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Mathematics

Quarter 3 – Module 15:
The Pythagorean Theorem

ALTERNATIVE DELIVERY MODE
ADM

Mathematics- Grade 9
Alternative Delivery Mode
Quarter 3- Module 15: **The Pythagorean Theorem**
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Mathematics

Quarter 3 – Module 15:
The Pythagorean Theorem

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

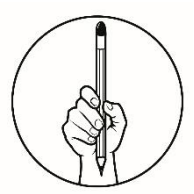
If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

In the previous lessons, you have learned about similarity between two triangles including right triangles. In this module, you will apply the right triangle similarity theorem to prove another theorem that we will use to find the measure of the sides of a right triangle.

After going through with this module, you are expected to be able to prove the conditions for similarity of right triangles.



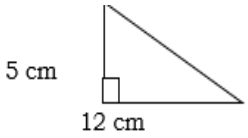
What I Know

Find out how much you already know about the module. Write the letter of the best answer to each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answers as you go through this module.

1. To what plane figure is Pythagorean Theorem commonly applied?
   A. Circle B. Hexagon C. Square D. Triangle

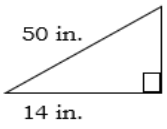
2. The Pythagorean Theorem is translated mathematically as $a^2 + b^3 = c^2$.
   A. True B. False C. It Depends D. None

3. Find the missing length of a side of the triangle at the right.
   A. 22 cm B. 17 cm C. 15 cm D. 13 cm



4. The Pythagorean Theorem ONLY works on which triangle?
   A. Obtuse B. Scalene
   C. Isosceles D. Right

5. Find the missing length of a side of the triangle at the right.
   A. 48 in. B. 42 in. C. 24 in. D. 16 in.

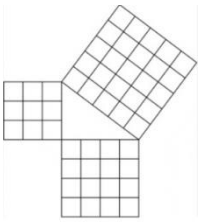


6. What kind of triangle according to angle is formed if the lengths of its sides are 15 cm, 12 cm, and 9 cm?
   A. Right B. Obtuse C. Acute D. Equiangular

7. A ladder leans against a vertical wall and reaches a crack 25 ft. from the ground. Its foot is 10 feet away from the wall. How long is the ladder to the *nearest whole number in foot*?
   A. 25 B. 26 C. 27 D. 28

8. A rectangular field is 20 meters wide and 40 meters long. Patrick walks through the diagonal of the field. How far does he walk?
   A. $20\sqrt{2}$ meters B. $20\sqrt{3}$ meters C. $20\sqrt{5}$ meters D. $20\sqrt{6}$ meters

9. Which of the following sentences describes the figure at the right?



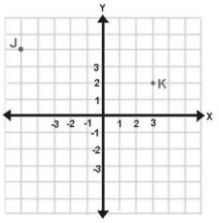
A. The sum of the areas of the two smaller squares is equal to the area of the largest square.

B. The sum of the side lengths of the two smaller squares is equal to the side length of the largest square.

C. The difference of the areas of the two smaller squares is equal to the area of the largest square.

D. The difference of the side lengths of the two smaller squares is equal to the side length of the largest square.

10. Find the distance between the points J and K as shown in the figure at the right.



A. 10 units B. $2\sqrt{17}$ units C. $2\sqrt{10}$ units D. $2\sqrt{2}$ units

11. Which set of numbers could **not b**e the lengths of the sides of a right triangle?

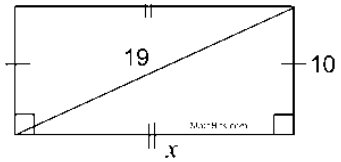
A. {5,12,13} B. {3,4,5} C. {2,3,4} D. {8,15,17}

12. In ΔABC, $\angle C$ is a right angle, $|AC| = 5\sqrt{2}$ cm and $|AB| = 2\sqrt{15}$ cm. Find $|BC|$.

A. $\sqrt{10}$ cm B. $\sqrt{20}$ cm C. $\sqrt{50}$ cm D. $\sqrt{110}$ cm

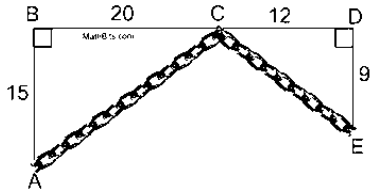
13. Given the rectangle at the right with diagonal 19 inches long and 10 inches wide. Find the length of the rectangle to the nearest whole number.

