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Mathematics

Quarter 2 – Module 9:
Solving Linear Equations and Inequalities in One Variable

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Quarter 2 – Module 9: Solving Linear Equations and Inequalities in One Variable
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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you master Solving Linear Equations and Inequalities in One Variable. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

The module is comprised of only one lesson:

- Solving Linear Equations and Inequalities in One Variable

After going through this module, you are expected to:

1. find the solution of linear equation or inequality in one variable;
2. solve linear equation or inequality in one variable involving absolute value by: (a) graphing; and (b) algebraic methods; and,
3. solve problems involving equations and inequalities in one variable.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What do you call the set of values that would make an equation or inequality a true statement?
    A. Equation set
    B. Inequality value
    C. Set value
    D. Solution set

2. Which of the following values of x will satisfy the equation $6x - 10 = 8$?
    A. 2
    B. 3
    C. 4
    D. 5

3. Which of the following values would make the inequality $7x + 5 \geq 9$ correct?
    A. -2
    B. -1
    C. 0
    D. 1

4. Which of the following values of x will satisfy the inequality $-6x - 4 \leq 10$?
    A. -5, 4
    B. -4, -3
    C. -3, -2
    D. -2, -1

5. Which of the following statements is NOT correct?
    A. $7x - 2x = 15$ if $x = 3$.
    B. $4x + 2 \geq 8$ if $x = 2$.
    C. $-3(x + 2) < 10$ if $x = -6$.
    D. $4x + 11 = 15$ if $x = 1$.

6. What is/are the possible value/s of x in the equation $|x| = 9$?
    A. 9
    B. -9
    C. 9 0r -9
    D. 99

7. What are the values of x in the equation? $|x - 7| = 4$?
   A. x = -3 and x = -11
   B. x = 3 and x = 11
   C. x = -3 and x = 11
   D. x = 3 and x = -11

8. Which of the following are possible values of x in the equation $|3x - 5| - 4 = 10$?
   A. x = 3 and x = $\frac{19}{3}$
   B. x = 3 and x = $-\frac{19}{3}$
   C. x = -3 and x = $-\frac{19}{3}$
   D. x = -3 and x = $\frac{19}{3}$

9. What is the solution set of the inequality $|x + 1| < 5$?
   A.$\{x|\ -6 < x < 4\}$
   B.$\{x|\ -6 < x > 4\}$
   C.$\{x|\ -6 > x > 4\}$
   D.$\{x|\ -6 > x < 4\}$

10. What values of a will satisfy the inequality $|6a + 3| > 9$?
   A.$\{a|\ a < -2 \text{or } a > 1\}$
   B.$\{a|\ a > -2 \text{or } a > 1\}$
   C.$\{a|\ a < -2 \text{or } a < 1\}$
   D.$\{a|\ a < -2 \text{or } a < 1\}$

11. The product of a number and -8 gives eight times the sum of that number and 36. What is the number?
   A. −36
   B. −18
   C. 8
   D. 18

12. The sum of two numbers is twelve and their difference is 2. What are the numbers?
   A. 10 and 2
   B. 8 and 4
   C. 7 and 5
   D. 6 and 6

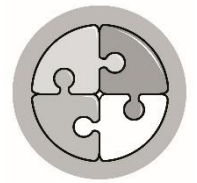
13. If 5 times a number is increased by 4, the result is at least 19. What is the least possible number that satisfies these conditions?
   A. 1
   B. 2
   C. 3
   D. 4

14. The sum of twice a number and 5 is at most 15. What is the largest possible value for the number?
    A. 4
    B. 5
    C. 6
    D. 7

15. The cost of a gallon of orange juice is Php 175.00? What is the maximum number of containers can you buy for Php 750.00?
    A. 3
    B. 4
    C. 5
    D. 6

Lesson 1 Solving Linear Equations and Inequalities in One Variable

In the previous lessons, you have learned that relationships between two quantities which can either be an equation or an inequality. Do you know that many practical problems in the real world require knowledge and skills in dealing with equations and inequalities?



What's In

Before finding the solutions of linear equation or inequality in one variable, let us review the basic concepts of equations or inequalities.

Recall that:

A **linear equation in one variable** is an equation that can be written in the form $ax + b = c$, where a, b and c are real numbers and $a \neq 0$. Linear equations are also first-degree equations because the exponent on the variable is always 1.

Examples: $x + 4 = 10$, $2y - 3 = 7$, $3p + 1 = 4$

A linear inequality in one variable is a mathematical statement that relates a linear expression as either less than or greater than another. It can be written in one of the following forms where a, b and c are real numbers and $a \neq 0$.
$ax + b < c$; $ax + b > c$; $ax + b \leq c$; $ax + b \geq c$;

Examples: $x + 4 > 10$ $2y - 3 < 7$ $3p + 1 \geq 4$

The **absolute value** of a number is the distance on the number line between the number and zero without any regards to its direction. Thus, the absolute value of any number is a non-negative.

Examples: $|0| = 0$ $|4| = 4$ $|-12| = 12$
$|7 - 2| = 5$ $|2\text{-}7| = 5$

Try this!

A. Identify which of the following is an equation or an inequality. Write **E** if it is an equation and **I** if it is an inequality.

_____ 1. x – 5 = 15

_____ 2. 12 + y ≠ 10

_____ 3. y + 8 > 10

_____ 4. 5x – 1 ≥ 24

_____ 5. 2y – y = 4

_____6. 3x > -1

B. Give the absolute value of the following values:

_____ 1. $|-17|$

_____2. $|10-2|$

_____3. $|5-10|$

_____4. $|9|+|-6|$

What's New

Complete the following table by placing a check mark on the cells that correspond to x values that will make the given statement true.

Expressions	$x = -3$	$x = -2$	$x = -1$	$x = 2$	$x = 3$
1. $x + 10 = 7$					
2. $3x - 4 \leq 2$					
3. $-4x \geq 1$					
4. $3x - 5 = 4$					
5. $7x = -14$					

1. In the table, are there any examples of linear equations that have more than one solution?

2. Do you think that there can be more than one solution to a linear inequality in one variable? Explain.

What is It

There are three (3) different ways to solve an equation or inequality by inspection.

A. Guess and Check

In this method, one guesses the solution and then substitute the guessed value into the problem to see if the answer is correct.

