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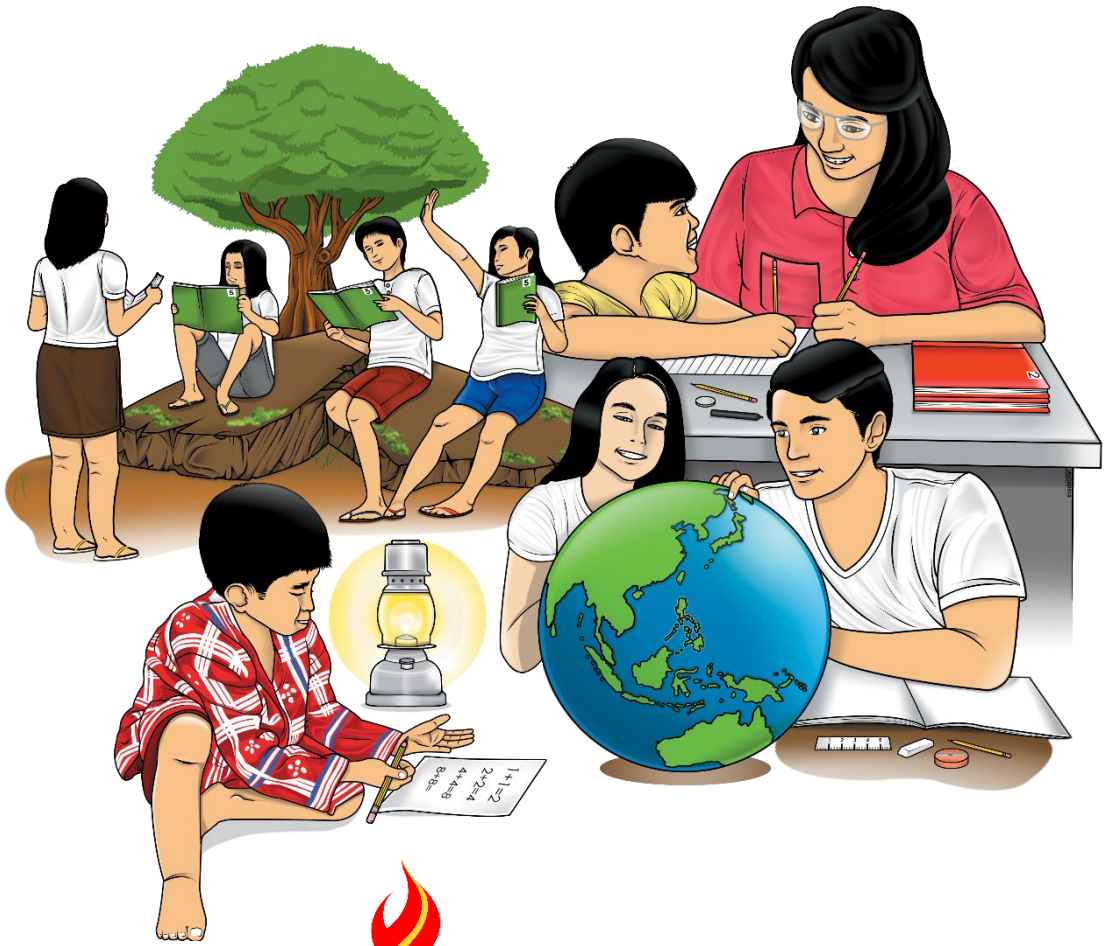
Mathematics

Quarter 4-Module 8
The Law of Cosines and Its Applications



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Development Team of the Module

Authors	:	Amparo B. Baniqued and Marissa S. Penaflor
Editors	:	Edwin M. Yap and Maita G. Camilon
Reviewers	:	Remylinda T. Soriano and Angelita Z. Modesto, and George B. Borromeo
Illustrators	:	Amparo B. Baniqued and Marissa S. Penaflor
Layout Artists	:	Amparo B. Baniqued, Marissa S. Penaflor, and Darven G. Cinchez
Management Team	:	Malcolm S. Garma, Genia V. Santos, Dennis M. Mendoza, Maria Magdalena M. Lim, Aida H. Rondilla, Lucky S. Carpio

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Department of Education - National Capital Region (NCR)

Office Address: Misamis St., Brgy. Bago Bantay, Quezon City
Telefax: (632) 8926-2213 /8929-4330 /8920-1490 and 8929-4348
E-mail Address: ncr@deped.gov.ph

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Mathematics

Quarter 4-Module 8
The Law of Cosines and Its Applications

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

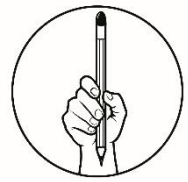
If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

In the last module you already learned how to solve an oblique triangle given *the two angles and one side* (SAA Case and ASA Case) or given *two sides and an angle opposite a given side* (SSA Case) using the Law of Sines. But not all oblique triangles can be solved using the Law of Sines. What if you are given two sides and an included angle (SAS case) or the three sides (SSS case)? Can you solve the oblique triangle using the Law of Sines? In this module you will learn how to solve oblique triangles using the Law of Cosines.

After going through with this module, you are expected to be able to illustrate law of cosines.



What I Know

Read and answer each of the questions carefully. Encircle the letter of the correct answer.

1. Three sides of a triangle are known. To find an angle, you should use _____.
   a. Law of Sines
   b. Law of Cosines
   c. Heron's Formula
   d. None of above

For items 2 and 3. In each ΔABC sides $a, b,$ and c are opposite sides of angles $A, B,$ and C, respectively.