A. 10 inches C. 16 inches
B. 14 inches D. 19 inches



14. Given the diagram at the right, as labeled in foot with points *B, C, and D* collinear. A chain is laid along the hypotenuse in each triangle. Find the total length of the chain.

A. 15 feet C. 30 feet
B. 25 feet D. 40 feet



15. Find the area of an equilateral triangle to the *nearest tenth* of a square inch whose side measures 7 inches.

A. 42.4 sq. in. B. 42.6 sq. in. C. 21.2 sq. in. D. 21.3 sq. in.

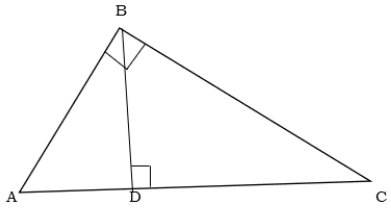
Lesson 1 The Pythagorean Theorem

What's In

Before going further, let us try to recall the Right Triangle Similarity.

- In a right triangle, the altitude to the hypotenuse divides the triangle into similar triangles each similar to the given triangle.

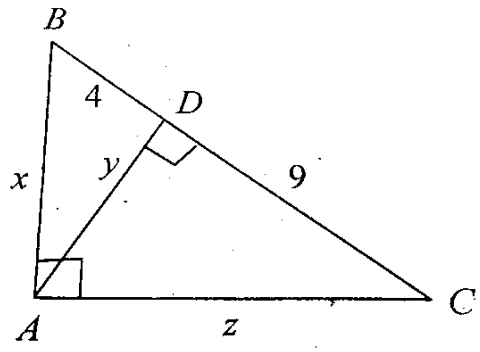
Given: $\Delta ABC, is\ a\ right\ triangle\ with\ \angle B\ as\ the\ right\ angle.$
$\overline{BD}\ is\ the\ altitude\ to\ \overline{AC}.$



- In a right triangle the altitude to the hypotenuse is the geometric mean of the lengths of the segments along the hypotenuse.
  So, $|BD|^2 = |AD||DC|$
  $|BD| = \sqrt{|AD||DC|}$

- The length of each leg is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to it.
  $|AB|^2 = |AD||AC|$ and $|BC|^2 = |DC||AC|$
  $|AB| = \sqrt{|AD||AC|}$ and $|BC| = \sqrt{|DC||AC|}$

Answer the following.
In $\Delta ABC\ at\ the\ right, \angle A\ is\ a\ right\ angle, \overline{AD}\ is\ the\ altitude\ to\ \overline{BC}.$
1. Name the similar triangles
2. Find the values of x, y, and z.



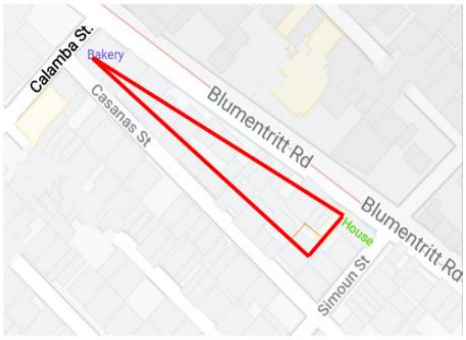
What's New

Directions: Read the selection below.

THE "NEW NORMAL" MODE OF TRANSPORTATION

The "new normal" demands social distancing – but with the state of our current public transport systems, there is no way to avoid a crowded commute. That's why biking is now being championed as an ideal alternative: efficient, affordable, and sustainable mode of transportation. With or without pandemic, biking is beneficial– including lessening the volume of cars on the road, lowering pollution rate emitted in air and the like.

Suppose that Mario, a resident of the nearby barangay, is planning to buy bread at a bakeshop. He usually rides his bike along Simoun St. then up Casañas St. He knows that the two streets are perpendicular and are 3 and 4 km long, respectively. An alternative route is the Blumentritt road as show in the map at the right. Is Blumentritt Road shorter? Which way saves more time?



Going back to the problem, observe that the three streets form a right triangle. We have to find the distance from his house going to the bakery passing through Blumentritt Road to know if this is the shorter way. To solve the problem, we can apply the Pythagorean Theorem.

