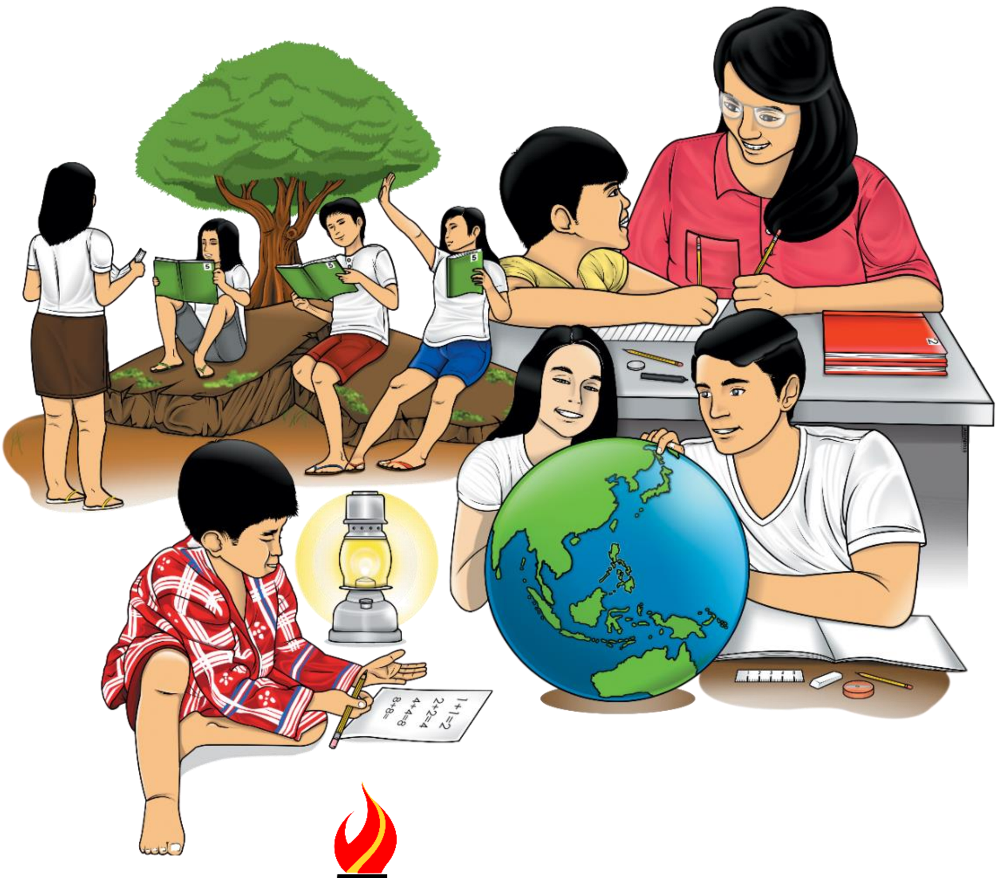
8

Mathematics

Quarter 3 – Module 2

Illustrating Axiomatic Structures of a Mathematical System



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Mathematics

Quarter 3 – Module 2

Illustrating Axiomatic Structures of a Mathematical System

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

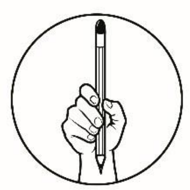
In this module, you will learn the axiomatic structure of a mathematical system and why there is a need to learn them. The scope of this module enables you to use it in many different learning situations. The lesson is arranged to follow the standard-sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains:

Lesson 1 – Illustrating Axiomatic Structures of a Mathematical System

Objectives: After going through this module, you are expected to:

1. define axiomatic system;
2. determine the importance of an axiomatic system in geometry;
3. illustrate the undefined terms; and
4. cite definitions, postulates, and theorems involving points, lines and planes.



What I Know

Pre-Assessment

Directions: Choose the letter of the correct answer. Write the chosen letter on a separate sheet of paper.

1. Which undefined term has a specific position but has no dimension or direction?
   A. point C. plane
   B. line D. angle

2. Which of the following represents a plane?
   A. tip of a pen C. top of the table
   B. stretched rope D. corner of a rectangular box

3. What do you call the lines that intersect at a point forming a right angle?
   A. diagonals C. parallel lines
   B. line segments D. perpendicular lines

4. Two angles with sum equal to 180°.
   A. acute angles C. nonadjacent angles
   B. vertical angles D. supplementary angles

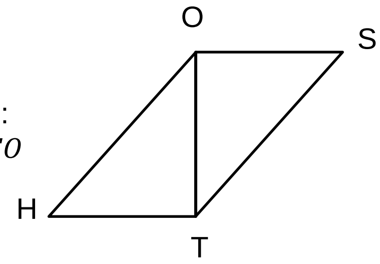
5. An axiomatic system is a set of what?
   I. Undefined and defined terms IV. Doubts
   II. Guesses V. Axioms
   III. Proof VI. Theorems
   A. I, II, III C. I, II, IV, V
   B. IV, V, VI D. I, III, V, VI

6. Which of the following statements is true?
   A. An axiomatic system is independent if there are no contradicting axioms or theorems.
   B. Consistency is a necessary requirement for an axiomatic system to be logically valid.
   C. Absolute consistency of the axiomatic system is established if an abstract model has been exhibited.
   D. In an axiomatic system, every statement, either itself or its negation, is capable of being proven true or false.

7. What undefined term is a one-dimensional figure that has infinite points and extends indefinitely in opposite direction?
   A. Point C. Plane
   B. Line D. Angle

8. What axiom of equality is represented by the illustration below?

Given:
$TO \cong TO$



A. transitive property
B. reflexive property
C. symmetric property
D. substitution property

9. What theorem states that the exterior angle of a triangle is equal to the sum of two remote interior angles of the triangle?
A. isosceles triangle theorem
B. exterior angle theorem
C. linear pair theorem
D. vertical angles theorem

10. A line which passes through the midpoint of segment at right angles.
A. diagonal
B. line segment
C. congruent segments
D. perpendicular bisector

11. What figure is formed when two noncollinear rays meet in a common point?
A. square
B. triangle
C. plane
D. angle

12. Which of the following statements represents the Segment Addition Postulate?
A. Points A, B, C are collinear and B is between A and B, then $AB + BC = AC$
B. Points A, B, C are collinear and B is between A and B, then $AB + AC = BC$
C. Points A, B, C are collinear and B is between A and B, then $AB = AC = BC$
D. Points A, B, C are collinear and B is between A and B, then $AB - BC = AC$

13. Which statement justifies congruent angles?
A. If two lines intersect
B. If it has a common side
C. If it has the same measure
D. If the sum of two angles is 180°

14. A triangular kite LMC has the following interior angles, $50°, 50°, and\ 80°$. Liza concluded that the kite is an isosceles triangle. What theorem would support her conclusion?
A. isosceles triangle theorem
B. exterior angle theorem
C. linear pair theorem
D. vertical angles theorem

15. You are tasked to make a triangular picture frame and your teacher gives you two sticks of the same length and one shorter length. What type of triangle can you make out of the materials given if you are not allowed to cut the stick?
A. scalene triangle
B. equilateral triangles
C. isosceles triangle
D. the materials cannot form a triangle

Lesson 1 Illustrating Axiomatic Structures of a Mathematical System

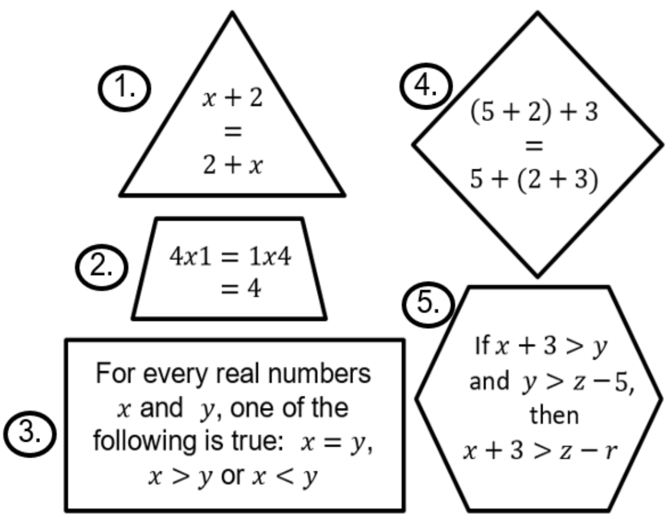
In this module you will learn about the terms in geometry which are said to be the bases in defining other geometric terms and formulating postulates which could be used to derive a logical result. These terms are said to be the building blocks of geometry. The discussion in this module will help you answer the question *"Why is there a need for an axiomatic structure in geometry?"*

Let us start this module by reviewing the axioms for real numbers.