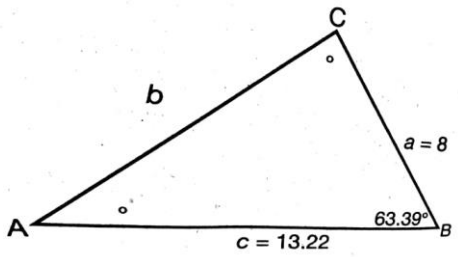
2. If $m\angle C = 100°, b = 5\ cm, and\ a = 4\ cm$, solve the triangle.
   a. c = 7.92 cm, $m\angle$A = 34.7°, $m\angle$B = 45.3°
   b. c = 5.92 cm, $m\angle$A =45.3°, $m\angle$B = 34.7°
   c. c = 6.92 cm, $m\angle$A = 45.3°, $m\angle$B = 34.7°
   d. c = 6.92 cm, $m\angle$A = 34.7°, $m\angle$B = 45.3°

3. $If\ a = 9\ cm, b = 6\ cm, and\ c = 5\ cm.$ solve the triangle.
   a. $m\angle$A = 38.9°, $m\angle$B = 31.6°, $m\angle$C = 109.5°
   b. $m\angle$A = 109.5°, $m\angle$B = 38.9°, $m\angle$C = 31.6°
   c. $m\angle$A = 38.9°, $m\angle$B = 109.5°, $m\angle$C = 31.6°
   d. $m\angle$A = 109.5°, $m\angle$B = 31.6°, $m\angle$C = 38.9°

4. ΔXYZ is a non-right triangle. If $|XY|$ = 20 cm, $|YZ|$ = 15 cm and $m\angle Y$ is 55°, then what is $|XZ|$?
   a. 16.8 cm b. 15.8 cm c. 14.5 cm d. 13.5 cm

5. The Law of Cosines is appropriate to use in solving oblique triangles when you know the following information.
   a. SAA b. ASA c. SSA d. SAS

6. Which of the following equations can be used to solve for the value of b?



a. $b^2 = \sqrt{8^2 + 13.22^2 - 2(8)(13.22)\cos 63.39°}$
b. $b^2 = 8^2 + 13.22^2 - 2(8)(13.22)\cos 63.39°$
c. $b = 8^2 + 13.22^2 - 2(8)(13.22)\cos 63.39°$
d. $b = 8^2 + 13.22^2 + 2(8)(13.22)\cos 63.39°$

For items 7 to 9, find the measure of the third side of triangle ABC.

7. $b = 12\ cm, c = 22\ cm, and\ m\angle A = 55°$

8. $a = 122\ cm, c = 55.9\ cm, and\ m\angle B = 44.2°$

9. $a = 120\ cm, b = 180\ cm, and\ m\angle C = 61°$

For items 10 to 12, given the three sides of triangle ABC, find the measure of the specified angle.

10. $a = 34\ cm, b = 30\ cm, and\ c = 60\ cm, find\ m\angle C.$

11. $a = 80\ cm, b = 78\ cm, and\ c = 70\ cm,$ find the measure of the largest angle.

12. $a = 19\ cm, b = 16\ cm, and\ c = 26\ cm,$ find the measure of the smallest angle.

13. What classification of triangle according to sides is given in item 8?

14. What classification of triangle according to angles is given in item 11?

15. The lengths of two sides of a parallelogram are 12 cm and 15 cm, and one angle of the parallelogram measures 40°. Find the lengths of the diagonals of the parallelogram.

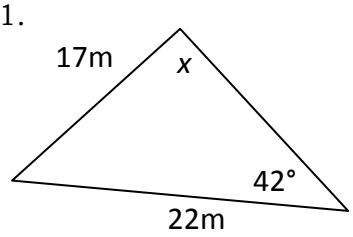
Lesson 1

Law of Cosines and Its Applications

What's In

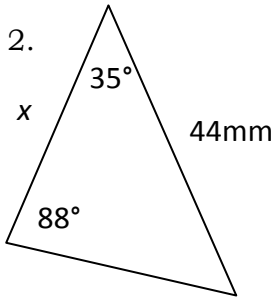
Solve for the value of x in each triangle using the Law of Sines. Round answer to the nearest tenth.

1.



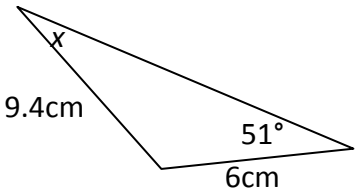
x = _____

2.



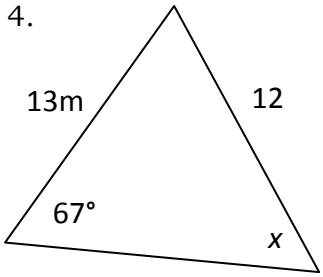
x = _____

3.



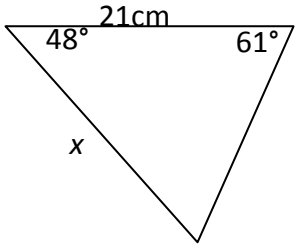
x = _____

4.



