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DEPARTMENT OF EDUCATION

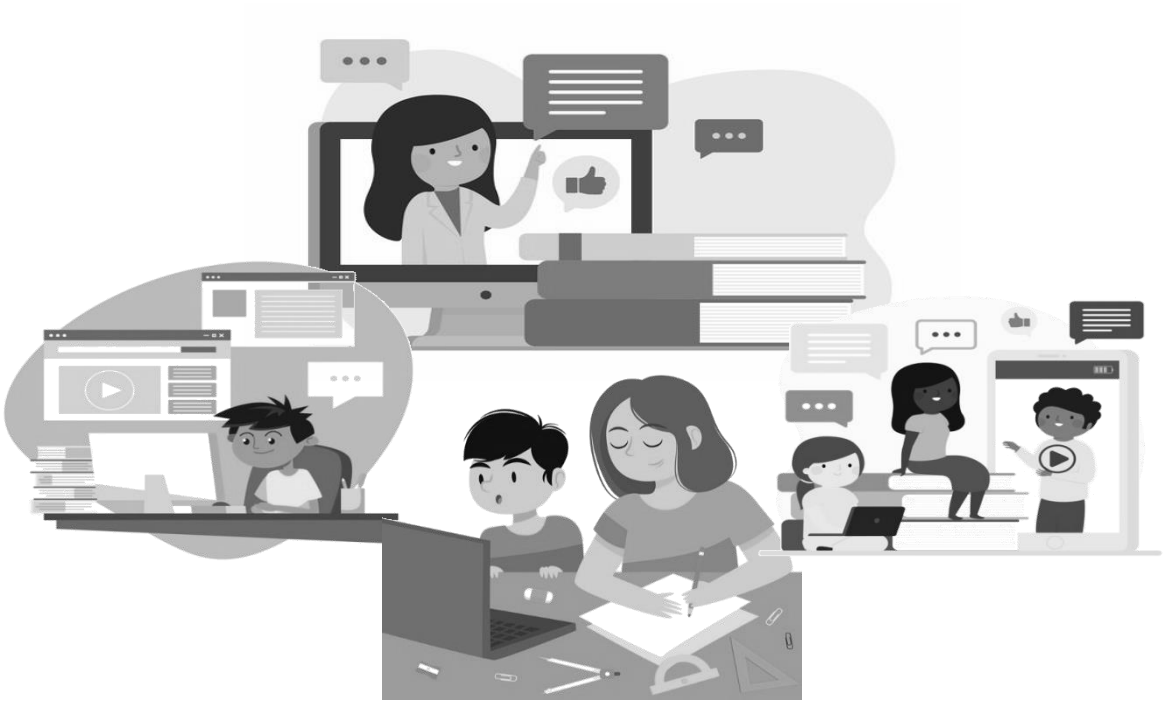
Mathematics

Quarter 1-Module 7

Relation of Roots and Coefficient of a Quadratic Equation

Week 2

Learning Code - M9AL-Ib-4



ALTERNATIVE DELIVERY MODE
ADM

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Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writers: Rowena F. Reyes - TI Analynn M. Argel - MTII

Editor: Sally C. Caleja – Head Teacher VI
Ma. Teresa M. Mallari – Head Teacher VI
Maita G. Camilon – Head Teacher VI

Validators: Remylinda T. Soriano, EPS, Math
Angelita Z. Modesto, PSDS
George B. Borromeo, PSDS

Illustrator: Writers

Layout Artist: Writers

Management Team: Malcolm S. Garma, Regional Director
Genia V. Santos, CLMD Chief
Dennis M. Mendoza, Regional EPS in Charge of LRMS and Regional ADM Coordinator
Maria Magdalena M. Lim, CESO V, Schools Division Superintendent
Aida H. Rondilla, Chief-CID
Lucky S. Carpio, Division EPS in Charge of LRMS and Division ADM Coordinator

MODULE 7

RELATION OF THE ROOTS AND COEFFICIENTS OF A QUADRATIC EQUATION

It is common that if we check the solutions of a quadratic equation, we simply substitute the solution to the original equation. But there is another method for this. In this module, you will discover the relationship of the roots and coefficients of a quadratic equation and apply this concept in checking the roots and in constructing a quadratic equation.