In the previous activity, we saw that linear equation in one variable may have a unique solution, but linear inequality in one variable may have many solutions. The following examples further illustrate this idea.

Example 1. Determine if the given values of x are solutions to the equation $2x - 6 = 4$.

$2x - 6 = 4$				
$For\ x = 3$ $2x - 6 = 4$ $2(3) - 6 = 4$ $6 - 6 = 4$ $0 \neq 4$ FALSE	$For\ x = 4$ $2x - 6 = 4$ $2(4) - 6 = 4$ $8 - 6 = 4$ $2 \neq 4$ FALSE	$For\ x = 5$ $2x - 6 = 4$ $2(5) - 6 = 4$ $10 - 6 = 4$ $4 = 4$ TRUE	$For\ x = 6$ $2x - 6 = 4$ $2(6) - 6 = 4$ $12 - 6 = 4$ $6 \neq 4$ FALSE	$For\ x = 7$ $2x - 6 = 4$ $2(7) - 6 = 4$ $14 - 6 = 4$ $8 \neq 4$ FALSE

Based on the evaluation, the equation is *true* if $x = 5$, while the rest are not a solution. Therefore, we can say that the equation has a unique solution.

Example 2. Verify if the given values of x are solutions to $6x - 2 \leq 4$.

$6x - 2 \leq 4.$				
$For\ x = 3$ $6x - 2 \leq 4$ $6(3) - 2 \leq 4$ $18 - 2 \leq 4$ $16 \leq 4$ FALSE	$For\ x = 2$ $6x - 2 \leq 4$ $6(2) - 2 \leq 4$ $12 - 2 \leq 4$ $10 \leq 4$ FALSE	$For\ x = 1$ $6x - 2 \leq 4$ $6(1) - 2 \leq 4$ $6 - 2 \leq 4$ $4 \leq 4$ TRUE	$For\ x = -1$ $6x - 2 \leq 4$ $6(-1) - 2 \leq 4$ $-6 - 2 \leq 4$ $-8 \leq 4$ TRUE	$For\ x = -2$ $6x - 2 \leq 4$ $6(-2) - 2 \leq 4$ $-12 - 2 \leq 4$ $-14 \leq 4$ TRUE

Based on the evaluation, the values 1, -1 and -2 are solutions to the inequality.

What happens if x is substituted with any negative number, or a number that is less than or equal to 1? Can we consider all numbers less than or equal to 1 solution to the given inequality?

B. Cover –up

In this method, one covers up the term with the variable.

Example1. $2x - 6 = 4$

$\square - 6 = 4$

Thus, to result in a true statement, the $\square$ must be 10.

Therefore, 2x must be 10, and x must be 5, since $2(5) = 10$.

Example2. $x - 4 > 2$

$\square - 4 > 2$

Thus, to result in a true statement, the $\square$ must be greater than 6.

Since 7 is a number greater than 6, $7 - 4 > 2$ is true. Thus all real numbers greater than 6 is the solution.

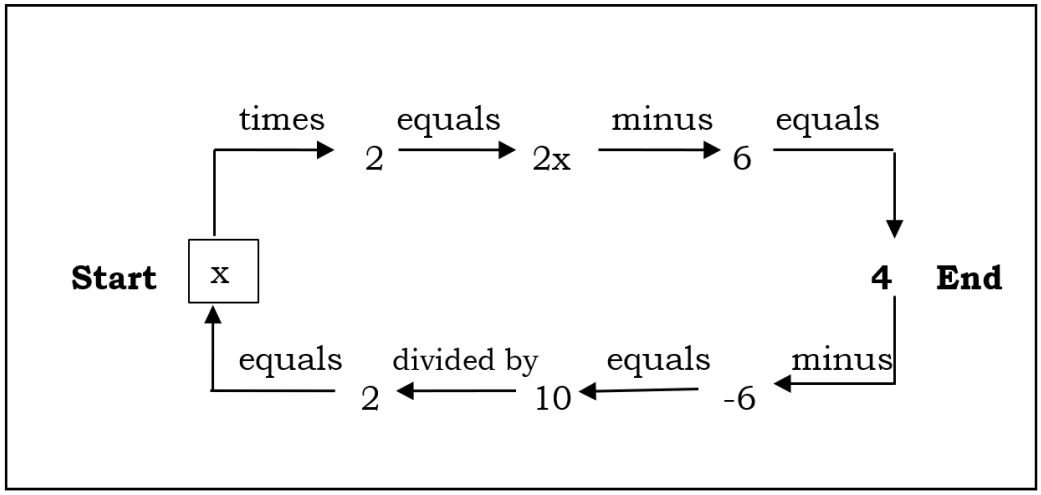
C. Working Backwards

In this method, the reverse procedure is used.

Example1. $2x - 6 = 4$

The equation tells that x is *multiplied* by 2 and 6 is *subtracted* to the product will give 4. Thus, if -6 is *subtracted* from 4 and *divided* by 2, the value of x can be obtained, so x = 5.

The diagram below illustrates this.



Linear equations and inequalities in one variable can also be solved algebraically by using the properties below.

Properties of Equality

Let a, b and c be any real numbers.

1. Addition Property of Equality (APE)
   If $a = b$, then $a + c = b + c$.
2. Subtraction Property of Equality (SPE)
   If $a = b$, then $a - c = b - c$.
3. Multiplication Property of Equality (MPE)
   If $a = b$, then $ac = bc$.
4. Division Property of Equality (DPE)
   If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
5. Substitution Law
   If $a + b = c$ and $a = b$, then $b + b = c$ or $a + a = c$.

Note: The Subtraction Property of Equality and the Division Property of Equality are special cases of the Addition and Multiplication Properties of Equality, respectively. Think of subtracting 1 both sides of an equation as adding -1 on both sides. Also, dividing on both sides by 2 of the equation is also the same as multiplying both sides by $\frac{1}{2}$. To complete the properties of equality, we state the following properties.

Other Properties of Equality

For any real value of a, b, and c,

1. Reflexive Property. Any number is equal to itself.
   $a = a, \quad b = b, \quad c = c$
2. Symmetric Property. The expressions on the both sides of the equation may be interchanged.
   If a = b, then b = a.
3. Transitive Property
   If a = b and b = c, then a = c.