What's In

Activity: Color me!

Directions: Determine the axiom illustrated by the statement in each of the following polygons. Color the polygon based on the legend given in a box at the right, then answer the questions that follow.



Color Legend:

Red	Trichotomy Axiom
Green	Distributive Axiom
Blue	Transitive Axiom
Yellow	Associative Axiom
Orange	Distributive Axiom
White	Commutative Axiom
Brown	Additon or Multiplication Axiom
Pink	Existence of Additive or Multiplicative Inverse
Violet	Existence of Additive or Multiplicative Identity

Questions:

1. Where you able to recall the axioms for real numbers illustrated in each polygon?
2. What axiom will justify the given statement, $10 + (-10) = 0$?
3. Describe your feeling as you color each polygon.
4. Do you know that psychological reactions to color could alter attention, mode, and motivation to learn?
5. Colors have effects on children's bodily functions, mind and emotions (Renk Etkis, 2017). Blue and green colors are associated to calmness, comfort, happiness and contentment, pink reduces aggressive behavior, while little red and yellow is helpful in drawing children's attention. What about you? What color do you think would motivate you to learn?

Now that you had picked your color, have it with you so it will get you motivated to learn and will make you ready to perform the next activity.

What's New

Activity 1: Visualize me!

Directions: Complete the table below by writing the appropriate representation of point, line, and plane found in the box.

Point	Line	Plane

top of the table	edge of the cabinet	tip of a pencil
edge of a book	tip of a marker	wall of a classroom
stretched rope	edge of a ruler	cellphone screen
corner of a rectangular tray		

What is It

Robots are increasingly used in many industries in the whole world, from healthcare and manufacturing to defense and education.

For instance, a certain robot is being used in an industry or establishment to do a particular activity. Suppose that the statements below describe the routine for such robot to control activity in a warehouse:

Set of statements:

Statement 1: Every robot has at least two paths.

Statement 2: Every path has at least two robots.

Statement 3: A minimum of one robot exist.

In this set of statements, which do you think are terms that need to be defined?

Suppose you are asked to prove another statement, say, *"a minimum of one robot exists"*, can you use these three statements to prove it?

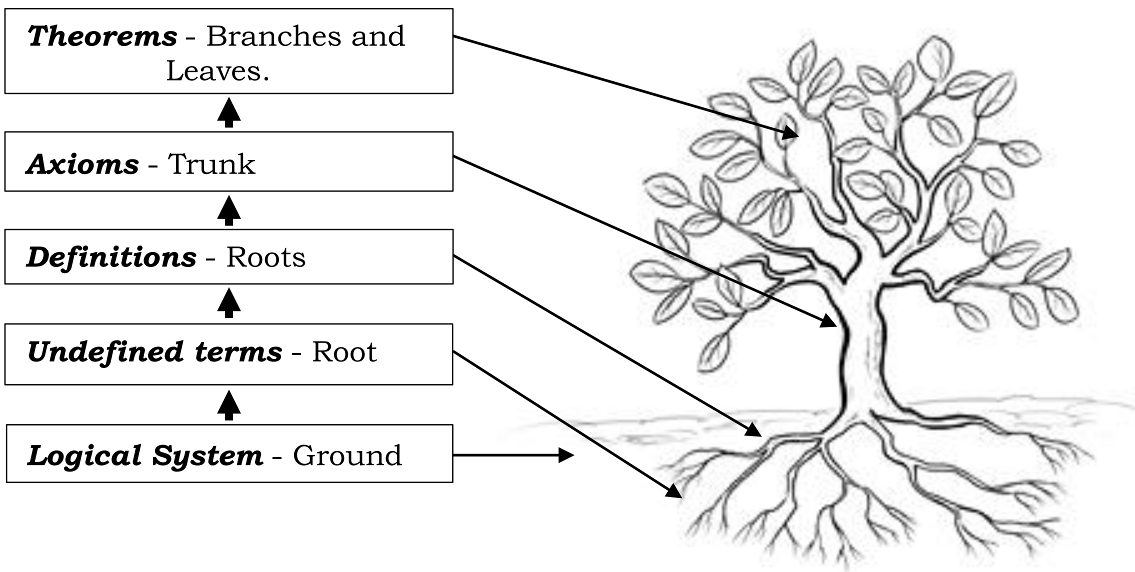
The set of statements above are true and contain terms that are undefined or needs to be defined. These statements can be used to create and to prove another statement. These set of statements are examples of an axiomatic system.

> An **axiomatic system** is a *logical system* which possesses an explicitly stated *set of axioms* from which *theorems can be derived.*

From the definition, you could say that axiomatic system consists of some ***undefined terms*** (also called the ***primitive terms***), ***defined terms***, list of ***axioms*** or ***postulates*** concerning the undefined terms, a ***system of logic*** *(or proofs)* to be used in deducing new statements called ***theorems***.

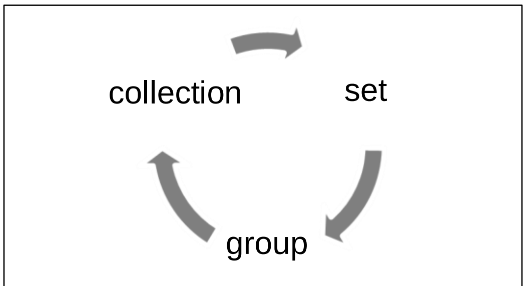
The axiomatic structure of a mathematical system can be compared to a tree.

The ground is not part of the tree but it is necessary for the tree to be planted and to grow. Like the ground, an axiomatic system needs a logical system of rules that allows one to make inferences.



The roots at the base of the tree correspond to the undefined and defined terms of the system. These are the basic term from which statements in the axiomatic system are made. Like the tip roots combined to build up roots of the tree, undefined terms are the starting point for every definition and statement of the system. They are combined in various ways into a statement called the ***definition***.

Terms are defined for us to be precise and concise on its meaning. But there are some basic terms of the system that are necessarily left undefined. When we define a term, we will be using different terms that we also need to define. For instance, we want to define the word **"set"**. Looking up for the word "set" in the dictionary, you will find that "set is a ***group*** of objects or numbers." Then you also have to define the term ***"group"***, which means ***"collection"***, and ***collection means "set"***. The process is a circular definition. Thus, there are basic terms left undefined to prevent circular definition.



Circular definition

The trunk of the tree corresponds to the ***axioms or postulates*** while the branches growing out of that trunk are the ***theorems***. ***Axioms*** are the statements that serve as a starting point for the system. Axioms are the basic truths and we used them to prove other statements. ***Theorems***, on the other hand, are statements deduced from the axioms.

Axiomatic system has three properties.

1. **Consistency.**

   An axiomatic system is said to be ***consistent*** if there are no axiom or theorem that contradict each other. This means that it is impossible to derive both a statement and its negation from the axiom set of system.

   Example:

   Axiom statement: ***There exist two lines that are parallel.***

   Negation: ***No two lines are parallel.***

   Notice that the negation is not an axiom nor a theorem.

   The system where a statement and its negation are both true is said to be ***inconsistent***.

An axiomatic system should be ***consistent*** for it to be logically valid. This means that there are no axiom or theorems that contradict each other. Otherwise, the axiomatic system is faulty or inaccurate.

