

GENERAL MATHEMATICS

Quarter 2: Module 3

Annuities



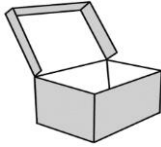
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What I Need to Know

Hello Grade 11 learners! In this module, you will learn how to:

- illustrate simple and general annuities (**M11GM-IIc-1**);
- distinguish between simple and general annuities (**M11GM-IIc-2**);
- find the future value and present value of both simple annuities and general annuities (**M11GM-IIc-d-1**);
- calculate the fair market value of a cash flow stream that includes an annuity. **M11GM-IId-2**; and,
- calculate the present value and period of deferral of a deferred annuity (**M11GM-IId-2**).

You can say that you have understood the lesson in this module if you can already:

1. define annuity;
2. identify the different classification of annuities;
3. illustrate simple and general annuity;
4. distinguish simple annuities from general annuities;
5. determine the future and present values of simple annuities;
6. calculate the periodic payment of simple annuity.
7. determine the future and present values of general annuities;
8. calculate the periodic payment of general annuity.
9. define fair market value or economic value;
10. illustrate the cash flow of the two offers using time diagrams;
11. calculate the fair market value of a cash flow stream that includes an annuity;
12. define deferred annuity; and
13. calculate the present value and period of deferral of a deferred annuity.



What I Know

Choose the letter that corresponds to the exact answer.

1. It is an annuity where the length of the payment is not the same as the length of the interest compounding period.
A. Annuity Certain C. General Ordinary Annuity
B. General Annuity D. Simple Annuity
2. A general annuity in which the periodic payment is made at the end of the payment interval.
A. Annuity Certain C. General Ordinary Annuity
B. General Annuity D. Simple Annuity

3. If payment made is yearly and the interest rate is compounded yearly, then what type of annuities does it illustrate?

- A. Annuity Certain C. General Ordinary Annuity
B. General Annuity D. Simple Annuity

4. Which problem illustrates general annuity?

- A. The payment is monthly and the interest rate is compounded monthly.
B. The payments are done quarterly and the interest rate being charged is also compounded quarterly.
C. The payment is annually and the interest rate is compounded annually.
D. The payments are done quarterly but the interest rate being charges is monthly.

5. Mr. Dela Cruz deposited ₱3,500.00.00 every six months to an account that earns 8% interest compounded semi-annually. What type of annuity does it illustrate?

- A. Annuity Certain C. General Ordinary Annuity
B. General Annuity D. Simple Annuity

For item number 6-7, please refer to the problem below.

Suppose Mrs. Taleno would like to save ₱4,000.00 every month in a fund that gives 8% compounded monthly.

6. What is the value of the interest rate per period (j)?

- A. 0.0015 B. 0.0025 C. 0.0055 D. 0.0067

7. How much is the future value of her saving after 6 months?

- A. ₱24,405.61 B. ₱34,405.61 C. ₱44,405.61 D. ₱54,405.61

8. The buyer of a house and lot pays ₱150,000.00 cash and ₱6,000.00 every month for 15 years. If it is compounded monthly at 8%, then solve for the present value.

- A. ₱534,246.80 B. ₱626,329.18 C. ₱705,890.09 D. ₱865,233.34

9. Ted deposit ₱3, 000.00 each month into an account earning 2.5% interest compounded monthly. How much will he earn in 10 years?

- A. ₱208 515.82 B. ₱308 515.82 C. ₱408 515.82 D. ₱508 515.82

10. Mang Celso borrowed a certain amount from Mang Kanor. He agrees to pay the interest plus the principal by paying ₱25,000.00 each year for 2 years. How much money did he borrow if the interest is 6% compounded quarterly?

- A. ₱44, 433.40 B. ₱45, 433.40 C. ₱46, 433.40 D. ₱47, 433.40

11. It is a cash flow (payment stream) on a particular date refers to a single amount that is equivalent to the value of the payment stream at that date.

- A. Cash flow C. Focal date
B. Fair market value D. Time diagram

For item 12 - 15, please refer to the problem below.

Mrs. Palma received two offers on a house and lot that she wants to sell. Mr. Tan has offered ₱40,000.00 and ₱800,000.00 lump sum payment 5 years from now. Mr. Lim has offered ₱40,000.00 plus ₱30,000.00 every quarter for 5 years. The money can earn 5% compounded annually. (Focal date is at $t = 0$)

12. Which of the following illustrates the correct cash flow using the time diagram of Mr. Tan?

A. **40,000**

					800,000
0	1	2	3	4	5

B. **40,000**

	30,000	30,000	30,000	30,000	30,000
0	1	2	3	4	5

C.

	40,000	40,000	...		800,000
0	1	2	...		20

D.

	30,000	30,000	...		30,000
0	1	2	...		5

13. Find the present value of Mr Tan's offer.

A. ₱529,180.75 B. ₱626,820.93 C. ₱666,820.93 D. ₱824,766.07

14. Solve for the fair market value of Mr. Tan's offer.

A. ₱529,180.75 B. ₱626,820.93 C. ₱ 666,820.93 D. ₱824,766.07

15. Calculate the fair market value of Mr. Lim's offer.

A. ₱ 529,180.75 B. ₱626,820.93 C. ₱666,820.93 D. ₱824,766.07

16. An annuity in which the periodic payment does not begin until a given time interval has passed.

A. Annuity due C. General annuity
B. Deferred annuity D. Ordinary annuity

17. Which of the following is the formula for computing the present value of a deferred annuity?

A. $P = F(1+j)^n$ C. $P = R \frac{1-(1+j)^{-(k+n)}}{j}$
B. $P = R \frac{1-(1+j)^{-n}}{j}$ D. $R \frac{1-(1+j)^{-(k+n)}}{j} - R \frac{1-(1+j)^{-k}}{j}$

18. What does R represent in the formula of finding the present value of a deferred annuity?

A. interest rate per period C. number of payments
B. number of conversion periods D. regular payment

19. Marwin availed of a loan from a bank that gave him an option to pay ₱30,000.00 monthly for 2 years. The first payment is due after 4 months. How much is the present value of the loan if the interest rate is 10% compounded monthly?

A. ₱88,520.58 B. ₱634,139.67 C. ₱722,660.25 D. ₱832,901.07

20. Monthly payment of ₱20,000.00 for 3 years that will start 8 months from now. Find the period of deferral in the deferred annuity.

A. 4 months B. 5 months C. 6 months D. 7 months





What's In

A. Solve the following problems that involve simple and compound interest.

1. A bank offers 0.30% annual simple interest rate of a particular deposit. How much interest will be earned if ₱1.5 million is deposited in this savings account for 2 years?
2. Find the maturity value and the compound interest if ₱12,000.00 is compounded annually at an interest rate of 3% in 4 years.

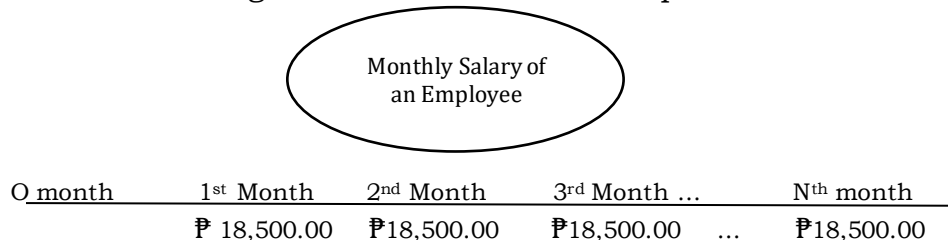
B. Complete the table by computing the interest rate per period and total number of conversion periods.