Examples. Solve each equation using the properties.

a. $2x - 4 = x + 6$ c. $3(x + 4) = -3$

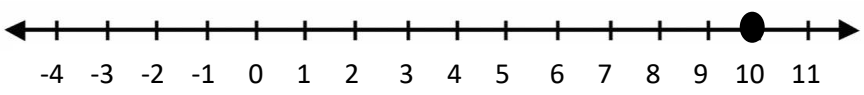
b. $\frac{3}{4}x + 5 = 8$ d. $2(x - 3) = x + 1$

Solution:

a.

$2x - 4 = x + 6$	Write the original equation
$2x - 4 + \mathbf{4} = x + 6 + \mathbf{4}$	APE (Add 4 on both sides)
$2x = x + 10$	Simplify
$2x - \boldsymbol{x} = x - \boldsymbol{x} + 10$	SPE (Subtract x from both sides)
$x = 10$	Simplify

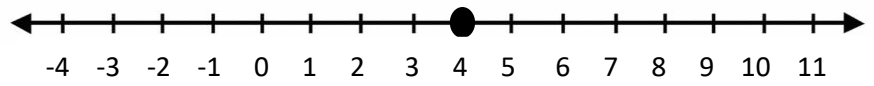
The graph of the solution of the equation $2x - 4 = x + 6$ which is $x = 10$ is shown below.



b.

$\frac{3}{4}x + 5 = 8$	Write the original equation
$\frac{3}{4}x + 5 - \mathbf{5} = 8 - \mathbf{5}$	SPE (Subtract 5 from both sides)
$\frac{3}{4}x = 3$	Simplify
$\left(\frac{4}{3}\right)\frac{3}{4}x = 3\left(\frac{4}{3}\right)$	MPE (Multiply both sides by $\frac{4}{3}$, the inverse of $\frac{3}{4}$)
$x = 4$	Simplify

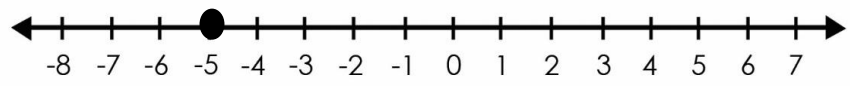
The graph of the solution of the equation $\frac{3}{4}x + 5 = 8$ which is x = 4 is shown below.



c.

$3(x + 4) = -3$	Write the original equation
$3x + 12 = -3$	Distributive Property (Multiply 3 to x and 3 to 4)
$3x + 12 - 12 = -3 - 12$	SPE (Subtract 12 from both sides)
$3x = -15$	Simplify
$x = -5$	DPE (Divide both sides by 3)

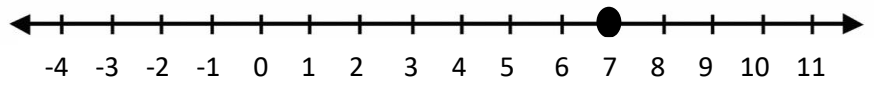
The graph of the solution of the equation $3(x + 4) = -3$ which is $x = -5$ is shown below.



d.

$2(x - 3) = x + 1$	Write the original equation
$2x - 6 = x + 1$	Distributive Property (Multiply 2 to x and 2 to - 3)
$2x - 6 + 6 = x + 1 + 6$	APE (Add 6 to both sides)
$2x = x + 7$	Simplify
$2x - x = x - x + 7$	SPE (Subtract x from both sides)
$x = 7$	Simplify

The graph of the solution of the equation $2(x - 3) = x + 1$ which is $x = 7$ is shown below.



Properties of Inequality

1. Trichotomy Property

   For all real number a and b, only one of the following is true:

   a < b, a =b, or a > b

   Example: Let a = 1 and b = 2, by substitution we have 1 < 2, 1 = 2 , 1 > 2.
   Of these 3, only one is true which is 1 < 2.

2. Transitive Property of Inequality

   For all real numbers a, b and c:

   If a < b and b< c, then a < c.
   If c > b and b > a, then c > a.

   Examples: Let a = 1 and b = 2 and c = 3:

   If 1 < 2 and 2 < 3, then 1 < 3.
   If 3 > 2 and 2 > 1, then 3 > 1.

3. Addition Property of Inequality (API)

   For all real numbers a, b and c:

   If a > b, then a + c > b + c.
   If a < b, then a + c < b + c.

   Examples:

   If 3 > 2, then 3+ 1 > 2 + 1.
   If 1 < 2, then 1+ 3 < 2 + 3.

4. Subtraction Property of Inequality (SPI)

   For all real numbers a, b and c:

   If a > b, then a - c > b - c.
   If a < b, then a - c < b - c.

   Examples:

   If 3 > 2, then 3 - 1 > 2 - 1.
   If 3 < 5, then 3 - 2 < 5 - 2.

5. Multiplication and Division Properties of Inequality by Positive Numbers

   For all numbers a, b and c, with c positive,

   If a > b then ac >bc and $\frac{a}{c} > \frac{b}{c}$.
   If a < b then ac <bc and $\frac{a}{c} < \frac{b}{c}$.

   Examples:

   If 5 > 3, then (5)(2) > (3)(2).
   If 3 < 5, then (3)(2) < (5)(2).

6. Multiplication and Division Properties of Inequality by Negative Numbers

   For all numbers a, b and c, with c negative,

   If a > b then ac <bc and $\frac{a}{c} < \frac{b}{c}$.
   If a < b then ac >bc and $\frac{a}{c} > \frac{b}{c}$

   Examples:

   If 4 > 2, then (4)(-1) < (2)(-1) and $\frac{4}{-1} < \frac{2}{-1}$.
   If 1 < 3, then (1)(-1) > (3)(-1) and $\frac{1}{-1} > \frac{3}{-1}$.

These properties can be used to solve inequalities. When you substitute a number for the variable, you can determine whether that value will make the inequality a true or a false statement. Any value that makes the sentence true is called a solution of the open sentence. Since solutions of open sentences are real numbers, you can graph them on a number line.

