

GENERAL MATHEMATICS

Quarter 1: Module 1

FUNCTIONS



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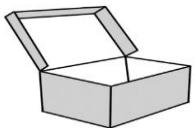
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What I Need to Know

This module was designed and written with you in mind. This will help you master **Functions**. The scope of this module is to represent real-life situations using functions, including piece-wise functions, evaluate a function, perform addition, subtraction, multiplication, division, composition of functions, and solve problems involving functions.

This module is divided into 5 lessons:

Lesson 1: Functions as Representations of Real-Life Situations

Lesson 2: Evaluation of Functions

Lesson 3: Operations on Functions

Lesson 4: Composition of Functions

Lesson 5: Problems Involving Functions

After going through this module, you are expected to:

1. give real-life situations using functions, including piece-wise functions,
2. evaluate a function,
3. perform addition, subtraction, multiplication, division of functions,
4. perform composition of functions and
5. solve problems involving functions.



What I Know

Choose the letter of the correct answer.

1. Determine which of the following represents a function.

A.

Input	Output
-1	1
-1	2
-1	5

B.

Input	Output
3	9
3	-9
4	16

C.

Input	Output
2	4
2	4
3	5

D.

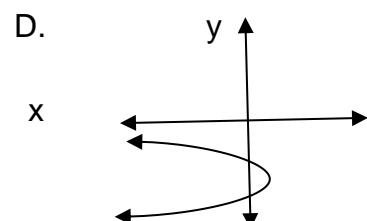
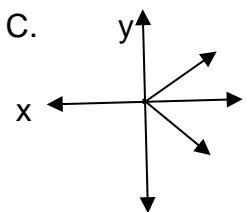
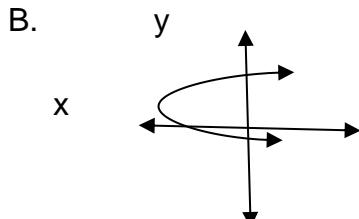
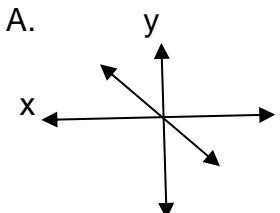
Input	Output
1	4
2	6
3	5



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2. Which of the following graph represents a function?



3. If $f(x) = x^2 - 2x + 3$ then $f(1)$ equals

A. -2

B. 0

C. 2

D. 3

4. Evaluate: $f(x) = 3x - 1$, if $x = 2$.

A. -4

B. -5

C. 5

D. 6

5. What is the sum of the given expressions $2x$ and $4x$?

A. $6x$

B. $6x^2$

C. $x + 6$

D. $2x+4$

6. Given $f(x) = x + 5$ and $g(x) = x - 5$. What is $(f \circ g)(x)$?

A. 0

B. 1

C. $\frac{x+5}{x-5}$

D. $x^2 - 25$

7. If $f(x) = 2x + 3$ and $g(x) = 2x$, find $(f \circ g)(x)$.

A. 3

B. $4x$

C. $4x - 3$

D. $4x + 3$

8. If $f(x) = 2x + 3$, find $f(-2)$.

A. -1

B. 1

C. 2

D. 3

9. Suppose a company sells bicycles. The company's profit can be represented by the function $p(x) = x^2 + 4x - 6300$, where x is the number of bikes sold. If the company sells 130 bicycles, what is the company's profit?

A. Php. 10, 120 B. Php. 12, 120 C. Php. 11, 120 D. Php. 13, 120

10. In Sumulong St., Parang, there are twice as many dogs as cats. How do you represent this using equation? Let D = number of dogs and C = number of cats.

A. $D = 2(D - C)$ B. $C = 2(D - C)$ C. $D = 2C$ D. $2D = C$

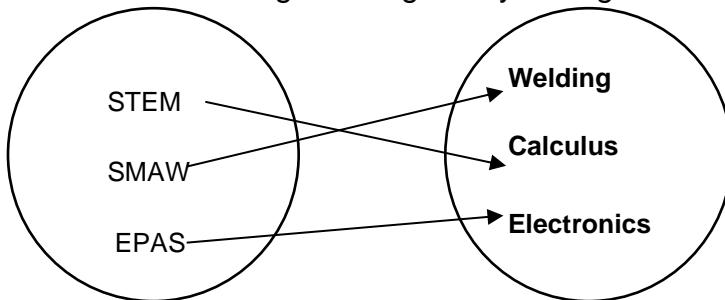


LESSON 1: Function as Representations of Real-Life Situations.



What's In

A. **Describe Me!** Describe the given diagram by writing the set of ordered pairs.



Guide Questions:

1. How many elements are there in the set of ordered pairs that you have made?
2. Is there a repetition of the first coordinates? How about the second coordinates?
2. Does the set of ordered pairs represent a function?

B. **Function or Not Function.** State whether each of the following is a function or not. Write a brief explanation of your answer.

1. $\{(2,3),(4,5),(-3,-1),(0,1)\}$

2. $\{(1,-1),(0,0),(1,1),(3,5)\}$

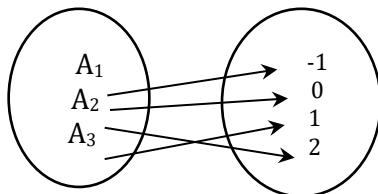
3.

x	1	2	3	4	5
y	± 1	± 4	± 9	± 16	± 25

4.

x	-2	-1	0	1	2
y	4	5	5	3	3

5.





What's New

Here are different presentations of functions:

A. Functions H and K as set of Ordered Pairs:

$$H = \{(2,3), (4,5), (-3,-1), (0,1)\}$$

$$K = \{(-1,3), (0,5), (1,-1), (2,1)\}$$

B. Functions as Equations:

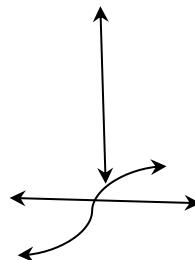
$$f(x) = x - 2$$

$$g(x) = 2x + 5$$

C. Function p as Table of values:

x	1	2	3	4	5
y	5	4	3	2	1

D. Function p as graph:



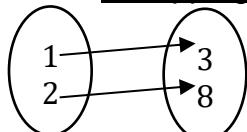
E. Functions as a Descriptive Rule (in words)

Examples:

F: a relation assigning a person to his/her height

G: a relation assigning a student to his/her strand (section)

F. Functions as a mapping diagram



What is It

Here you will learn about how functions, including piece-wise function represent real-life situations.