2. **Independence.**

   In an axiomatic system, an axiom or postulate is said to be ***independent*** if it is not a theorem that follows the other axioms. It is not a theorem that can be derived or cannot be proven true using other axioms in the system.

   For instance, you have four different axioms. If you can make a model showing that one axiom is independent of the other, that is, you cannot use the other three axioms to prove such axiom, then the axiom is independent.

   An example to this is Euclid's fifth postulate. Many people tried to prove this axiom using the other four postulates but either failed or used faulty reasoning. This problem led to the development of other geometries where the fifth theorem of Euclid was shown to be independent of the other postulates.

Independence is not a necessary requirement for an axiomatic system.

3. Completeness.

An axiomatic system is ***complete*** if for every statement, either itself or its negation, is derivable in that system. In other words, every statement is capable of being proven true or false.

Now that you have learned the axiomatic system, let us try to apply it in the situation mentioned earlier about the artificial axiomatic system describing a routine for a computer to control activity of the robot in a warehouse. The set of axioms given were:

Axiom 1. Every robot has at least two paths.
Axiom 2. Every path has at least two robots.
Axiom 3. There exists at least one robot.

a. What are the undefined terms in this axiom set?

Answer:

In this system, the undefined terms are *"robot"*, *"path"* and *"has"*. The terms *"robot"* and *"path"* are ***elements*** and the term *"has"* is a ***relation***. This indicates that there is some relationship between *robot* and *path*. These undefined terms can be used to build construct to various proofs.

b. If you are asked to prove say, "Theorem 1. There exists at least one path.", how would you do it?

Answer:

Notice that Axiom 3 guarantees that a robot exists but no axiom clearly states that there is a path. The sequence of proof could be as follow:

Proof:
1. By the third axiom, there is an existence of a robot.
2. By the first axiom, each robot must have at least two paths.
3. Therefore, there exist at least one path.

Notice that Axiom 3 is a consequence of Axiom 1 and Axiom 2.

c. *What is the minimum number of paths? Prove it.*

Answer: Notice that Axiom 1 states that every robot has at least two paths. Hence, the minimum number of paths is two.

Proof:

1. By the third axiom, a robot exists, call it $\boldsymbol{R_1}$.
2. By the first axiom, $\boldsymbol{R_1}$ must have at least two paths call them $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$.
3. Therefore, at least two paths exist.

> The example above clearly shows that an **axiomatic system** is a collection of axioms, or statement about undefined terms, from which proofs and theorems or logical arguments are built.

The following are some examples and illustrations of each part in the axiomatic structure.

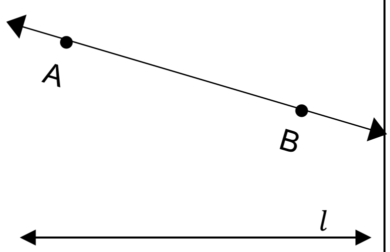
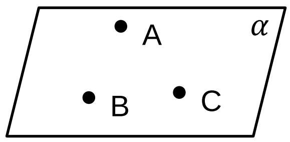
Undefined terms

Axiomatic structure started with ***three undefined terms*** (or primitive terms): ***point, line***, and ***plane***. These terms are the bases in defining new terms, hence they are called the *building blocks of geometry*. Even though they are called undefined terms, it does not really mean that we are restricted to describe or represent them.

The table below shows the different ways of describing these three undefined terms.

Point	Line	Plane
Something having specific position but it has no dimension (no length, no width, and no thickness) or direction.	A one-dimensional figure with infinite numbers of points, no specific length, without width nor thickness. It is always straight that extends indefinitely in two opposite directions.	A flat surface where infinite numbers of lines can lie. It has no specific length and width and without thickness. It extends indefinitely in all directions.
There are objects that illustrates point, line, and plane in real-world. In your previous activity, they are:		
1. tip of a pencil; 2. tip of a marker; 3. corner of a rectangular tray;	1. edge of the cabinet; 2. edge of a book; 3. stretched rope; 4. edge of a ruler.	1. top of the table; 2. wall of a classroom; 3. cellphone screen;

<table>
<tr><td colspan="3">Now look around you, can you name something in your place that would illustrate point, line, and plane?</td></tr>
<tr>
<td>A point can be represented with a dot and is denoted by a capital letter.
The two points below are point A and point B.

</td>
<td>The line below is line AB, denoted by the symbol $\overleftrightarrow{AB}$, which is named after the two points that are on the line.
Lines can also be denoted by a lower case letter like line l below.

</td>
<td>The parallelogram below is a plane denoted by the Greek Letter α, read as ‘alpha’. This plane can also be named as plane ABC.

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Defined terms

From these three undefined terms, important concepts in geometry will be defined. Remember that we need ***defined terms*** because we want to be precise and concise on the meaning of a term. Definitions will enable us to understand each other and to make sure we mean the same thing about a certain term.

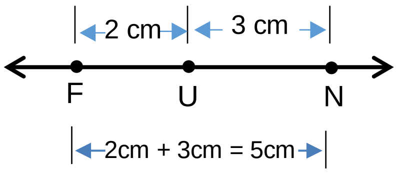
Below are some definitions derived from the undefined terms, point, line, and plane.

1. *Definition of a Segment*

Segment AB, denoted by $\overline{AB}$ or $\overline{BA}$, *is the union of points A, B and all the points between them.* A and B are called the ***endpoints*** of the segments.

2. *Definition of Between*

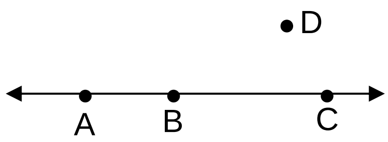
Point U is said to be between F and N if and only if F, U, and N are distinct points of the same line and $FU + UN = FN$.



In $\overleftrightarrow{FN}$, ***U* is between *F* and *N*** since F, U, and N are ***distinct points*** on the same line and $FU + UN = FN$ and $2cm + 3cm = 5cm$.

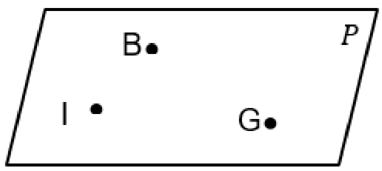
3. *Definition of Collinear Points and Coplanar Points*

When points are on the same line, they are called ***collinear points***.



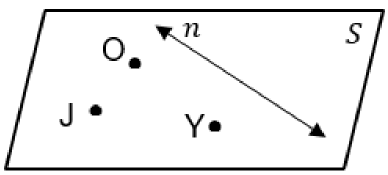
Note that points A, B, and C are on the same line, hence they are said to be collinear, while point D is **not** on the same line with the other three, thus, these four points A, B, C, and D are *noncollinear.*

When points are on the same plane, they are called ***coplanar points***.



Notice that points B, I, and G are on the same plane P, hence they are said to be coplanar.

Can points and lines be coplanar? The answer is yes. As long as they are on the same plane, they are said to be coplanar like the one illustrated below.



In the figure, points J, O, Y and line n are all on the same plane S, hence they are coplanar.

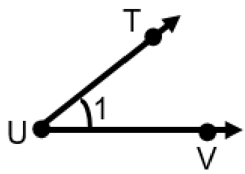
4. *Definition of a Ray*

Ray is a part of a line that has one endpoint and goes on infinitely in one direction.

In the figure above, ray PQ starts from point P and goes on to the right without bound. P is called the *endpoint* of $\overrightarrow{PQ}$. Can you call it ray PR? The answer is yes.

5. *Definition of an Angle*

An ***angle*** is the union of two noncollinear rays with a common endpoint.



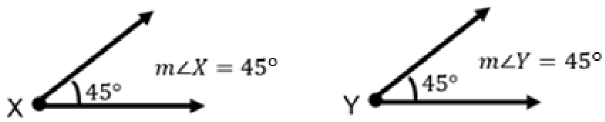
The common endpoint being shared by $\overrightarrow{UT}$ and $\overrightarrow{UV}$ is point U called the vertex. The angle formed could be named as $\angle 1$ or $\angle TUV$ or $\angle VUT$.