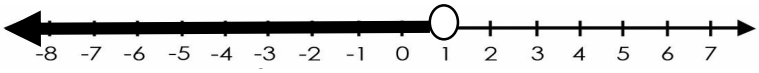
To graph an inequality:

1. If the symbol uses (≥ or ≤), draw a solid circle on a number line at the indicated number. If the symbol uses > or <, draw an open circle on the number line at the indicated number.
2. If the variable is greater than the indicated number, shade the area on the number line on the right side of the indicated number. If the variable is less than the indicated number, shade the area on the number line on the left side of the indicated number.

Example:

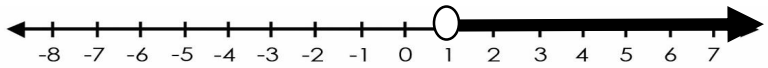
Let us consider a point on the number line whose coordinate is 1.

1. The shaded portion of the graph shows all points whose coordinates are less than 1, that is x < 1, are solution to the inequality. The small unshaded circle on top of 1 means that 1 is not included in the solution of the inequality.



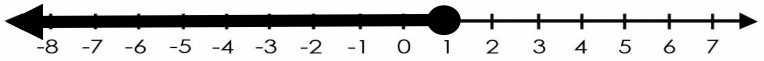
In symbols, x < 1.

2. The shaded portion on the graph which represents all numbers greater than 1 are solution of the inequality x > 1, except for 1.



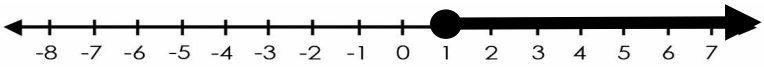
In symbols, x > 1.

3. The shaded circle on top of 1 and the arrow from 1 to left on the number line indicate that all numbers to the left of 1, including 1, are solutions of the inequality.



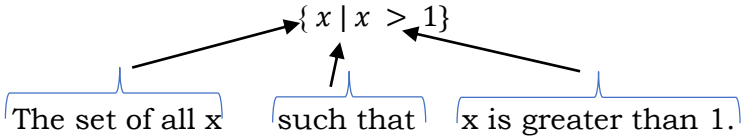
In symbols, x ≤ 1.

4. The graph shows all numbers to the right of 1, including 1, are solutions of the inequali



In symbols, x ≥ 1.

Solutions of inequalities can be represented using set notations. For example, the inequality x > 1 can be written in set notation as



Example: Solve each inequality then graph the solution.

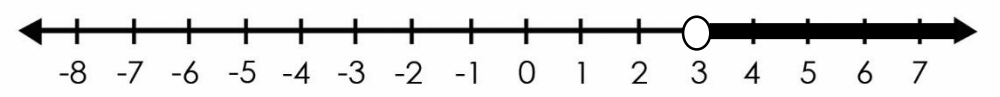
a. $x + 2 > 5$ b. $-8 \geq x - 5$

Solution:

a.

$x + 2 > 5$	Write the original inequality
$x + 2 - 2 > 5 - 2$	Subtract 2 from both sides
$x > 3$	Simplify

Any number greater than 3 will make the statement true. Therefore, the solution set is $\{x | x > 3\}$.



To check the solution, try any number greater than 3.

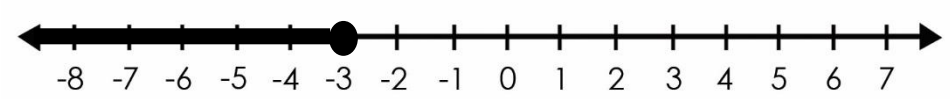
Check:

$x + 2 > 5$	Write the inequality
$4 + 2 > 5$	Replace x with 4
$6 > 5$	This statement is true.

b.

$-8 \geq x - 5$	Write the original inequality
$-8 + 5 \geq x - 5 + 5$	Add 5 to both sides
$-3 \geq x$	Simplify

The solution set is $\{x | -3 \geq x\}$ or $\{x | x \leq -3\}$.



To check the solution, try any number less than -3.

Check:

$-8 \geq x - 5$	Write the inequality
$-8 \geq -4 - 5$	Replace x with -4
$-8 \geq -9$	This statement is true.

Many absolute value equations and inequalities are not easy to solve by the guess and check, cover-up and working backwards methods. These can be solved by graphing and applying the properties of absolute value.

Properties of Absolute Value

Let $a > 0$.

1) If $|x| = a$, then $x = a$ or $x = -a$.
2) If $|x| < a$, then $-a < x < a$.
3) If $|x| > a$, then $x < -a$ or $x > a$.

Example 1. Solve each equation.

a. $|3a - 4| - 11 = 15$ b. $|c - 7| = |2c - 2|$

Solution:

a.

$\|3a - 4\| - 11 = 15$	Write the original equation
$\|3a - 4\| - 11 + 11 = 15 + 11$	Add 11 on both sides of the equation
$\|3a - 4\| = 26$	Simplify

or

$3a - 4 = 26$	$3a - 4 = -26$	Apply Property 1
$3a - 4 = 26$	$3a - 4 = -26$	
$3a = 30$	$3a = -22$	
$a = 10$	$a = -\frac{22}{3}$	

The solution set is $\{x \mid x = 10, x = -\frac{22}{3}\}$.

b.

$\|c - 7\| = \|2c - 2\|$		Write the original equation
$c - 7 = +(2c - 2)$	$c - 7 = -(2c - 2)$	Apply Property 1
$c - 7 = +(2c - 2)$	$c - 7 = -(2c - 2)$	
$c - 7 = 2c - 2$	$c - 7 = -2c + 2$	
$-c - 7 = -2$	$3c - 7 = 2$	
$-c = 5$	$3c = 9$	
$c = -5$	$c = 3$	

The solution set is $\{x \mid x = -5, x = 3\}$.

Example 2. Solve and graph each inequality.

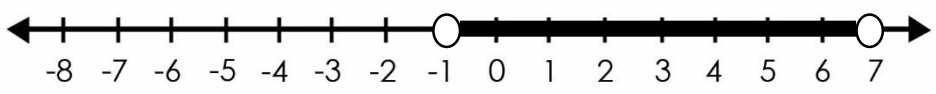
a. $|x - 3| < 4$ b. $|-2x + 3| \geq 7$

Solution:

a. Use the fact that $|x - 3| < 4$ is equivalent to $-4 < x - 3 < 4$.