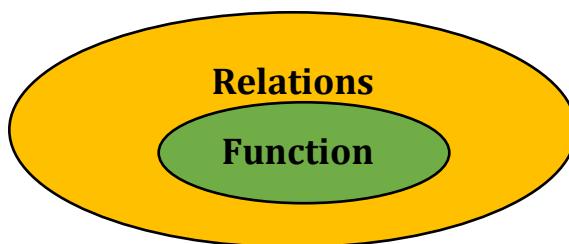
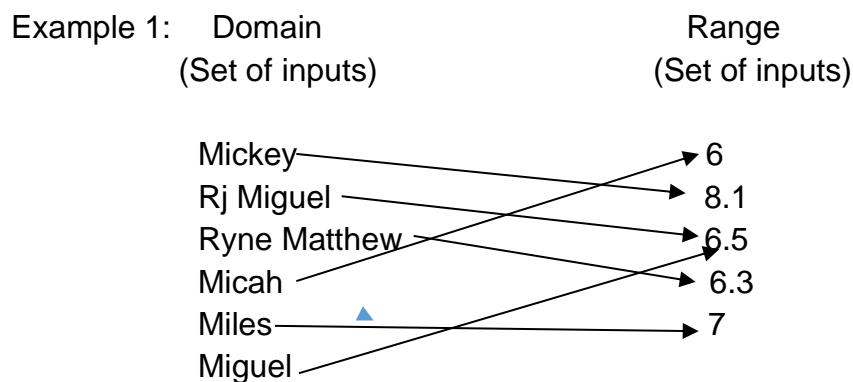


Figure 1: Relations and Functions



A **Relation** shows the correspondence between two non-empty sets. A **function** is a relation which derives one OUTPUT for each given INPUT. The members of the domain can be called inputs and the range can be called as outputs.



This relation is a function because each element in the domain $\{Mickey, Rj\ Miguel, Ryne\ Matthew, Micah, Miles, Miguel\}$ is assigned to one element in the range. Although the range 6.5 is assigned to two elements of the domain, Rj Miguel and Miguel, it is still a function since each element of the domain is assigned to each element of the domain.

Example 2: Are the following relations considered to be a function?

Height(x)	Weight(y)
77in	185 lb
65in	168 lb
65in	140 lb
62in	160 lb
70in	182 lb

Input	Output
Bread	(Price)
Pandesal	150.00
Monay	150.00
Ensaymada	180.00
Pan De Coco	180.00
Pan De Leche	200.00

The given table of values in example 2.a is not a function since there are two ordered pairs that have the same height. Example 2.b on the other hand, represents a function.

The next example shows the relationship between two variables that can be associated as a function.



Example 3: The relationship between time and temperature.

Answer: Definitely there is a relation between time and temperature. In specific location or area, temperature is a function of time. For example, the brewing pot temperature of coffee is 180°F , the room temperature is 77°F . After 5 minutes the temperature of the coffee is 168°F .

Here are some examples on how to deal with problem solving involving functions and piece-wise functions.

Example 4: A student who wants to earn extra income while studying got a job at a certain food chain where he/she earns Php.450.00. Write the total income/salary H as a function of the number n days that the student works.

Solution: $H(n)$ ----- Total salary in terms of number of days you work

450.00-----Salary in a day

n-----Number of days you work

$$H(n) = 450n$$

A **piece-wise or compound function** is a function defined by multiple sub-functions, where each sub-function applies a certain portion of the main function's domain.

Example of piece-wise function

$$a. g(x) = \begin{cases} 2 & \text{if } x \leq 1 \\ x & \text{if } x > 1 \end{cases}$$

b. Ms.Weng availed of a phone subscription plan that provides a monthly allowance for calls. For a base rate of P300 per month, she can do a maximum of 300 minutes of calls to all networks. Beyond this limit, she is charged P 5 per minute of call. Represent the **total amount** that she will pay per month as a function of the number of call minutes x made in calls.

Solution:

Let $f(x)$ represents the amount paid by Ms. Weng each month.

It can be expressed by the piece-wise function

$$f(x) = \begin{cases} 300 & \text{if } 0 < x \leq 300 \\ 300 + x & \text{if } x > 300 \end{cases}$$

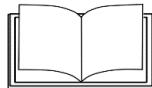
c. The minimum fare for a tricycle ride costs P 20.00 for the first five kilometers and each additional kilometer adds P 2.00 to the fare. Represent the tricycle fare in terms of the distance d in kilometers using piece-wise function.

Solution:



Let the input value is distance and the output as the cost of the tricycle fare. If $T(fd)$ represents the fare , then

$$T(fd) = \begin{cases} 20 & \text{if } 0 < fd \leq 5 \\ 20 + 2.00fd & \text{if } fd > 5 \end{cases}$$



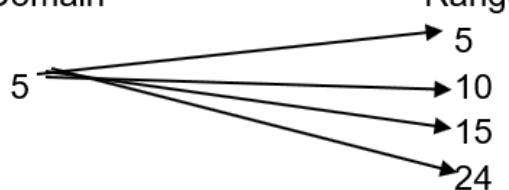
What's More

A. Indicate whether each relation represents a function. Write **F** if it is a function and **N** if not.

1. Domain



2. Domain



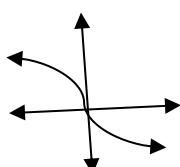
B. Determine if each ordered pair is a **Function** or a mere **Relation**. Give the **domain** and **range**.

3. $\{(-1,2), (-3,4), (-5,6), (-7,8)\}$
 Domain: _____ Range: _____

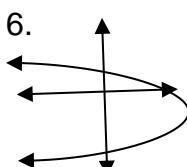
4. $\{(1,5), (3,7), (1,7), (9,8)\}$
 Domain: _____ Range: _____

C. Tell whether the given graph describes a function or a relation.

5.



6.





What I Have Learned

Fill in the blanks with the correct answers to make the given statement true.

1. “ _____ can be is a set of ordered pairs”. The _____ of the relation is the set of the first coordinates and the _____ is the set of the second coordinates.
2. _____ is a relation in which each element of the domain corresponds to exactly _____ element of the range.
3. The members of the domain can be called as _____ and the members of the range can be called as _____.
4. A _____ function or a compound function is a function defined by multiple functions where each function applies to a certain interval of the main function's domain.