The **Pythagorean Theorem** is one of the earliest theorems known to ancient civilizations. This famous theorem is named after the Greek mathematician and philosopher, Pythagoras.

THE PYTHAGOREAN THEOREM

The Pythagorean Theorem is a statement about right triangles. The Pythagorean Theorem states that:

"The area of the square built upon the hypotenuse of a right triangle is equal to the sum of the areas of the squares upon the remaining sides."

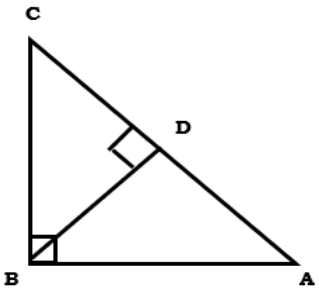
There are many ways to prove the Pythagorean Theorem and one way is by using the Right Triangle Similarity Theorem.

Here's the example.

Given: Right triangle ABC.

Prove: $|AC|^2 = |AB|^2 + |BC|^2$

Construction: Draw a perpendicular $\overline{BD}$ intersecting $\overline{AC}$ at D.



Proof: First, we draw $\overline{BD}$ perpendicular to side$\overline{AC}$. We know,$\Delta ADB \sim \Delta ABC$, therefore, $\frac{|AD|}{|AB|} = \frac{|AB|}{|AC|}$ (Condition for similarity) or $|AB|^2 = |AD||AC|$. (Eq. 1)
Also,$\Delta BDC \sim \Delta ABC$, therefore, $\frac{|CD|}{|BC|} = \frac{|BC|}{|AC|}$ (Condition for similarity) or $|BC|^2 = |CD||AC|$. (Eq.2)

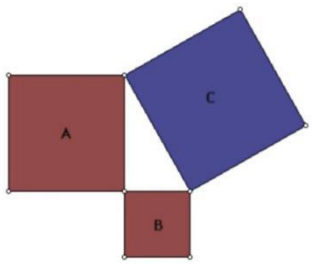
Adding equations (1) and (2) we get,

$$|AB|^2 + |BC|^2 = |AD||AC| + |CD||AC|$$
$$|AB|^2 + |BC|^2 = |AC|\,(|AD| + |CD|)$$

Since, $|AD| + |CD| = |AC|$, then $|AB|^2 + |BC|^2 = |AC||AC|$

Therefore, $|AC|^2 = |AB|^2 + |BC|^2$

Hence, the Pythagorean Theorem is proved.



Note: Pythagorean Theorem is only applicable to Right triangles.

According to the Pythagorean Theorem, the sum of the areas of the two red squares A and B, is equal to the area of the blue square C.

Let Area of Square A = a^2

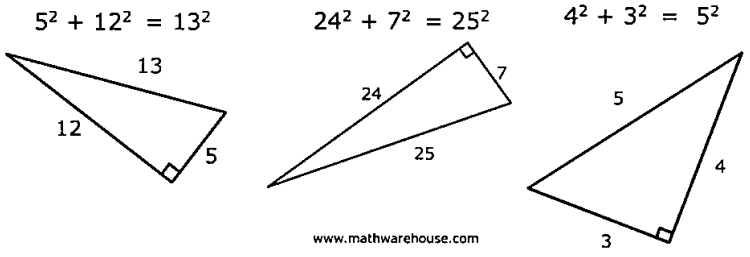
Area of Square B = b^2

Area of Square C = c^2

Thus, the Pythagorean Theorem is stated algebraically as $\boldsymbol{a^2 + b^2 = c^2}$ for a right triangle with sides of lengths a, b, and c, where c is the length of the hypotenuse.

EXAMPLES OF THE PYTHAGOREAN THEOREM

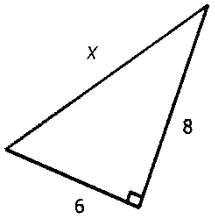
When you use the Pythagorean Theorem, just remember that the length of the hypotenuse is always 'c' in the formula above. Look at the following examples



What is It

Finding The Length Of A Side Of A Right Triangle Using Pythagorean Theorem

Example 1. (Solving for the length of the hypotenuse)
Use the Pythagorean Theorem to determine the value of x in the figure at the right.