Nominal Rate $i^{(m)}$	Interest Compounded	Frequency of conversion (m)	Interest rate for each conversion period (j)
10%	Semi-annually	(1)	(2)
(3)	Quarterly	(4)	0.015
12%	Monthly	(5)	(6)



What's New

Consider the diagram below and answer the questions that follow.



1. How much salary was received by the employee monthly?
2. What have you notice on the monthly salary of the employee?
3. What do you call a fixed amount of money receive by an employee?
4. Can you cite other examples of fixed amount of payments?

Answer:

1. ₱18,500.00
2. The employee receives fixed monthly salary payments.
3. Annuity
4. Apartment rentals, Motorcycle loan, housing loan, and personal loan.



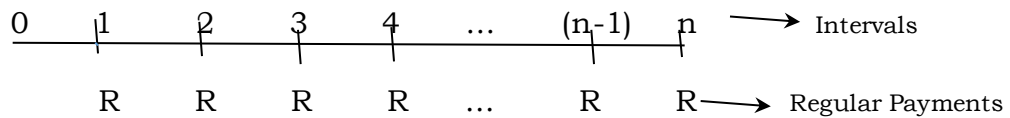
What is It

In this lesson, we will illustrate and distinguish simple and general annuity.

Definition:

Annuity is a sequence of payments made at equal (fixed) intervals or period of time.

Let us consider this cash flow diagram representing an annuity



where R represent a fixed amount of money made a regular interval. The intervals are not always yearly; the intervals can be monthly, quarterly, semi-annually or annually.

Here are some examples of Annuity:

1. Monthly salaries of workers/employees where they receive a fixed amount of money every month
2. Apartment rentals where the boarders pay a fixed amount of rent every month

Classifications of Annuity Based on Interest Period and Payment Interval Annuity

A. Simple Annuity – is an annuity where the payment interval is the same as the interest period.

Example: 1. The payment is made yearly and the interest rate is also compounded yearly.

2. The payments are done quarterly and the interest rate being charged is also compounded quarterly.

B. General Annuity – is an annuity in which the annuity payments and compounding periods are not the same.

Example: 1. The payments are done yearly but the interest rate is compounded semi-annually.

2. The payments are done quarterly but the interest rate being charged is monthly.

Classifications of Annuity Based on Payment Schedule

A. Ordinary Annuity – is an annuity that is paid or received at the end of the time period.

Example: Monthly Salaries

B. Annuity Due – is an annuity that is paid or received at the beginning of the time period.

Example: Apartment rentals where the boarders must pay first before they can occupy the apartment

Classifications of Annuity Based on Duration

A. Annuity Certain – an annuity where the payments are made for a fixed period of time.

Example: Yearly payments for 10 years

Quarterly payments for 3 years

B. Contingent Annuity- an annuity in which the payments extend over an indefinite length of time.

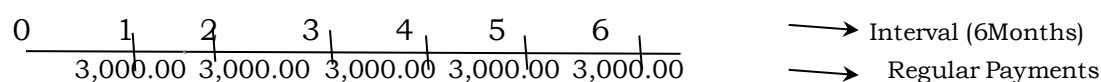
Example: Monthly pension for a person until he/she dies

Let us now illustrate using the cash flow in a time diagram and determine if the given problem is a simple annuity or general annuity.

Example 1: Mrs. Remoto would like to save ₱3,000.00 every month in a fund that gives 9% compounded monthly for 6 months.

Solution: The cash flow in a time diagram is shown below.

Given: $R = 3,000$, $n = 6$ payments

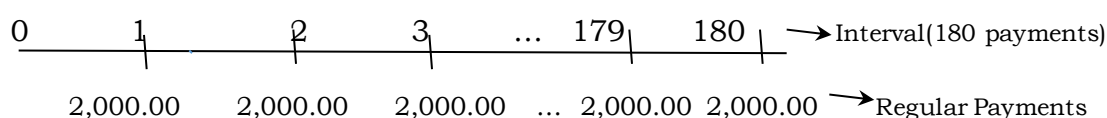


Since the payment interval and interest period are the same, therefore example 1 is an example of simple annuity.

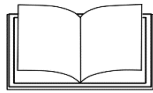
Example 2: Cris started to deposit ₱2,000.00 monthly in a fund that pays 6% compounded quarterly for 15 years.

Solution: The cash flow in a time diagram is shown below

Given: $R = 2,000$, $n = 12(15) = 180$ payments



Since the payment interval and interest period are not the same, therefore example 2 is an example of general annuity.



What's More

Given the problems below, illustrate the cash flow using a time diagram and determine if it is simple or general annuity.

1. Steve, a high school student would like to save ₱4 000.00 every year in a fund that gives 10% compounded annually for 4 years.
2. Mr. Castro deposited ₱3 000.00 every month in the bank that pays 3% compounded quarterly for 4 years.



What I Have Learned

Complete the statement/s. Write your answer on the space provided.

1. Annuity is a sequence of payments made at _____ or period of time.
2. _____ is an annuity where the payment interval is the same as the interest period.
3. The payments are done quarterly and the interest rate being charged is also compounded quarterly. This statement is an example of _____ annuity.
4. _____ is an annuity in which the annuity payments and compounding periods are not the same.
5. The classifications of annuity based on payment schedule are _____ and _____.



What I Can Do

Determine if the following statements illustrate simple annuity or general annuity.

1. The payments are done yearly and the interest rate is compounded annually.
2. The payments are made quarterly but the interest rate being charged is semi-annually.
3. The payments are done monthly and the interest rate is compounded quarterly.
4. The payments are made semi-annually and the interest rate being charged is compounded semi-annually



Additional Activities

Illustrate the cash flow using a time diagram and identify if the problem shows simple or general annuity.

1. Teodoro wants to earn his first million in 20 years from now. To do this, he will save money at the end of every 2 months and put it on an investment company where his money will grow by 7% semi-annually.
2. Maricar intends to borrow money as capital for her business. To pay it off, she will pay ₱4,000.00 at the end of every month for 5 years. The money is compounded quarterly.

Lesson 2

Future Value and Present Value of Simple and General Annuity



What's In

Re-arrange the jumbled letters to get the correct answer and write it on the space provided.

1. UITYANN – a sequence of payments made at equal (fixed) intervals or periods of time. _____
2. GENTCONTIN UITYANN – an annuity in which the payments extend over an indefinite (or indeterminate) length of time. _____
3. IMSELP UITYANN – an annuity where the payment interval is the same as the interest period. _____
4. DIARYNOR UITYANN – a type of annuity in which the payments are made at the end of each payment interval. _____
5. EEGNRLA UITYANN – an annuity where the payment interval is not the same as the interest period. _____



What's New

Let us study this situation and answer the questions that follow.

Mr. Kevin would like to save ₱2,500.00 every month in a fund that gives 6% compounded monthly. How much is the amount or future value of his savings after 6 months?