What I Can Do

Problem solving: Solve and show your complete solutions.

In Barangay Magsaysay, a washing machine can be rented for P500 for one day, but for the exceeding days, an additional of P200 per day is added.

- a. Represent the cost of renting a washing machine as a piece-wise function of the number of days it is rented.
- b. If the renter was able to return the washing machine on the fifth day, how much will he/she pay?
- c. What if the washing machine was broken and the owner demands the renter to pay for P15,000 in lieu of the machine. Would you agree to pay if you were the renter? Why? Elaborate your answer.

Rubrics for problem solving

Score	Description
15 points	Complete solutions with 90-100% correct answer
10 points	Complete solutions with 85-90% correct answer
5 points	Incomplete solution with 50% incorrect answer
No point earned	No output at all



Assessment

Choose the letter of the correct answer.

1. Determine which of the following represents a function.

A.	<table border="1"> <thead> <tr> <th>Input</th><th>Output</th></tr> </thead> <tbody> <tr> <td>-1</td><td>1</td></tr> <tr> <td>-1</td><td>2</td></tr> <tr> <td>-1</td><td>5</td></tr> </tbody> </table>	Input	Output	-1	1	-1	2	-1	5	C.	<table border="1"> <thead> <tr> <th>Input</th><th>Output</th></tr> </thead> <tbody> <tr> <td>2</td><td>4</td></tr> <tr> <td>2</td><td>4</td></tr> <tr> <td>3</td><td>5</td></tr> </tbody> </table>	Input	Output	2	4	2	4	3	5
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3	9																		
3	-9																		
4	16																		
Input	Output																		
1	4																		
2	6																		
3	5																		

2. Which of the following sets of ordered pairs defines a function?

- A. $\{(-1,4), (2,4), (2,5), (-4,4), (5,4)\}$ B. $\{(1,4), (2,4), (1,8), (1,14)\}$
 C. $\{(-1,4), (-1,4), (-3,4), (-3,6)\}$ D. $\{(1,-1), (2,4), (3,5)\}$

For item number 3– 4. Refer to the given data below.

Consider the following sets of ordered pairs below:

Set A $\{(3,4), (4,5), (5,6), (6,7), (7,8)\}$

Set B $\{(2,2), (2,-2), (3,-3), (3,-3), (4,-4)\}$

Set C $\{(0,1), (1,1), (2,1), (3,1), (5,1)\}$

3. What is the domain of Set A?

- A. $\{3,4,5,6,7\}$ B. $\{3,4,5,6,\}$ C. $\{3,4,5,\}$ D. $\{3,4,\}$

4. What is the range of Set B?

- A. $\{2,4,5,6,7\}$ B. $\{3,4,5,6,\}$ C. $\{2,-2,-3,-4,\}$ D. $\{-3,3,\}$

5. Rona is working as an online tutor and is offered a rate P100 per hour. Her earnings are related to the number of hours of work. How much will Rona earn if she works for 5 hours a day?

- A. P400.00 B. P500.00 C. P 600.00 D. P700.00



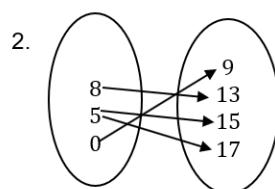
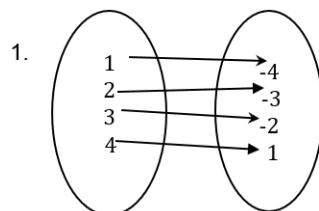
Additional Activities

- A. Which of the following are functions?

$$G(x) = \{(-1,0),(0,1),(1,2),(2,3)\} \quad H(x) = \{(2,-1),(2,0),(3,4),(3,-3)\}$$

$$P(x) = \{(1,2),(2,4),(3,6), \dots (n,2n)\}$$

- B. Which of the following mapping diagrams represents functions?



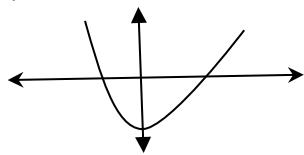
LESSON 2: Evaluation of Functions



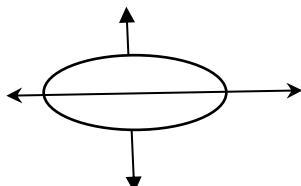
What's In

A. Determine each graph if it is a Function or Not a function.

1.



2.



B. List the domain and range of each of the following:

1. $\{(1,2), (2,3), (4,5), (6,7)\}$.

Domain: _____

Range: _____

2.

x	-10	-8	-6	-4	-2	0
y	3	5	7	9	11	13

Domain: _____

Range: _____

Follow Up Questions:

- How will you know if the graph is a function or not a function?
- What if the domain is being mapped twice from the range? Is it a function?
- How will you complete the table of values if there is something missing in the domain and range? Explain your answer.

Given: $y = \pm\sqrt{x}$ which defines a relation in the set of positive rational numbers.

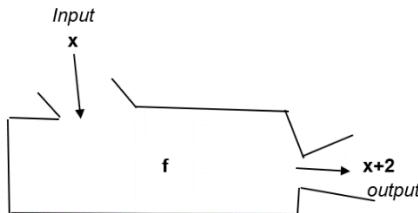
- What are the values of y if $x = 4$? Write the ordered pairs having 4 as the first element.
- Give two more ordered pairs that belong to this relation?
- What did you do to find the value of y?





What is It

What you will learn here is all about evaluating a function. The input – output process can also be thought of in terms of a function machine. Inputs from the domain are put into the machine and the machine gives the proper output.



This function machine, for the relation of f assigns to each input x the output $x+2$. It adds 2 to each input. The outputs, for the inputs 8, -3, 0 and 5 are as follows:

$$8 \rightarrow 10 \quad -3 \rightarrow 1 \quad 0 \rightarrow -2 \quad 5 \rightarrow 7$$

The symbol $f(x)$ denotes the number assigned to x by the function f . If x is the input, $f(x)$ is the output. We can write the above results as follows where $f(x) = x + 2$.

$$f(8) = 8 + 2 = 10$$

$$f(-3) = -3 + 2 = -1$$

$$f(0) = 0 + 2 = 2$$

$$f(5) = 5 + 2 = 7$$

The outputs of a function are also called **function values**. For example above, we have $f(8) = 10$. We can say that “10 is the value of the function $f(x) = x + 2$ when $x = 8$.”

Note: The symbol $f(x)$ is read as “*f of x*”. It is the mathematical shorthand for “function of x ”.