WHAT I NEED TO KNOW

LEARNING COMPETENCY

The learners will be able to:

- describe the relationship between the coefficients and the roots of a quadratic equation. **M9AL-Ib-4**

WHAT I KNOW

Find out how much you already know about the relation of the roots and coefficients of a quadratic equation. Write the letter that you think is the best answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. Given the quadratic equation $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$, which of the following is the sum of its roots?
   A. a
   B. c
   C. $-\frac{b}{a}$
   D. $\frac{c}{a}$

2. Which of the following is the product of the roots of $ax^2 + bx + c = 0$?
   A. $\frac{c}{a}$
   B. $-\frac{b}{a}$
   C. c
   D. a

3. What is the sum of the roots of the quadratic equation $2x^2 + 10x - 14 = 0$?
   A. -5
   B. 5
   C. 7
   D. -7

4. In the quadratic equation $2x^2 + 10x - 14 = 0$, which of the following is the product of its roots?
   A. -5
   B. 5
   C. 7
   D. -7

5. The roots of a quadratic equation are -9 and 3. Which of the following quadratic equations has these roots?
   A. $x^2 - 9x + 3 = 0$
   B. $x^2 + 6x - 27 = 0$
   C. $9x^2 - 3x + 27 = 0$
   D. $3x^2 - 9x + 27 = 0$

6. The sum of the roots of a quadratic equation is -8. If one of the roots is 7, which of the following is the quadratic equation?
   A. $x^2 + 8x - 105 = 0$
   B. $x^2 - 8x + 105 = 0$
   C. $x^2 + 8x + 105 = 0$
   D. $x^2 - 8x - 105 = 0$

7. The product of the roots of a quadratic equation is 48. If one of the roots is -16, which of the following could be the equation?
   A. $x^2 - 48 = 0$
   B. $x^2 + 16 = 0$
   C. $x^2 + 19x + 48 = 0$
   D. $x^2 - 19x + 48 = 0$

8. $\frac{3+\sqrt{7}}{4}$ and $\frac{3-\sqrt{7}}{4}$ are the roots of which of the following quadratic equations?
   A. $8x^2 - 12x + 1 = 0$
   B. $8x^2 + 12x + 1 = 0$
   C. $12x^2 - 8x + 1 = 0$
   D. $12x^2 - 8x - 1 = 0$

9. Let s and p be the roots of the equation $4x^2 - tx = n$.What is $s + p$?
   A. $\frac{t}{4}$
   B. $\frac{n}{4}$
   C. 4
   D. $-t$

10. What is the value k if one root of $3kx^2 - 2kx + 9 = 0$ is 1?
   A. -18
   B. -9
   C. 9

WHAT'S IN

Critical Thinking

Refer to the two quadratic equations below. Use the concepts learned in the previous module to fill in the blanks with the correct answer. Then, answer the question at the end.

$x^2 + 4x - 21 = 0$	$x^2 + 8x - 20 = 0$
What are the values of : 1. $a =$ _____, 2. $b =$ _____, 3. $c =$ _____?	**What are the values of :** 1. $a =$ _____, 2. $b =$ _____, 3. $c =$ _____?
Using any method, what are the roots? 1. $x_1 =$ _____ 2. $x_2 =$ _____	**Using any method, what are the roots?** 1. $x_1 =$ _____ 2. $x_2 =$ _____
Perform the indicated operation. 1. $x_1 + x_2 =$ _____ 2. $x_1 \cdot x_2 =$ _____	**Perform the indicated operation.** 1. $x_1 + x_2 =$ _____ 2. $x_1 \cdot x_2 =$ _____

Question:

1. What do you observe about the sum and product of the roots in relation to the values of a, b, and c?

WHAT'S NEW

Communication

The Author of Analytic Art

- ✓ He was a French Mathematician, lawyer, and consultant to Kings Henry III and IV of France.
- ✓ He made some significant developments in Algebra
- ✓ He wrote some books about geometry and trigonometry.
- ✓ He calculated π to decimal places using a polygon with 393 216 sides.
- ✓ He discovered the *Viète's formula* which is the connection between the roots and coefficients of polynomial.

Since a quadratic expression is considered as a polynomial, hence, the person behind in this module's topic is **François Viète**.

WHAT IS IT

The generalization about the sum and product of the roots of a quadratic equation can be derived by using the quadratic formula as a method in finding the roots.