1. Give the values of the following:

Periodic payment: $R =$ _____ Term: $t =$ _____

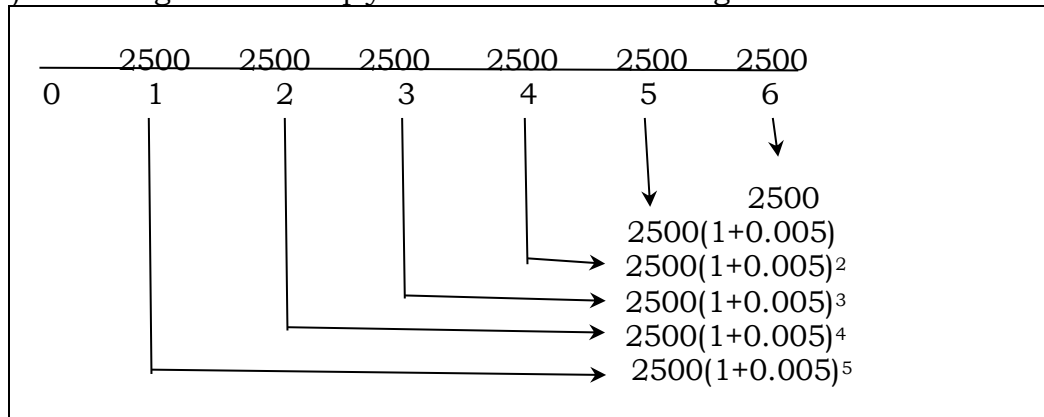
Interest rate per annum: $i^{12} =$ _____

Number of conversion per year: $m =$ _____

Interest rate per period: $j =$ _____

2. What type of annuity does this problem illustrate?

The problem above requires you to find the future value at the end the term (F). This diagram will help you to solve the following.



3. Calculate the amount to which the first ₱2500.00 will grow using the compound interest formula: $F = P(1+j)^n$

$$F = 2,500(1 + 0.005)^5$$

4. Calculate the amount to which the second ₱2500.00 will grow using the compound interest formula: $F = P(1+j)^n$

$$F = 2,500(1 + 0.005)^4$$

5. Calculate the amount to which the third ₱2500.00 will grow using the compound interest formula: $F = P(1+j)^n$

$$F = 2,500(1 + 0.005)^3$$

6. Calculate the amount to which the fourth ₱2500.00 will grow using the compound interest formula: $F = P(1+j)^n$

$$F = 2,500(1 + 0.005)^2$$

7. Calculate the amount to which the fifth ₱2500.00 will grow using the compound interest formula: $F = P(1+j)^n$

$$F = 2,500(1 + 0.005)^1$$

8. How much is the amount or future value of his savings after 6 months?



What is It

Definition of Terms

Term of an annuity (t) – time between the first payment interval and last payment interval

Regular or Periodic payment (R) – the amount of each payment

Amount (Future Value) of an annuity (F) – sum of the future values of all payments to be made during the entire term of the annuity



Present value of an annuity (P) – sum of present values of all the payments to be made during the entire term of the annuity

Future Value of ordinary annuity:

The future value F of an ordinary annuity is given by

$$F = R \frac{(1+j)^n - 1}{j}$$

Where: R is the regular payment;
j is the interest rate per period; and
n is the number of payments

Example 1: In order to save for his high school graduation, Jonathan decided to save ₱250.00 at the end of each month. If the bank pays 0.30% compounded monthly, how much will his money be at the end of 5 years?

Given: $R = 250$ $m = 12$ $i^{12} = 0.30\% = 0.0030$
 $t = 6 \text{ years}$ $j = \frac{0.0030}{12} = 0.00025$ $n = (5)(12) = 60 \text{ periods}$

Find: F

Solution:

$$F = R \frac{(1+j)^n - 1}{j}$$

$$F = 250 \frac{(1+0.00025)^{60} - 1}{0.00025}$$

$$F = 15,111.16$$

Present Value of ordinary annuity:

The present value P of an ordinary annuity is given by

$$P = R \frac{1 - (1+j)^{-n}}{j}$$

Where: R is the regular payment;
j is the interest rate per period; and
n is the number of payments

The **cash value** or **cash price** of a purchase is equal to the down payment (if there is any) plus the present value of the installment payments.

Example 2: Mr. Ribaya paid ₱200,000.00 as down payment for a car. The remaining amount is to be settled by paying ₱16,200.00 at the end of the month for 5 years. If the interest is 10.5% compounded monthly, what is the cash price of his car?

Given: Down payment = ₱200,000.00

$R = 16,200$ $i^{12} = 0.105$ $m = 12$

$j = \frac{0.105}{12} = 0.00875$ $t = 5 \text{ years}$ $n = m(t) = 12(5) = 60 \text{ periods}$



Find: Cash value or cash price of the car

Solution:

The time diagram for the installment payments is given by:

P = ?

	16,200	16,200	16,200	...	16,200
0	1	2	3		60

The **present value** of this ordinary annuity is given by

$$P = R \frac{1-(1+j)^{-n}}{j}$$

$$P = 16,200 \frac{1-(1+0.00875)^{-60}}{0.00875}$$

$$P = 753,702.20$$

$$\begin{aligned} \text{Cash value} &= \text{Down payment} + \text{present value} \\ &= 200,000 + 753,702.20 \\ &= 953,702.20 \end{aligned}$$

Hence, the cash price of the car is ₱953,702.20.

Periodic payment R of an annuity:

Periodic payment R can also be solved using the formula for amount F or present value P of an annuity.

$$\begin{aligned} F &= R \frac{(1+j)^n - 1}{j} \quad \longrightarrow \quad R = \frac{F}{\frac{(1+j)^n - 1}{j}} \\ P &= R \frac{1-(1+j)^{-n}}{j} \quad \longrightarrow \quad R = \frac{P}{\frac{1-(1+j)^{-n}}{j}} \end{aligned}$$

Where: R is the regular payment
P is the present value of an annuity
F is the future value of an annuity
j is the interest rate per period; and
n is the number of payments

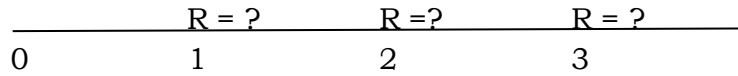
Example 3: Paolo borrowed ₱100,000.00. He agrees to pay the principal plus interest by paying an equal amount of money each year for 3 years. What should be his annual payment if interest is 8% compounded annually?

$$\begin{aligned} \text{Given: } P &= 100,000 & i^1 &= 0.08 & m &= 1 \\ j &= 0.08 & t &= 3 \text{ years} & n &= mt = (1)(3) = 3 \text{ periods} \end{aligned}$$



Solution: The cash flow of this annuity is illustrated in the time diagram given below.

$$P = 100,000$$



$$R = \frac{P}{\frac{1-(1+j)^{-n}}{j}}$$

$$R = \frac{100,000}{\frac{1-(1+0.08)^{-3}}{0.08}}$$

$$R = 38,803.35$$

Thus, the man should pay ₱38,803.35 every year for 3 years.

Future and Present Value of a General Ordinary Annuity

The future value F and present value P of a general ordinary annuity is given by

$$F = R \frac{(1+j)^n - 1}{j} \quad \text{and} \quad P = R \frac{1-(1+j)^{-n}}{j}$$

where R is the regular payment;

j is the equivalent interest rate per payment interval converted from the interest rate per period; and

n is the number of payments.

The extra step occurs in finding j: the given interest rate per period must be converted to an equivalent rate per payment interval.

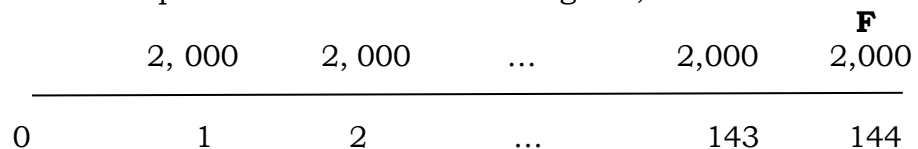
Example 1: Finding the Future Value of General Ordinary Annuity

Mr. Cruz started to deposit ₱2,000.00 monthly in a fund that pays 5% compounded quarterly. How much will be in the fund after 12 years?

Given: R = 2,000 n = 12(12) = 144 payments $i^4 = 0.05$ m = 4

Solution:

The cash flow for this problem is shown in the diagram, below.