Examples 1:

Consider the $f(x) = 2x^2 + 5$; find the following:

a. $f(-2)$

b. $f(0)$

c. $f(3)$

Solution:

a. $f(-2)$ $f(x) = 2x^2 + 5$
 $f(-2) = 2(-2)^2 + 5$
 = $2(4) + 5$
 = $8 + 5$
 $f(-2) = 13$

b. $f(0)$ $f(x) = 2x^2 + 5$
 $f(0) = 2(0)^2 + 5$
 = $0 + 5$
 $f(0) = 5$

c. $f(3)$ $f(x) = 2x^2 + 5$
 $f(3) = 2(3)^2 + 5$
 = $2(9) + 5$
 = $18 + 5$
 $f(3) = 23$



Example 2: Evaluate the function $f(x) = x + 8$, if:

a. $f(-4)$ b. $f(2)$ c. $f(5)$

Solution:

a. $f(-4) \quad f(x) = x + 8$
 $f(-4) = (-4) + 8$ Replace x with -4
 $f(-4) = 4$

b. $f(2) \quad f(x) = x + 8$
 $f(2) = (2) + 8$ Replace x with 2
 $f(2) = 10$

c. $f(5) \quad f(x) = x + 8$
 $f(5) = (5) + 8$ Replace x with 5
 $f(5) = 13$
$$f(x) = \begin{cases} x^2 + 6, & x < 0 \\ 5x - 7, & x \geq 0 \end{cases}$$

Example 3: If $g(x) = \begin{cases} x^2 + 6, & x < 0 \\ 5x - 7, & x \geq 0 \end{cases}$

Find the following: a. $f(-5)$ b. $f(5)$

Solution:

- a. To find $f(-5)$, we let $x = -5$ and since -5 is less than 0, then, we have to use the first line of the function $g(x) = x^2 + 6$. Thus,

$$\begin{aligned} g(x) &= x^2 + 6 \\ &= (-5)^2 + 6 \\ &= 25 + 6 \end{aligned}$$

$$g(-5) = 31$$

- b. To find $f(5)$, we let $x = 5$ and since 5 is greater than 0, then, we have to use the second line of the function $g(x) = 5x - 7$. Thus,

$$\begin{aligned} g(5) &= 5x - 7 \\ &= 5(5) - 7 \\ &= 25 - 7 \end{aligned}$$

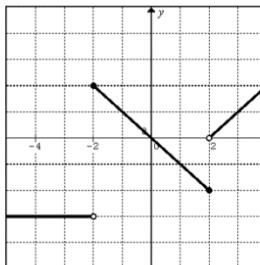
$$g(5) = 18$$

Let F, G, and H be represented as follows:

Graph of $F(x)$

$G(x)$ is represented by ordered pairs.

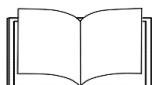




$$G(x) = \{(1, -2), (0, 3), (1, -4)\}$$

$H(x)$ is represented by table of values.

x	-2	-1	0	1	2
$H(x)$	5	-2	3	0	-4



What's More

Complete the following table:

x	0	1	2
$p(x) = 2x^2 - x + 1$			

x	-4	-2	-1
$s(x) = \frac{x}{2}$			



What I Have Learned

Fill in the blanks with the correct answers to make the given statement true.

1. The outputs of a function are also called _____.
 2. The symbol $f(x)$ is read as “____ of ____.” It is a mathematical shorthand for _____ of x .
 3. To find the value of $f(2) = 4x + 1$, replace the value of x by _____ and add _____ by 1 that will give the result of _____.
 4. If $f(x) = 12 - 7x$, then $f(2) =$ _____.
 5. If $f(x) = \sqrt{x + 1}$, then $f(3) =$ _____.



What I Can Do

Solve the given problem.

1. Mark started selling snacks in the nearby school. In one day he spends P 200 for rent and P 25 for each snack item he prepares. His expenses in a single day can be expressed as the function $C(x) = 25x + 200$, where x is the number of items and $C(x)$ is his daily expenses in pesos. How much are his expenses if he prepares 100 snack items?



2. The function for the height of an object dropped from 100 meter tall platform at time t in seconds is approximated by $s(t) = 5t^2 + 100$. (This formula is based on an approximated value for acceleration $g = 10\text{m/s}^2$ due to gravity).

- A. What is the height of the object after 2 seconds?
- B. At what time will the object reach 50 meters?

Rubrics for the Performance Task

Score	Description
15 points	Complete solutions with correct answers
10 points	Correct answers with incomplete solutions.
5 points	50% correct answer with incomplete solution
No point earned	No output at all



Assessment

Choose the letter of the correct answer.

1. What is $f(1)$ if $f(x) = x^2 - 2x + 3$?
A. -2 B. 0 C. 2 D. 3
2. The ordered pair $(0, 1)$ belongs to which of the following functions?
A. $f(x) = x^2 + 1$ B. $f(x) = x^3 - 2$ C. $f(x) = x^2 - 1$ D. $f(x) = 2x - 1$
3. What is the value of the function $f(x) = 3x - 1$ when $x = 2$?
A. -4 B. -5 C. 5 D. 6
4. Translate to mathematical sentence: the function $f(x)$ is equal to thrice the quotient of x and 5.
A. $f(x) = 3x + 5$ B. $f(x) = 3\left(\frac{5}{x}\right)$ C. $f(x) = \left(\frac{x}{5}\right) + 3$ D. $f(x) = 3\left(\frac{x}{5}\right)$
5. Consider the function $g(s) = 2s + 4$. Find $g(1)$.
A. 6 B. 5 C. 4 D. 3



LESSON 3: Operations on Functions



What's In

Given two functions, $f(x) = 3x - 1$ and $g(x) = x + 2$:

1. Construct a table of values for functions **f** and **g**. *Note:* $x = \{-2, -1, 0, 2\}$

2a: Add the results to come up with a table of values for $f(x) + g(x)$. Will the results be the same if we add the equations defining **f** and **g** and then plug in the values from the domain?

2b: Subtract the results to come up with a table of values for $f(x) - g(x)$. Will the results be the same if we subtract the equations defining **f** and **g** and then plug in the values from the domain?

2c: Multiply the results to come up with a table of values for $f(x) \bullet g(x)$. Will the results be the same if we multiply the equations defining **f** and **g** and then plug in the values from the domain?