$\|x - 3\| < 4$	Write the original equation
$-4 < x - 3 < 4$	Apply Property 2
$-4 + 3 < x < 4 + 3$	Add 3 to both sides
$-1 < x < 7$	Simplify

The solution set is $(-1, 7)$, or the real numbers between -1 and 7 or $\{x \mid -1 < x < 7\}$. The graph is shown below.



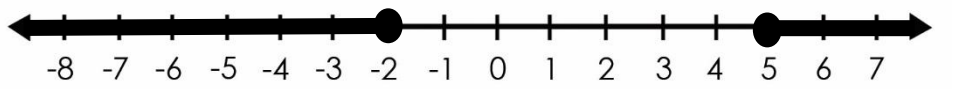
Note that -1 and 7 are not included in the solution set since an open circle is being used in the graph.

b. $|-2x+3| \geq 7$ Write the original equation

	or		
$-2x + 3 \leq -7$		$-2x + 3 \geq 7$	Apply Property 3
$-2x + 3 - 3 \leq -7 - 3$		$-2x + 3 - 3 \geq 7 - 3$	Subtract 3 from both sides
$-2x \leq -10$		$-2x \geq 4$	Simplify
$x \geq 5$		$x \leq -2$	Divide both sides by -2

Note that multiplying and dividing an inequality with a negative number means you must **reverse** the inequality sign. Recall that when you multiply or divide both sides by a negative value, the sign of the resulting product or quotient changes. This means that there is now a change in value of the numbers in both sides. Here, you make the side that is of greater value to have a "bigger" negative number, which actually means it is now less than the value on the other side. To make the statement true, you must reverse the inequality sign whenever you multiply or divide by a negative number.

Thus, the solution set is $\{x | x \leq -2 \text{ or } x \geq 5\}$ and the graph is shown below.



Note that -2 and 5 are included in the solution set that will satisfy the given equation since a solid circle is being used in the graph.

To solve word problem, one must know how to write equations. There are steps to follow in writing the equation and finding the solution.

Steps in Problem Solving

1. Read and explore the problem. Choose the variable to represent the unknown number in the problem. This is called defining the variable.
2. Use this variable in writing expressions for other unknown numbers in the problem.
3. Read the problem again and decide how the unknown relates to the other information in the problem.
4. Write an equation to represent the relationship.
5. Solve for the unknown.
6. Check by going back to the original statement.

Illustrative Examples

A. *Number Relation Problem*

A number is 3 less than another number. If their sum is 49, find the two numbers.

Step 1: Let x be the first number.
Step 2: $x - 3$ is the other number.
Step 3: The sum of the two numbers is 49.
Step 4: $x + (x - 3) = 49$

Step 5: $x + x - 3 = 49$ *Remove the parentheses*

$2x - 3 = 49$ *Combine like terms*

$2x - 3 + 3 = 49 + 3$ *Add 3 to both sides*

$2x = 52$ *Simplify*

$\boldsymbol{x} = 26$ *Divide both sides by 2*

Since $x = 26$, substitute 26 in the second number, that is $x - 3$

second number $= x - 3$

$= 26 - 3$

$= 23$

Step 6: *Check:* The two numbers, 26 and 23, when added is equal to 49, and 23 is 3 less than 26.

Answer: The first number is 26 and the other number is 23.

B. Consecutive Numbers Problem

The sum of three consecutive even integers is 96. Find the largest of these integers.

Step 1: Let x be the first even integer.

Step 2: $x + 2$ is the second even integer.

$x + 4$ is the third and largest even integer.

Step 3: The sum of the three consecutive even integers is 96.

Step 4: $(x) + (x + 2) + (x + 4) = 96$

Step 5: $x + x + 2 + x + 4 = 96$

$3x = 96 - 6$

$3x = 90$

$\boldsymbol{x} = 30$ the first even integer

Since $x = 30$, substitute 30 in $x + 2$ and $x + 4$

second even integer $= x + 2$

$= 30 + 2$

$= 32$

third even integer $= x + 4$

$= 30 + 4$

$= 34$

Step 6: *Check:* The consecutive even integers, 30, 32 and 34 when added is equal to 96.

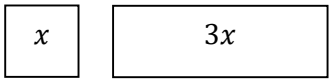
Answer: The largest of the three even integers is 34.

C. Geometric Relation Problem

The area of a rectangle is three times the area of a square. If four times the area of the square is twelve square centimeters bigger than the area of the rectangle, what is the area of the square and the area of the rectangle in square centimeters?

Step 1: Let x be the area of the square.

Step 2: $3x$ is the area of the rectangle.



Step 3: $4x$ is 12 cm^2 bigger than $3x$.

Step 4: $4x = 3x + 12$

Step 5: $4x - 3x = 12$

$\boldsymbol{x} = 12\ cm^2$ *The area of the square*

$3x = 3(12cm^2)$

$\boldsymbol{3x} = 36\ cm^2$ *The area of the rectangle*

Step 6: *Check:* The area of the rectangle, $36\,cm^2$, is three times the area of the square $12\,cm^2$.

Answer: The area of the square is $12\,cm^2$ and the area of the rectangle is $36cm^2$.

D. Age Problem

Mark is now 25 years older than his son. In 7 years, he will be 3 years more than thrice as his son's age. What are their present ages?

Step 1: Let x be the age of Mark's son.

Step 2: Let $x + 25$ be Mark's present age

Step 3: After 7 years, each age is increased by 7

Son's age: $\boldsymbol{x + 7}$ Mark's age: $x + 25 + 7 =$ **x + 32**

Step 4: In 7 years, Mark's age will be 3 years more than thrice his son's age, we have $x + 32 = 3(x + 7) + 3$

Step 5:
$$x + 32 = 3(x + 7) + 3$$
$$x + 32 = 3x + 21 + 3$$
$$x + 32 = 3x + 24$$
$$32 - 24 = 3x - x$$
$$8 = 2x$$
$$4 = x \qquad \textit{son's age at present}$$
$$x + 25 = 4 + 25 = 29 \qquad \textit{Mark's age at present}$$

Step 6: *Check:* Mark's age is 25 years more than his son's age: $29 = 4 + 25$. In 7 years, their ages will be 36 and 11, respectively. Thus, Mark's age is 3 years more than thrice his son's age: $36 = 3(11) + 3$

Answer: Mark's present age is 29 and his son's age is 4.

