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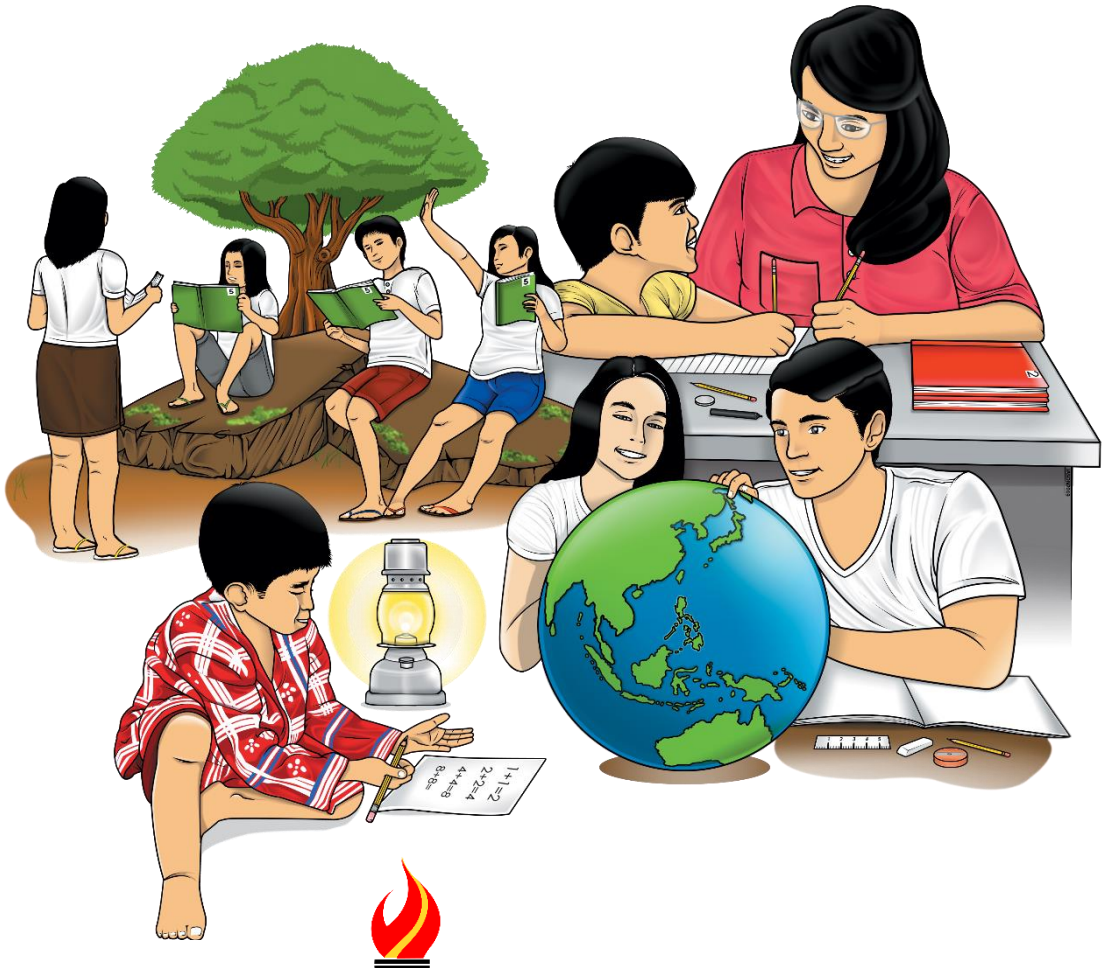
DepED
DEPARTMENT OF EDUCATION

Mathematics

Quarter 4 – Module 5:
Angle of Depression



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Alternative Delivery Mode
Quarter 4 – Module 5: Angle of Depression
First Edition, 2020

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Published by the Department of Education
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Undersecretary: Diosdado M. San Antonio

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Printed in the Philippines by _____

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Mathematics

Quarter 4 – Module 5:
Angle of Depression

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

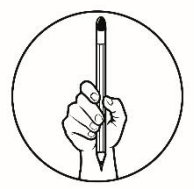
Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you master **Solving Problems Involving Angle of Depression.** The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

- Solve problems involving Angle of Depression. **M9GE-IVe-45.2**



What I Know

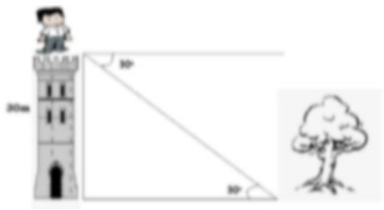
Read and answer each of the questions carefully. Write your answers on a separate sheet of paper.