Step 1: Convert 5% compounded quarterly to its equivalent interest rate for monthly payment interval.

$$F_1 = F_2$$

$$P\left(1+\frac{i^{12}}{12}\right)^{(12t)} = P\left(1+\frac{i^4}{4}\right)^{(4t)}$$

$$\left(1+\frac{i^{12}}{12}\right)^{12} = \left(1+\frac{0.05}{4}\right)^{(4)}$$

$$\left(1+\frac{i^{12}}{12}\right)^{12} = (1.0125)^4$$



$$\begin{aligned} \left(1 + \frac{i^{12}}{12}\right)^{12} &= (1.0125)^4 \\ \left(1 + \frac{i^{12}}{12}\right)^{12} &= [(1.0125)^4]^{(1/12)} \\ \frac{i^{12}}{12} &= (1.0125)^{1/3} - 1 \\ \frac{i^{12}}{12} &= 0.004149425 \text{ or } j = 0.004149425 \end{aligned}$$

Thus, the interest rate per monthly payment interval is 0.004149425 or 0.4149425%.

Step 2: Apply the formula in finding the future value of an ordinary annuity using the computed equivalent rate.

$$\begin{aligned} F &= R \frac{(1+j)^n - 1}{j} \\ F &= 2000 \frac{(1+0.004149425)^{144} - 1}{0.004149425} \\ F &= 392\,996.53 \end{aligned}$$

Thus, Mr. Cruz will have ₱392 996.53 in the fund after 12 years.

Number of Decimal Places

When solving for an equivalent rate, say $j = (1.0125)^{(1/3)} - 1$, six or more decimal places will be used. If you use fewer or more decimal places, your answer may differ from the answer provided. You can ignore these discrepancies, but it is suggested that you use at least six decimal places or the exact value.

Example 2: Finding the Present Value of General Ordinary Annuity

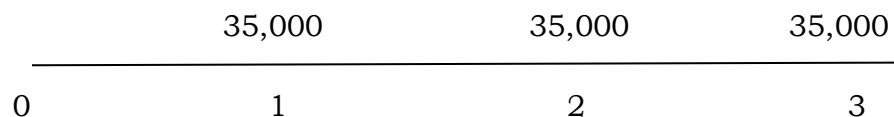
Jennifer borrowed money from Lea. She agrees to pay the principal plus interest by paying ₱35,000.00 each for 3 years. How much money did she borrow if interest is 6% compounded quarterly?

Given: $R = 35,000$ $i^4 = 0.06$ $m = 4$ $n = 3$ payments
Find: Present Value

Solution:

The cash flow for this problem is shown in the diagram, below.

P = ?



Step 1: Convert 6% compounded quarterly to its equivalent interest rate for each payment interval.

$$\begin{aligned} F_1 &= F_2 \\ P\left(1 + \frac{i^1}{1}\right)^{(1t)} &= P\left(1 + \frac{i^4}{4}\right)^{(4t)} \\ \left(1 + \frac{i^1}{1}\right)^1 &= \left(1 + \frac{0.06}{4}\right)^4 \\ \left(1 + \frac{i^1}{1}\right) &= (1 + 0.02)^4 \\ \frac{i^1}{1} &= (1.02)^4 - 1 \end{aligned}$$



$$\frac{i^1}{1} = 0.061364 \text{ or } j = 0.061364$$

Thus, the interest rate per payment interval is 0.061364 or 6.1364%.

Step 2: Apply the formula in finding the present value of an ordinary annuity using the computed equivalent rate $j = 0.061364$.

$$P = R \frac{1 - (1 + j)^{-n}}{j}$$

$$P = 35,000 \frac{1 - (1 + 0.061364)^{-3}}{0.061364}$$

$$P = 93,319.82$$

Hence, Jennifer borrowed ₱93,319.83 from Lea.

Example 3: Calculating the Periodic payment of General Ordinary Annuity

To accumulate a fund of ₱500,000.00 in 3 years, how much should Aling Paring deposit in her account every 3 months if it pays an interest of 5.5% compounded annually?

Given: $F = 500,000$ $i^{(1)} = 0.055$
 $m = 4$ $n = (4)(3) = 12 \text{ deposits}$

Find: Quarterly deposit (R)

Solution:

Step 1: Convert 5.5% compounded annually to its equivalent interest rate for quarterly deposit.

$$F_1 = F_2$$

$$P(1 + \frac{i^4}{4})^{(4t)} = P(1 + \frac{i^1}{1})^{(1t)}$$

$$(1 + \frac{i^4}{4})^4 = (1 + \frac{0.055}{1})^1$$

$$(1 + \frac{i^4}{4})^4 = (1 + 0.055)$$

$$\frac{i^4}{4} = (1.055)^{1/4} - 1$$

$$\frac{i^4}{4} = j = 0.013475$$

Thus, the interest rate per payment interval is 0.013475 or 1.3475%.

Step 2: Find the periodic deposit (R) of an ordinary annuity using the equivalent rate $j = 0.013475$.

$$R = \frac{F}{\frac{(1 + j)^n - 1}{j}}$$

$$R = \frac{500,000}{\frac{(1 + 0.013475)^{12} - 1}{0.013475}}$$

$$R = 38,668.16$$

Hence, the periodic payment or deposit is ₱38,668.16.



Example 4: Nadine is the beneficiary of a ₱1,000,000.00 insurance policy. Instead of taking the money as lump sum, she opted to receive a monthly stipend over a period of 10 years. If the insurance policy pays an interest of 5% compounded annually, what will be her monthly stipend?

Given: $P = 1,000,000$ $i^{(1)} = 0.05$ $m = 1$
 $n = (10)(12) = 120$ monthly stipends

Find Quarterly deposit (R)

Solution:

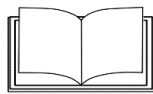
Step 1: Convert 5% compounded annually to its equivalent interest rate for monthly interval.

$$\begin{aligned} F_1 &= F_2 \\ P\left(1 + \frac{i^{12}}{12}\right)^{(12t)} &= P\left(1 + \frac{i^1}{1}\right)^{(1t)} \\ \left(1 + \frac{i^{12}}{12}\right)^{12} &= \left(1 + \frac{0.05}{1}\right)^1 \\ \left(1 + \frac{i^{12}}{12}\right)^{12} &= (1 + 0.05) \\ \frac{i^{12}}{12} &= (1.05)^{(1/12)} - 1 \\ \frac{i^{12}}{12} = j &= 0.004074 \end{aligned}$$

Step 2: Find the monthly stipend R of an ordinary annuity using the equivalent rate $j = 0.004074$.

$$\begin{aligned} R &= \frac{\frac{P}{j}}{\frac{1 - (1+j)^{-n}}{j}} \\ R &= \frac{1,000,000}{\frac{1 - (1 + 0.004074)^{-120}}{0.004074}} \\ R &= 10,552.28 \end{aligned}$$

Hence, the monthly stipend R is ₱10,552.28.



What's More

Solve the following problems:

- A. Find the future value F of the ordinary annuity.
 1. Monthly payments of ₱3,000.00 for 4 years with interest rate of 3% compounded monthly
- B. Find the present value P of the ordinary annuity.
 1. Monthly payments of ₱2,000 for 5 years with interest rate of 12% compounded monthly
- C. Find the periodic payments of the ordinary annuity.
 1. Monthly payment of the future value of ₱50,000.00 for 1 year with an interest rate of 10% compounded monthly
- D. Find the future value (F) of the general annuity.
 1. Monthly payments of ₱3,000.00 for 4 years with interest rate of 3% compounded quarterly

- E. Find the present value (R) of the general annuity.
1. Monthly payments of ₱2,000.00 for 5 years with interest rate of 12% compounded quarterly
- F. Find the periodic payments of the following general annuity.
1. Monthly payment of the future value of ₱50,000.00 for 1 year with an interest rate of 10% compounded quarterly



What I Have Learned

Analyze the given problem below and answer the questions that follow. Write your answer on the space provided

Problem: A teacher saves ₱3,000.00 every 6 months in a bank that pays 2% compounded monthly. How much will be her savings after 10 years?