2d: Divide the results to come up with a table of values for $\frac{f(x)}{g(x)}$. Will the results be the same if we divide the equations defining **f** and **g** and then plug in the values from the domain? Are there any additional conditions in the domain so that the quotient/answer is defined?



What's New

This module focuses on learning operations on functions. Operations on Functions are the same with operation on numbers and polynomials. For sure you have learned how to add, subtract, multiply and divide numbers. This time, you will learn how to find the sum, difference, product and quotient of functions.



What is It

Operations on Functions

Functions with overlapping **domains** can be added, subtracted, multiplied and divided. If $f(x)$ and $g(x)$ are two functions, then for all x in the domain of both functions the sum, difference, product and quotient are defined as follows:

The Sum of **f** and **g**: $(f + g)(x) = f(x) + g(x)$

The Difference of **f** and **g**: $(f - g)(x) = f(x) - g(x)$

The Product of **f** and **g**: $(f \bullet g)(x) = f(x) \bullet g(x)$

The Quotient of **f** and **g**: $\frac{f(x)}{g(x)}$ is not equal to 0.

Example 1: Let $f(x) = x + 4$ and $g(x) = x - 3$. Determine the following functions.

a. $(f + g)(x)$

b. $(f - g)(x)$

c. $(f \bullet g)(x)$

d. $(f/g)(x)$

Solution:

a. The Sum of f and g: $(f + g)(x) = (x+4) + (x - 3)$
 $= 2x - 1$

b. The Difference of f and g: $(f - g)(x) = (x+4) - (x - 3)$
 $= x+4 - x + 3$
 $= 7$

c. The Product of f and g: $(f \bullet g)(x) = f(x) \bullet g(x) = (x+4)(x - 3)$
 $= x^2 + x - 12$

d. The Quotient of f and g: $\frac{f}{g}(x) = \frac{x+4}{x-3}$

Use the following functions for Example 2.1 – 2.2 :

- $p(x) = 3x + 1$
- $q(x) = x^2 - 2x + 1$

Example 2.1. Express the $f_1(x) = x^2 + x + 1$ as a sum or difference of the function above.

Note: To express the function $f_1(x) = x^2 + x + 1$ involves **trial** and **error** in getting the sum and difference of the functions above.

Let's check if the sum of $p(x)$ and $q(x) = x^2 + x + 1$.

$$\begin{aligned} q(x) + p(x) &= (x^2 - 2x + 1) + (3x + 1) \\ &= x^2 + x + 2 = f_1(x) \end{aligned}$$

Therefore, $f_1(x) = q(x) + p(x)$.

Example 2. 2. Express the function $f_2 = x^2 - 5x$ as the sum or difference of the given functions above.

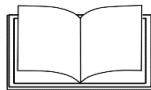
Note: To express the function $f_2(x) = x^2 - 5x$, it involves **trial** and **error** in getting the sum and difference of the functions above.

Again, Let's check if the difference of $q(x)$ and $p(x) = x^2 - 5x$

$$\begin{aligned} q(x) - p(x) &= (x^2 - 2x + 1) - (3x + 1) \\ &= x^2 - 5x = f_2(x) \end{aligned}$$

Therefore, $f_2(x) = q(x) - p(x)$.





What's More

A. Perform the indicated operations on functions to the following.

Given the following functions: $f(x) = 3x + 2$, $g(x) = x^2 - 1$ and $p(x) = x^2 + 4x + 3$		
1. What is the sum of $g(x)$ and $p(x)$? _____	2. Find $f(x) + p(x)$. _____	3. Find the difference of $g(x)$ and $f(x)$. _____
4. The product of $p(x)$ and $f(x)$. _____	5. The quotient of $p(x)$ and $g(x)$. _____	6. Express $f_1(x) = x^2 + 3x + 1$ as the sum and difference. _____

B. Put Check mark (\checkmark) if the statement or solution is correct otherwise put cross mark (\times), then write the correct answer.

(\checkmark) (\times) Correct

Answer

1. The $(g \circ f)(x) = f(g(x))$. _____
2. Given the following $f(x) = 3x^2 - 2x$ and $g(x) = x - 1$.
 - a. the sum of $f(x)$ and $g(x)$ is equal to $3x^2 - x - 1$. _____
 - b. the difference of the function is $3x^2 - 3x$. _____
 - c. the product of $f(x)$ and $g(x) = 3x^3 - 4x^2 + 2x$. _____
 - d. The quotient of $f(x)$ and $g(x) = x(3x-2)$. _____



What I Have Learned

Fill in the Blanks. Write the correct term/s or expression/s that will complete the sentences.

Operations with functions are just like we add, subtract, multiply and divide _____.

Definition. Let f and g be functions. The sum of the two functions is denoted by _____ 2 _____. Their difference is denoted by $f - g$, is the function denoted by $(f - g)(x) =$ _____ 3 _____. The product of the functions $f \bullet g$ is denoted by _____ 4 _____ = _____ 5 _____. Their quotient, denoted by f/g , is the function denoted by $(f/g)(x) =$ _____ 6 _____, if $g(x) = 0$ the quotient is _____ 7 _____.





What I Can Do

A. Perform the indicated operations on functions given the function f, g, and h below.

$$f(x) = x - 3 \quad g(x) = x^2 - 9 \quad h(x) = 2x + 4$$

$$1. (f + g)(x) \quad 2. (g - h)(x) \quad 3. (f + g + h)(x)$$

$$4. (f \bullet g)(x) \quad 5. \left(\frac{g}{f}\right)(x)$$

Rubrics

Score	Description
15 points	Complete solutions with correct answers
10 points	Correct answers with incomplete solutions.
5 points	50% correct answer with incomplete solution
No point earned	No output at all



Assessment

Choose the letter that corresponds to the exact answer.

1. Given the functions $f(x) = x^2 - 8$ and $g(x) = 4 - x$, determine the equation for the combined function $y = f(x) - g(x)$.

- A. $y = x^2 + x - 12$ C. $y = x^2 + 32x - 4$
B. $y = x^2 - 32x - 12$ D. $y = x^2 + x - 4$

2. Given the functions $f(x) = x^3 - 25x$ and $g(x) = x+5$, determine the simplified equation for the combined function $y = \frac{f(x)}{g(x)}$.