1) It is an acute angle formed by the eye level (horizontal line) and the line of sight when an observer looks at an object that is located at a lower level.
   a. Angle of elevation
   b. Angle of depression
   c. Inclination
   d. Reference angle

2) What do you call the line segment drawn from the eyes of the observer to the object being observed?
   a. eye level
   b. line of sight
   c. opposite side
   d. adjacent side

3) Which ratio of the lengths of the sides of a right triangle defines the tangent function of one of its acute angles?
   a. $\frac{opposite}{adjacent}$
   b. $\frac{opposite}{hypotenuse}$
   c. $\frac{adjacent}{opposite}$
   d. $\frac{hypotenuse}{adjacent}$

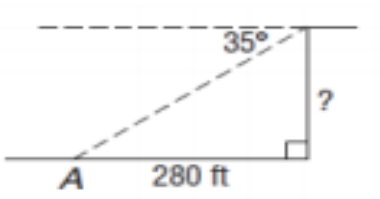
4) From the top of a 30-m tower, a man observes that the angle of depression of the foot of a tree is 30^0. How far is the tree from the tower?
   a. 82.95 m
   b. 78.13 m
   c. 51.96 m
   d. 43.05 m



5) The man's path as he dives into the sea from his boat forms a 23.5-degree angle with the water surface. How far did he swim if he reached a depth of 150 ft?
   a. 59.81 feet
   b. 137.56 feet
   c. 163.57 feet
   d. 376.18 feet

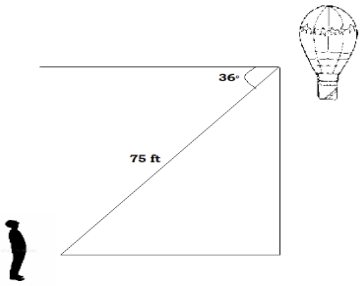
6) The pilot of an airplane over the Pacific Ocean notices that the angle of depression of an atoll to be 23 degrees. If the plane is 445 m above the sea level, how far is the plane from the atoll?
   a. 1,138.89 m
   b. 1,048.35 m
   c. 483.43 m
   d.173.88 m

7) The angle of depression of point A on the ground from the top of a sheer cliff is 35°. If point A is 280 feet from the base of the cliff, how tall is the cliff?



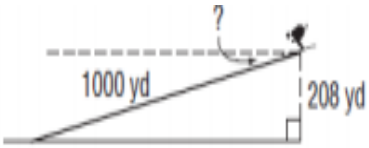
a. 399.88 feet
b. 229.36 feet
c. 196.06 feet
d. 160.6 feet

8) From a hot air balloon, the angle of depression of a man on the ground is 36°. If the hot air balloon is 75 feet away from the man, how high is the balloon?



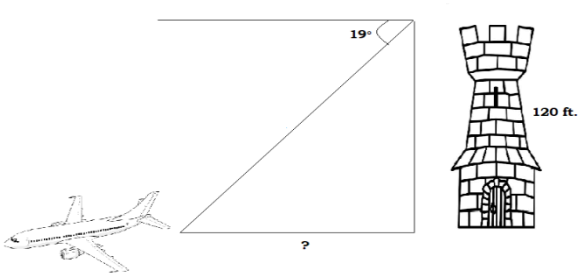
a. 44.08 feet
b. 54.49 feet
c. 60.67 feet
d. 103.23 feet

9) A ski run is 1,000 yards long with a vertical drop of 208 yards. Find the angle of depression of the ski run.



a. 78.25°
b. 78°
c. 12°
d. 11.75°

10) From the top of a 120-foot high airport control tower, an air traffic controller observes that the angle of depression of an airplane on the runway is 19°. How far from the base of the tower is the airplane?



a. 368.59 feet
b. 348.51 feet
c. 126.91 feet
d. 113.46 feet

Lesson 1

Solving Problems Involving Angle of Depression

When you look down at something on the ground, you are creating an angle called Angle of Depression. When we look down, we create right triangle all around us that will help us find distances and angles. Many real- life problems on right triangle can be solved using this. At the end of this module, there is an assurance that you can appreciate and utilize this concept in solving real-life problems.