1. What type of annuity is being illustrated from the problem?

2. What are the values of the following?
 $R =$ _____ $i^{12} =$ _____
 $n =$ _____ $m =$ _____
3. What is being asked in the problem? Is it the Future value, Present Value, or Periodic Payment?

4. Illustrate the cash flow for this problem.
5. Find j . Convert 2% compounded monthly to its equivalent rate for each semi-annual payment interval.
 $j =$ _____
6. How much is the savings of the teacher after 10 years. Apply the formula in finding the future value of an ordinary annuity using the computed equivalent rate.
 $F =$ _____



What I Can Do

Solve the following problems.

1. Sherly started to deposit ₱18,000.00 semi-annually in a fund that pays 5% compounded semi-annually. How much will be in the fund after 10 years?
2. A furniture set is for sale at ₱18,000.00 in cash or on monthly installment of ₱3,200.00 for 6 months at 8% compounded semi-annually. Which is lower: the cash price or the present value of the installment term?



3. Find the periodic payments of monthly installment of appliances cash prize of ₱20,000.00 for 6 months with an interest rate of 6% compounded semi-annually.

Rubrics for problem solving

Score	Description
15 points	Complete solutions and correct answer
10 points	Incomplete solutions and correct answer
5 points	Incomplete solutions and incorrect answer
No point earned	No output at all



Additional Activities

Solve the following problems.

1. Teacher Kaye is saving ₱2,000.00 every month by depositing it in a bank that gives an interest of 1% compounded quarterly. How much will she save in 5 years?
2. Vladimir purchased a new car of ₱99,000.00 down payment and ₱15,000.00 every month. If the payments are based on 7% compounded quarterly what is the total cash price of his car?

Lesson 3

Fair Market Value of a Cash Flow Stream that includes an Annuity



What's In

A. Answer the following questions.

1. What do you call the amount of each payment in the annuities?
2. In the given formula $F = R \frac{(1+j)^n - 1}{j}$, what does **n** represent?
3. Give the formula in calculating the present value of an ordinary annuity.

Problem: Aling Marie started to deposit ₱3,000.00 quarterly in a fund that pays 6% compounded quarterly for 6 years.

4. Give the values of the following:

R = _____ m = _____ i^4 = _____ j = _____
t = _____ n = _____

5. How much will be in the fund after 6 years?

What's New

Consider the two offers below and answer the questions that follow.

Mr. Dela Cruz's offer	Mr. Onahon's offer
₱20,000.00 down payment	₱20,000.00 down payment
₱200,000.00 after 5 years	₱10,000.00 every quarter for 5 years
2% compounded annually	2 % compounded annually

1. How can you illustrate Mr. Ocampo's offer using a time diagram?
2. How can you illustrate Mr. Cruz's offer using a time diagram?
3. What is the number of payments of Mr. Ocampo's offer? How about Mr. Cruz's offer?
4. In your own opinion, which do you think has a higher offer or higher market value and why?



What is It

Let us look with the following definitions.

A **cash flow** is a term that refers to payments received (cash inflows) or payments or deposits made (cash outflows). Cash inflows can be represented by positive numbers and cash outflows can be represented by negative numbers.

The **Fair market value** or **economic value** of a cash flow (payment stream) on a particular date refers to a single amount that is equivalent to the value of the payment stream at that date. This particular date is called the **focal date**.

Here are the following examples in finding the fair market value of each offer.

Example 1. Mr. Ribaya received two offers on a lot that he wants to sell.

Mr. Ocampo has offered ₱50,000.00 and 1 million lump sum payment 5 years from now. Mr. Cruz has offered ₱50,000.00 plus ₱40,000.00 every quarter for five years. Compare the fair market value of the two offers if the money can earn 5% compounded annually.

Given:

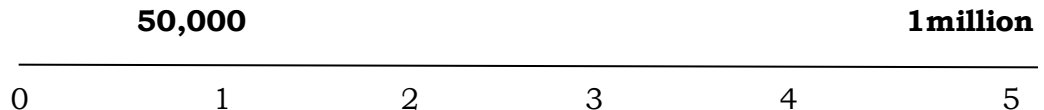
Mr. Ocampo's offer	Mr. Cruz's offer
₱50,000.00 down payment	

₱1,000,000.00 after 5 years	₱50,000.00 down payment ₱10,000.00 every quarter for 5 years
-----------------------------	---

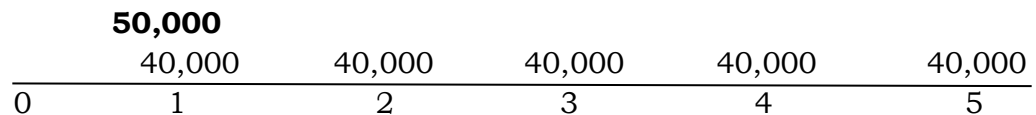
Find: fair market value of each other

Solution: We illustrate the cash flows of the two offers using time diagrams.

Mr. Ocampo's offer:



Mr. Cruz's offer:



Choose a focal date and determine the values of the two offers at the focal date. For example, the focal date can be the date at the start of the term.

Since the focal date is at $t = 0$, compute for the present value of each offer.

Mr. Ocampo's offer: Since ₱50,000.00 is offered today, then its present value is still ₱50,000.00. The present value of ₱1,000,000.00 offered for 5 years from now is

$$P = F(1+j)^{-n}$$

$$P = 1,000,000(1+0.05)^{-5}$$

$$P = 783,526.20$$

Hence, the present value is ₱783,526.20.

Fair Market value (FMV) = Down payment + Present Value

$$\text{FMV} = 50,000 + 783,526.20$$

$$\text{FMV} = 833,526.20$$

Thus, the fair market value of Mr. Ocampo's offer is ₱833,526.20.

Mr. Cruz's offer: We first compute for the present value of a general annuity with quarterly payments but with annual compounding at 5%.

Solve the equivalent rate, compounded quarterly of 5% compounded annually.

$$F_1 = F_2$$

$$P\left(1+\frac{i^4}{4}\right)^{(4)(5)} = P\left(1+\frac{i^1}{1}\right)^{(1)(5)}$$

$$\left(1+\frac{i^4}{4}\right)^{20} = \left(1+\frac{0.05}{1}\right)^5$$



$$1 + \frac{i^4}{4} = (1.05)^{\left(\frac{1}{4}\right)}$$

$$\frac{i^4}{4} = (1.05)^{\left(\frac{1}{4}\right)} - 1$$

$$\frac{i^4}{4} \text{ or } j = 0.012272$$

The present value of an annuity is given by:

$$P = R \frac{1 - (1+j)^{-n}}{j}$$

$$P = 40,000 \frac{1 - (1 + 0.012272)^{-20}}{0.012272}$$

$$P = 705,572.70$$

Hence, the present value is ₱705,572.70.

Fair Market Value = Down payment + Present value

Fair Market Value = 50,000 + 705,572.70

Fair Market Value = 755,572.70

Thus, the fair market value of Mr. Cruz's offer is ₱755,572.70.

Hence, Mr. Ocampo's offer has a higher market value. The difference between the market values of the two offers at the starts of the term is
 $833,526.20 - 755,572.70 = 77,953.50$.