- A. $\frac{x}{x+5}, x \neq -5$ C. $x(x-5), x \neq -5$
B. $x(x+5), x \neq -5$ D. $\frac{x}{x-5}, x \neq -5\sqrt{x-1}$

3. Let $f(x) = x - 4$ and $g(x) = x^2 - 1$, what is $(f \bullet g)(x)$?

- A. $x^2 - 4x + 4$ B. $x^3 + 4x^2 + x + 4$ C. $x^3 - 4x^2 - x + 4$ D. $x^3 + 4x^2 - x - 4$

4. Given $p(x) = 12x^2 + 8x$ and $v(x) = 4x$. Determine the function $(p/v)(x)$.

- A. $3x$ B. $4x$ C. $3x + 2$ D. $3x - 2$

5. What is the sum of the given expressions $2x$ and $4x$?

- A. $6x$ B. $6x^2$ C. $x + 6$ D. $2x+4$





Additional Activities

Perform the indicated operations on the following functions.

Given: $p(x) = x - 3$ $r(x) = x^2 - 1$ $u(x) = 2x - 6$

1. $(p + r)(x)$
2. $(r - u)(x)$
3. $(p + r + u)(x)$
4. $(r \cdot u)(x)$
5. $(\frac{u}{p})(x)$

Lesson 4: Composition of Functions



What's In

A. Perform the indicated operation on functions.

Given: $f(x) = x + 2$ $g(x) = x^2 - 4$ $h(x) = 2x - 1$

1. Find $(f + g)(x)$.
2. What is the difference of $g(x)$ and $h(x)$?
3. Divide: $g(x)$ and $f(x)$
4. Multiply: $h(x)$ and $f(x)$
5. Evaluate: $(f + g + h)(2)$

B. Evaluate the following:

Given $P(x) = 2x^2 + 4x - 1$ $v(x) = \frac{x+1}{x-1}$

1. Find $P(1)$.
 2. What is the sum of $P(1)$ and $P(-1)$?
 3. Evaluate $v(0)$.
 4. What is the product of $P(1)$ and $v(0)$?
 5. Find $v(1)$.
- Guide Questions.
1. What is evaluating a function?
 2. What are the steps in evaluating a function?



What's New

Given $f(x) = x^2 - 3$, determine the following:

- A. $f(x + 1)$
- B. $f(2x - 2)$
- C. $f(3x)$





What is It

Another way of combining two functions is the so called **composite function**. Composite function can be denoted by $f \circ g$. The notation of the function f with g is $(f \circ g)(x) = f(g(x))$ and is read f of g of x . It means that wherever there is an x in the function f , it is replaced with the function $g(x)$.

Example 1: Let $f(x) = x^2$ and $g(x) = x - 5$. Find $f(g(x))$.

$$\begin{aligned} \text{Solution: } f(g(x)) &= f(x - 5) \\ &= (x - 5)^2 \\ &= x^2 - 10x + 25 \end{aligned}$$

Example 2: Let $f(x) = 2x + 1$ and $g(x) = 3x - 4$. Find $f(g(x))$ and $g(f(x))$.

$$\begin{aligned} \text{Solution: } f(g(x)) &= f(3x - 4) \\ &= 2(3x - 4) + 1 \\ &= 6x - 8 + 1 \\ &= 6x - 7 \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(2x + 1) \\ &= 3(2x + 1) - 4 \\ &= 6x + 3 - 4 \\ &= 6x - 1 \end{aligned}$$

Note: Since $6x - 7 \neq 6x - 1$, therefore $f(g(x)) \neq g(f(x))$.

Example 3: Let $f(x) = \sqrt{2x}$, $g(x) = 3x^2 - 2x$, and $h(x) = x + 1$. Find the following and determine also the domain.

$$\text{a. } (g \circ h)(x) \quad \text{b. } (f \circ g)(x) \quad \text{c. } (h \circ f)(x)$$

Solutions:

$$\begin{aligned} \text{a. } (g \circ h)(x) &= g(h(x)) \\ &= 3(x+1)^2 - 2(x+1) \\ &= 3(x^2 + 2x + 1) - 2x - 2 \\ &= 3x^2 + 6x + 3 - 2x - 2 \\ &= 3x^2 + 4x + 1 \end{aligned}$$

The domain is $\{x/x \text{ is a real number}\}$.

$$\begin{aligned} \text{b. } (f \circ g)(x) &= f(g(x)) \\ &= \sqrt{2(3x^2 - 2x)} \\ &= \sqrt{6x^2 - 4x} \\ &= \sqrt{2x(3x - 2)} \end{aligned}$$

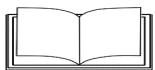
The domain is $\{x/x \text{ is an element of the interval } [0, \frac{2}{3}]\}$.



$$\begin{aligned}
 c. (h \circ h)(x) &= h(h(x)) \\
 &= h(x+1) \\
 &= x + 1 + 1 \\
 &= x + 2
 \end{aligned}$$

The domain is $\{x/x \text{ is any real number}\}$.

The domain of a composition $(f \circ g)(x) = f(g(x))$ must be the set of elements in the domain of g whose results are included in the domain of f . To get the domain of a composite function, check the inner function and select elements whose results can be found in the domain of the outer function.



What's More

Composite Functions:

- A. 1. Find $f(g(x))$ when $f(x) = x - 7$ and $g(x) = 5x + 3$.
 2. Find $h(g(n))$ when $h(n) = 3n + 5$ and $g(n) = n + 2$.

B. Find the composite function given the following functions, and give its domain.

$f(x) = 4x - 5$	$g(x) = \frac{1}{x}$	$h(x) = 2x^2$
1. $(f \circ g)(x)$	2. $(g \circ f)(x)$	3. $(f \circ h)(x)$
4. $(h \circ g)(x)$	5. $(h \circ f)(x)$	6. $(f + (h \circ g))(x)$



What I Have Learned

Complete the sentences by filling the blanks with the appropriate expression/term/s.

It is a way of combining two functions and it is called _____ 1 _____. Composite function can be denoted by _____ 2 _____.

The notation of the function f with g is _____ 3 _____ and is read _____ 4 _____. It means that wherever there is an x in the function f , it is replaced with the function $g(x)$. The domain of _____ 5 _____ is the set of all x in the domain of g such that _____ 6 _____ is in the domain of f .





What I Can Do

Word Problem:

Before Novel Corona Virus (COVID19) struck our country, you bought canned sardines for Php.16.00 each from the supermarket and a shopping cart which costs Php 50 to transport the cans of sardines. Now, you are selling the cans of sardines for Php.28 each to the survivors of this pandemic.