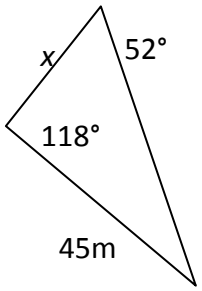
x = _____

5.



x = _____

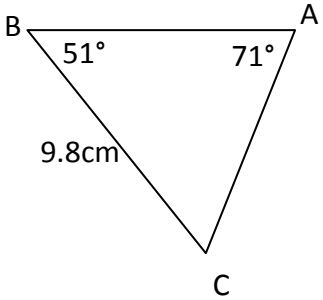
6.



x = _____

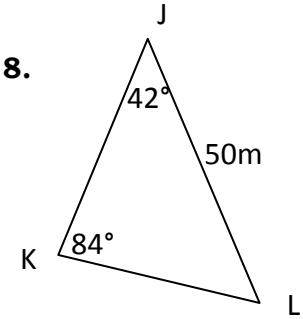
B. Solve for <u>all</u> missing measures of sides and angles in each triangle using Law of Sines. Round off answers to the nearest tenths.

7.



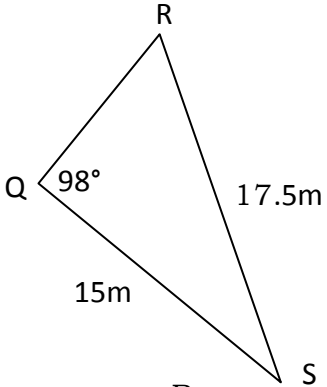
C = _____
b = _____
c = _____

8.



L = _____
l = _____
j = _____

9.

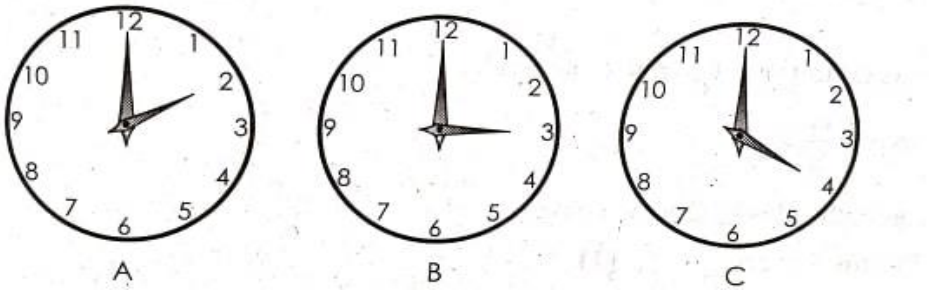


R = _____
S = _____
s = _____

What's New

Getting Ready:

You think of the hour and minute hands of a clock as two sides of a triangle and the third side connects the tips of the hands. The shape of the triangle changes as the hands move and the measure of the angle they formed changes.



On each clock shown above, the minute hand is 8 cm. long and the shorter hand is 5 cm. long. Use this information to answer questions 1-4.

1. What is the measure of the angle formed by the hands in clock A? How far apart are the tips of the hands?
2. Compare clock A with clock B.
   a. Is the measure of the angle formed by the hands in clock A greater than or less than the measure of the angle formed by the hands in clock B?
   b. Is the distance between the tips of the hands in clock A greater than or less than the distance between the tips of the hands in clock B?
3. Compare clock B and clock C using same questions (a) and (b) in item 2.
4. Can you find the distance between the tips of the hands in clock A or in clock B? Why or why not?

Developing Skills:

You already know how to use the Pythagorean Theorem to find the measure of the third side of a right triangle. The Law of Sines and the Law of Cosines are general laws you can apply to solve any triangle.

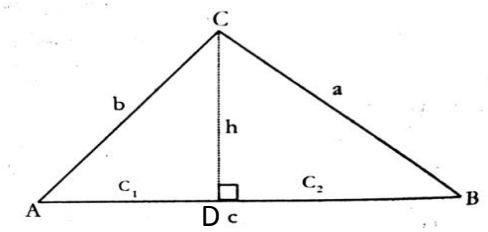
To solve an oblique triangle, you need to be given with the measure of one side and the measures of any two other parts of the triangle such as two sides, two angles, or one angle and one side.

Specifically, to solve an oblique triangle, any of the following four cases must be given.

1. Two angles and any side (AAS or ASA)
2. Two sides and an angle opposite one of them (SSA or Angle Side Side)
3. Three sides (SSS)
4. Two sides and their included angle (SAS)

The first two cases can be solved using the Law of Sines as you have learned in the previous lesson. How about the last two cases?

To derive a formula to solve the last two cases, consider triangle ABC, with angles A, B, and C and corresponding opposite sides a, b, and c as shown below.



Given:
The measure of an altitude from C to D is h .
Segment AB is divided into two segments with lengths c_1 ($|AD|$) *and* c_2 ($|DB|$).

From the definition of the cosine of an angle, in ΔCDA we get

$$\cos A = \frac{c_1}{b}$$

Multiplying both sides by $2bc$, we have

$$2bc\cos A = 2cc_1 \rightarrow \text{Eq'n (1)}$$

Using the Pythagorean Theorem, in ΔCDB, we get

$$a^2 = h^2 + {c_2}^2 \rightarrow \text{Eq'n (2)}$$

Since $c_2 = c - c_1$

$a^2 = h^2 + (c - c_1)^2$	(**1**) Substitute c_2 by $c - c_1$ in eq. 2
$a^2 = h^2 + (c^2 - 2cc_1 + {c_1}^2)$	(**2**) Expand $(c - c_1)^2$
$a^2 = (h^2 + c_1^2) + c^2 - 2cc_1$	(**3**) Apply Associative Property

Since $b^2 = h^2 + {c_1}^2$ in ΔCDA, then

$a^2 = b^2 + c^2 - 2cc_1$	(**4**) Substitute $h^2 + {c_1}^2$ by b^2 in Equation 3
$a^2 = b^2 + c^2 - 2bc\ cos\ A$	(**5**) Substitute $2cc_1$ by $2bc\cos A$ in Equation 4

This gives the Law of Cosines.

