Department of Education
National Capital Region
**SCHOOLS DIVISION OFFICE
MARIKINA CITY**

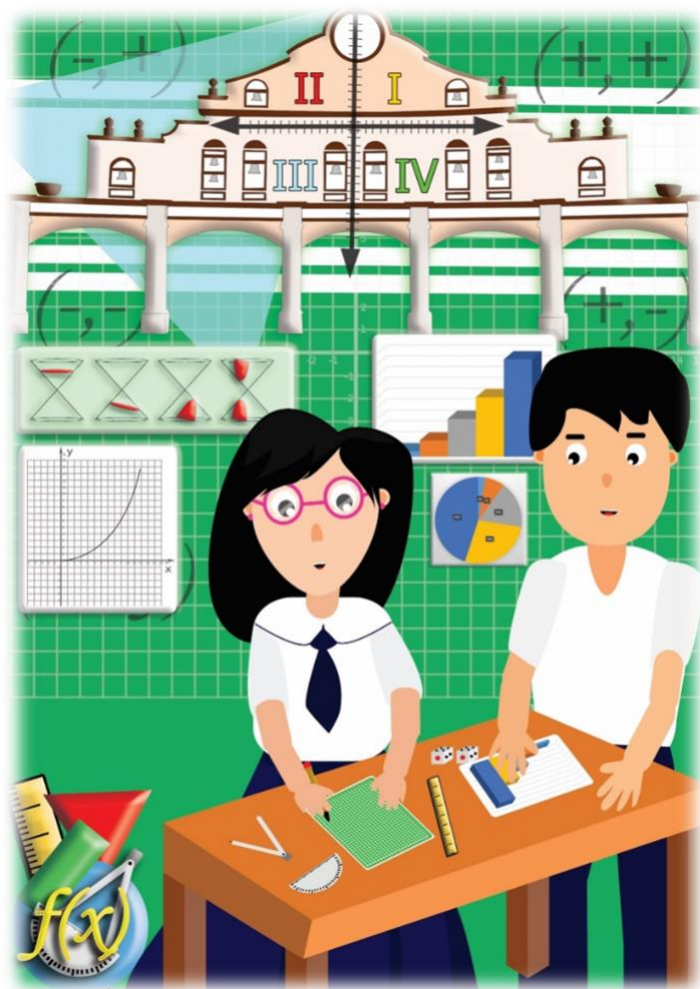# MATHEMATICS
Quarter 3: Module 3
Interpreting and Solving Problems Involving Mean and Variance of Probability Distribution

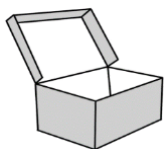|	
Writer:	Emelito James D. Evangelio (SNNHS)
Cover Illustrator:	Joel J. Esdtudillo (SNNHS)

City of Good Character

DISCIPLINE • GOOD TASTE • EXCELLENCE

What I Need to Know

Hello Grade 11 learners! In this module, you will learn how to:

Interpret the mean and the variance of a discrete random variable **M11/12SP-IIIb-3** and

Solve problems involving mean and variance of probability distributions **M11/12SP-IIIb-4**

You can say that you have understood the lesson in this module if you can already:

1. solve problems involving mean and variance of probability distributions and
2. interpret the mean and the variance of a discrete random variable.

What I Know

Before studying this module, take this simple test to determine what you already know about the topic covered. Please write the letter that corresponds to the correct answer. If there are no options provided, do as directed.

1. You buy one ₱500.00 raffle ticket for a prize of new car valued at ₱700, 000.00. Two thousand tickets are sold. If X denotes the net gain from the purchase of a randomly selected ticket, which of the following tables of values shows the probability distribution of X?

A.

x	700,000	-500
P(x)	$\dfrac{1}{2000}$	$\dfrac{1999}{2000}$

C.

x	699,500	-500
P(x)	$\dfrac{1}{2000}$	$\dfrac{1999}{2000}$

B.

x	700,000	500
P(x)	$\dfrac{1}{2000}$	$\dfrac{1999}{2000}$

D.

x	699,500	500
P(x)	$\dfrac{1}{2000}$	$\dfrac{1999}{2000}$

2. Open question: Would you bet your money in the raffle? Why or why not?

3. One thousand tickets are sold for ₱10.00 each. One ticket will win ₱2,000.00, two tickets will win ₱1,000.00 each and four tickets will win ₱500.00 each. What is the probability of winning any amount in the purchase of one ticket?