Alternate Solution (Focal date at the end of the term):

Mr. Ocampo's offer:

The future value of ₱1,000,000.00 at the end of the term at 5% compounded annually is given by

$$F = P(1+j)^n$$

$$F = 50,000(1+0.05)^5$$

$$F = 63,814.08$$

The fair market value of this offer at the end of the term is ₱63,814.08 plus 1,000,000 pesos amounting to ₱1,063,814.08.

Mr. Cruz's offer:

The future value of this ordinary general annuity is given by:

$$F = R \frac{(1+j)^n - 1}{j}$$

$$F = 40,000 \frac{(1 + 0.012272)^{20} - 1}{0.012272}$$

$$F = 900,509.40$$

The future of ₱50,000.00 at the end of the term is ₱63,814.08, which was already determined earlier.

Fair Market Value = 900,509.40 + 63,814.08
 = ₱964,323.48



As expected, Mr. Ocampo's offer still has a higher market value, even if the focal date was chosen to be at the end of the term. The difference between the market values of the two offers at the end of the term is

$$₱1,063,814.08 - ₱964,323.48 = ₱99,490.60.$$

You can also check that the present value of the difference is the same as the difference computed when the focal date was the start of the term:

$$\begin{aligned} P &= 99,490.60(1+0.05)^{-5} \\ &= ₱77,953.49. \end{aligned}$$

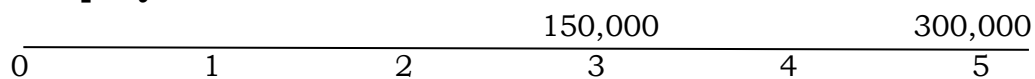
Example 2: Company A offers ₱150,000.00 at the end of 3 years plus ₱300,000.00 at the end of 5 years. Company B offers ₱25,000.00 at the end of each quarter to the next 5 years. Assume that money is worth 8% compounded semi-annually. Which offer has a better market value?

Given:

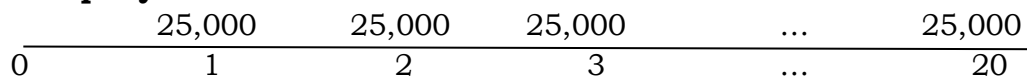
Company A	Company B
₱150,000.00 at the end of 3 years ₱300,000.00 at the end of 5 years	₱25,000.00 at the end of each quarter for the next 5 years

Solution: Illustrate the cash flow of the two offers using the time diagrams.

Company A offer:



Company B offer:



Suppose that the selected focal date is the start of the term. Since the focal date is the start of the term, compute for the present value of each offer.

Company A offer:

The present value of ₱150,000.00 three years from now is

$$\begin{aligned} P_1 &= F(1+j)^{-n} \\ P_1 &= 150,000(1+0.04)^{-6} \\ P_1 &= 118,547.18 \end{aligned}$$

The present value of 300,000 five years from now is

$$\begin{aligned} P_2 &= F(1+j)^{-n} \\ P_2 &= 300,000(1+0.04)^{-10} \\ P_2 &= 202,669.25 \end{aligned}$$



$$\text{Fair Market Value (FMV)} = P_1 + P_2$$

$$\text{Fair Market Value (FMV)} = 118,547.18 + 202,669.25$$

$$\text{Fair Market Value (FMV)} = 321,216.43$$

Hence, the fair market value of Company A is ₱321,216.43.

Company B offer: Compute for the present value of a general annuity with quarterly payments but with semi-annual compounding at 8%.

Solve the equivalent rate, compounded quarterly, of 8% compounded semi-annually.

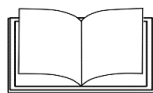
$$\begin{aligned} F_1 &= F_2 \\ P\left(1 + \frac{i^4}{4}\right)^{(4)(5)} &= P\left(1 + \frac{i^2}{2}\right)^{(2)(5)} \\ \left(1 + \frac{i^4}{4}\right)^{20} &= \left(1 + \frac{0.08}{2}\right)^{10} \\ 1 + \frac{i^4}{4} &= (1.04)^{(1/2)} \\ \frac{i^4}{4} &= (1.04)^{(1/2)} - 1 \\ \frac{i^4}{4} = j &= 0.019803903 \end{aligned}$$

The present value of an annuity is given by

$$\begin{aligned} P &= R \frac{1 - (1+j)^{-n}}{j} \\ P &= 25,000 \frac{1 - (1 + 0.019803903)^{-20}}{0.019803903} \\ P &= 409,560.4726 \end{aligned}$$

Hence, the fair market value of Company B is ₱409,560.4726.

Therefore, Company B offer is preferable since its market value is larger.



What's More

Solve the following problems:

Problem 1: Mr. Laconsay offers ₱200,000.00 at the end 4 years plus ₱400,000.00 at the end of 6 years. Mr. Ladesma offers ₱35,000.00 at the end of each quarter for the next 6 years. Assume that money is worth 6% compounded semi-annually, which offer has a better market value? (Focal date is the start of the term)

Problem 2: Mrs. Lopez received two offers on a house and lot that she wants to sell. Mrs. Taleno has offered ₱60,000.00 and ₱1,200,000.00 lump sum payment 6 years from now. Mrs. Daves has offered ₱60,000.00 plus ₱50,000.00 every quarter for 6 years. Compare the fair market values of the two offers if the money can earn 8% compounded annually. Which offer has a higher market value? (Focal date is the start of the term)



What I Have Learned

Consider the problem below and answer the questions that follow:

Problem: Company X has offered ₱30,000.00 and ₱400,000.00 lump sum payment 4 years from now. Company Y has offered 30,000.00 plus ₱20,000.00 every quarter for 4 years. Compare the fair market values of the two offers if the money can earn 5% compounded annually. (Focal date is the start of the term).

1. Illustrate the cash flow of the two offers using time diagrams.
2. Company X: Calculate the present value of ₱400,000.00 offered 4 years from now.
3. What is the market value of Company X?
4. Company Y: Calculate the present value of a general annuity with quarterly payments but with annual compounding at 5%.
5. What is the fair market value of Company Y?
6. Which offer has a higher market value?



What I Can Do

Solve the given problem:

Investment in Sunrise Company is ₱100,000.00 at the end of 5 years plus ₱24,000.00 annually for 4 years afterwards. Investment in XYZ Company B offers ₱50,000.00 semi-annually ₱15,000.00 every 6 months after 6 years. Assume that money is worth 9% compounded annually. Which investment is preferable?

Rubrics:

Score	Description
15 points	Complete solutions and correct answer
10 points	Incomplete solutions and correct answer
5 points	Incomplete solutions and incorrect answer
No point earned	No output at all





Additional Activities

Solve.

Mr. Melendes offers ₱180,000.00 at the end of 5 years plus ₱260,000.00 at the end of 7 years. Mr. Santos offers ₱40,000.00 at the end of each quarter for the next 7 years. Assume that money is worth 9% compounded semi-annually. Which offer has a better market value?

Lesson 4

Deferred Annuity



What's In

Consider the two offers below and answer the questions that follow.

Problem: Mr. Donato wants to sell his car. He received two offers from Buyer A and Buyer B. Buyer A has offered ₱30,000.00 and ₱300,000.00 lump sum, payment 5 years from now. Buyer B has offered ₱30,000.00 plus ₱25,000 every quarter for 5 years. Assume that the money can earn 5% compounded annually.

1. Illustrate the cash flows of the two offers using time diagrams.
2. If the focal date can be the date at the start of the term, find the present value and market value of the two buyers.
3. Which offers has a highest market value?
4. What is the difference between the market values of the two offers?



What's New

Let us study this situation and answer the questions that follow.