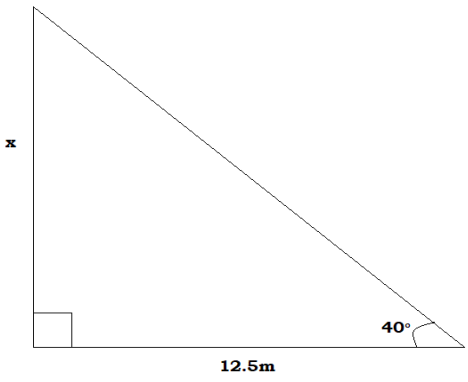
What's In

Preliminary Exercises:

A. Answer the following questions. Write your answer on your answer sheet.
   1) Sine function is the ratio of what sides of a right triangle?
   2) Secant function is the ratio of what sides of a right triangle?
   3) Tangent function is the ratio of what sides of a right triangle?
   4) Cosine function is the ratio of what sides of a right triangle?
   5) What does SOH-CAH-TOA stand for?

B. Solve for the value of x or θ in the following triangles below. Write your complete solution on your answer sheet.

1.



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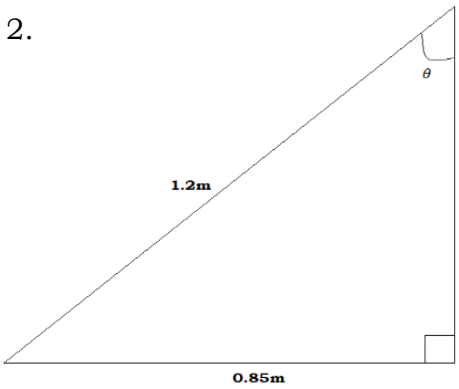

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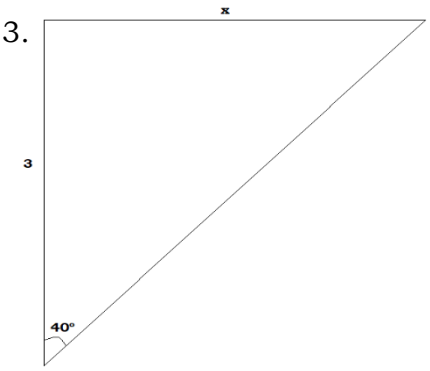

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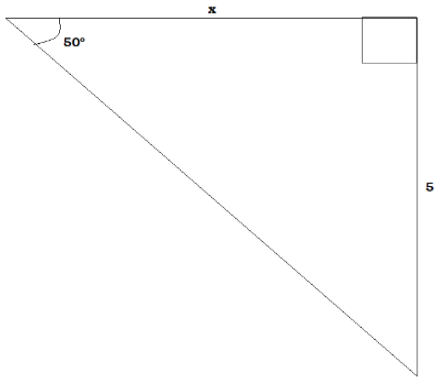
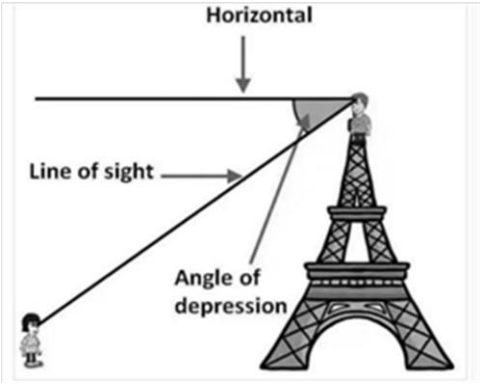


What's New

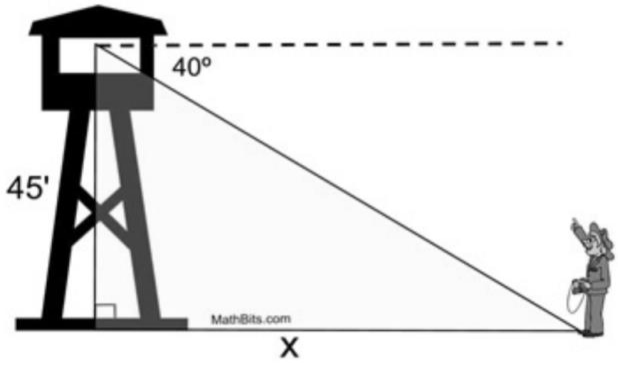
Angle of depression is an acute angle formed by a horizontal line (eye level) and the line of sight of the observer when looking at an object that is located lower than the observer.