    A. $\dfrac{1}{1000}$ B. $\dfrac{2}{1000}$ C. $\dfrac{5}{1000}$ D. $\dfrac{7}{1000}$

4. A roulette wheel in a fiesta carnival has the numbers 1 through 50. If you bet ₱5.00, you will have a chance to win a kitchen utensil worth ₱200.00. How much the organizer will earn if 100 games will be played?

    A. ₱100.00   B. ₱200.00   C. ₱300.00   D. ₱400.00

5. A life insurance company will sell a ₱500,000.00 five-year term life insurance policy exclusive for police enforcers for a premium of ₱1,000.00. Find the expected value to the company of a single policy if a police enforcer has a 99.95% chance of surviving five years?

    A. ₱550.00   B. ₱650.00   C. ₱750.00   D. ₱850.00

Alfred North Whitehead said *"The only use of knowledge of the past is to equip us for the*

What's In

present." This part will help you to activate your prior knowledge that will help you get through this lesson.

Activity: Complete Me

Distribution of the number of heads in three independent tosses of one fair coin

X=Number of Heads	P(x)	(X)P(X)	(X-μ)²	(X-μ)²·P(X)
0				
1				
2				
3				
Total				

Questions:

1. How did you get the probability of each cases? _____

_____

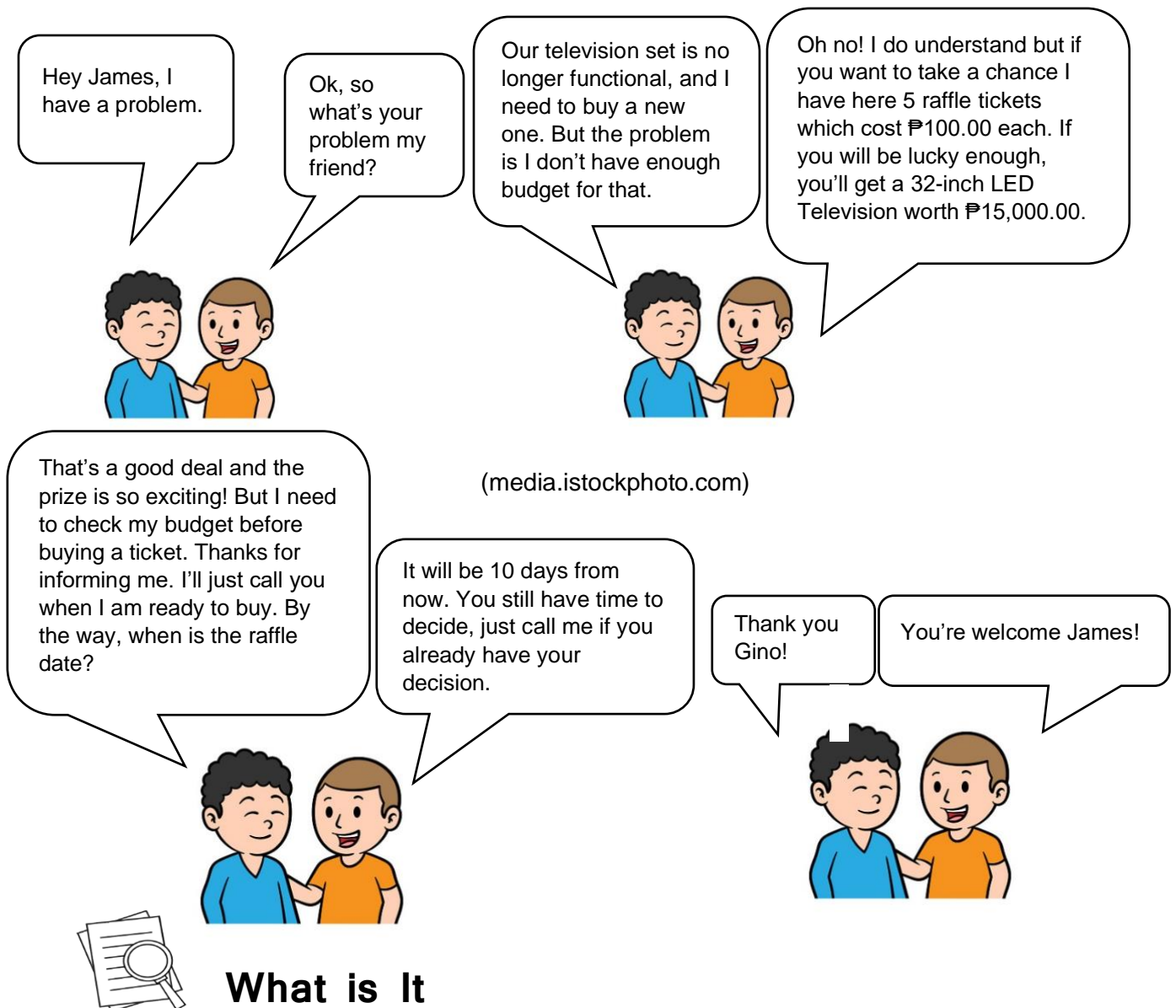2. How did you get the expected value or the mean, what is the value?