There will be monthly payments of ₱35,000.00 for 3 years that will start 10 months from now.

1. How much is the regular payment from the given situation?
2. Based from the given situation above, when is the first payment?
3. What do you call the periods or months that is not included from the first payment?

Answer.

1. $R = 35,000$
2. The first payment is at time 10.
3. Period of deferral



What is It

Definition of Terms

Deferred Annuity is an annuity that does not begin until a given time interval has passed.

Period of Deferral is a time between the purchase of an annuity and the start of the payments for the deferred annuity.

Time Diagram for a Deferred Annuity

	R^*	$R^* \dots$		R^*	R	$R \dots$	R
0	1	2 ...		k	k+1	k+2	k+n

In this time diagram the period of deferral is k because the regular payments of R start at time K+1.

The notation R^* represent k “artificial payments”, each equal to R but are not actually paid during the period of deferral.

To determine the present value of a deferred annuity, find the present value of all k+n payments (including the artificial payments).

Present Value of a Deferred Annuity

The present value of a deferred annuity is given by

$$P = R \frac{1 - (1+j)^{-(k+n)}}{j} - R \frac{1 - (1+j)^{-k}}{j}$$

Where R is the regular payment;
 j is the interest rate per period;
 n is the number of payments
 K is the number of conversion periods in the deferral

Example 1: On his 40th birthday, Mr. Ramos decided to buy a pension plan for himself. This plan will allow him to claim ₱10,000.00 quarterly for 5 years starting 3 months after his 60th birthday. What one-time payment should he make on his 40th birthday to pay off this pension plan, if the interest rate is 8% compounded quarterly?

Given: $R = 10,000$ $m = 4$ $i^4 = 0.08$

Find: P

Solution:

The annuity is deferred for 20 years and it will go on for 5 years. The first payment is due three months (one quarter) after his 60th birthday, or at the end of the 81st conversion period. Thus, there are 80 artificial payments.

Number of artificial payments: $k = mt = 4(20) = 80$

Number of actual payments: $n = mt = 4(5) = 20$

Interest rate per period: $j = \frac{i^4}{m} = \frac{0.08}{4} = 0.02$

If you assume that there are payments in the period of deferral, there would be a total of $k + n = 80 + 20 = 100$ payments.

Time diagram:

				10,000	10,000	10,000	...	10,000
0	1	2 ...	80	81	82	...	100	

Thus, the present value of the deferred annuity can be solved as

$$P = R \frac{1-(1+j)^{-(k+n)}}{j} - R \frac{1-(1+j)^{-k}}{j}$$

$$P = 10,000 \frac{1-(1+0.02)^{-100}}{0.02} - 10,000 \frac{1-(1+0.02)^{-80}}{0.02}$$

$$P = 33,538.38$$

Therefore, the present value of these monthly pensions is ₱33,538.38.

Example 2: A credit card company offers a deferred payment option for the purchase of any appliances. Rose plans to buy a smart television set with monthly payments of ₱4,000.00 for 2 years. The payments will start at the end of 3 months. How much is the cash price of the TV set if the interest rate is 10% compounded monthly?

Given: $R = 4,000$ $m = 12$ $i^{12} = 0.10$

Find: P

Solution: The annuity is deferred for 2 months and it will go on for 2 years. The first payment is due at the end of 3 months, or at the end of the 3rd conversion period. Thus, there are 2 artificial payments.

Number of artificial payments: $k = 2$

Number of actual payments: $n = mt = 12(2) = 24$

Interest rate per period: $j = \frac{i^{12}}{m} = \frac{0.10}{12} = 0.00833$

If you assume that there are payments in the period of deferral, there would be a total of $k+n = 2+24 = 26$ payments.

Time Diagram:

			4,000	4,000	...	4,000
0	1	2	3	4	...	26

Thus, the present value of the deferred annuity can be solved as

$$P = R \frac{1-(1+j)^{-(k+n)}}{j} - R \frac{1-(1+j)^{-k}}{j}$$

$$P = 4,000 \frac{1-(1+0.00833)^{-26}}{0.00833} - 4,000 \frac{1-(1+0.00833)^{-2}}{0.00833}$$

$$P = 85,260.53$$



Therefore, the present value of these monthly pensions is ₱85,260.53.

Other examples:

In number 1 to 3, find the period of deferral in the deferred annuity.

1. Monthly payments of ₱50,00.00 for 3 years that will start 8 months from now

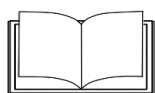
Solution. The first payment is at time 8. The period of deferral is from time 0 to 7, which is equivalent to 7 periods or 7 months.

2. Annual payments of ₱2,500.00 for 24 years that will start 12 years from now

Solution. The first payment is at time 12. The period of deferral is from time 0 to 11, which is equivalent to 11 periods or 11 years.

3. Quarterly payments of 300 for 9 years that will start 1 year from now

Solution. The first payment is at time 4 because there are 1 quarter in 1 year. The period of deferral is from time 0 to 3, which is equivalent to 3 periods or 3 quarters.



What's More

A. Solve the given problem:

1. Melwin availed of a loan from a bank that gave him an option to pay ₱20,000.00 monthly for 2 years. The first payment is due after 4 months. How much is the present value of the loan if the interest rate is 10% compounded monthly?

B. Find the period of deferral in the deferred annuity.

1. Semi-annual payments of ₱8,000.00 for 12 years that will start 5 years from now

2. Payments of ₱10,000.00 every 2 years for 30 years starting at the end of 16 years



What I Have Learned

Complete the following statements with the correct word/s.

1. _____ an annuity that does not begin until a given time interval has passed.

2. _____ is a time between the purchase of an annuity and the start of the payments for the deferred annuity.

3. The formula in finding the present value of deferred annuity is given by _____

4. In finding the present value of deferred annuity, R represent the _____.

5. Number of actual payments: $n =$ _____.



What I Can Do

A. Find the present value of deferred annuity.

1. Mrs. Cruz purchased a smart television set through the credit cooperative of their company. The cooperative provides an option for deferred payment. Mrs. Cruz decided to pay after 2 months of purchase. Her monthly payment is computed as ₱3,600.00 payable in 12 months. How much is the cash value of the television set if the interest rate is 12% convertible monthly?

B. Find the period of deferral in each of the following deferred annuity problem.

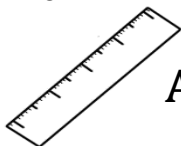
1. Monthly payments of ₱1,000.00 for 9 years that will start 9 months from now
2. Semi-annual payment of ₱12,700.00 for 5 years that will start 2 years from now



Additional Activities

Find the period of deferral in each of the following deferred annuity problem.

1. Payments of ₱13,000.00 every 3 months for 18 years that will start four years from now
2. Monthly payments of ₱200.00 for 6 years that will start 10 months from now



Assessment

Choose the letter that corresponds to the exact answer.

1. An annuity in which the periodic payment is made at the end of each payment interval.
A. Annuity due
B. General annuity
C. Ordinary annuity
D. Simple annuity
2. An annuity in which interest conversion or compounding period is the same as the payment interval.
A. Annuity due
B. General annuity
C. Ordinary annuity
D. Simple annuity
3. A deposit of ₱2,000.00 was made every 6 months to an account that earns 6% interest compounded semi-annually. What type of annuity does this problem illustrate?
A. Annuity due
B. General annuity
C. Ordinary annuity
D. Simple annuity
4. Which of the following problems below illustrate simple annuity?