Line of sight is a line that connects the eye of an observer to the object being observed.



The angle of depression can be found using any of the trigonometric functions.

1) From the top of a fire tower, a forest ranger sees his partner on the ground at an angle of depression of 40°. If the tower is 45 feet in height, how far is the partner from the base of the tower?



Solution: Let x be the distance of the partner from the base of the tower.

$\tan 40° = \frac{45}{x}$

$0.8390 = \frac{45}{x}$

$(0.8390)\ x = 45$

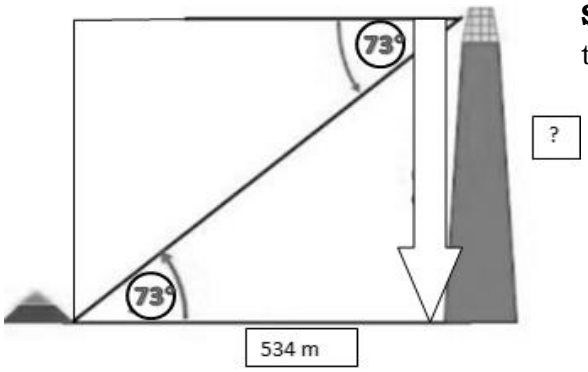
$x = \frac{45}{0.8390}$

$x = 53.6353$

$\mathbf{x \approx 53.64}$

53.64 ft is the distance of the partner from the base of the tower.

2) From the top of an observation tower, the angle of depression of a campfire is 73˚. If the base of tower is 534 meters from the campfire, how tall is the tower?



Solution: Let y be the height of the tower.

$\tan 73° = \frac{y}{534}$

$3.2709 = \frac{y}{534}$

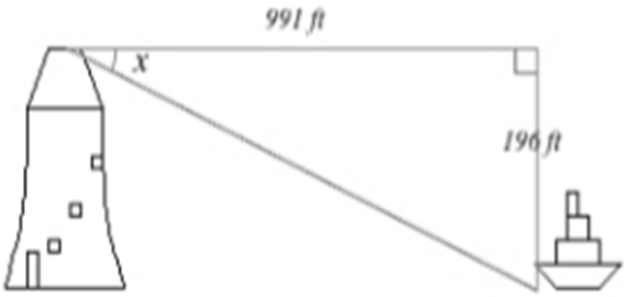
$y = (3.2709)\ (534)$

$y = 1{,}746.635$

$\mathbf{y \approx 1{,}746.64}$

The height of the tower is 1,746.64 m.

3) From the top of a 196-ft lighthouse, a man observes a ship that is 991 ft offshore. Find the angle of depression of the ship.



Solution:

Let **θ** the angle of depression of the ship from the top of the tower.

$\tan \theta = \frac{196}{991}$

$\tan \theta = 0.1978$

$\theta = \tan^{-1}(0.1978)$

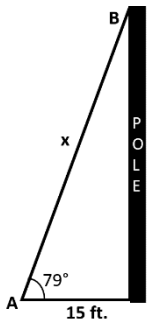
$\theta = 11.1887$

$\theta = 11° 11' 19.24"$

$\boldsymbol{\theta \approx 11°}$

The angle of depression of the ship from the top of the lighthouse is 11°.

4) Find the length of the wire connecting point A to point B if it makes an angle of 79° with the ground and if the distance from point A to the base of the pole is 15 ft. Refer to the figure at the right.



Solution: Let x be the distance from point A to point B.