Given the standard form, $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, the roots can be expressed as:

$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

So,

SUM	**PRODUCT**
$x_1 + x_2$ $= \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$ $= \frac{-2b}{2a}$ $= -\frac{b}{a}$	$x_1 \cdot x_2$ $= \frac{-b+\sqrt{b^2-4ac}}{2a} \cdot \frac{-b-\sqrt{b^2-4ac}}{2a}$ $= \frac{b^2-(b^2-4ac)}{4a^2}$ $= \frac{b^2-b^2+4ac}{4a^2}$ $= \frac{4ac}{4a^2}$ $= \frac{c}{a}$

Therefore, the sum of the roots of a quadratic equation in terms of its coeffcients is denoted by: $x_1 + x_2 = -\frac{b}{a}$	On the otherhand, the product of the roots of a quadratic equation in terms of its cofficients is denoted by: $x_1 \cdot x_2 = \frac{c}{a}$

Now let's go back to the standard form of a quadratic equation, $ax^2 + bx + c = 0$.

If we divide both sides of the equation by a, the cofficient of x^2, the equation becomes

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

What do you notice?

This implies that, in a quadratic equation of the form $ax^2 + bx + c = 0$, where a = 1, the negative of the sum of the roots is the cofficient of x, and the constant term is the product of the roots. Similarly , given the sum and poduct of the roots, we can obtain the quadratic equation.

Example 1: Without solving for the solutions of the quadratic equation, $4x^2 + 2x - 5 = 0$, find the sum and product of its roots.

Solution: Given the equation $4x^2 + 2x - 5 = 0$, we have $a = 4$, $b = 2$ and $c = -5$.

Hence,

the sum of the roots is $-\frac{b}{a} = -\frac{2}{4} =$ $\boxed{-\frac{1}{2}}$

the product of the roots is $\frac{c}{a} =$ $\boxed{-\frac{5}{4}}$

Example 2: Using the concept of the sum and product of roots, form the quadratic equation whose roots are $\frac{1}{5}$ *and* $\frac{3}{7}$.

Solution:

Sum of the roots: $\frac{1}{5} + \frac{3}{7} = \frac{7+15}{35} =$ $\boxed{\frac{22}{35}}$ $-\frac{b}{a}$

Product of the roots: $\frac{1}{5} \cdot \frac{3}{7} =$ $\boxed{\frac{3}{35}}$ $\frac{c}{a}$

Hence, following the equation form $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, we have:

$\boxed{x^2 - \frac{22}{35}x + \frac{3}{35} = 0}$ → $\boxed{35x^2 - 22x + 3 = 0}$

WHAT'S MORE

Collaboration, Critical Thinking

I. Find the sum and product of the roots of each equation placed inside the rectangle:

$2x^2 - 5x + 1 = 0$ **SUM:** _____ **PRODUCT:** _____	$4x^2 + 6x + 3 = 0$ **SUM:** _____ **PRODUCT:** _____	$2x^2 + 11x - 15$ **SUM:** _____ **PRODUCT:** _____

II. Find the quadratic equation whose roots are given below. You may discuss your idea to your housemate.

$x_1 = 2$ $x_1 = 3$ **QUADRATIC EQUATION:** _____	$x_1 = \frac{1}{2}$ $x_1 = \frac{2}{5}$ **QUADRATIC EQUATION:** _____	$x_1 = \sqrt{5}$ $x_1 = -\sqrt{5}$ **QUADRATIC EQUATION:** _____

III. If s and p are the roots of the equation $5x^2 - mx = n$, what is $s + p$?

WHAT I HAVE LEARNED

Given the quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, the following are the relations regarding its roots $x_1, and\ x_2$ and its coefficients.

(a) The **sum of the roots** is the additive inverse of the quotient of the coefficients b and a denoted by:

$$x_1 + x_2 = -\frac{b}{a}$$

(b) The **product of the roots** is the quotient of the coefficients c and a denoted by:

$$x_1 \cdot x_2 = \frac{c}{a}$$

WHAT I CAN DO

I. Without solving, find the sum and product of the roots of each of the following equations.

1. $4x^2 + 3x + 8 = 0$
2. $d^2 + 7d - 5 = 0$
3. $6x^2 - 5x + 8 = 0$
4. $2z^2 + 7z - 3 = 0$
5. $10m^2 - 8m + 3 = 0$

II. Find the quadratic equations whose root are:

1. $\frac{1}{2}, 3$
2. $7, -3$
3. $-4, 7$
4. $\frac{1}{2}, \frac{2}{3}$
5. $\frac{11}{2}, \frac{5}{2}$

III. If r and t are the roots of the equation $7x^2 - gx = n$, what is $r + t$?

ASSESSMENT

Write the letter of the correct answer on your answer sheet. If your answer is not among the choices, write E together with your final answer.