E. Distance Problem

Two cars leave Manila and travel in opposite directions. One of the cars' average speed is12 km/h less than the other. After 3 hours, they are 396 km apart. What is the average speed of each car?

Note: The distance covered by a moving body/object is the product of the time spent and the rate of the moving body.

$$d = rt$$

Step 1: Let r be the rate of the first car in km/h.

$r - 12$ is the rate of the second car.

Step 2: $3r$ is the distance covered by the first car after 3 hours.

$3(r - 12)$ is the distance covered by the second car after 3 hours.

Step 3: The total distance covered by the car is 396.

Step 4: $3r + 3(r - 12) = 396$

Step 5:
$$3r + 3(r - 12) = 396$$
$$3r + 3r - 36 = 396$$
$$6r = 396 + 36$$
$$6r = 432$$
$$r = 72\ km/h \qquad \textit{rate of the first car}$$
$$r - 12 = 60\,km/h \qquad \textit{rate of the second car}$$

Step 6: *Check:* The sum of the distances covered by the two cars should be 396 km. Thus,

$$3r + 3(r - 12) = 396$$
$$3(72) + 3(60) = 396$$
$$216 + 180 = 396$$
$$396 = 396$$

Answer: The average speed of the first car and second car are $72\ km/h$ and $60\ km/h$, repectively.

F. Money Problem

Jane has some 1-peso coins and 4 more 5-peso coins than 1-peso coins, making a total of 22 coins. How much money does she have?

Step 1: Let x be the number of 1-peso coins.
$x + 4$ is the number of 5-peso coins.
Step 2: $1.00x$ is the amount of 1-peso coins.
$5.00(x + 4)$ is the amount of 5-peso coins.
Step 3: The total number of 1-peso and 5-peso coins is 22.
Step 4: $x + (x + 4) = 22$
Step 5:

$$x + x + 4 = 22$$
$$2x = 22 - 4$$
$$2x = 18$$
$$x = 9 \quad \textit{number of 1-peso coins}$$
$$x + 4 = 9 + 4 = 13 \quad \textit{number of 5-peso coins}$$
$$1.00(9) = \text{Php } 9.00 \quad \textit{amount of 1-peso coins}$$
$$5.00(13) = \text{Php } 65.00 \quad \textit{amount of 5-peso coins}$$
$$\text{Php } 9.00 + \text{Php } 65.00 = \text{Php } 74.00 \quad \textit{total amount of money Jane has}$$

Step 6: *Check:* The nine 1-peso coins added to thirteen 5-peso coins is equal to 22 coins. Also, nine 1-peso coins is equal to Php 9.00 and the thirteen 5-peso coins is equal to Php 65.00, making a total of Php 74.00.
Answer: Jane has a total money of Php 74.00.

G. Inequality Problems

1. Khiff has Php 300 to ride a roller coaster and play games in the amusement park. If a ticket to a roller coaster ride costs Php 60.00, what is most amount he can spend on games?

Step 1: Let x be the amount of money Khiff can spend on games.
Step 2: $60 + x$ is the amount he can spend in the amusement park
Step 3: At most means less than or equal to
Step 4: $60 + x \leq 300$
Step 5:

$$60 + x \leq 300$$
$$60 - 60 + x \leq 300 - 60 \quad \textit{Subtract } 60 \textit{ from both sides}$$
$$x \leq 240 \quad \textit{Simplify}$$

Step 6: Check by choosing an amount less than or equal to Php 240 or an amount greater than Php 240.

Php 200 – *amount less than Php* 240	Php 250 – *amount greater than Php* 240
Let's check if Php 200 is a possible solution if $x \leq 240$. $60 + x \leq 300$ $60 + 200 \leq 300$ $260 \leq 300.$ CORRECT	Let's check if Php 250 is a possible solution if $x \leq 240$. $60 + x \leq 300$ $60 + 300 \leq 300$ $360 \leq 300$ INCORRECT

Comparing the two solutions above, we can say that the most amount Khiff can spend on games is Php 240.00.

Answer: Khiff can spend at most Php 240.00 on games.

2. Khaye is saving money for a field trip. She has Php 1, 500 but her goal is to save at least Php 8,000. What is the least amount Khaye needs to save to reach her goal?

Step 1: Let x be the amount of money Khaye still needs to save.
Step 2: $1500 + x$ is the total amount Khaye needs to save to reach her goal
Step 3: At least means greater than or equal to
Step 4: $1500 + x \geq 8000$
Step 5: $1500 + x \geq 8000$
$1500 - 1500 + x \geq 8000 - 1500$ *Subtract* 1500 *from both sides*
$x \geq 6500$ *Simplify*
Step 6: Check by choosing an amount greater than or equal to Php 6500 or an amount less than Php 6500, then compare its result.

Php 6500 – *amount equal to Php* 6500	Php 6000 – *amount less than Php* 6500
Let's check if Php 6500 is a possible solution if $x \geq 6500$. $1500 + x \geq 8000$ $1500 + 6500 \geq 8000$ $8000 \geq 8000.$ CORRECT	Let's check if Php 6000 is a possible solution if $x \geq 6500$. $1500 + x \geq 8000$ $1500 + 6000 \geq 8000$ $7500 \geq 8000.$ INCORRECT

Comparing the two solutions above, we can say that the least amount Khaye needs to save is Php 6500.00.

Answer: The least amount Khaye needs to save to reach her goal is Php 6500.00.

Points to Remember:

- The values of the variable that result in a true statement are called ***solutions*** or ***roots*** of the equation.
- To ***solve an equation*** means to find all the solutions of the equation.
- The ***solution set*** of an inequality is the set of values that makes the inequality a true statement.
- **Solving Inequalities** is the same as solving an equation. You see the same steps as in solving equations, but the only difference is if you *multiply* or *divide by a negative number.* Multiplying and dividing an inequality with a negative number means you must **reverse** the inequality sign.

Example: $-4x < 12$
$x > -3$

What's More

Let us try to answer more challenging set of problems.

A. Match the solutions under Column B to each equation or inequality in one variable under Column A. Remember that inequalities can have more than one solution. List all acceptable solutions.