6. *Definition of Congruent Angles*

Two angles are ***congruent*** *if and only if their measures are equal.*

In symbol:

$\angle X \cong \angle Y$, *if and only if* $m\angle X = m\angle Y$.

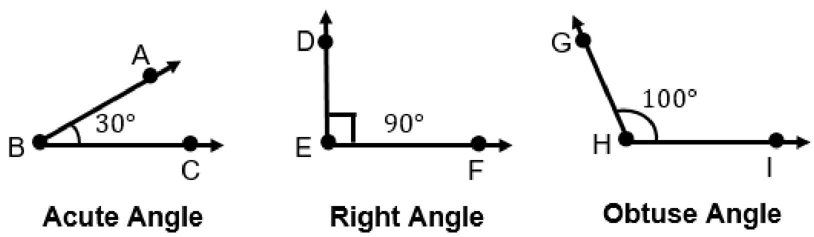


7. *Definitions of Acute Angle, Right Angle, and Obtuse Angle*

An ***acute angle*** is an angle with a measure greater than 0° but less than 90°.

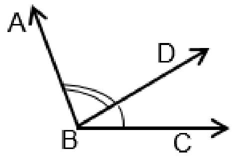
A ***right angle*** is an angle with a measure of 90° .

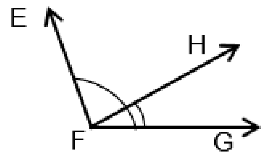
An ***obtuse angle*** is an angle with a measure greater than 90° but less than 180°.



8. *Definition of Adjacent Angles*

Adjacent angles share a common vertex and a common side, but do not overlap.



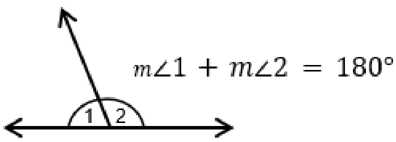


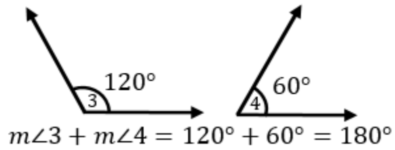
∠ABD and ∠CBD are adjacent angles which have a common vertex B and a common side $\overrightarrow{BD}$. The interiors of ∠ABD and ∠CBD do not intersect and therefore, the two angles have no interior points in common.

∠EFG and ∠GFH have F as common vertex, and $\overrightarrow{FG}$ as common side, but the interiors of the two angles intersect, this means that the two angles have common interior points. Thus, ∠EFG and ∠GFH are not adjacent angles.

9. *Definition of Supplementary Angles*

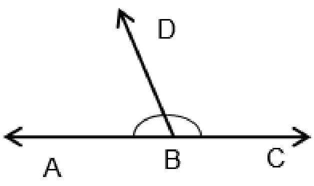
Two angles are ***supplementary*** when the sum of their angles is 180°.





10. *Definition of Linear Pairs*

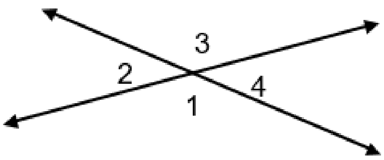
A ***linear pair of angles*** is formed when two lines intersect. Two angles are said to be **linear** if they are adjacent angles formed by two intersecting lines and are supplementary.



$\overrightarrow{BD}$ is the common side, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays, $\angle ABD$ and $\angle CBD$ forms a linear pair.

11. *Definition of Vertical Angles*

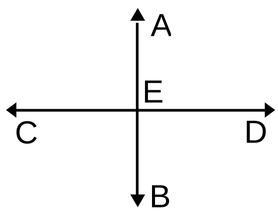
Opposite angles formed by two intersecting lines are ***vertical angles***.



∠1 and ∠3 are vertical angles. ∠2 and ∠4 are also vertical angles. Noticed that these angles are opposite each other.

12. Definition of Perpendicular Lines

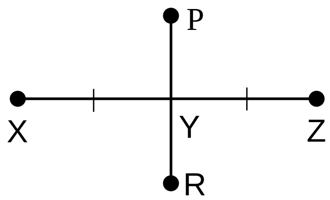
Perpendicular lines are two lines that intersect to form a right angle.



$\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at point E. $\angle AED$, $\angle AEC$, $\angle BEC$, and $\angle BED$ are right angles form by these two intersecting lines, hence, $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$. In symbol, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$.

13. Definition of Perpendicular Bisector

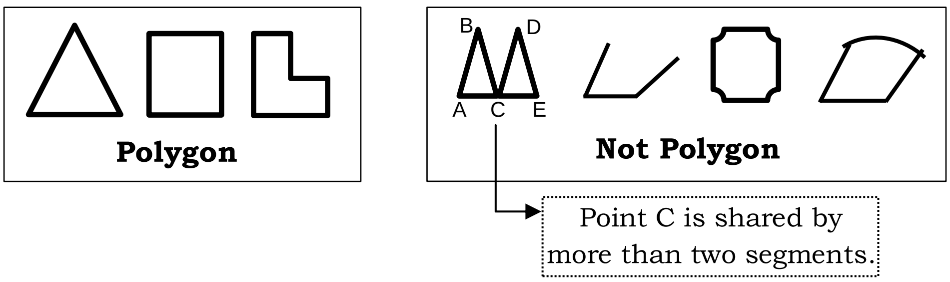
A ***perpendicular bisector*** PR of a line segment XZ is a line segment perpendicular to XZ and passing through the **midpoint**.



$\overline{PR} \perp \overline{XZ}$ at point Y. $\overline{PR}$ divides $\overline{XZ}$ into two equal parts. Thus, $XY = ZY$. Furthermore, $\overline{XY} \cong \overline{ZY}$.

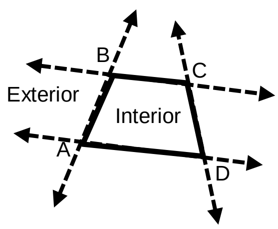
14. Definition of Polygon

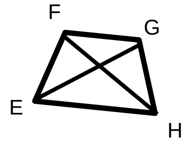
A ***polygon*** is a closed figure such that the union of three or more coplanar segments, which intersect at endpoints, with each endpoint shared by exactly two noncollinear segments.



15. Definition of Convex Polygon

A polygon is ***convex*** if and only if the lines containing the sides of the polygon do not contain points in its interior.

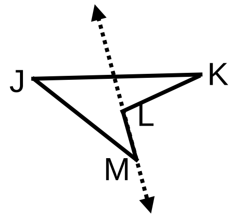




If each diagonal, except its endpoints, is entirely in the interior of the polygon, then the polygon is convex, like polygon EFGH.

16. Definition of Nonconvex (Concave) Polygon

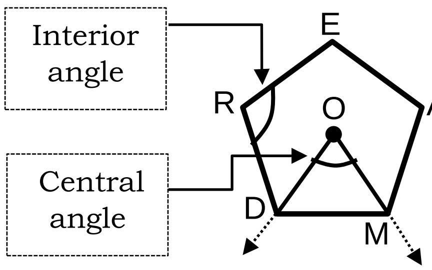
A polygon is ***nonconvex (concave)*** if and only if at least one of its sides is contained in a line, which contains also points in the interior of the polygon.

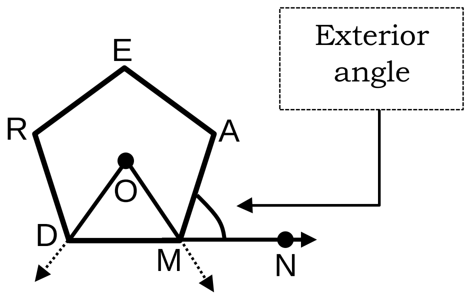


$\overleftrightarrow{LM}$ which contains $\overline{LM}$ also contains points in the interior of the polygon, hence, polygon JKLM

17. Definition of Regular Polygon

A ***regular polygon*** is a polygon that is both equilateral and equiangular.