Two other formulas may be derived using a similar procedure, namely:

$$\boldsymbol{b^2 = a^2 + c^2 - 2ac\cos B}$$

$$\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos C.}$$

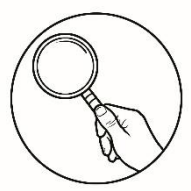
The Law of Cosines

In triangle ABC, with angles A, B, and C and corresponding opposite sides a, b, and c,

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$$
$$\boldsymbol{b^2 = a^2 + c^2 - 2ac\cos B}$$
$$\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos C.}$$

To help you remember the Law of Cosines, notice that it has the basic pattern.

$$\begin{pmatrix}\text{side}\\ \text{opposite}\\ \text{angle}\end{pmatrix}^2 = \begin{pmatrix}\text{side}\\ \text{adjacent}\\ \text{to angle}\end{pmatrix}^2 + \begin{pmatrix}\text{other side}\\ \text{adjacent}\\ \text{to angle}\end{pmatrix}^2 - 2\begin{pmatrix}\text{one}\\ \text{adjacent}\\ \text{side}\end{pmatrix}\begin{pmatrix}\text{other}\\ \text{adjacent}\\ \text{side}\end{pmatrix}\cos(\text{angle})$$



What is It

The Law of Cosines will now enable us to solve the last two cases.

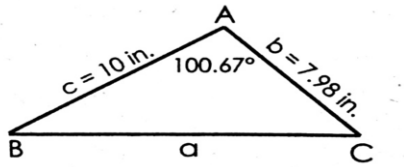
Strategies in solving the **SAS** case

1. To find the measure of the side opposite the given angle, use the Law of Cosines.
2. To find the measure of a second angle, use the Law of Sines or the Law of Cosines.
3. To find the measure of the third angle, subtract the sum of the measures of the given angle and the second angle from 180°.

Example 1:

Solve the ΔABC *given* $b = 7.98\ cm,\ c = 10\ cm,$ and $m\angle A = 100.67°$. (SAS case)

Solution:



Find the value of a

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 7.98^2 + 10^2$

$- 2(7.98)(10)cos100.67°$

a

Solve for $m\angle B$.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\sin B = \frac{b\sin A}{a}$$

$$\sin B = \frac{7.98\sin 100.67°}{13.9} = 0.564$$

$$m\angle B = 34.34°$$

Solve for $m\angle C$.

$m\angle C = 180° - (100.67° + 34.34°)$

$m\angle C = 180° - 135° = 45°$

Thus, $m\angle B = 34.34°, m\angle C = 45°,$
$and\ a = 13.9\ cm$

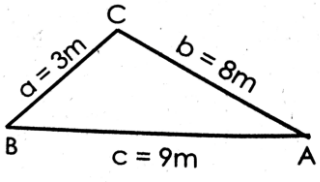
Strategies in solving the SSS case

1. To find the measure of one angle, use the Law of Cosines.
2. To find the measure of either of the remaining angles, use the Law of Sines or the Law of Cosines.
3. To find the measure of the third angle, subtract the sum of the measures of the angles obtained in Steps 1 and 2 from 180°.

Example 2

A triangular piece of land has side lengths $a = 3\ m, b = 8\ m, and\ c = 9\ m$. Find the measures of the interior angles of the triangle.

Solution:



Find the measure of the angle opposite the longest side.
Solve for $m\angle C$

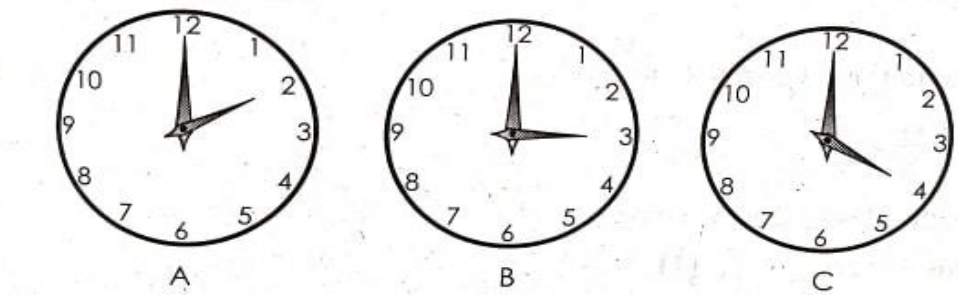
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$2ab\cos C = a^2 + b^2 - c^2$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\cos C = \frac{3^2 + 8^2 - 9^2}{2(3)(8)}$$
$$\cos C = -0.1667$$
$$m\angle C = 99.59°$$
$$m\angle C \approx 100°$$

What's More

A. Answer all questions in Getting Ready

Getting Ready:

You think of the hands of a clock as two sides of a triangle and the third side is the line segment that connects the tips of the hands. The shape of the triangle changes as the hands move and the measure of the angle they formed changes.



On each clock shown above, the minute hand is 8 cm. long and the hour hand is 5 cm. long.