Step 1. Identify the legs and the hypotenuse of the right triangle. The legs have lengths 6 cm and 8 cm and x is the length of the hypotenuse, the side opposite the right angle.

Step 2. Substitute the variables by the given values into the formula (remember 'c' is the length of the hypotenuse).

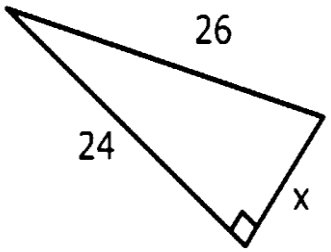
$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = x^2$$

Step 3. Solve for the unknown.

$$6^2 + 8^2 = x^2$$
$$36 + 64 = x^2$$
$$100 = x^2$$
$$\sqrt{100} = x$$
$$10 \text{ cm} = x$$

Therefore, the length of the hypotenuse of the right triangle is 10 cm.

Example 2. (Solving for length of the leg)
Use the Pythagorean Theorem to determine the value of x in the figure at the right.



Step 1. Identify the legs and the hypotenuse of the right triangle. The legs have lengths 24 and x. The length of the hypotenuse is 26.

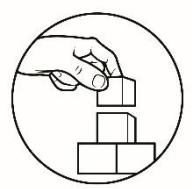
Step 2. Substitute the variables by the given values into the formula (remember 'c' is the length of the hypotenuse)

$$a^2 + b^2 = c^2$$
$$x^2 + 24^2 = 26^2$$

Step 3. Solve for the unknown.

$$x^2 + 24^2 = 26^2$$
$$x^2 + 576 = 676$$
$$x^2 = 676 - 576$$
$$x^2 = 100$$
$$x = \sqrt{100}$$
$$\mathbf{x = 10}$$

Therefore, the length of the other leg of the right triangle is 10 cm.



What's More

Activity 1: Given below are sets of the measures of the sides of triangles in cm. Determine the kind of triangle formed according to angle given the lengths of the sides.

1.) 5, 3, 6 _____

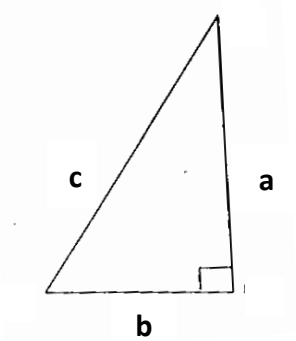
2.) 4, 7, 8 _____

3.) 17, 8, 18 _____

4.) 13, 8, 6 _____

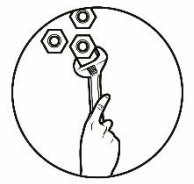
5.) $\sqrt{15}, \sqrt{8}, \sqrt{7}$ _____

Activity 2: Use the Pythagorean Theorem to find the unknown side of the given right triangle if the lengths of its two sides are given.

Figure	**Right Triangle**	**Shorter Leg *A***	**Longer Leg *b***	**Hypotenuse *c***
	A	3		5
	B	5	12	
	C		24	25
	D	8	15	
	E	9		41

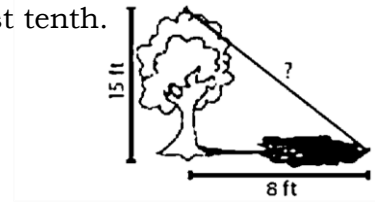
What I Have Learned

- The formula to find the length of the hypotenuse of a right triangle is $a^2 + b^2 = c^2$.
- The formula to find the lengths of the legs of a right triangle are $a^2 = c^2 - b^2$ and $b^2 = c^2 - a^2$.
- A triangle is right if the square of the measure of the longest side is *equal* to the sum of the squares of the measures of the shorter sides.
- A triangle is obtuse if the square of the measure of the longest side is *greater than* the sum of the squares of the measures of the shorter sides.
- A triangle is acute if the square of the measure of the longest side is *less than* the sum of the squares of the measures of the shorter sides.

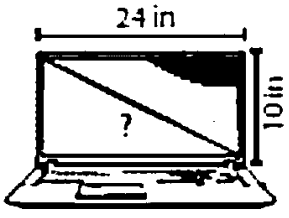


What I Can Do

Solve the word problems. Round the answer to the nearest tenth.