The regular polygon DREAM has five interior angles $\angle RDM$, $\angle ERD$, $\angle AER$, $\angle EAM$, $\angle AMD$. These angles are equal,

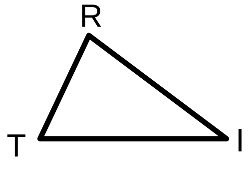
$$m\angle RDM = m\angle ERD = m\angle AER = m\angle EAM = m\angle AMD.$$

The polygon DREAM has five equal sides, $DR = RE = EA = AM = MD$. Point O is the center of the given polygon. $\angle O$ is the central angle.

Regular polygon DREAM has also five exterior angles. These angles are obtained when one of the intersecting sides is extended such as $\angle AMN$. The outside angle along with the vertex is an exterior angle.

18. Definition of a Triangle

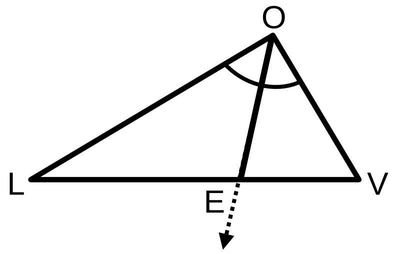
A ***triangle*** is a three-sided polygon.



The symbol "Δ" followed by the three letters representing the noncollinear points (or the vertices) is used to name the triangle. Every triangle, like ΔTRI has three sides ($\overline{TR}, \overline{RI}, \overline{TI}$), three angles ($\angle T$, $\angle R, \angle I$), and three vertices (T, R, I)

19. Definition of Angle Bisector of a Triangle

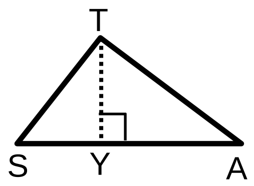
An ***angle bisector of a triangle*** is a segment contained in the ray, which bisects the angle of the triangle, and whose endpoints are the vertex of this angle and a point on the opposite side.



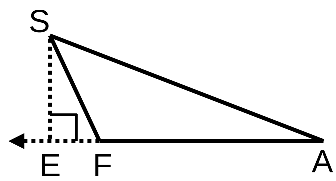
$\overline{OE}$ is an angle bisector of ΔLOV. It is the bisector of $\angle LOV$. The endpoint O of the angle bisector is the vertex of ΔLOV and the other endpoint E is on the opposite side. Thus, $\angle LOE \cong \angle EOV$.

20. Definition of an Altitude of a Triangle

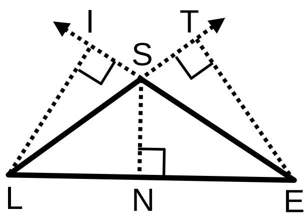
A segment is an ***altitude of a triangle*** if and only if it is perpendicular from a vertex of the triangle to the line that contains the opposite side.



Every triangle has three altitudes. In ΔSTA, $\overline{TY}$ is one of the three altitudes.



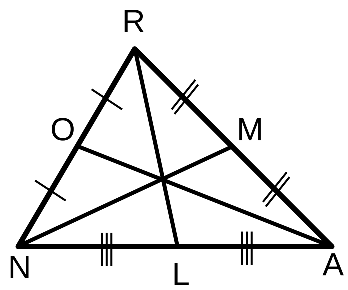
In ΔSAF, $\overline{SE}$ is one of the three altitudes.



In ΔLSE, $\overline{LI}$, $\overline{SN}$, and $\overline{TE}$ are the three altitudes.

21. Definition of Median

A segment is a ***median*** *of a triangle if and only if its endpoints are a vertex and the midpoint of the opposite side.*



Every triangle has also three medians. Median, except its endpoints, is always in the triangle's interior. Unlike altitude that can be drawn from the exterior of the triangle. $\overline{AO}$ is the median to $\overline{NR}$, $\overline{RL}$ is the median to $\overline{NA}$, and $\overline{NM}$ is the median to $\overline{RA}$.

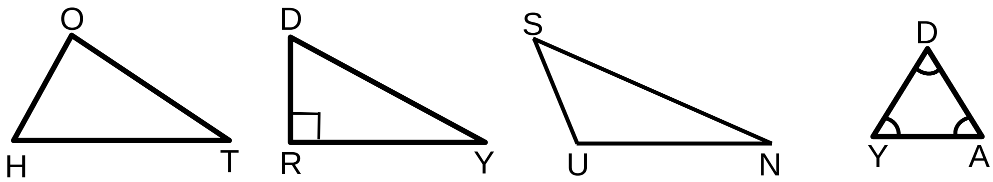
22. Definitions of Acute, Right, Obtuse, and Equiangular Triangle

An ***acute triangle*** is a triangle in which all angles are acute.

A ***right triangle*** is a triangle in which one of the angles is a right angle.

An ***obtuse triangle*** is a triangle in which one of the angles is obtuse.

An ***equiangular triangle*** is a triangle in which all angles are congruent.



Acute Triangle

Right Triangle

Obtuse Triangle

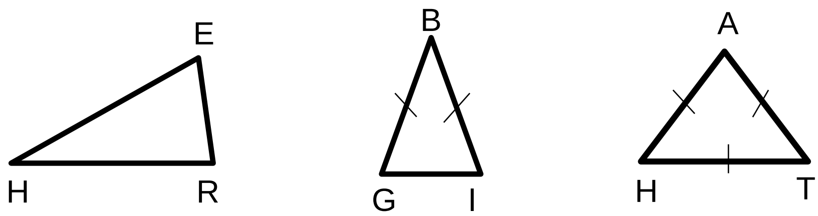
Equiangular Triangle

23. *Definitions of Scalene, Isosceles, and Equilateral Triangle*

A ***scalene triangle*** is a triangle with no congruent sides.

An ***isosceles triangle*** is a triangle with at least two congruent sides.

An ***equilateral triangle*** is a triangle with all sides congruent.



Scalene Triangle

Isosceles Triangle $\overline{BG} \cong \overline{BI}$

Equilateral Triangle $\overline{HA} \cong \overline{AT} \cong \overline{HT}$

Take note of these definitions because you will be using them in your future lessons.

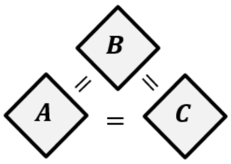
Axioms

In the axiomatic structure of a mathematical system, **axiom** is defined as a *logical statement accepted to be true without proof.* Axioms can be used as a premise in a deductive argument. In the *Elements*, Euclid presented 10 assumptions, five of which are not specific to geometry, and he called them **common notions (axioms)**, while the other five are specifically geometric in which he called them **postulates.**

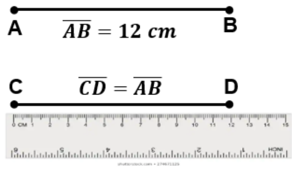
The following are examples:

Common Notions (Axioms):

Axiom 1. Things which are equal to the same thing are also equal to one another. This is ***transitive property of equality***.

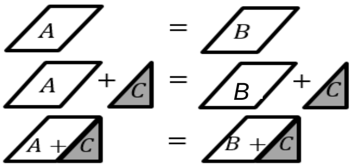


If $A = B$ and $B = C$, then $A = C$.

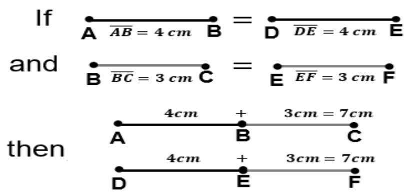


Since $\overline{AB} = 12\ cm$ and $\overline{CD} = \overline{AB}$, then $\overline{CD} = 12\ cm$.

Axiom 2. If equals are added to equals, the wholes are equal. This is ***addition property of equality***.



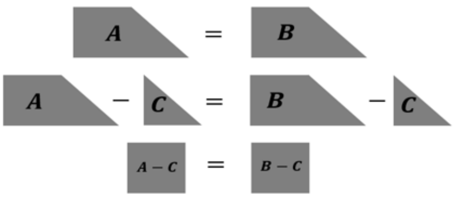
If $A = B$ and $C = C$, then
$A + C = B + C$.