1. What is the measure of the angle formed by the hands of clock A? How far apart are the tips of the hands?

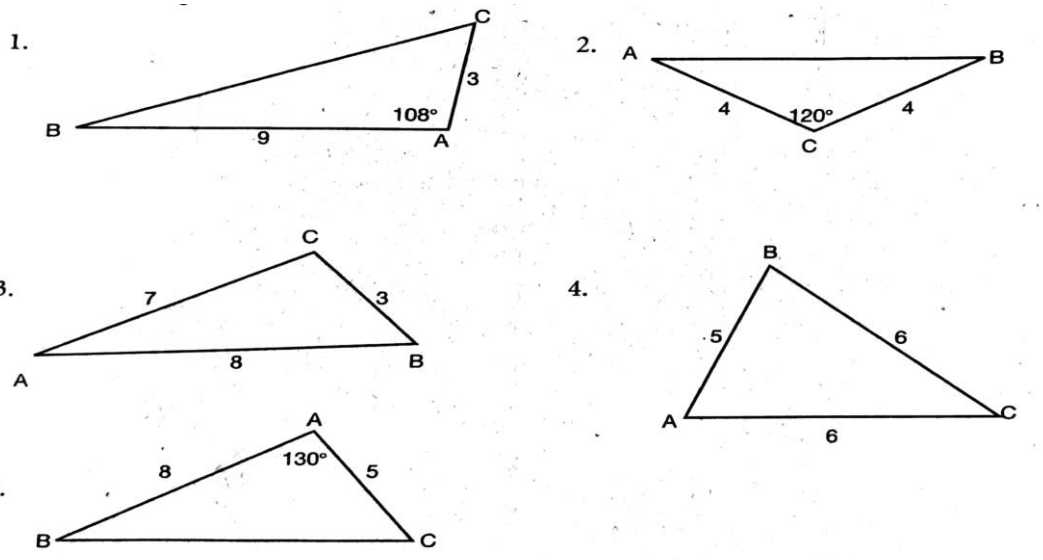
2. Compare clock A with clock B.
   a. Is the measure of the angle formed by the hands in clock A greater than or less than the measure of the angle formed by the hands in clock B?

   b. Is the distance between the ends of the hands in clock A greater than or less than the distance formed in clock B?

3. Compare clock B and clock C using same questions (a) and (b) in item 2.

4. Can you find the distance between the tips of the hands in clock A or clock B? Why or why not?

B. Solve each triangle. Round off the answers to the nearest hundredth.



C. For each of the given situational problems,

   a. sketch the required triangle to illustrate the situation, and

   b. use the Law of Cosines to solve the problem.

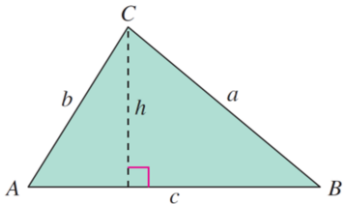
1. Peter has three sticks measuring 19 cm, 23 cm, and 27 cm. He lays them down on the floor to form a triangle. Find the measure of the angle formed by the 19 cm, and 23 cm sides to the nearest degree.

2. An aircraft tracking station determines the distance from common point O to each aircraft and the angle between the aircrafts. If the angle *0* between the two aircrafts is 49° and the distances from point O to the two aircrafts are 50 km and 72 km, find the distance between the two aircrafts. (Round off the answer to the nearest tenths)

3. A triangular parcel of land has sides 50 ft., 40 ft., and 35 ft. What are the measures of the interior angles of the triangle? Express answer to the nearest degree.

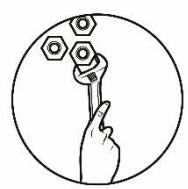
What I Have Learned

The Law of Cosines can be used to find the measures of the other parts of an oblique (non-right) triangle when either the lengths of two sides and the measure of the included angle are known (SAS) or the lengths of the three sides (SSS) are known.



The Law of Cosines states that

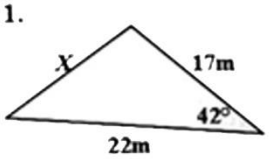
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C.$$



What I Can Do

A. Solve for the unknown (x or θ) in each triangle. Round off to the nearest hundredth.

1.



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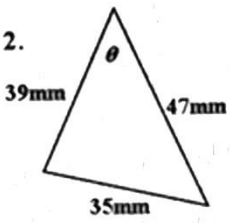

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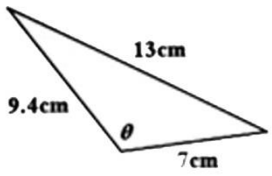

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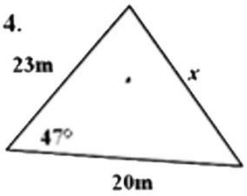

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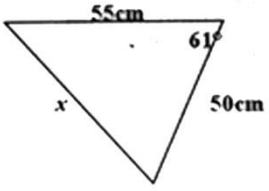
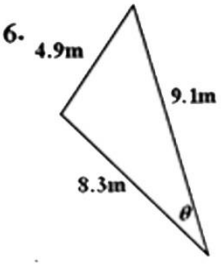


6.



B. Draw the triangles given the indicated measures, then solve for <u>all</u> missing measures of the sides and angles in each triangle. Round off answers to the nearest hundredth. ** USE PROPER VARIABLES

7. $\Delta XYZ: x = 29\,m, y = 15\,m, and\ m\angle Z = 122°$

8. $\Delta GHI: g = 13\,cm, h = 8\,cm,\ and\ i = 15\,cm.$

9. $\Delta MNO: n = 31\,m, o = 28\,m, and\ m\angle M = 62°$

C. Solve each problem.

10. The diagonals of a parallelogram are 58 inches and 36 inches long and intersect at an angle of 120°. Find the length of the shorter side.