$\cos 79° = \frac{15}{x}$

$0.1908 = \frac{15}{x}$

$(0.1908)\ x = 15$

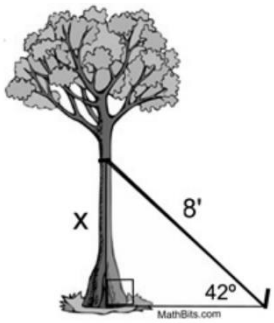
$x = \frac{15}{0.1908}$

$x = 78.6163$

$\mathbf{x \approx 78.62}$

The length of wire connecting point A to point B is 78.62 ft.

5) Michael plants a new tree and attaches a guy wire to help support the tree while its roots take hold. An eight-foot wire is attached to the tree and to a stake on the ground. From the stake, the guy wire makes an angle of 42º with the ground. Find to the *nearest tenth of a foot*, the height of the connection point on the tree.



Solution: Let x be the height of the connection point on the tree.

$\sin 42° = \frac{x}{8}$

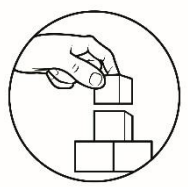
$0.6691 = \frac{x}{8}$

$(0.6691)\ 8 = x$

$x = 5.3528$

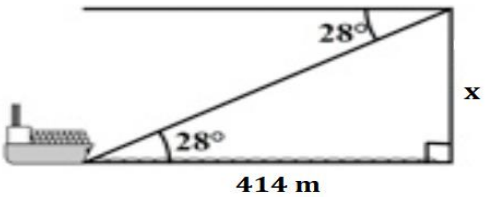
$\mathbf{x \approx 5.4}$

The connection point on the tree is 5.4 ft high.

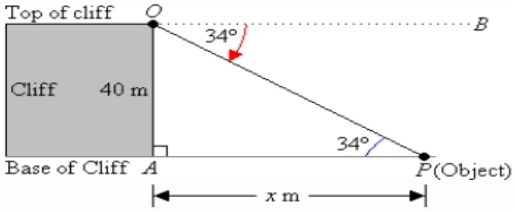


What's More

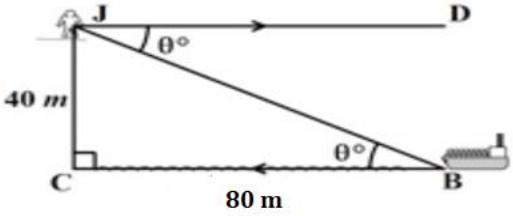
1. The angle of depression of a ship from the top of the vertical cliff is 28°. If the distance of the ship from the base of the cliff is 414 m, find the height of the cliff.



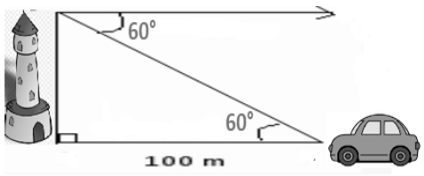
2. From the top of a 40-m high vertical cliff, the angle of depression of an object that is levelled with the base of the cliff is 34°. How far is the object from the base of the cliff?



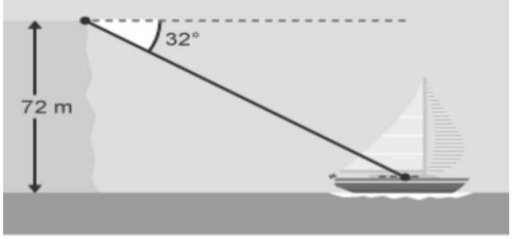
3. Jason is on top of a 40-m cliff. He observes a boat is 80 m away from the base of the cliff. Find the angle of depression of the boat.

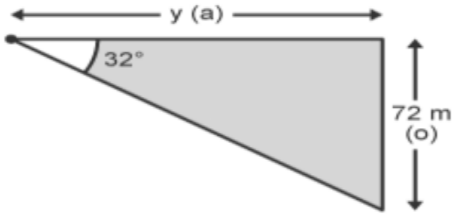


4. From the top of a tower, the angle of depression of a vehicle on the ground is 60°. If the vehicle is 100 meters away from the building, find the height of the tower.

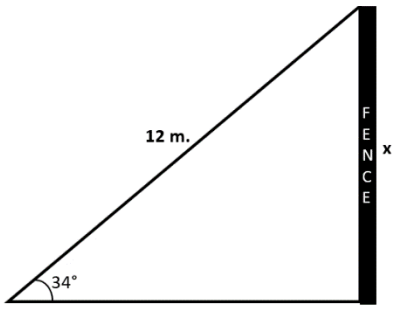


5. From the top of a 72-m high vertical cliff, a boat has an angle of depression of 32°. How far is the boat from the base of the cliff?

