.00	-0.0005	-13.37	1064.49	189.86	1315.16	332.91		
15.52	-0.0024	-60.00	-0.0005	-14.13	1075.56	185.72	1328.85	330.00		
15.68	-0.0024	-60.00	-0.0005	-14.87	1086.59	181.50	1342.48	327.02		
15.84	-0.0024	-60.00	-0.0005	-15.60	1097.56	177.21	1356.07	323.96		
16.00	**-0.0024**	**-60.00**	**-0.0006**	**-16.31**	**1108.50**	**172.85**	**1369.62**	**320.82**	**c = h (Last point that stress block assumption holds)**	
N/A	**-0.0021**	**-60.00**	**-0.0021**	**-60.00**	**1435.88**	**0.00**	**2072.77**	**0.00**	**P_0 (Pure Comperession)**	

P-M Interaction Diagram:



Discussion:

From the P-M interaction diagram, it is observed that increasing compressive strength of concrete significantly increases the axial load capacity when the column is under compression control, especially when the bending moment is small. However, there is no significant difference for the moment capacity when the column is under tension control (almost no different in the moment capacity when the section is under pure bending). Therefore, if concrete with compressive strength of 8000 psi is appropriate for the lower stories (with high axial force) of a high-rise structure, concrete with compressive strength of 5000 psi may work for the upper stories since upper stories has lower axial load demand.