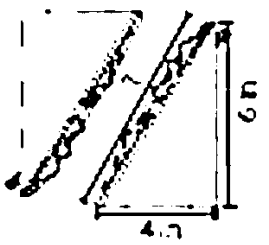


1.) A 15- foot tree casts a shadow that is 8 feet long. What is the distance from the tip of the tree to the tip of its shadow?

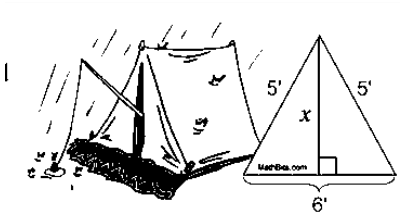


2.) Joshua won a laptop in a school raffle. The laptop screen measures 10 inches in height and 24 inches in width. Find the diagonal length of the laptop screen.

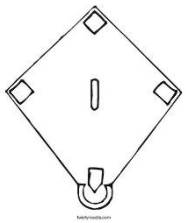
3.) Joey made a sandwich that was 4 inches wide and 6 inches long. If he cuts the sandwich in half as shown in the figure, what would be the diagonal length of the sandwich?



4.) The front view of a tent measures 6 feet across the bottom and 5 feet on the slant sides. What is the height of the tent?



5.) A baseball diamond is a square with sides of 90 feet. What is the shortest distance from the first base to the third base?



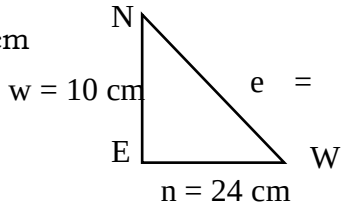
Assessment

Read and answer each of the following items accurately. Write the letter of the correct answer on your answer sheet.

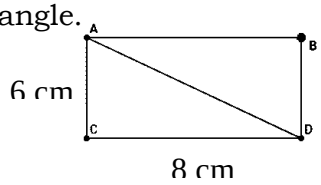
1. What do you call the longest side of a right triangle?
   A. Hypotenuse B. Leg C. Base D. Median

2. What triangle is formed if the lengths of the sides are 9 cm, 12 cm and 15 cm respectively?
   A. Acute B. Obtuse C. Right D. Equilateral

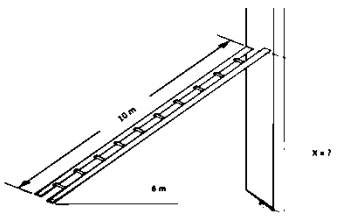
3. Given at the right is ΔNEW with right ∠ E. If w =10 cm and n = 24 cm, find the value of e.
   A. 26 cm C. 30 cm
   B. 28 cm D.40 cm



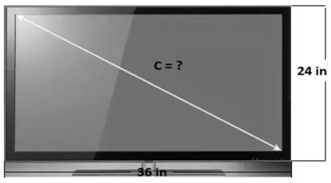
4. The length and width of a rectangle are 8cm and 6cm, respectively. Find the length of the diagonal of the rectangle.
   A. 30 cm C. 15 cm
   B. 20 cm D. 10 cm



5. If the foot of a 10-meter ladder is placed 6 meters from a building, how high up the building will the ladder reach?
   A. 24 m C. 16 m
   B. 18 m D. 8 m

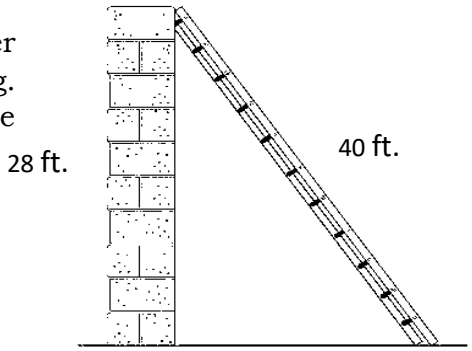


6. Dr. Young finds that his TV is 36 inches wide and 24 inches tall. What is the length of the diagonal of the TV?



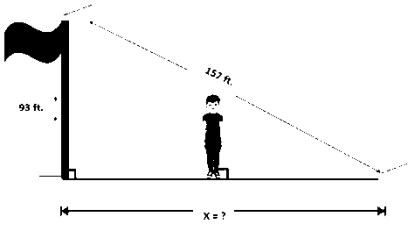
A. 40.27 inches
B. 43.27 inches
C. 50.27 inches
D. 27.43 inches

7. Firefighters have a 40-foot extension ladder in order to reach 28 feet up the building. How far away from the building should the foot of the ladder be placed?