1. Given the quadratic equation $ax^2 + bx = -c$ where a, b, c are real numbers and $a \neq 0$, which of the following is the product of its roots?
   A. a
   B. c
   C. $-\frac{b}{a}$
   D. $\frac{c}{a}$

2. In the quadratic equation $ax^2 + bx = -c$, which of the following is the sum of its roots?
   A. $\frac{c}{a}$
   B. $-\frac{b}{a}$
   C. c
   D. a

3. What is the sum of the roots of the quadratic equation $8x^2 + 5x - 14 = 0$?
   A. $-\frac{5}{8}$
   B. $-\frac{5}{14}$
   C. 5
   D. 14

4. In the quadratic equation $8x^2 + 5x - 14 = 0$, which of the following is the product of its roots?
   A. -5
   B. $-\frac{7}{4}$
   C. 5
   D. $\frac{7}{4}$

5. The roots of a quadratic equation are 6 and -3. Which of the following quadratic equations has these roots?
   A. $x^2 - 3x - 18 = 0$
   B. $x^2 + 3x + 18 = 0$
   C. $x^2 - 3x + 18 = 0$
   D. $x^2 + 3x - 18 = 0$

6. Let s and p be the roots of the equation $x^2 - tx = n$.What is sp?
   A. n B. $-n$ C. t D. $-t$

7. If $\frac{2+\sqrt{3}}{5}$ and $\frac{2-\sqrt{3}}{5}$ are the roots, which of the following is the quadratic equation?
   A. $25x^2 - 20x + 1 = 0$
   B. $25x^2 + 20x - 1 = 0$
   C. $25x^2 - 20x - 1 = 0$
   D. $25x^2 + 20x + 1 = 0$

8. The product of the roots of a quadratic equation is 36. If one of the roots is 9, which of the following could be the equation?
   A. $x^2 + 13x + 48 = 0$
   B. $x^2 - 13x - 48 = 0$
   C. $x^2 - 13x + 36 = 0$
   D. $x^2 + 13x - 48 = 0$

9. What is the value k if one root of $(3k+1)x^2 - 5kx - 5 = 0$ is 2?
   A. $-\frac{1}{2}$
   B. $\frac{1}{2}$
   C. 5
   D. -5

10. The sum of the roots of a quadratic equation is 5. If one of the roots is 1, which of the following is the quadratic equation?
    A. $x^2 + 5x - 4 = 0$
    B. $x^2 - 5x + 4 = 0$
    C. $x^2 + 5x + 4 = 0$
    D. $x^2 - 5x - 4 = 0$

ADDITIONAL ACTIVITIES

Critical Thinking and Creativity

ENCLOSED! Mang Kevin owns a rectangular lot. The perimeter of the lot is 100 meters and the area is 450 square meters.

1source: https://www.thisiscolossal.com/tags/assemblage/

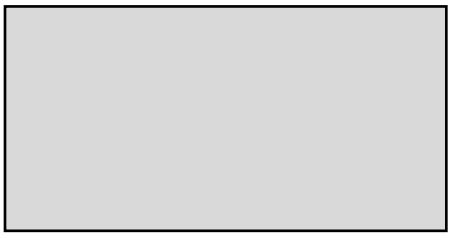
Using the concept of the sum and product of roots of a quadratic equation, how would you determine the length and the width of the rectangular lot? Provide a quadratic equation representing this scenario.

PROBLEM – BASED WORKSHEET

Read and study the given scenario. Then answer the questions that follow.

The Rectangular Garden

Ms. Ann bought a rectangular lot with a perimeter of 38m and an area of $84m^2$.