1. Find the function that represents the cost for the cans of sardines and the cart.
2. Give the function that represents how much revenue you make in selling the cans of sardines.
3. Write the function that represents your profit from selling the cans of sardines to the pandemic survivors.
4. Determine how much profit you make from selling 20 cans of sardines.

Note: C----represent the number of cost of cans of sardines

X ----Number of cans of sardines

R-----Represent the revenue

P-----Represent the profit

Rubrics for problem solving.

Score	Description
15 points	Complete solutions with correct answers
10 points	Correct answers with incomplete solutions.
5 points	50% correct answer with incomplete solution
No point earned	No output at all



Assessment

Choose the letter that corresponds to the exact answer.

1. Given $p(x) = 4x - 5$ and $g(x) = x - 3$, find $p(g(3))$.
A. -4 B. 4 C. -5 D. 5
2. Find $g(p(0))$ if $p(x) = 4x - 5$ and $g(x) = x - 3$.
A. -8 B. 8 C. -17 D. 17
3. Find $(f \circ f)(x)$ if $f(x) = 3x - 4$.
A. $3x + 8$ B. $3x - 8$ C. $9x + 16$ D. $9x - 16$
4. Which expression represents $f(g(x))$, if $f(x) = x^2 + x$ and $g(x) = 3x + 1$?
A. $3x^2 + 3x + 1$ B. $3x^2 + 3x$ C. $(3x^2 + 1)$ D. $(3x+1)^2 + 3x + 1$
5. If $h(x) = x^2$ and $p(x) = 3x - 1$, find $h(p(x))$.
A. $(3x - 1)$ B. $(3x - 1)^2$ C. $9x^2 - 1$ D. $12x^2 + 1$



Additional Activities

Determine the composite function given the following functions below.

$$F(x) = 5x - 2 \quad G(x) = \sqrt{x - 3}$$

1. $(F \circ G)(x)$
2. $(G \circ F)(x)$
3. $(F \circ F)(x)$
4. $(G \circ G)(x)$

Lesson 5: Problems Involving Functions



What's In

A. Mapping Function or Not Function

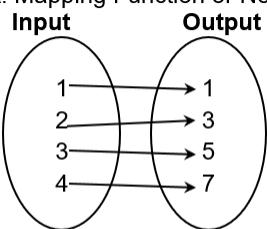


Figure 1

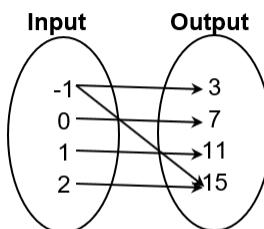


Figure 2

Write a brief explanation on the difference between a mapping that is a function and one that is not.

B. Map bank logos to their respective tag-lines or slogan.



Banco de Oro-Wikipedia



Manila Bulletin Business



GetGo.com.ph



Wikipedia



Manila Bulletin Business

Make the Best Happen

We Help You Grow

You're Life Your Bank

You're in Good Hands

We Find Ways



City of Good Character
DISCIPLINE • GOOD TASTE • EXCELLENCE

Guide Questions:

1. How will you determine if the mapping diagram is a function or not?
2. How will you determine if the graph is a one-to-one function?
3. In your own opinion, how important is the term function in your life?



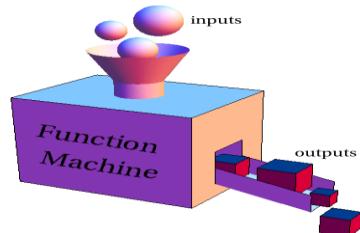
What's New

A function is described as a machine that generates output in correlation to a given input. So, if $f(x) = 3x - 1$, then $f(4) = 11$. Understanding this behavior is very important in recognizing the difference of input/output correlations in the real world. Again, function provides a mathematical way to model or represent situations where a certain input will provide a certain output.

Since, functions can be seen anywhere, a simple application of function is technology, where machine learning is more on learning functions.

Examples:

1. Mapping of images to the owner of Facebook account users
2. Mapping of the names of account holders to their ATM account numbers
3. Mapping of numbers of positive Covid-19 patients to their respective regions in the NCR



The function Machine-Math Insight



What is It

First, let us define once again the term function.

Function, in mathematics, is an expression, rule, or law that defines a relationship between one variable (the independent variable) and another variable (the dependent variable). Functions are present in mathematics and are essential for formulating physical relationships in the sciences. The modern definition of function was first given in 1837 by the German mathematician Peter Dirichlet:

If a variable y is so related to a variable x that whenever a numerical value is assigned to x , there is a rule according to which a unique value of y is determined, then y is said to be a function of the independent variable x .

This relationship is commonly symbolized as $y = f(x)$. In addition to $f(x)$, other abbreviated symbols such as $g(x)$ and $P(x)$ are often used to represent functions of the independent variable x , especially when the nature of the function is unknown or unspecified.

Now, let us study the various applications of functions to the real world. Here are some examples of solving problems in real world involving functions.

1. Circle: The formula in getting the area of a circle, $A = \pi r^2$, where the dependent variable is A (Area) as a function to the independent variable r (radius).

Example 1: Mr. Marvin is a talented painter and he is about to paint a large circle on the wall of his new house. He plans that the largest distance across the circle will be 10 feet. How many square feet will the circle cover on the wall of his house?

Solution: The area of a circle is $A = \pi r^2$.

Note: If you are asked to compute the number of square feet covered, it only means you are to find the area.

$$A = \pi r^2.$$

$$A = 25\pi$$

$$A \approx 78.54$$

Thus, Mr. Marvin's circle will cover about 78.54 square feet of his wall.

2. Distance Travelled

Example 2: Jogging at the rate of 8 km/h, a woman covered certain distance in 1 hr 30 min. Jogging at a speed of 20 km/h the woman will cover the same distance.

Solution: Distance = Speed x times

Where: $t = 1 \text{ hr } 30 \text{ min} = 3/2 \text{ hr}$

$$\text{Distance} = 8 \times \frac{3}{2} = 12 \text{ km}$$

$$\text{New Speed} = 20 \text{ km/h}$$

$$\text{Therefore, time} = \frac{D}{S} = \frac{12}{20} \times 60 = 36 \text{ min.}$$

3. Compound Interest: Here, the inputs are the principal amount, interest rate, and the duration of time of the loan/investment.