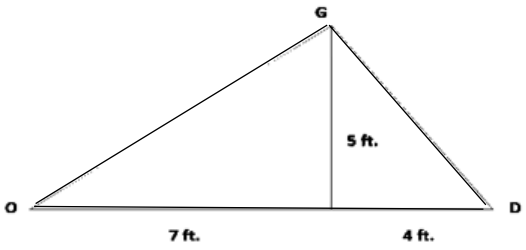
A. 40.57ft.
B. 38.57 ft.
C. 30.57 ft.
D. 28.57 ft.

8. A flagpole stands 93 feet tall and is supported by a guy wire measuring 157 feet long that is attached to the top of the pole and to the ground some distance from the base of the pole. Find the distance of the wire's ground attachment point from the base of the pole to the nearest whole number.



A. 156 feet B. 136 feet C. 126 feet D.116 feet

9. The A-frame of a house is not drawn to scale. Find the respective lengths of $\overline{GO}$ and $\overline{GD}$.
   A. 8.60 ft. and 6.40 ft.
   B. 5.60 ft. and 8.40 ft.
   C. 10.60 ft. and 12.40ft.
   D. 12.60 ft. and 16.40ft



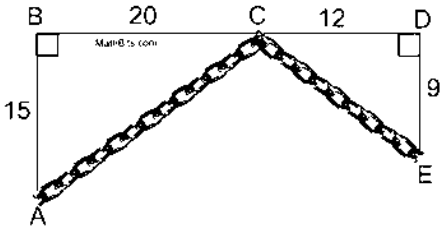
10. Jake plans to use a ramp to make it easier to move a piano out at the back of his truck. The back of the truck is 83 cm high, and the ramp is 158 cm long. What is the horizontal distance from the end of the ramp to the back of his truck?
    A. 120.44 cm B. 134.44 cm C. 140.45 cm D. 154.45 cm

11. Which of the following sets of measures of segments CAN be sides of a right triangle?
    A. 6 in, 12 in, 13 in
    B. 15 in, 21 in, 29 in
    C. 10 in, 24 in, 26 in
    D. 9 in, 39 in, 41 in

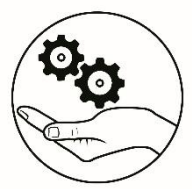
12. At late afternoon, the shadow of an object is exceedingly long due to the low position of the Sun. A 20-m high lamp post casts a 99-m shadow. What is the distance from the top of the pole to the tip of its shadow?
    A. 79 m B. 97 m C. 101 m D. 119 m

13. How long is the diagonal of a monitor with height and width of 27 in and 36 in, respectively?
    A. 90 in B. 63 in C. 54 in D. 45 in

14. Given the diagram shown at the right, as labeled in foot and points *B, C, and D* are collinear. A chain is laid along the hypotenuse in each triangle. Find the total length of the chain.
    A. 15 feet C. 30 feet
    B. 25 feet D. 40 feet



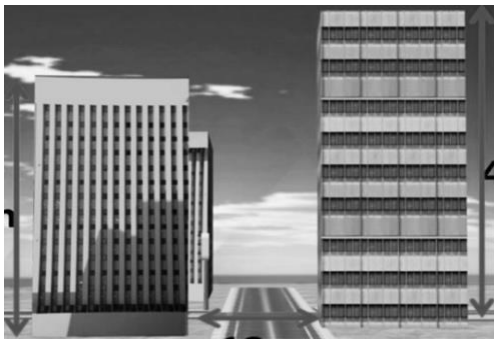
15.) Using the Pythagorean Theorem, find the area of an equilateral triangle whose side measures 7 inches to the *nearest tenth* of a square inch.
    A. 42.4 sq. in. B. 42.6 sq. in. C. 21.2 sq. in. D. 21.3 sq. in.



Additional Activities

Consider the given situation and answer the questions that follow. Use your knowledge of the Pythagorean Theorem.