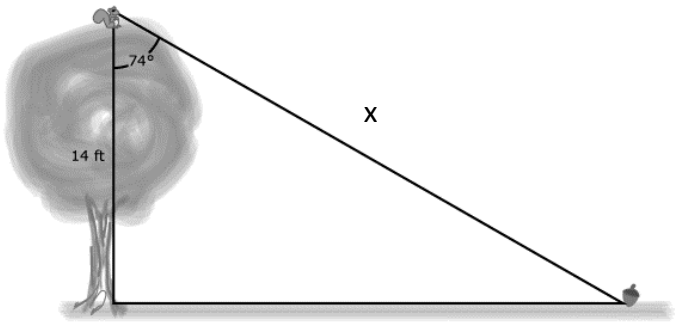




6. A 12-meter ladder leans against a fence at its top and forms an angle of 34° with the ground. How high is the fence?



7. A squirrel sits at the top a 14-foot-tall tree. He spies a nut on the ground at some distance away, and his line of sight makes a 74-degree with the tree. How far away is the nut from the squirrel?

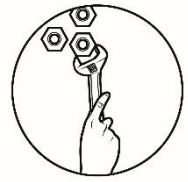


B. Solve each of the following problems in column A and match the result in Column B. Show your complete solution.

COLUMN A	COLUMN B
1. At an altitude of 12,000 feet, an airplane is 78,000 feet away from the runway as measured along the ground. What is the angle of depression of the runway from the plane?	A. 885 feet B. 858 feet
2. An observer in a lighthouse 60 ft above sea level finds that the angle of depression of a small boat on the water is 4°. Find the distance of the boat from the base of the lighthouse.	C. 8.65 feet D. 5.77 feet E. 2.89 feet
3. The trunk of a tree breaks into two parts during a typhoon, with the upper part forming an angle of 30° with the horizontal. If the top of the tree hit the ground 5 feet away from the base of the tree, what was the original height of the tree?	F. 8.85° G. 8.75° H. 81.25°

What I Have Learned

- ✓ Angle of depression is an acute angle formed by a horizontal line (eye level) and the line of sight of the observer when looking at an object that is located lower than the observer.
- ✓ Line of sight is a line that connects the eye of an observer to the object being observed.
- ✓ The angle of depression can be found using any of the trigonometric functions.



What I Can Do

Illustrate and solve the following problems accurately. Write your answers on your answer sheet.

1) From a window of a house, a child sees a cat on the ground that is 20 m away from the house. If the angle of depression of the cat is 75°, how far is the widow from the ground?

2) From the top of a tower, the angle of depression of a stone on the ground is 45^0. If the stone is 120 meters away from the building, find the height of the tower.

3) A 4-ft tall child is standing 3 feet away from a toy on the floor. What is the angle of depression of the toy?

4) From the top of a 200-ft building, a man sees the top of the 50-ft tree which is 100 away from the building. What is the angle of depression of the top of the tree?

5) The top of a vertical pole is connected to a rock on the ground by 60-meter wire. If the angle of depression of the rock from the top of the pole is 40°, A) how far is the rock from the foot of the tower? B) what is the height of the pole?

Assessment

Read and answer each of the following questions carefully. Write the letter of the correct answer on your paper.

1) From an airplane at an altitude of 1200 m, the angle of depression of a building on the ground is 28°. Find the distance from the plane to the building.
   a. 563.37 m b. 1,359.08 m c. 2,256.87 m d. 2,556.07 m

2) From a plane flying due east at 265 m above sea level, the angles of depression of two ships sailing due east are 35° and 25°. If the plane and the two ships lie on a vertical plane, how far apart are the two ships?
   a. 189.83 m b. 378.46 m c. 568.29 m d. 946.75 m

3) Standing on a cliff 380 meters above the sea, Pat sees an approaching ship and measures its angle of depression to be 9°. How far is the ship from shore?
   a. 2,429.13 m b. 2399.23 m c. 384.74 m d. 60.19 m