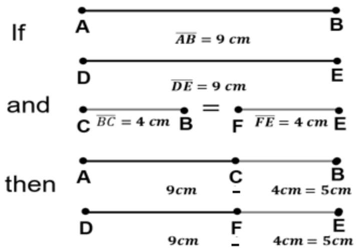
Since $\overline{AB} = \overline{DE}$ and $\overline{BC} = \overline{EF}$, and if $\overline{AB} + \overline{BC} = \overline{DE} + \overline{EF}$, then $\overline{AC} = \overline{DF}$, that is

$$4cm + 3cm = 4cm + 3cm$$
$$7cm = 7cm$$

Axiom 3. If equals are subtracted from equals, the remainders are equal. This is ***subtraction property of equality***.



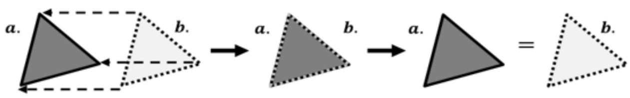
If $A = B$ and $C = C$, then
$A - C = B - C$.



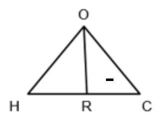
Since $\overline{AB} = \overline{DE}$ and $\overline{CB} = \overline{FE}$, and if $\overline{AB} - \overline{CB} = \overline{DE} - \overline{FE}$, then $\overline{AC} = \overline{DF}$, that is

$$9cm - 4cm = 9cm - 4cm$$
$$5cm = 5cm$$

Axiom 4. Things which coincide with one another are equal to one another. ***This is reflexive property***.



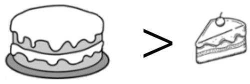
If the triangle in b fits to the triangle in a, then the triangle in a is equal to the triangle in b.



OR ≅ OR, then OR = OR

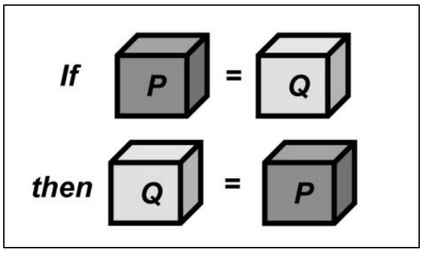
Any segments, angles and polygons are always equal to themselves.

Axiom 5. The whole is greater than the part.

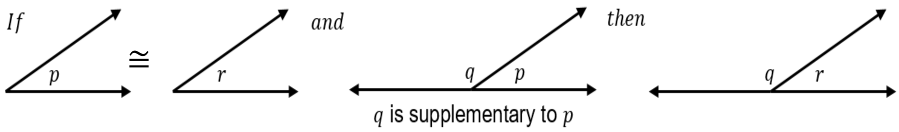


The following axioms below are both used in geometry and other field of mathematics.

- **Symmetric Property of Equality**
  - ➢ For all real numbers p and q, if $p = q$. then $q = p$.

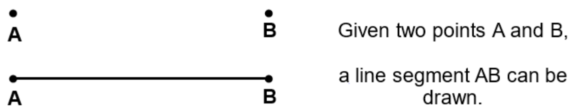


- **Substitution Property of Equality**
  - ➢ For all real numbers p and q, if $p = q$, then q can be substituted for p in any expression.

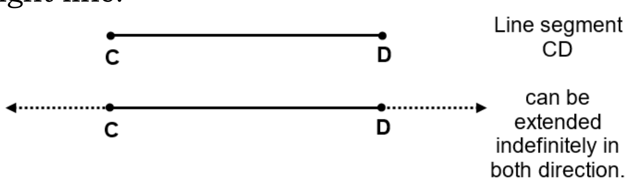


Geometrical Postulates:

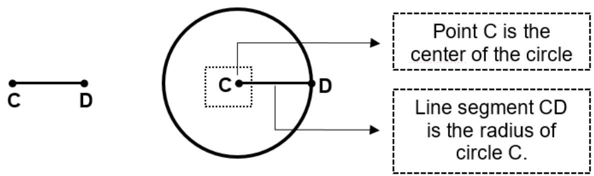
Postulate 1. A straight line segment can be drawn joining any two distinct points.



Postulate 2. Any straight line segment can be extended indefinitely in a straight line.

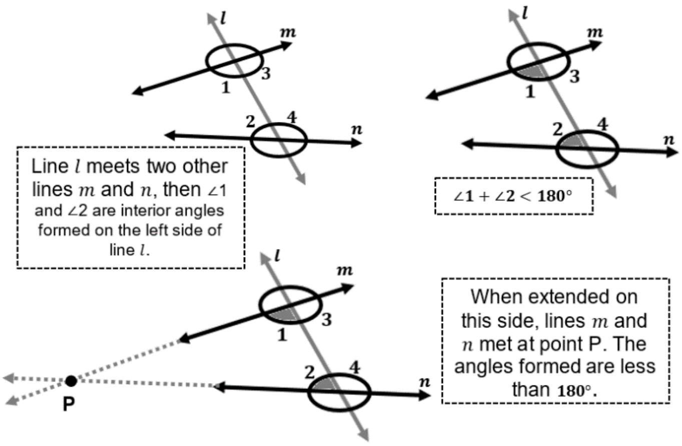


Postulate 3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.

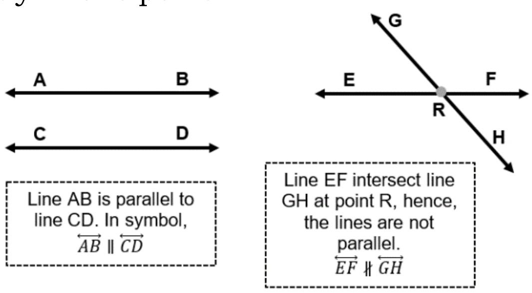


Postulate 4. All right angles are congruent.

Postulate 5. If a straight line meets two other lines, so as to make the two interior angles on one side of it together less than two right angles, the other straight lines will meet if produced on that side which the angles are less than two right angles.



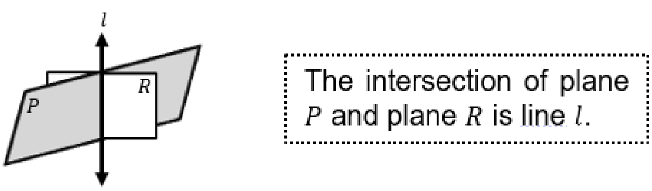
Postulate 5 asserts that two distinct straight lines in a plane are either parallel or meet exactly in one point.



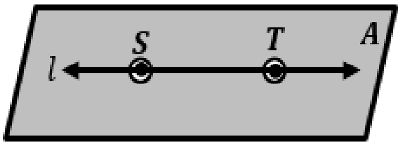
Postulate 5 is referred to as the ***parallel line postulate***. This postulate was questions by many mathematicians. To them, the postulate seemed less obvious that the others and claimed that it should be proven rather than simply accepting it as a fact.

The following are other postulates about point, line, plane, and figures formed by these and the basic postulates.

- *If two distinct planes intersect, then their intersection is a line.*



- *If two points of a line are in a plane, then the line containing these points is in the plane.*

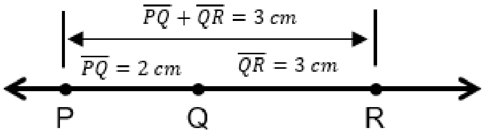


A line that lies in a plane divides the plane into two subsets, each of which is called a ***half plane***. The dividing line is called the ***edge***.

Points S and T of line l are on plane A, and therefore line l is in plane A.

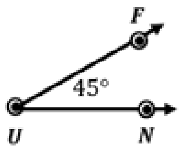
- **Segment Addition Postulate**

  ➢ *If points P, Q and R are collinear and point Q is between points P and R, then $PQ + QR = PR$*



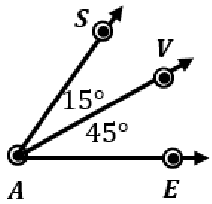
- **Angle Measurement Postulate**

  ➢ *To every angle there corresponds a unique real number r where* $0 < r < 180°$.