- A. The payment is quarterly and the interest rate is compounded monthly.
 B. The payments are done annually and the interest rate being charges is also compounded quarterly.
 C. The payment is quarterly and the interest rate is compounded annually.
 D. The payments are done monthly but the interest rate being charges is monthly.
5. Mr. Delima deposited ₱5,500.00 every six months to an account that earns 8% interest compounded annually. What types of annuity does this problem illustrated?
- A. Annuity Certain C. General Ordinary Annuity
 B. General Annuity D. Simple Annuity
6. ₱50,000.00 is invested for 5 years at 6% compounded quarterly. What is the value of n?
- A. 20 B. 15 C. 10 D. 5
7. The buyer of a house and lot pays ₱140,000.00 cash and ₱4,000.00 every month for 12 years. If it is compounded monthly at 8%, solve for the present value.
- A. ₱368,783.57 B. ₱426,329.18 C. ₱535,890.09 D. ₱655,203.34
8. You deposit ₱6, 000.00 each month into an account earning 2% interest compounded monthly. How much will you have in the account in 10 years?
- A. ₱208 515.82 B. ₱308 515.82 C. ₱408 515.82 D. ₱797,949.79
9. To accumulate a fund of ₱400,000.00 in 3 years, how much should Aling Nena deposit in her account every 3 months if it pays an interest of 5.5% compounded annually?
- A. ₱38,668.16 B. ₱48,788.90 C. ₱58,888.65 D. ₱68,763.89
10. Mang Celso borrowed an amount of money from Mang Kanor. He agrees to pay the interest plus the principal by paying ₱35,000.00 each year for 3 years. How much money did he borrow if interest is 8% compounded quarterly?
- A. ₱54, 433.40 B. ₱65, 433.40 C. ₱76, 433.40 D. ₱89,804.04
11. It is a term that refers to payments received (cash inflows) or payments or deposit made (cash outflows).
- A. Cash flow C. Focal date
 B. Fair market value D. Time diagram

For item 12 - 15, please refer to the problem below.

Mrs. Daves received two offers on a house and lot that she wants to sell. Mr. X has offered ₱30,000.00 and ₱600,000.00 lump sum payment 5 years from now. Mr. Y has offered ₱30,000.00 plus ₱25,000.00 every quarter for 5 years. The money can earn 5% compounded annually. (Focal date is at $t = 0$)

12. Which of the following illustrates the correct cash flow using time diagram of Mr. X?

- A. **30,000**
- | | | | | | |
|---|---|---|---|---|---------|
| | | | | | 600,000 |
| 0 | 1 | 2 | 3 | 4 | 5 |
- B. **30,000**
- | | | | | | |
|---|---|---|---|---|--------|
| | | | | | 30,000 |
| 0 | 1 | 2 | 3 | 4 | 5 |

C.

	25,000	25,000	...	600,000
0	1	2	...	20

D.

	25,000	25,000	...	25,000
0	1	2	...	5

13. Find the present value of Mr. X's offer.

- A. ₱470,115.70 B. ₱726,820.93 C. ₱500,115.70 D. ₱824,766.07

14. Solve for the fair market value of Mr. X's offer.

- A. ₱470,115.70 B. ₱500,115.70 C. ₱666,820.93 D. ₱824,766.07

15. Calculate the fair market value of Mr. Y's offer.

- A. ₱229,180.75 B. ₱346,520.63 C. ₱446,820.93 D. ₱470,983.96

16. A time between the purchase of an annuity and the start of the payments for deferred annuity.

- A. Cash flow C. periods
B. Deferred annuity D. Period of deferral

17. What is the formula in computing the present value of a deferred annuity?

- A. $P = R \frac{1-(1+j)^{-(k+n)}}{j} - R \frac{1-(1+j)^{-k}}{j}$ C. $P = R \frac{1-(1+j)^{-(k+n)}}{j}$
B. $P = R \frac{1-(1+j)^{-n}}{j}$ D. $P = F(1+j)^n$

18. What does j represent in the formula of finding the present value of a deferred annuity?

- A. interest rate per period C. number of payments
B. number of conversion periods D. regular payment

19. Joshua availed of a loan from a bank that gave him an option to pay ₱60,000.00 monthly for 2 years. The first payment is due after 4 months. How much is the present value of the loan if the interest rate is 10% compounded monthly?

- A. ₱98,820.88 B. ₱765,155.67 C. ₱1,268,279.35 D. ₱1,532,325.07

20. Monthly payment of ₱50,000.00 for 3 years that will start 7 months from now. Find the period of deferral in the deferred annuity.

- A. 4 months B. 5 months C. 6 months D. 7 months

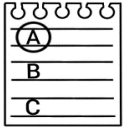


References

Lynie Dimanasuay, Jeric Alcala, and Jane Palacio. **General Mathematics** by C & E Publishing, Inc.

General Mathematics Learner's Materials: Published by Department of Education





Answer Key

What I Know:
1. C 2. C 3. D 4. D 5. D 6. D 7. A 8. B 9. C 10. A 11. B 12. A 13. B 14. C 15. A 16. B 17. D 18. D 19. B 20. D

Lesson 1:

What's In

A. 1,900.00 2. F = 13,506.12 I_c = 1,506.12
B. 1. m = 2 2. 0.1 3. 6% 4. M = 4 5. 12 6. 0.01

What's More

1. Simple Annuity 2. General Annuity

What I Have Learned

1. equal (fixed) 2. Simple Annuity 3. Simple Annuity 4. General Annuity 5. Ordinary annuity and Annuity Due

What I can Do

1. Simple Annuity 2. General Annuity 3. General Annuity 4. Simple Annuity

Additional Activity

1. Simple Annuity 2. General Annuity

Lesson 2:

What's In

1. Annuity 2. Contingent Annuity 3. Simple Annuity 4. Ordinary Annuity 5. General Annuity

What's More

A. P152,793.63 B. P89,910.08 C. P3,979.13 D. P152,770.88

What I have Learned

1. General Annuity 2. R = 3,000 n = 20 i² = 0.02 m = 12

3. Future Value of General Annuity 5. J = 0.01004176 6. F = 66,083.87

What I can do

1. P459,803.80 2. P18,776.88 3. P3,448.11

Additional Activity

1. P122,995.56 2. P857,260.30

Lesson 3:

What's In

A. 1. Regular payment 2. Number of payment 3. P = R $\frac{f}{1-(1+i)^{-n}}$

4. R = 3,000 m = 4 i⁴ = 0.06 t = 6yrs n = 24 payments/periods

5. F = 85,900.56

What's More

1. Mr. Laconsay's offer: F₁ = 438,433.80, Mr. Ladema's offer: P = 601,559.46 2. Mrs. Taleno's offer: F₂ = 816,203.55, Mrs. Dave's offer: F₃ = 1,011,868.53

What I can Do

Sunrise Investment: F₄ = 109,755.10 (larger), XYZ: F₅ = 106,542.70

Additional Activity

Mr. Melendez's offer: F₆ = 256,299.92, Mr. Santos: P = 826,925.32

Lesson 4:

What's More

A. P = 422,759.78 B. 1. 9 periods or 9 semi-annual intervals

2. 7 periods or 7 two-year intervals

What I have Learned

1. deferred annuity 2. Period of deferral

3. P = R $\frac{f}{1-(1+i)^{-k+n}} - R \frac{f}{1-(1+i)^{-k}}$

4. Regular payment 5. (m)(t)

What I can do

A. 40,117.12 B. 1. 8 periods or 8 months 2. 3 periods or semi-annual intervals

1. 15 periods or 15 quarters

2. 9 periods or 9 months

Assessment

1. C 2. B 3. D

6. A 7. A 8. D

11. A 12. A 13. A

16. D 17. A 18. A

4. D 5. C 9. A 10. A 14. B 15. D 19. C 20. C

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