4) Lindsey is on a terrace of a building 9.2 meters high and observes that the angle of depression of Pete on the ground is 79°. Find the distance of Pete from the base of the building.
   a. 1.75 m b. 1.79 m c. 9.03 m d. 9.37 m

5) Aga who is directly below Leah on a terrace of a building, walks out into the street 4.3 meters. Find the angle of depression of Aga from Leah if the terrace is 9.2 meters from the ground.
   a. 25.05° b. 27.87° c. 62.13° d. 64.95°

6) Anita is in a plane that has an elevation of 5,000 feet. She spots a person sunbathing on the roof of a 2,250-foot building. The distance from the plane to the person is 3,600 feet. Find the angle of depression of the sunbather from the plane.
   a. 52.62° b. 49.81° c. 40.19° d. 37.38°

7) An airplane is flying at a height of 2 miles above the level ground. The angle of depression of a tree from the plane is 15°. What is the distance the plane must fly to be directly above the tree?
   a. 7.73 miles b. 7.46 miles c. 2.07 miles d. 1.93 miles

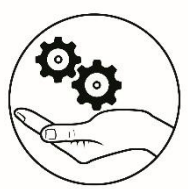
8) From the top of an 80-foot cliff, the angle of depression to a boat is 35°. How far is the boat from the base of the cliff?
   a. 56.01 $feet$ b. 97.66 $feet$ c. 114.25 $feet$ d. 139.48 $feet$

9) A swimming pool is 3 feet deep in the shallow end. The floor of the pool has a steady downward drop of 12° towards the deep end. If the pool is 50 feet long, how deep is the deep end?

a. 10.39 $feet$ b. 10.63 $feet$ c. 13.39 $feet$ d. 13.63 $feet$

10) A buoy in the ocean is observed from the top of a 40-meter-high oil rig. The angle of depression of the buoy from the top of the tower is 6°. How far is the buoy from the base of the oil rig?

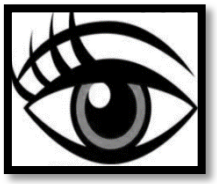
a. 382.67 m b. 380.57 m c. 40.22 m d. 4.20 m



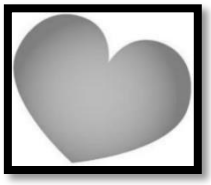
Additional Activities

"Guess the words"

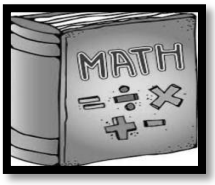
Directions: Solve the following problems and guess the correct word in a given picture in every item. Write your answers on your answer sheet.



1. A boy at the top of a tower, observes that the angle of depression of a ball on the ground to be 30˚. The ball is located 60 meters from the foot of the tower. Find the height of the tower. (eye)



2. Suppose you are a lifeguard looking down at a swimmer in a swimming pool. Your line of sight to the swimmer forms a 55-degree with the horizontal. You are 10 ft. up in your seat. How far is the swimmer from the base of your lifeguard stand? (heart)



3. If an airplane is 15,600 ft away from the runway and its altitude is 2,400 ft, what is the angle of depression of the runway from the plane? (operations)

"GUESS THE WORDS"

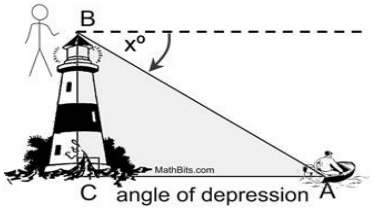
_____ _____ _____

1. 2. 3.

PROBLEM – BASED LEARNING WORKSHEET

Angle of Depression

You can think of the angle of depression in relation to the direction of your eyesight. Imagine that you are at the top of a lighthouse and you are looking straight horizontally (eye level). When you lower down (*depress*) your eyesight, you may see a boat on the water.

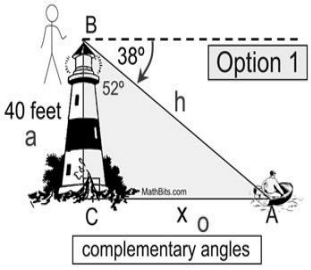


In this diagram, x^o marks the angle of depression of the boat at sea from the top of the lighthouse.