In geometry, angles are measures in units called ***degrees***. In symbol °. Angle ***FUN*** is 45 degrees. In notation, $m\angle FUN = 45°$

- **Angle Addition Postulate**



In the figure,

$$m\angle SAE = m\angle SAV + m\angle VAE$$
$$m\angle SAE = 15° + 45°$$
$$m\angle SAE = 60°$$

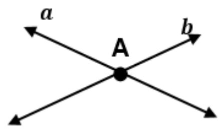
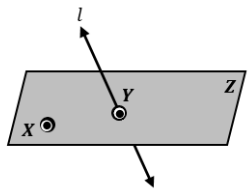
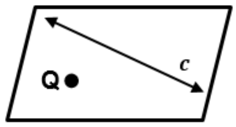
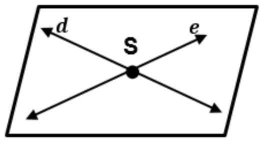
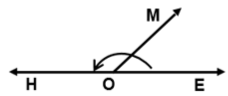
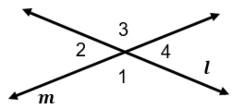
A System of Logic (Proof)

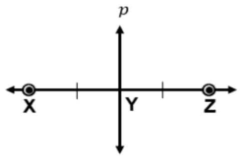
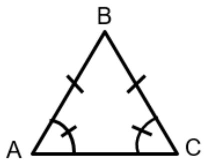
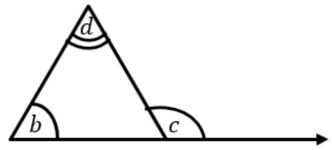
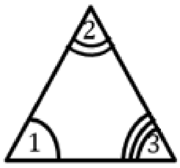
The sets of axioms (or postulates) you just learned were used to deduce new propositions or to prove other statements using the rules of inference of a system of logic. In other words, the system of logic is your proof. Euclid used deductive reasoning in organizing the Euclidian geometry.

Theorems

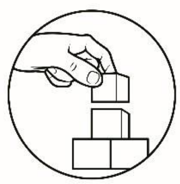
Theorems are the new statements which are deduced or proved using sets of axioms, system of logic, and previous theorems. These are statements accepted after proven deductively.

The following are some theorems about points, lines, and planes.

Theorems	Descriptions	Illustrative Example
Theorem 1	If two different lines intersect, then they intersect at exactly one point.	
Theorem 2	If a line not contained in a plane intersects the plane, then the intersection contains only one point.	
Theorem 3	If a point lies outside a line, then exactly one plane contains both the line and the point.	
Theorem 4	If two distinct (different) lines intersect, then exactly one plane contains both lines.	
Linear Pair Theorem	If two angles form a linear pair, then they are supplementary.	 $\angle HOM$ and $\angle MOE$ form a linear pair. The sum of the measure of supplementary angles is 180 degrees. $m\angle HOM + m\angle MOE = 180°$
Vertical Angles Theorem	Vertical angles are congruent.	 Two lines m and l intersect forming two pairs of vertical angles, $\angle 1$ and $\angle 3$, and $\angle 2$ and $\angle 4$, hence, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$.

Perpendicular Bisector Theorem	Point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.	 Since $\overline{XY} \cong \overline{ZY}$. Then, Y is equidistant from the endpoints X and Z.
Isosceles Triangle Theorem	Base angles of isosceles triangles are congruent.	 If $AB \cong CB$, then $\angle$ A $\cong$ $\angle$ C
Exterior Angle Theorem	The exterior angle of a triangle is equal to the sum of the two remote interior angles of the triangle. The exterior angle of a triangle is greater than either of the measures of the remote interior angles.	 $\angle b$ and $\angle d$ are the remote interior angles of the triangle. $m\angle c = m\angle b + m\angle d$ $m\angle c > m\angle b$ $m\angle c > m\angle d$
Triangle Angle Sum Theorem	The sum of the measures of the angles of a triangle is 180°.	 $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Now that you have learned definitions, postulates and theorem, you are now ready for the next activities.



What's More

Activity 1: Make me meaningful!

Directions: The following is an axiomatic system. Answer each question as required.

Axiom Set:

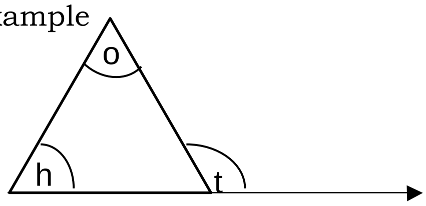
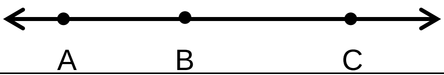
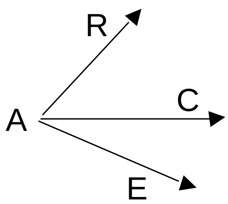
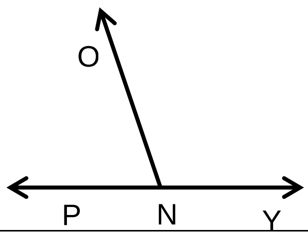
Axiom 1: Each line is a set of three points.
Axiom 2: Each point is contained by two lines.
Axiom 3: Two distinct lines intersect at exactly one point.

Questions:

1. What are the undefined terms in this axiom set?
2. Is the axiomatic system consistent? Why? Why Not? State what specific property is the given axiomatic system.

Activity 2: "Who am I"

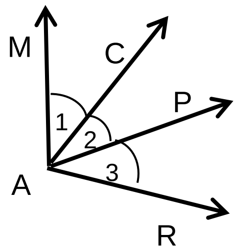
Directions: Write the definition, postulate, or theorem that supports each statement.

Illustration	Statement	What Definition, Postulate or theorem
Example 	$m\angle t = m\angle h + m\angle o$	Exterior Angle Theorem
1. 	$AB + BC = AC$	
2. 	$m\angle RAC + CAE$ $=$ $m\angle RAE$	
3. 	$m\angle PNO + m\angle ONY$	

Activity 3: "Write the missing reasons"

Directions: Using the figure below write the missing reasons corresponding to its statement. Number six is done for you.

Given: $m\angle MAP = m\angle CAR$
Prove: $m\angle 1 = m\angle 3$



Statement Figure	Reason
1. $m\angle MAP = m\angle CAR$	1.
2. $m\angle MAP = m\angle 1 + m\angle 2$	2.
3. $m\angle CAR = m\angle 2 + m\angle 3$	3.
4. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	4.
5. $m\angle 2 = m\angle 2$	5.
6. $m\angle 1 = m\angle 3$	6. Subtraction Property

Activity 4: "Identify and Illustrate"

Directions: Given the properties, identify whether it is a theorem or postulate and illustrate.

Properties	**Theorem or Postulate**	**Illustration**
1. It states that two angles form a linear pair are supplementary. ➢ $\angle PNO$ and $\angle YNO$ form a linear pair		
2. It states that angles opposite to each other and formed by two intersecting straight lines are congruent. ➢ Line m intersect line n ➢ $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$ are congruent		
3. It states that points L, M and N are collinear and M is between points L and N, then $LM + MN = LN$.		

What I Have Learned

After going through with this module, it's now time to check what you have learned from the activities. Read carefully and answer the items that follow.

Directions: Tell whether the following statement is true or false.

__________ 1. An axiomatic system consists of undefined terms, defined terms, axioms, and theorems.

__________ 2. Theorems are proved using undefined terms, defined terms, axioms, a logical system, and/or previous theorems.

__________ 3. A model of an axiomatic system is obtained by assigning meaning to the undefined terms of the axiomatic system in such a way that the axioms are true statements about the assigned concepts.

__________ 4. Euclid's postulates 1, 2, 3, and 4 can be used to prove postulate 5 (or Euclidean Parallel Postulate).

__________ 5. An axiomatic system should be consistent for it to be logically valid.

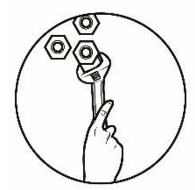
__________ 6. Independence and completeness are necessary requirements of an axiomatic system.