11. A triangular lot has side lengths 20.6 m, 31.4 m, and 38.3 m. Find the angles at the corners of the property.

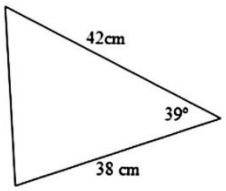
12. The lengths of two adjacent sides of a parallelogram are 2 cm and 8 cm. If the measure of the angle formed by these two sides is 30°, find the length of the shorter diagonal.

Assessment

A. Answer each of the following items accurately. Write the letter of the correct answer on your answer sheet.

1. To solve a triangle, if the only given information are the measures of the three sides of the triangle, you should use _____.
   a. Law of Sines
   b. Heron's Formula
   c. Law of Cosines
   d. All of the above

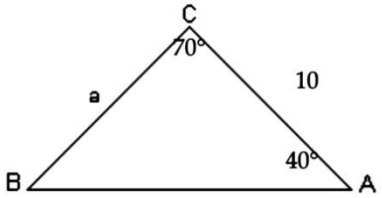
2. To find the missing length of a side of the triangle below, which of the following is the Law to use and how long is the side?



   a. Law of Cosines, 27 cm
   b. Law of Cosines, 727.4 cm
   c. Law of Sines, 1199.2 cm
   d. Law of Sines, 34.6 cm

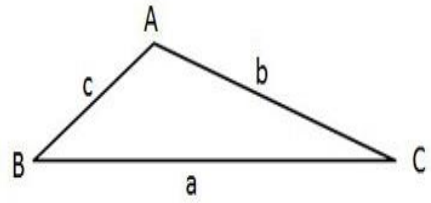
3. ■$ABCD$ is a parallelogram, if $|AB|$ is 8 cm long, $|BC|$ is 5 cm and their included angle measures 100°, how long is the diagonal AC?
   a. 12.95 cm b. 12.59 cm c. 10.40 cm d. 10.14 cm.

4. Solve the triangle ABC below.



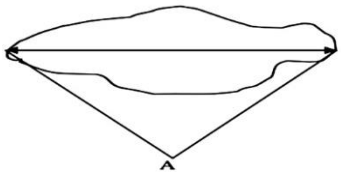
a. $m\angle B = 70°, a = 10, and\ c = 6.84$
b. $m\angle B = 75°, a = 6.84, and\ c = 10$
c. $m\angle B = 65°, a = 10, and\ c = 6.84$
d. $m\angle B = 70°, a = 6.84, and\ c = 10$

5. In the given triangle below, $b = 25\ cm, c = 12\ cm, and\ m\angle A = 110°$, approximately how long is side a?



a. 21 cm
b. 31 cm
c. 41 cm
d. 51 cm

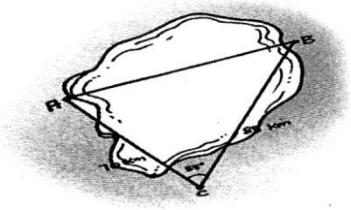
6. Erwin and Wesley wanted to measure the length of an alligator infested pond. From point A as shown below, the measured distances to each end of the pond are 300 ft. and 216 ft. The angle between these two side measurements is 78°. Find the length of the pond.



a. 331 ft.
b. 356 ft
c. 375 ft.
d. 389 ft.

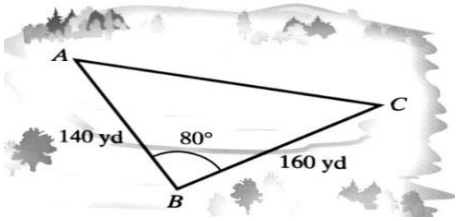
7. A triangular-shaped cornfield has sides of lengths 400 m, 200 m, and 320 m.
What is the measure of the largest angle of the triangle?
a. 89.9° b. 97.9° c. 102.5° d. 110.5°

8. Find the distance across the lake using the given measures below.
In the figure: Given: $a = 88\ km, b = 70\ km\ and\ m\angle C = 55°$



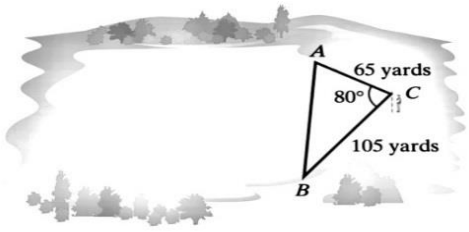
a. 74.7 km
b. 64.5 km
c. 57.4 km
d. 44.4 km

9. Find the distance across the lake from A to C, to the nearest whole number of yard, using the measurements shown in the figure.



a. 210 yards
b. 205 yards
c. 195 yards
d. 193 yards

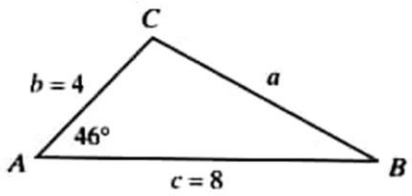
10. You want to find the distance across a protected cove at a lake., A surveyor makes the measurements shown in the figure. Use these measurements to find the distance from A to B to the nearest whole number of yard.



a. 105 yards
b. 113 yards
c. 133 yards
d. 150 yards

B. In each ΔABC sides $a, b,$ and c are opposite sides of angles $A, B,$ and C, respectively. Solve each triangle.

1.