COLUMN A	COLUMN B
_____1. $4x - 4 = 12$	A. -1
_____2. $2x - 1 = 3$	B. 0
_____3. $x - 2 > 7$	C. 1
_____4. $4x > 12$	D. 2
_____5. $x - 5 > 3$	E. 3
_____6. $x + 4 = 5$	F. 4
_____7. $7x - 5 = 16$	G. 5
_____8. $5x = 30$	H. 6
_____9. $3x - 1 < -1$	I. 9
_____10. $2x - 5 = 13$	J. 10

B. Match the following inequalities with its graph. Write the letter that corresponds to the graph of the given equations and inequalities.

_____1. $|x| \leq 3$

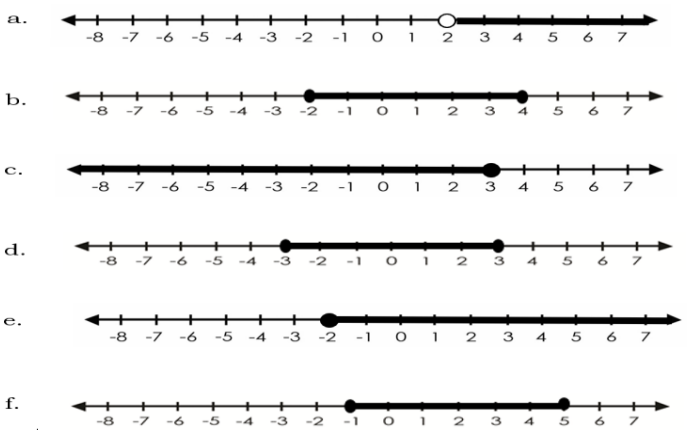
_____2. $|x - 2| \leq 3$

_____3. $|x - 1| \leq 3$

_____4. $x - 4 \leq -1$

_____5. $x + 3 > 5$

_____ 6. $2x - 1 \geq -5$



C. Use the variable x to represent the unknown.

1. Julie is 15 years old now. Represent her age x years ago.
2. Represent "four less than a given number" if the given number is represented by $3x$.
3. The number of days is at most than 14.
4. Express 72 as the sum of three consecutive even integers.
5. A number is 9 times the other and their sum is 100.

Good job! Get ready for another learning battle ahead!

What I Have Learned

Here is another activity that will let you apply what you have learned about solving linear equations and inequalities in one variable. Choose the word/expressions from the box to fill in each blank to make a statement true.

solution set	divide	$x < -a$ or $x > a$
subtract	working backward	defining the variable
multiply	–a<x<a	cover-up
guess and check	-3	Trichotomy Property of Inequality
add	$\frac{1}{3}$	Symmetric Property of Equality

1. There are three (3) different ways to solve an equation or inequality in one variable by inspection. These include the guess and check method, cover-up method and the _____ method.
2. The _____ method will let one guesses and substitutes values to see if the value being substituted will make the equation true.
3. The _____ of an inequality is the set of values that makes the inequality a true statement.
4. Subtracting 3 from both sides of the equation is the same as adding _____ to both sides of the equation.
5. To solve x + 8 = 12, _____ 8 from both sides of the equation.
6. To solve -2x – 5 = -x, _____ x on both sides of the equation.
7. The _____ tells us that the expressions on the both sides of the equation may be interchanged.
8. If $|x| < a$, then _____.
9. To solve $10x = 140$, _____ both sides of the equation by 10.
10. The first step in solving word problems is choosing the variable to represent the unknown number in the problem. This is called _____.

Nice work! Now you're up for the next challenge of this lesson.

What I Can Do

Apply what you have learned about solving linear equations and inequalities in one variable.

A. Find the solution of the equation or inequality using the three methods.

For items 1-3, use Guess and Check Method.

1. $4(x + 2) = 6x + 6$ if $x = -1$, $x = 1$, $x = 2$
2. $-3(x + 2) < 11$ if $x = 5$, $x = 6$, $x = -6$
3. $2x + 5 \geq -1$ if $x = -3$, $x = 0$, $x = 3$

For items 4-5, use Cover-up Method.

4. $x + 9 = 14$
5. $2x - 5 = 13$

For items 6-7, use Working Backwards method by illustrating a diagram.

6. $4x + 7 = 15$
7. $6x - 4 = 20$

B. Solve the following equation and inequality. Then show the graph of each inequality.

1. $-3|x + 3| + 6 = -3$
2. $|2x + 5| = 2$
3. $|2x + 4| \geq 12$
4. $|x + 1| < 3$
5. $|x - 3| > 2$

C. Solve the following word problems on linear equations and inequalities in one variable.

1. The sum of two numbers is 25. The second number is 5 more than the first number. Find the numbers.

2. Find three consecutive even numbers whose sum is 72.

3. One angle of a triangle is thrice as large as another. The measure of the third angle is 40^0 greater than that of the second angle. Find the measure of each angle.

4. The difference between two numbers is less than or equal to 96. The larger number is 245. Find the smallest possible number.

5. The sum of Ben's age and Jul's age is 53. Five years ago, Ben was 7 years more than one-half as old as Jul then. How old is Jul?

Great work! You did a good job in applying what you have learned!

Assessment

Multiple Choice. Choose the letter of the correct answer. Write the chosen letter on a separate sheet of paper.

1. What is the value of x that would make the equation $4x - 8 = 12$ correct?
   A. 3
   B. 5
   C. 7
   D. 9

2. Which of the following is a possible solution of $-7x - 4 \leq 10$?
   A. -5
   B. -4
   C. -3
   D. -2

3. Which of the following values will satisfy the equation $12x - 14 = 10$?
   A. -3
   B. -2
   C. 2
   D. 3

4. What is a possible solution of $7x + 5 \geq 9$?
   A. -2
   B. -1
   C. 0
   D. 1

5. What are the possible solutions of $6(x - 5) > 13$?
   A. 1, 3
   B. 4, 5
   C. 6, 7
   D. 8, 9

6. What are the values of x in the equation $|x + 5| = 4$?
   A. x = -1 and x = -9
   B. x = 1 and x = -9
   C. x = -1 and x = 9
   D. x = 1 and x = 9