The heights of two buildings are 41 m and 32 m and they are 12 m apart. A man went to the roof top of the shorter building and wondered looking at the roof top of the taller building.



a.) What could be the distance between the roof tops?
b.) What could be the equation to use to represent the situation?
c.) How much taller is one building than the other?
d.) Try to do an experiment with a partner or group. Choose two buildings and find the distance between their roof tops. Record the necessary measurements and write the equation that represents the situation. Then determine the distance between the roof tops.

E-Search

You may also check the following link for your reference and further learnings on solving quadratic equation using completing square.

- Introduction to Pythagorean Theorem https://mathbitsnotebook.com/Geometry/RightTriangles/RTpythagorean.html
- Ways to prove the Pythagorean Theorem https://www.youtube.com/watch?v=YompsDlEdtc
- Application of Pythagorean Theorem in Real-Life Situationhttps://mathbitsnotebook.com/Geometry/RightTriangles/RTPythPractice.html

PROBLEM – BASED LEARNING WORKSHEET

The History of the Concrete Building

Concrete is one of the most important and widely used materials in construction. It is both practical as well as flexible, and can be used in the construction of all sorts of things, from roads and bridges to concrete buildings and other forms of architecture. An early form of concrete was first discovered by Nabataea traders or Bedouins in around 700 BC. The concrete used in most buildings today is made from Portland cement which is the most common type of cement used around the world. Portland cement was developed from natural cements made in Britain during the 18th century.

Some of the notable concrete buildings are: The Ingalls Building in Cincinnati, Ohio; Hoover Dam in Colorado River and borders Arizona and Nevada; and 311 South Wacker Drive in Chicago Illinois. Often, when builders want to lay the foundation for the corners of a building, one of the methods they use is based on the Pythagorean Theorem. Also, it can be used to build staircases, roofs, and can even be used to calculate the angle for safely placing a ladder when you need to work in high areas.

Imagine, if a worker has an h feet extension ladder in order to reach x feet up a building, how far away from the building should the foot of the ladder be placed?

Let's Analyze

1. What is the title of the story?
2. What is one of the most important and widely used materials in construction?
3. Who discovered the early form of concrete?
4. What is the most common type of cement used around the world?
5. Name three notable buildings in the article?
6. How do builders make use of the Pythagorean Theorem in construction?
7. Write an equation that describes the distance of the foot of the ladder from the building?
8. If a worker has a 50-foot ladder to reach 30 feet up a building, how far away from the building should the foot of the ladder be placed?

Answer Key

WHAT I KNOW

1. D	4. D	7. C	10. B	13. C
2. B	5. A	8. C	11. C	14. D
3. D	6. A	9. A	12. A	15. C

WHATS IN

1. $\Delta BAC, \Delta ADC, and\ \Delta BDA$ 2. x = $2\sqrt{13}$, y = 6, z = $3\sqrt{13}$

ACTIVITY 1

1. Obtuse 2. Acute 3. Acute 4. Obtuse 5. Right

ACTIVITY 2

WHAT I CAN DO

1. 17 ft. 2. 26 inches 3. 7.2 or $2\sqrt{13}$ inches 4. 4 ft. 5. 127.3 or $90\sqrt{2}$ ft.

ASSESSMENT

1. A	4. D	7. D	10. B	13. D
2. C	5. D	8. C	11. C	14. D
3. A	6. B	9. A	12. C	15. C

ADDITIONAL ACTIVITIES

a. 15 meters
b. Let x be the distance between the tops of the buildings

$$c^2 = 9^2 + 12^2$$
$$x^2 = 9^2 + 12^2$$
$$x = \sqrt{9^2 + 12^2}$$

c. 9 meters
d. Answer may vary

PROBLEM – BASED LEARNING WORKSHEET

1. The History of the Concrete Building
2. Concrete
3. Nabataea traders or Bedouins
4. Portland Cement
5. Ingalls Building in Cincinnati, Ohio; Hoover Dam in Colorado River

References:

A. Book

1. Bernabe, Julieta G. Geometry Textbook for Third Year. JTW Corporation, 2002

2. Oronce, O and Mendoza, M. *E-Math 9 Worktext in Mathematics.* Philippines: Vibal Group Inc., 2016.

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