Example 3: A principal of Php.3000 is placed in a savings account at 5% per annum compounded annually. How much is in the account after one year, two years and three years?

Solution:

When interest is compounded annually, total amount A after t years is given by: $A = P(1 + r)^t$, where P is the initial amount (principal), r is the rate and t is time in years.

$$1 \text{ year: } A = 3000(1 + 0.05)^1 = \text{Php.}3150$$

$$2 \text{ years: } A = 3000(1 + 0.05)^2 = \text{Php.}3307.5$$

$$3 \text{ years: } A = 3000(1 + 0.05)^3 = \text{Php.}3472.88$$



4. Temperature: If you look at how temperature varied over a time period during the day, you will see temperature as a function over time.

Example 4. For a room temperature of 76°F , the temperature T of a mug initially at 180°F decreases in x in minutes according to the formula:

$$T(x) = 76 + 104e^{-0.025x}$$

Here are the other functions as representations of real-life situations:

Example 5. My officemate wants to buy another meal as an addition to his food. The cost of buying one meal is Php.50.00. Use function P in representing the cost of buying x meals.

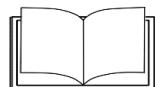
Solution: Note: The meal costs Php.50.00

Thus, the function is $P(x) = 50x$

Example 6. The mother of my cousin bought a cellphone for him as a gift for his coming birthday. The cellphone is actually a mobile plan that charges Php.350 monthly that includes 100 free text messages. Each message sent exceeding this allocation is charged P1. What is the monthly cost for text messages? Use function $f(x)$, where x is the number of messages sent in a month.

Solution: this problem can be expressed using piece-wise function.

$$F(x) = \begin{cases} 350, & \text{if } 0 < x \leq 100 \\ 350 + x, & \text{if } m > 100 \end{cases}$$



What's More

A. Illustrate the function H , a relation that relates the names of Philippine Presidents with their birth months.

$H = \{ \underline{\hspace{10cm}} \}.$

B. Identify which are functions and which are merely relations. Explain your answers.

1. The rule which assigns to each students to his/her LRN.

Explanation:

2. The rule which assigns to each teacher to his/her employee number.

Explanation:



3. Chemical elements to its chemical symbol.

Explanation:

C. Problem Solving:

In Barangay Fortune, Mang Roberto plans to make dish-washing products as a means of livelihood. The weekly profit P (in pesos) is a function of number of bottles b of dish-washing products per week. The function approximating the profit is defined as

$$P(b) = 5b - 5, \text{ where } 0 \leq b \leq 20$$

1. Construct the table showing the relation between the number of dish-washing bottles and weekly profit.

b									
P(b)									

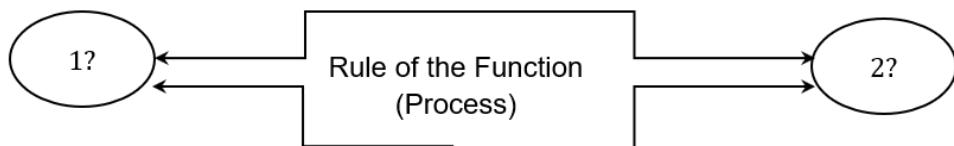
2. How much is Mang Roberto's profit if he made 10 dish-washing bottles in a week?
3. Estimate his profit if he made 50 bottles of dish-washing products in a week?
4. What is $P(0)$? Interpret the value in the context of the problem?



What I Have Learned

Activity 2: “Bridge the Gap”

Fill the spaces with the correct missing terms.



Function Machine:

A _____ 3 _____ is a “machine” that generates some _____ 4 _____ in correlation to a given _____ 5 _____. Function is also an _____ 6 _____, _____ 7 _____, or law that defines a relationship between one variable (the _____ 8 _____ variable) and another variable (the _____ 9 _____ variable).





What I Can Do

Read, analyze, and solve the following problems. Write your answers in a separate sheet of paper.

Problem Solving:

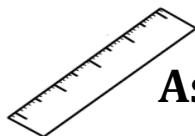
1. Mr. Radec's Pansit Palabok costs Php.25 with the first topping, and then an additional Php.5 for each additional topping. If n represents the number of toppings on the Pansit Palabok, what function represents the **total cost** of an order of Pansit Palabok with at least one topping?

What is your answer?

2. What would a savings account of Php 15,000 become if it has a non-compounded interest of 5% per year for 5 years?

Rubrics for problem solving.

Score	Description
15 points	Complete solutions with correct answers
10 points	Correct answers with incomplete solutions.
5 points	50% correct answer with incomplete solution
No point earned	No output at all



Assessment

Choose the letter that corresponds to the exact answer.

1. In the function $y = f(x)$, "y" is classified as
 - Dependent variable
 - Independent variable
 - Both a and b
 - Neither a nor b
2. Which of the following can be used to represent a function?
 - equation
 - graph
 - table
 - all of the above
3. Suppose a company sells bicycles. The company's profit can be represented by the function $p(x) = x^2 + 4x - 6300$, where x is the number of bikes sold. If the company sells 130 bicycles, what is the company's profit?
 - Php. 10, 120
 - Php. 12, 120
 - Php. 11, 120
 - Php. 13, 120

4. A certain bottled soda drink costs Php.25.00 per bottle. If you buy 12 bottles more, they will be marked down to a price of Php.22.00. Present the cost in terms of the number of bottled soda drinks bought. Use piece-wise function.

A. $f(n) = \{25n, \text{ if } 0 < n \leq 12\}$ C. $f(n) = \{25n, \text{ if } 0 < n \leq 12, 22n, \text{ if } n > 12\}$

B. $f(n) = \{22n, \text{ if } n > 12\}$ D. $f(n) = \{22n, \text{ if } 0 < n \leq 12, 25n, \text{ if } n > 12\}$

5. A working student is earning Php 250 per day as a part time job. Express the total salary T as a function of the number n of days that the working student works.

A. $T(n) = 250n$ B. $T(n) = 250n^2$ C. $T(n) = 250 - n$ D. $T(n) = 250 + n$



Additional Activities

1. List at least 5 different applications of functions in real life situations. Make a brief explanation for each.

2. As dry air moves upward, it expands and cools. The air temperature $A(x)$ in degrees Celsius at an altitude of x kilometers above sea level is given by the equation $A(x) = 27 - 9x$.

- At what specific altitude will dry air have a temperature of 3°C ?
- Evaluate $A(0)$. What does this figure mean in the context of the problem?



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