Supposing the width and the length are x and y, respectively,

1. What equations represent the area and perimeter of the lot?

2. Use the sum and product of the roots to find the quadratic equation that describes the situation.

3. Find the dimensions of the rectangular lot.

E-Search

You may also check the following link for your reference and further learnings on solving quadratic equation using quadratic formula.

- https://www.khanacademy.org/math/math-for-fun-and-glory/aime/2003-aime/v/sum-of-polynomial-roots-proof
- https://www.youtube.com/watch?v=D57Gg7CGsTI

REFERENCES

Morgan, F. M., & Paige, B. L. (n.d.). *Algebra 2.* America: Henry Holt and Company.

Oronce, O. A., Santos, G. C., & Ona, M. I. (n.d.). *Interactive Mathematics III (Concepts,Structures, adn Metods for High School.* Manila: Rex Book Store

https://www.britannica.com/topic/paddy

https://www.freepik.com/free-vector/woman-with-long-hair-teaching-online_7707557.htm

https://www.freepik.com/free-vector/kids-having-online-lessons_7560046.htm

https://www.freepik.com/free-vector/illustration-with-kids-taking-lessons-online-design_7574030.htm

https://www.thisiscolossal.com/tags/assemblage/

WHAT I KNOW	ASSESSMENT
1. C	1. D
2. A	2. B
3. A	3. A
4. D	4. B
5. B	5. A
6. A	6. B
7. C	7. A
8. A	8. C
9. A	9. B
10. B	10. B

PROBLEM – BASED WORKSHEET

Solutions:

1. For the area of the lot: $xy = 84$

For the perimeter of the lot: $2x + 2y = 38$

2. The area represents the product of the roots while the semi perimeter, $x + y = 19$, represents the sum of the roots. Thus, the equation will be:

$$x^2 - 19x + 84 = 0$$

Solving for x using quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(84)}}{2(1)}$$

$$x = \frac{19 \pm \sqrt{25}}{2}$$

$$x = \frac{19 \pm 5}{2}$$

Therefore:

$$x = \frac{19+5}{2} \quad \text{or} \quad x = \frac{19-5}{2}$$

$$x = 12 \qquad\qquad x = 7$$

Thus, the length of the lot is 12m and the width of the lot is 7m.

ANSWER KEY

RELAY!

For $x^2 + 4x - 21 = 0$:	For $x^2 + 8x - 20 = 0$:
I. 1. $a = 1$ 2. $b = 4$ 3. $c = -21$	I. 1. $a = 1$ 2. $b = 8$ 3. $c = -20$
II. 1. $x_1 = -7$ 2. $x_2 = 3$	II. 1. $x_1 = -10$ 2. $x_2 = 2$
III. 1. $x_1 + x_2 = -4$ 2. $x_1 \cdot x_2 = -21$	III. 1. $x_1 + x_2 = -8$ 2. $x_1 \cdot x_2 = -20$

WHAT'S MORE

I.

1. $\frac{5}{2}, \frac{1}{2}$
2. $-\frac{3}{2}, \frac{3}{4}$
3. $-\frac{11}{2}, -\frac{15}{2}$

II. C

1. $x^2 - 5x + 6 = 0$
2. $10x^2 - 9x + 2 = 0$
3. $x^2 - 5 = 0$

III. $5x^2 - mx = n$

$a = 5,\ b = -m$

$$x_1 + x_2 = -\frac{b}{a}$$

$$s + p = -\frac{(-m)}{5}$$

$$\boldsymbol{s + p = \frac{m}{5}}$$

WHAT I CAN DO

I.

1. sum: $-\frac{3}{4}$;product: 2
2. sum: -7 ;product:$--5$
3. sum: $\frac{5}{6}$;product: $\frac{4}{3}$
4. sum: $-\frac{7}{2}$;product:$-\frac{3}{2}$
5. sum: $\frac{4}{5}$;product: $\frac{3}{10}$

II.

1. $2x^2 - 7x + 3 = 0$
2. $x^2 - 4x - 21 = 0$
3. $x^2 - 3x - 28 = 0$
4. $6x^2 - 7x + 2 = 0$
5. $4x^2 - 32x + 55 = 0$

III. $7x^2 - gx = n$

$a = 7,\ b = -g$

$$x_1 + x_2 = -\frac{b}{a}$$

$$r + t = -\frac{(-g)}{7}$$

$$\boldsymbol{r + t = \frac{g}{7}}$$