Notice that the horizontal line in the angle of depression is PARALLEL to the level ground. The fact that horizontal lines are always parallel guarantees that the alternate interior angles are equal in measure when cut by a transversal. In the diagram, the angle marked x° is equal in measure to m∠BAC. Simply stated, this means that ...

→ the angle of elevation = the angle of depression ←.

When solving a problem with an angle of depression you need to find the measure of an angle **INSIDE** the triangle. There are two options:



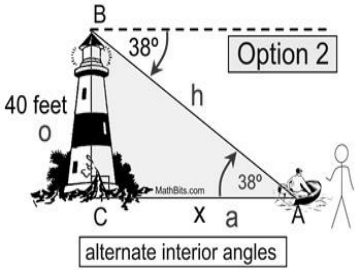
Option 1: Find the angle **inside** the triangle that is adjacent (next door) to the angle of depression. This adjacent angle will always be the complement of the angle of depression since the horizontal line and the vertical line are perpendicular (90°). In the diagram below, the adjacent angle measures 52°.

$$\tan 52^\circ = \frac{opposite}{adjacent} = \frac{x}{40}; \quad 1.279941632 = \frac{x}{40}; \quad x \approx 51 \text{ ft.}$$

Option 2: Utilize the fact that the angle of depression = the angle of elevation and label ∠BAC as 38° inside the triangle.

$$\tan 38^\circ = \frac{opposite}{adjacent} = \frac{40}{x}; \quad 0.7812856265 = \frac{40}{x}; \quad x \approx 51 \text{ ft.}$$

Notice that both options yielded the same answer.



LET'S ANALYZE

1. *It is an acute angle formed by the eye level (horizontal line) and the line of sight when looking at an object located lower than the observer.*
2. *What relationship does the horizontal line in the angle of depression have with the level ground?*
3. *What mathematical concept guarantees the fact that horizontal lines are always parallel?*

Answer Key

WHAT I KNOW

1) B
2) B
3) A
4) C
5) D
6) A
7) C
8) A
9) C
10) B

WHAT I CAN DO

1) h = 74.64 m
2) h = 120 m
3) $\theta = 53.13^0$
4) $\theta = 56.31^0$
5) A. x = 45.96 m
   B. y = 38.57 m

ASSESSMENT

1) D
2) A
3) B
4) B
5) D
6) B
7) B
8) C
9) D
10) B

PROBLEM-BASED LEARNING WORKSHEET

1) Angle of Depression
2) Parallel
3) Alternate interior angles are equal in measure

WHAT'S IN

A.

1) $\sin\theta = \frac{opposite}{hypotenuse}$
2) $\sec\theta = \frac{hypotenuse}{adjacent}$
3) $\tan\theta = \frac{opposite}{adjacent}$
4) $\cos\theta = \frac{adjacent}{hypotenuse}$
5) SOH CAH TOA means
6) $\sin\theta = \frac{opposite}{hypotenuse}$
7) $\cos\theta = \frac{adjacent}{hypotenuse}$
8) $\tan\theta = \frac{opposite}{adjacent}$

B.

1) x = 10.49
2) $\theta = 45.1^0$
3) x = 2.52
4) x = 4.2

WHAT'S MORE

A.

1) The height of the cliff is 220.13m.
2) The distance of the object from the base of the cliff is 59.30m.
3) The angle of depression from Japan to the boat is 27^0.
4) The height of the tower is 173.21m.
5) The distance between the boat and the base of the cliff is 115.22m.
6) The height of the fence is 6.71 m.
7) The distance between the nut and the squirrel is 50.80 ft.

B.

1) G (8.75^0)
2) B (858 feet)
3) C (8.65 feet)

References

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Bryant, Merden L. et. Al., Mathematics Learner's Material 9, First Edition. Department of Education. Pasig City, Philippines.

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Figures

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E-Search

To further explore the concept learned today and if possible to connect the internet, you may visit the following links:

- https://study.com/academy/practice/quiz-worksheet-angles-of-elevation-depression.html
- https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-modeling-with-right-triangles/a/angles-of-elevation-and-depression
- https://www.youtube.com/watch?v=A3iuSkQYeVM
- https://www.youtube.com/watch?v=Pp6h4GMMXI4

  https://www.youtube.com/watch?v=vl90rqRjXZo

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