__________ 7. Some terms remain undefined to avoid circular definition.

__________ 8. Points, lines and planes are undefined terms in geometry.

__________ 9. Euclid's first five assumptions are not specific in geometry in which he called them postulates and the other five are specifically geometric which he called them common notions.

__________ 10. In our daily lives, we use true statements based on facts as our reasons in order to arrive at a conclusion. In this manner, we are applying axiomatic system.



What I Can Do

Let's get this real!

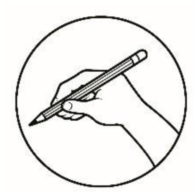
At this point, you will be given a practical task which will demonstrate your understanding on axiomatic system.

Directions: Read the situation below and answer the questions that follow.

Situation: Teacher Clarenda wanted to group here students into committees. She is grouping them according to the following axiom set:

Axiom 1: Each committee consists of exactly two members.
Axiom 2: There are exactly six committees.
Axiom 3: Each member serves on exactly two committees.

Questions:
1. What are the undefined terms in the axiom set?
2. Let a dot represent a member, and a line represent a committee, make at least one concrete model of the axiom set.
3. Deduce the theorem that "*There is a finite number of members.*"



Assessment

Directions: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What undefined term is represented by a dot?
   A. point C. plane
   B. line D. angle

2. What undefined term is represented by the edge of a ruler?
   A. point C. plane
   B. line D. angle

3. Which of the following does not represent a plane?
   A. tip of a pen C. cover page of a book
   B. top of the table D. faces of a rectangular box

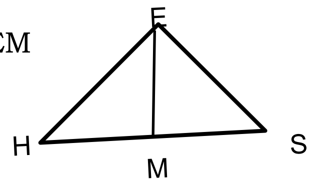
4. A consistent axiomatic system does NOT do what to itself?
   A. doubt C. affirm
   B. guess D. contradict

5. Which of the following statements best describe a perpendicular line?
   A. a straight line
   B. intersecting lines
   C. intersecting lines forming a linear pair
   D. intersecting lines forming 4 right angles

6. How do you call a line perpendicular to a given segment which divides it into two equal line segments?
   A. diagonals
   B. line segment
   C. congruent segments
   D. perpendicular bisector

7. What axioms of equality do the illustration below represent?

Given: EM ≅ EM



   A. reflexive property
   B. substitution property
   C. symmetric property
   D. transitive property

8. Which of the following represents a plane?
   A. cover page of ADM module
   B. edge of the of the book
   C. corner of a room
   D. door knob

9. What axioms of equality stated that for all real number p and q, if $p = q$ then $q = p$?
   A. reflexive property
   B. substitution property
   C. symmetric property
   D. transitive property

10. What is the sum of all interior angles of a triangle?
   A. 90°
   B. 160°
   C. 180°
   D. 360°

11. Which of the statements justify a supplementary angle?
   A. It has a common side.
   B. It has the same measures.
   C. The sum of two angles is 180°.
   D. Two lines intersect.

For items 12 to 13, consider the following axiom set below:

Axiom 1: There are at least two buildings on a school campus.
Axiom 2: There is exactly one sidewalk between any two buildings.
Axiom 3: Not all the buildings have the same sidewalk between them.

12. What are the undefined terms in given axiom set?
   A. building only
   B. sidewalk only
   C. building and campus
   D. building and sidewalk

13. Arrange the sequence of proof to prove that "there are at least two sidewalks on the school campus."

I. By Axiom 3, not all buildings have the same sidewalk between them, hence, there must be another building b_3 which does not have S_{12} between it and b_1 or b_2.
II. By axiom 1, there are two buildings denoted by b_1 and b_2.
III. With Axiom 2, there must be a sidewalk between either b_1 and b_3 or b_2 and b_3 which is not S_{12}.
IV. By axiom 2, there is exactly one sidewalk between b_1 and b_2 named S_{12}.

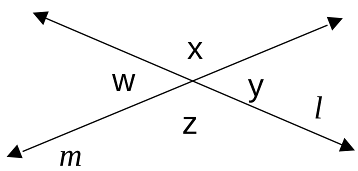
A. I, II, III, IV
B. II, IV, I, III
C. III, II, I, IV
D. IV, III, II, I

14. The angles inside the triangular garden ABC are all 60°. To be able to find the length of the sides of the garden, Mary measured the sides as follows; 2.5 m, 3 m and 5 m. Did Mary measure it correctly?
A. No, because an equiangular triangle is also equilateral.
B. No, because two sides of an equiangular triangle must be equal.
C. Yes, because a triangle is not equilateral if it is equiangular.
D. Yes, because not all equiangular triangles are equilateral.

15. You are tasked to make a triangular picture frame and your teacher gives you five sticks of the same length and one shorter length. What triangle can you make out of the materials given if you are not allowed to cut the stick?
A. equilateral triangles
B. scalene and isosceles triangles
C. isosceles and equilateral triangles
D. The materials cannot form a triangle

Additional Activities

Directions: Complete the table below with the facts associated with the given problems.

Facts	Application/ Illustration	Solution
Vertical angles are congruent.	Use the facts that linear pair forms supplementary angles to prove that vertical angles are congruent.  	1. 2. 3. 4. 5. 6. 7. 8. 9.

What I know

1.	A	9.	B
2.	C	10.	D
3.	D	11.	D
4.	D	12.	A
5.	D	13.	C
6.	B	14.	A
7.	B	15.	C
8.	B		

What's In

1. White color (Commutative)
2. Violet (Existence of Multiplicative Identity)
3. Red (Trichotomy Axiom)
4. Yellow (Associative Axiom)
5. Blue (Transitive Axiom)

What's New

Points
- ➢ Corner of a rectangular tray
- ➢ Tip of a marker
- ➢ Tip of a pencil

Lines
- ➢ Stretched rope
- ➢ Edge of the cabinet
- ➢ Edge of a ruler
- ➢ Edge of a book

Plane
- ➢ Top of the table
- ➢ Wall of a classroom
- ➢ Cellphone screen

What's More

Activity 1: Make me meaningful!

1. line and point
2. a.

Axioms	Model 1	Model 2
1	✓	✓
2	✓	✓
3	✓	✓

b. The two models are non-isomorphic because they are not of equivalent structure or they do not look exactly the same.

c. The axiomatic system is consistent because the two models are non-isomorphic making the system non-categorical, hence, it is a consistent system.

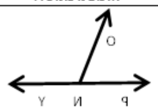
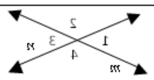
Activity 2

1. Segment Addition Postulate
2. Angle Addition Postulate
3. Definition of Supplementary angles

Activity 3

1. Given
2. Angle addition Postulate
3. Angle addition Postulate
4. Substitution property
5. Reflexive property

Activity 4

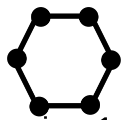
Illustration	Identify
	1. Theorem
	2. Theorem
	3. Postulate

What I Have Learned

1. T
2. F – using only axioms, a logical system, and previous theorems
3. T
4. F – cannot be used
5. T
6. F – are not necessary
7. T
8. T
9. F – common notions are none geometric, postulates are geometric
10. T

What I can Do

1. committee and member
2. Possible model



3. By axiom 1, a member exist. By axiom 2, there are exactly 6 committees. Therefore, there is a finite number of members.

Assessment

1. A
2. B
3. A
4. D
5. D
6. D
7. A
8. C
9. C
10. C
11. C
12. D
13. B
14. A
15. C

Additional Activities

1. m∠w + m∠x = 180°
2. m∠y + m∠z = 180°
3. m∠w + m∠x = m∠y + m∠z
4. m∠w = m∠y
5. m∠x + m∠y = 180°
6. m∠z + m∠y = 180°
7. m∠x + m∠y = m∠z + m∠y
8. m∠x + m∠y - m∠y = m∠z + m∠y - m∠y
9. m∠x = m∠z

Answer Key

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