7. What are the solutions to the equation $|3x - 5| - 4 = 10$?
   A. x = 3 and $x = \frac{19}{3}$
   B. x = 3 and $x = -\frac{19}{3}$
   C. x = -3 and $x = -\frac{19}{3}$
   D. x = -3 and $x = \frac{19}{3}$

8. What is/are the value/s of x in the equation $|x| = 2$?
   A. 2
   B. -2
   C. 2 or -2
   D. 22

9. What is the solution set of the inequality $|x + 2| < 6$?
   A. $\{x \mid -8 < x < 4\}$
   B. $\{x \mid -8 < x > 4\}$
   C. $\{x \mid -8 > x > 4\}$
   D. $\{x \mid -8 > x < 4\}$

10. What values of a will satisfy the inequality $|4a + 1| > 5$?
   A. $\{a \mid a < -\frac{3}{2} \text{ or } a > 1\}$
   B. $\{a \mid a > -\frac{3}{2} \text{ or } a > 1\}$
   C. $\{a \mid a < -\frac{3}{2} \text{ or } a < 1\}$
   D. $\{a \mid a < -\frac{3}{2} \text{ or } a > 1\}$

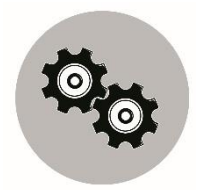
11. Ana weighs $3\,kg$ more than her sister Nina, and their total weight is $105\,kg$. What is the weight of Nina?
   A. $50\,kg$
   B. $51\,kg$
   C. $53\,kg$
   D. $54\,kg$

12. The lengths of a triangle are consecutive integers. If the perimeter of the triangle is $108\,cm$, what is the length of the longest side?
   A. 35 cm
   B. 36 cm
   C. 37 cm
   D. 38 cm

13. Jay's father is twice as old as Jay. In 20 years, Jay will be two-thirds as old as his father. How old is Jay's father?
   A. 35
   B. 40
   C. 42
   D. 44

14. Mark earned 6, 7 and 10 points out of 10 on the first three quizzes. What must he score on the fourth quiz to average at least 8?
   A. at least 6
   B. at least 7
   C. at least 8
   D. at least 9

15. Which statement can be modeled by $x + 3 \leq 12$?
   A. Joan has 3 bottles of water. Together, Joan and her friend have at most 12 bottles of water.
   B. Gayzel sold 3 facemasks. To reach her target sales, Gayzel must sell at least 12 facemasks.
   C. Glen has 2 baseball hats. Glen and his brothers have fewer than 12 baseball hats.
   D. Daphne swam 3 laps in the pool this week. She must swim more than 12 laps.



Additional Activities

This section includes supplementary activities related to translating and solving equations.

You have heard of number puzzles when someone asks you to choose numbers and perform some calculations, and then the person tells you what the result is, or is able to tell you the number you chose.

Example:

Choose a number.	x
Add three.	$x + 3$
Double the result.	$2(x + 3) = 2x + 6$
Subtract two.	$2x + 6 - 2 = 2x + 4$
Divide by two.	$\frac{2x+4}{2} = x + 2$
Subtract the number you chose.	$x + 2 - x = 2$
The result is 2.	

Try to use other numbers and follow the steps discussed in the example above. Check whether the answer results to 2.

To see why the result is always 2, look at the expression for each step.

Your turn!

Write the expressions that show how the following number puzzles work. Then explain how did you arrived with your answer.

1. Choose a number. _____
   Add the next smaller number. _____
   Add nine. _____
   Divide by two. _____
   Subtract the number you chose. _____
   The result is 4. _____

2. Choose a number. _____
   Double it. _____
   Add six. _____
   Double the result. _____
   Divide by four. _____
   Subtract the number you chose. _____
   The result is 3. _____

3. Create your own number puzzle. Then show how it works for a given number.

Answer Key

What I Know

1. D
2. B
3. D
4. D
5. C
6. C
7. B
8. D
9. A
10. A
11. B
12. C
13. C
14. B
15. B

What I Have Learned

1. working backward
2. guess and check
3. solution set
4. -3
5. subtract
6. add
7. Symmetric Property of Equality
8. –a<x<a
9. divide
10. defining the variable

What's In

A.		B.	
1.	E	1.	17
2.	I	2.	8
3.	I	3.	5
4.	I	4.	15
5.	E		
6.	I		

What's New

a. x = -3
b. x = -3 , x = -2 , x = -1 , x = 2
c. x = -3 , x = -2 , x = -1
d. x = 3
e. x = -2

1. No.
2. Yes.

What I Can Do

A. 1. x = -1 (not solution)
x = 1 (solution)
x = 2 (not solution)
2. x = 5 (solution)
x = 6 (solution)
x = -6 (not solution)
3. x = -3, 0 3 (solutions)
4. x = 5
5. x = 9
6. x = 2
7. x = 4

B.
1. x = -6, x = 0
2. $x = \frac{-7}{2}$, $x = \frac{-3}{2}$
3. $\{x \mid x \le -8 \text{ or } x \ge 4\}$
4. $\{x \mid -4 < x < 2\}$
5. $\{x \mid x < 1 \text{ or } x > 5\}$

C.
1. 10 and 15
2. 22, 24, and 26
3. 20^0, 60^0 and 100^0
4. 149
5. 29

What's More

A.		B.
1.	F	1. d
2.	D	2. f
3.	J	3. b
4.	F, G, H, I, J	4. c
5.	I, J	5. a
6.	C	6. e
7.	E	
8.	H	
9.	A	
10.	I	

C.
1. $15 - x$
2. $3x - 4$
3. $x \le 14$
4. $(x) + (x + 2) + (x + 4) = 72$
or
$2x + (2x + 2) + (2x + 4) = 72$
5. $9x + x = 100$

Assessment

1.	B	11.	B
2.	D	12.	C
3.	C	13.	B
4.	D	14.	D
5.	D	15.	A
6.	A		
7.	D		
8.	C		
9.	A		
10.	A		

Additional Activities

1. x
$x + x - 1 = 2x - 1$
$2x - 1 + 9 = 2x + 8$
$\frac{2x + 8}{2} = x + 4$
$x + 4 - x = 4$

2. x
$2x$
$2x + 6$
$\frac{4x + 12}{4} = x + 3$
$x + 3 - x = 3$

3. Answers may vary

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