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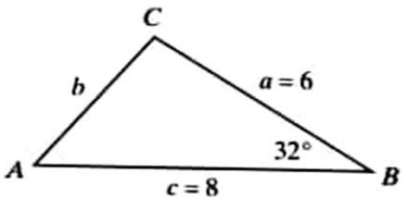

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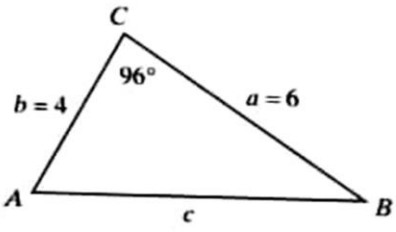

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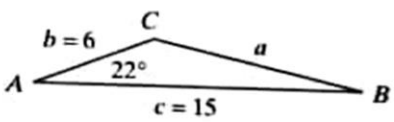

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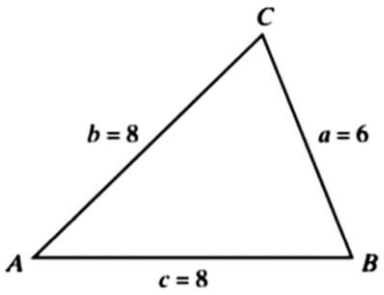

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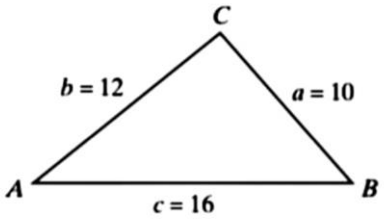

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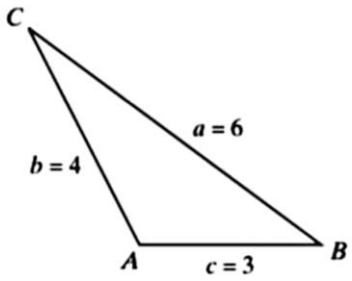
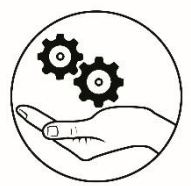


C. Solve each triangle. Round the lengths to the nearest tenth and angle measure to the nearest whole number of degree.

8. $a = 5\ cm, b = 7\ cm, and\ m\angle C = 42°$
9. $b = 5\ cm, c = 3\ cm, and\ m\angle A = 102°$
10. $a = 6\ cm, c = 5\ cm, and\ m\angle B = 50°$
11. $a = 5\ cm, b = 7\ cm, and\ c = 10\ cm$
12. $a = 3\ cm, b = 9\ cm, and\ c = 8\ cm$
13. $a = 66\ cm, b = 25\ cm, and\ c = 45\ cm$

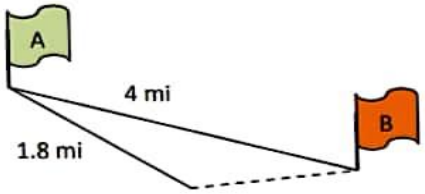


Additional Activities

Challenge Problems:

1. Peter has three sticks measuring 19 inches, 23 inches, and 27 inches. He lays them down on the floor to form a triangle. Find the measure of the angle included by the 19-inch and 23-inch sides to the nearest whole number of degree.

2. Aubrey is orienteering a large flat plain from Marker A to Marker B which are 4 miles apart. After walking 1.8 miles she realizes she is 6° off-course. To the nearest tenth of a mile, how far from Marker B is she when she realizes her error?



E-Search

To further explore the concept learned today and if it possible to connect the internet, you may visit the following links:

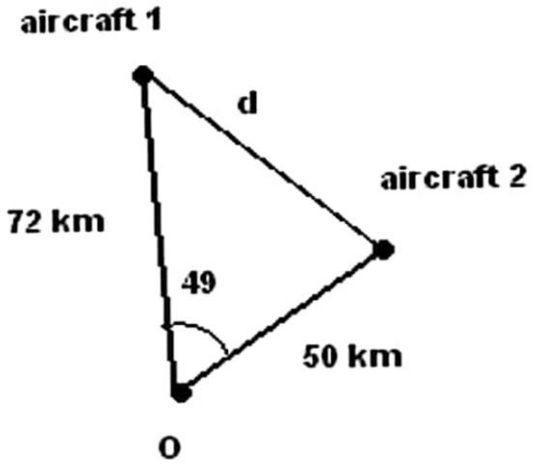
- www.onlinemath learning.com>law of sine 2
- www.math is fun .com>algebra>trig-sine-law
- www.teacherspayteachers .com>Browse>Search law of sines
- www.mathworksheetsgo.com
- www.study.com>academy>lesson>law of sine lesson-plan
- www.mathopenref.com/lawofcosinesproof.html
- https://cdn.kutasoftware.com
- https://www.buffaloschool.org

PROBLEM – BASED LEARNING WORKSHEET

Problem:

An aircraft tracking station determines the distance from a common point *O* to each aircraft and the angle between the two aircrafts. If the measure of angle *O* between the two aircrafts is equal to 49° and the distances from point *O* to the two aircrafts are 50 km and 72 km, find distance d between the two aircrafts (round answer to the nearest tenth)

The diagram to the above problem is shown below



Questions:

1. What law are you going to use to find d or the distance between the two aircrafts?
2. This problem is an example of what case using law of cosines?
3. Write the equation to solve for d.
4. What is the value of d?
5. What is the distance between the two aircrafts?