_____

3. What must be solve first in order to get the variance? _____

4. What is the value of variance? _____

5. What will you do in order to get the standard deviation, and what is value of the standard deviation in the table above?_____

_____

❓ What's New

Hey James, I have a problem.

Ok, so what's your problem my friend?

Our television set is no longer functional, and I need to buy a new one. But the problem is I don't have enough budget for that.

Oh no! I do understand but if you want to take a chance I have here 5 raffle tickets which cost ₱100.00 each. If you will be lucky enough, you'll get a 32-inch LED Television worth ₱15,000.00.

(media.istockphoto.com)

That's a good deal and the prize is so exciting! But I need to check my budget before buying a ticket. Thanks for informing me. I'll just call you when I am ready to buy. By the way, when is the raffle date?

It will be 10 days from now. You still have time to decide, just call me if you already have your decision.

Thank you Gino!

You're welcome James!

What is It

Consider the situation below.

Senior citizens of a barangay organize a Christmas raffle bonanza. One thousand raffle tickets are sold for ₱50.00 each. Each one has an equal chance of winning. The first prize is a 32-inch LED TV worth ₱10,000.00, the second prize is an electric oven worth ₱5,000.00 and the third prize is a grocery pack worth ₱2,500.00. Let X denotes the net gain from the purchase of <u>one ticket.</u>

a. Construct the probability distribution of X.

b. Find the expected value of X, then interpret.

Solution:

a. The Probability Distribution is given below.

	x	$P(X)$
x_0	-50	0.997
x_1	9950	0.001
x_2	4950	0.001
x_3	2450	0.001

Explanation:

x_0 = lose/not able to win the three prizes

x_1 = winning first prize

x_2 = winning second prize

x_3 = winning third prize

(Assume that the person bought only one ticket)

How to get the values of x_i

x_1 = 10000(amount of the prize) – 50(price of ticket bought) = 9950

x_2 = 5000(amount of the prize) – 50(price of ticket bought) = 4950

x_3 = 2500(amount of the prize) – 50(price of ticket bought) = 2450

x_0 = -50 because you lose

How to get the probability: use the concept of simple probability where in this problem, it is $\frac{\text{no. of ticket bought that have a chance to win a prize}}{\text{total no of tickets}}$.

$$x_1 = \frac{1}{1000} = 0.001 \qquad\qquad x_2 = \frac{1}{1000} = 0.001$$

$$x_3 = \frac{1}{1000} = 0.001 \qquad\qquad x_0 = \frac{997}{1000} = .997$$

b. Using the formula of expected value or mean.

	x	$P(X)$	$x \cdot P(X)$
x_0	-50	0.997	- 49.85
x_1	9950	0.001	9.95
x_2	4950	0.001	4.95
x_3	2450	0.001	2.45
Total			-32.50

$$E(X) = 9950(0.001) + 4950(0.001)$$
$$+ 2450(0.001)$$
$$+ (-50)(0.997)$$

$$E(X) = -32.50$$

You might be wondering why the answer is negative, and what does it mean? Worry not because the **negative value** means one loses money on the average. This does not mean that a person will always lose because obviously there is a chance of winning but on the average, on the long run, a person would lose thirty-two pesos and fifty cents per ticket purchased.

If you were that person would you take the risk to join the raffle? Why or why not?

Consider this another situation.

In a *perya*, a color wheel contains 38 numbers: 18 are red, 18 are black, and 2 are green. When the wheel is spun, the ball is equally likely to land on any of the 38 numbers. Suppose that you bet ₱20 on red. If the ball lands on a red number, you win ₱40; otherwise you lose your ₱20. Let X be the amount you win on your ₱20 bet.

a. Construct the probability distribution of X.

b. Find the expected value of X, then interpret

c. On average, how much will you lose per play?

d. Approximately how much would you expect to lose if you bet ₱20 on red 100 times? 1000 times?

a. Probability Distribution

X	$P(X)$
20	.474
-20	.526

Explanation:

20 = win : 40 (winning price) – 20 (your money bet) = 20

-20= lose

(Assume that the person bet on only one red.)

How to get the probability?

$20 = \frac{18}{38} = 0.001$ 18 because there are 18 red and 38 is total no of colors in a wheel

$-20= \frac{20}{38} = 0.001$ 20 because there are 20 non-red (black and green) and 38 is the total no. of colors in a wheel

b. Use the formula of expected value or mean.

X	P(X)	x · P(X)
20	.474	9.48
-20	.526	-10.52
Total		-1.04

$$E(X) = 20(0.474) + (-20)(0.526)$$

$$E(X) = -1.04$$

c. On average, how much will you lose per play?

You will lose ₱1.04 on average.

d. Approximately how much would you expect to lose if you bet ₱20 on red 100 times? 1000 times?

100 times = 100 (₱1.04)

= ₱104

1000 times = 1000 (₱1.04)

= ₱1040

The same manner can be used in insurance.

A life insurance company will sell a ₱500,000.00 one-year term life insurance policy for members of armed forces of the Philippines for a premium of ₱10,400. Find the expected value to the company of a single policy if a member of the armed forces police has a .01 % chance of not surviving in one year?

Let X denotes the net gain to the company from the sale of one policy. There are two possibilities: the insured person lives the whole year or the injured person dies before the year is finished. Applying the "income minus expenses" principle, in the first case the value of X = 200 – 0; in the latter case it is 200 – 250,000 = - 249,800. Since the probability in the first case is 99.98% or 0.9998 and in the latter case is 1 – 0.9998 = 0.0002, the probability distribution for X is:

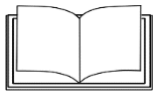| x	10,400	- 489,600
$P(x)$	0.999	0.001

Therefore

$$E(X) = \sum xP(x) = (10{,}400)(0.999) \ + \ (-489{,}600)(0.001)$$

$$E(x) = \ 9{,}900$$

Interpretation: In the long-run average, the company would be expected to make a total profit of Php9,900.

What's More

Activity: Give what is asked for.

1. Now that you are equipped, look back at the conversation of James and Gino and answer the following.
   If Cardo decided to buy one ticket, what is the probability that he would win the prize if 500 tickets were sold?
a. Make a probability distribution.
b. Find the expected value.
c. How much money will James lose?
d. If you were James, will you buy a ticket? Why and Why not?
2. Two thousand tickets are sold for ₱50.00 each. One ticket will win ₱10,000.00, two tickets will win ₱5,000.00 each and three tickets will win ₱1,000.00 each. Let X denotes the net gain from the purchase of a randomly selected ticket.
   a. Construct the probability distribution of X.
   b. Find the probability of winning any amount in the purchase of one ticket.
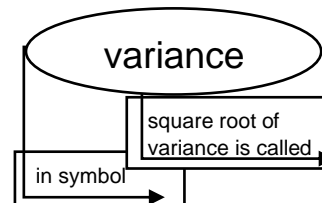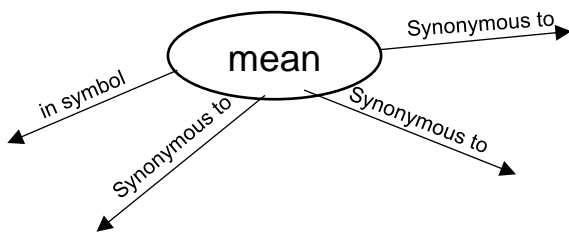   c. Find the expected value of X, then interpret.

What I Have Learned

Please share your thoughts about this:

"Take chances, make mistakes. That's how you grow."

– Mary Tyler Moore

> To obtain the _____ of a discrete random variable, _____ each possible value by its _____ and then _____ those products.

mean

Synonymous to →

in symbol →

Synonymous to →

Synonymous to →

variance

square root of variance is called →

in symbol →

What I Can Do

Solve

1. An insurance company will sell a ₱800,000.00 one-year term legs insurance policy for ramp models for a premium of ₱50,000.00. Find the expected value to the company of a single policy if a model has a 99.96% chance of being uninjured in one year?
   a. Construct the probability distribution of X.
   b. Find the expected value of X, then interpret.

Assessment

Solve the following real-life problems. Please write the letter that corresponds to the correct answer. If there are no options provided, do as directed.

1. One thousand tickets are sold for ₱50.00 each. One ticket will win ₱10,000.00, two tickets will win ₱5,000.00 each and three tickets will win ₱1,000.00 each. If X denotes the net gain from the purchase of a randomly selected ticket, which of the following tables of values shows the probability distribution of X?

A.

x	10,000	5,000	1,000	-50
P(x)	$\frac{1}{1000}$	$\frac{2}{1000}$	$\frac{3}{1000}$	$\frac{994}{1000}$

B.

x	10,000	5,000	1,000	-50
P(x)	$\frac{1}{999}$	$\frac{2}{998}$	$\frac{3}{997}$	$\frac{1}{999}$

C.

x	9,950	4,950	950	-50
P(x)	$\frac{1}{1000}$	$\frac{2}{1000}$	$\frac{3}{1000}$	$\frac{994}{1000}$

D.

x	9.950	4,950	1,000	-50
P(x)	$\frac{1}{999}$	$\frac{2}{998}$	$\frac{3}{997}$	$\frac{1}{999}$

2. Open question: Would you bet your money in the raffle? Why or why not?

3. You buy three ₱1,000.00 raffle tickets for a prize of a new 20-passenger Sarao jeepney valued at ₱800, 000.00. Two thousand tickets are sold. What is the probability of winning the prize in the purchase of three tickets?

A. $\frac{1}{2000}$ B. $\frac{2}{2000}$ C. $\frac{3}{2000}$ D. $\frac{4}{2000}$

City of Good Character

DISCIPLINE • GOOD TASTE • EXCELLENCE

4. A life insurance company will sell a 1 million three-year term life insurance policy exclusive in a particular risk group for a premium of ₱2,000.00. Find the expected value to the company of a single policy if a person in this risk group has a 99.95% chance of surviving three years? (round of the nearest whole number)

A. ₱1,600.00   C. ₱1,666.00

B. ₱1,660.00   D. ₱1,667.00

5. A roulette wheel in an amusement park has the numbers 1 through 60. If you bet ₱100.00 you will have a chance to win a cellular phone worth 5,000.00. How much the organizer will earn if 100 games will be played? Would you bet in the game assuming that you have only ₱250 left in you? Why or why not? (Open question)

Additional Activities

Solve the problem below.

An insurance company will sell a ₱500,000.00 one-year term legs insurance policy for ramp models for a premium of ₱500.00. Find the expected value to the company of a single policy if a model has a 99.96% chance of being uninjured in one year?
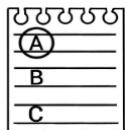
References

(n.d.). Retrieved from http://www.worldmayor.com/contest_2008/comments-
        marikina.html
Estevez, M. C. (2019, May 05). Retrieved from scribd.com:
        https://www.scribd.com/document/411783721/Chapter-I-Random-Variables-
        and-Probability-Distribution
media.istockphoto.com. (n.d.). Retrieved from
        https://media.istockphoto.com/vectors/cartoon-two-friends-talking-illustration-
        vector-
        id1226902816?k=6&m=1226902816&s=170667a&w=0&h=Y2CRFTLZStGTF
        O0m6tnqsseKQBDM-YzBbizKFDgtWRU=

What I Know

1. b
2. answer may vary
3. a
4. a
5. c

What's In

x	P(x)	(X)P(X)	(X-μ)^2	P(X)(X-μ)^2
0	0.125	0	2.25	0.28125
1	0.375	0.375	0.25	0.09375
2	0.375	0.75	0.25	0.09375
3	0.125	0.375	2.25	0.28125
		Mean = 1.5	Variance = 0.75	
				SD = 0.8660254

1. divide the number of cases to the total no. of cases.
2. by getting the sum of the product of cases and its corresponding probability.
3. solve first the mean
4. variance is equal to .75
5. just take the square root of the variance. In case of the problem, square root of .75 is equal to .8660 - the value of standard deviation.

What's More

1. a.

x	P(x)	(X)P(X)	(X-μ)^2	P(X)(X-μ)^2
14,900	0.002	29.8	2241009000	4482201.8
-100	0.998	-99.8	900.00	898.2
		Mean = -70	Variance = 449100	
				SD = 670.1492371

b. expected value / mean is equal to -70

c. James will lose P70 pesos

d. answer may vary

2. a.

x	P(x)	XP(X)	(X-μ)2	P(X)(X-μ)2
-50	0.997	-49.85	132.25	131.85325
9950	0.0005	4.975	99770132.25	49885.066013
4950	0.001	4.95	24885132.25	24885.13225
950	0.0015	1.425	977132.25	1465.698375
		Mean = -38.5		Variance = 76367.75
				SD = 276.3471549

b. 9950 − 1/2000
4950 − 2/2000
950 − 3/2000
-50 − 1996/2000

c. (-50)(.997)+(9950)(0.0005)+(4950)(0.001)+(950)(0.0015)=-38.5

Assessment

1. c
2. answer may vary
3. c
4. d
5. answer may vary

City of Good Character

DISCIPLINE • GOOD TASTE • EXCELLENCE