Answer Key

WHAT I KNOW

A. 1. b
2. d
3. b
4. a
5. d
6. b

B. 7. $a = 18$
8. $b = 90.7$
9. c =160.8
10. $m\angle C = 139.2°$
11. $m\angle A = 65.2°$
12. $m\angle B = 37.8°$
13. Scalene Δ
14. Acute Δ
15. 25.4 and 9.7

WHAT'S IN

A. 1. 60°
2. 36.9 mm
3. 29.7°
4. 85.7
5. 19.4 cm
6. 9.9 m

B. 7. $m\angle C = 58°, b = 8.1\ cm, and\ a = 8.8\ cm$
8. $m\angle L = 54°, l = 40.7\ m, and\ j = 33.6\ m$
9. $m\angle R = 58°,\ m\angle S = 24°, and\ s = 7.2\ m$

WHAT MORE

A. 1. 60°,. (7 cm)
2. a. less than
   b. less than
3. a. The measure of the angle formed by the hands of clock B is less than the measure of the angle formed by the hands of clock C?
   b. The distance between the ends of the hands of clock B is less than the distance formed in clock C.
4. We can find their distances using the Law of Cosines.

B. 1. $a = 10.33, m\angle B = 16.03°, m\angle C = 55.96°$
2. $m\angle A = 30°, m\angle B = 30°, and\ c = 6.93$
3. $m\angle A = 21.79°, m\angle B = 60.01°, and\ m\angle C = 98.20°$
4. $m\angle A = 65.38°, m\angle B = 65.38°\ and\ m\angle C = 49.25°$
5. $m\angle B = 18.86°, m\angle C = 31.14°, and\ c = 11.85$

C. 1. 79°, 44°, 57°
2. $54.4\ km$
3. $44°, 53°, and\ 83°$

WHAT I CAN DO

A. 1. $14.74\ m$
2. 46.90°
3. 103.91°
4. $17.37\ m$
5. $53.47\ cm$
6. 32.30°

B. 7. $m\angle X = 39°, m\angle Y = 19°, and\ z = 39.08$
8. $m\angle G = 59.99°, m\angle H = 32.20°, and\ m\angle I = 87.81°$
9. $m\angle N = 63.85°, m\angle O = 54.15°, and\ m = 30.5\ m$

C. 10. $25.36\ inches$
11. $32.51°, 55°, and\ 92.49°$
12. $6.35\ cm$

ASSESSMENT

A.

1. c	6. a
2. a	7. b
3. d	8. a
4. d	9. d
5. b	10. b

B.

1. $m\angle B = 28.66°,\ m\angle C = 105.34°, and\ a = 6\ cm$
2. $m\angle A = 47.54°,\ m\angle C = 100.46°, and\ b = 4.31\ cm$
3. $m\angle A = 52.22°,\ m\angle B = 31.8°, and\ c = 7.55\ cm$
4. $m\angle B = 13.39°,\ m\angle C = 144.61°, and\ a = 9.70$ cm
5. $m\angle A = 44°,\ m\angle B = 68°, and\ m\angle C = 68°$
6. $m\angle A = 38.62°,\ m\angle B = 48.50°, and\ m\angle C = 92.88°$
7. $m\angle A = 117.28°,\ m\angle B = 36.34°\ and\ m\angle C = 26.38°$
8. $m\angle A = 46°,\ m\angle B = 92°, and\ c = 4.7\ cm$
9. $m\angle B = 50°,\ m\angle C = 28°, and\ a = 6.3\ cm$
10. $m\angle A = 78°,\ m\angle C = 52°, and\ b = 4.7\ cm$
11. $m\angle B = 40°, and\ m\angle C = 112°,\ m\angle A = 28°$
12. $m\angle A = 19°,\ m\angle B = 100°\ and\ m\angle C = 61°$
13. $m\angle A = 140°,\ m\angle B = 14°, and\ m\angle C = 26°$

ADDITIONAL ACTIVITIES

1. 79°
2. 2.2 miles

PROBLEM-BASED LEARNING WORKSHEET

1. Law of Cosines
2. SAS Case
3. $d^2 = 72^2 + 50^2 - 2(72)(50)cos\ 49°$
4. 54.4
5. The distance between the two aircrafts is 54.4 km.

References:

A. Book

1. Gladys C. Nivera and Minie Rose C. Lapinid, *Grade 9 Mathematics, Patterns and Practicalities* (Makati City, Philippines: Salesiana BOOKS by Don Bosco Press, Inc., 2013).

2. Jose Dilao Soledad, Fernando B. Orines, and Julieta G. Bernabe, *Advances Algebra Trigonometry and Statistics* (Quezon City, Philippines: JTW Corporation, 2013).

3. Merden L. Bryant, et. al., *MATHEMATICS, Learner's Material 9* (Pasig City, Philippines: Vibal Group, Inc., 2014).

4. Orlando A. Oronce and Marilyn O. Mendoza, *Worktext in Mathematics, E-Math Advanced Algebra and Trigonometry* (Manila, Philippines: Rex Bookstore, 2013).

5. Regina Macarangal Tresvalles and Wilson Cordova, *Math Ideas and Applications Series Advanced Algebra, Trigonometry, and Statistics* (Quezon City, Philippines: Abiva Publishing House, Inc., 2010).

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph