9

Mathematics

Quarter 1-Module 8

<u>Equations in Quadratic Form</u>

Week 3

Learning Code - **M9AL-Ib-7**

Quarter 1 – Module 8 – **New Normal Math for G9**
First Edition 2020

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writers: Rowena F. Reyes - TI   Analynn M. Argel - MTII

Editor: Sally C. Caleja – Head Teacher VI
Ma. Theresa G. Mallari- Head Teacher VI
Judy Ann G. Gallo – Teacher I

Validators: Remylinda T. Soriano, EPS, Math
Angelita Z. Modesto, PSDS
George B. Borromeo, PSDS

Illustrator: Writers

Layout Artist: Writers

Management Team: Malcolm S. Garma, Regional Director
Genia V. Santos, CLMD Chief
Dennis M. Mendoza, Regional EPS in Charge of LRMS and Regional ADM Coordinator
Maria Magdalena M. Lim, CESO V, Schools Division Superintendent
Aida H. Rondilla, Chief-CID
Lucky S. Carpio, Division EPS in Charge of LRMS and Division ADM Coordinator

MODULE 8

EQUATIONS IN QUADRATIC FORM

In the previous modules, you have learned about the different methods of solving quadratic equations. In this module, you will find that these ways are also necessary to solve some rational and higher degree equations.

WHAT I NEED TO KNOW

LEARNING COMPETENCY

The learners will be able to:

- solve equations transformable to quadratic equations (including rational algebraic equations). **M9AL-Ib-7**

WHAT I KNOW

Find out how much you already know about equations transformable to quadratic form. Write the letter that you think is the best answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. Which of the following mathematical equations could be transformed to quadratic form?
   A. $r = 0$
   B. $x + \frac{8}{x-2} = 1 + \frac{4x}{x-2}$
   C. $7v - 8 = 3v - 3$
   D. $4x^2 - 4x + 11 = 3x^2 - 6x + x^2$

2. Which of the following is the quadratic transformation of the equation: $1 + \frac{x-1}{x+1} = \frac{x+1}{x-1}$?
   A. $x^2 + 4x + 1 = 0$
   B. $x^2 - 4x - 1 = 0$
   C. $x^2 - 4x + 1 = 0$
   D. $x^2 + 4x - 1 = 0$

3. Referring to the equation in item #2, which of the following are the solutions?
   A. $-2 \pm \sqrt{5}$
   B. $2 \pm \sqrt{5}$
   C. $-2 \pm \sqrt{3}$
   D. $2 \pm \sqrt{3}$

4. Which of the following are the solutions of $x(x + 3) - 9 = 0$?
   A. $\frac{3 \pm 3\sqrt{5}}{2}$
   B. $\frac{-3 \pm 3\sqrt{5}}{2}$
   C. $\frac{3 \pm 3\sqrt{3}}{2}$
   D. $\frac{3 \pm 3\sqrt{3}}{2}$

5. What are the solutions of the equation: $3(x+3)^2 + 5(x+3) - 2 = 0$?
   A. $-\frac{8}{3}, 5$
   B. $\frac{8}{3}, -5$
   C. $\frac{8}{3}, 5$
   D. $-\frac{8}{3} - 5$

6. Which of the following are the solutions of the 4th degree equation $x^4 + 13x^2 + 36 = 0$?
   A. $2i, -2i, 3i, -3i$
   B. $2i, 3i, 2, 3$
   C. $-2i, 3i, -2, 3$
   D. $2i, -3i, 2, -3$

7. Which of the following rational algebraic equations is transformable to a quadratic equation?
   A. $\frac{v+1}{2} - \frac{v+2}{4} = 7$
   B. $\frac{2}{r} + \frac{3}{r+1} = 5$
   C. $\frac{2b-1}{3} + \frac{1}{2} = \frac{3b}{4}$
   D. $\frac{3}{z-2} + \frac{4}{z+2} = \frac{7}{z}$

8. What are the solutions of the equation $(y^2 - 5y + 2)^2 + 6(y^2 - 5y + 2) + 8 = 0$?
   A. $1, 2, 3, 4$
   B. $-1, -2, -3, -4$
   C. $3, 4, 5, 6$
   D. $-3, -4, -5, -6$

9. What are the solutions of $x^6 - 9x^3 + 8 = 0$?
   A. $1, 1, 1, 2, 2, 2$
   B. $1, (\frac{-1 \pm i\sqrt{3}}{2}), 2, (-1 \pm i\sqrt{3})$
   C. $1, (\frac{1 \pm i\sqrt{3}}{2}), 2, (1 \pm i\sqrt{3})$
   D. $-1, -1, -1, 2, 2, 2$

10. The algebraic rational equation $1 + \frac{x^2 - 5x - 24}{3x} = \frac{x-6}{3x}$ is reducible into quadratic form. Which of the following are the solutions of this equation?
    A. $3, 6$
    B. $-3, -6$
    C. $-3, 6$
    D. $3, -6$

WHAT'S IN

LET'S RECALL

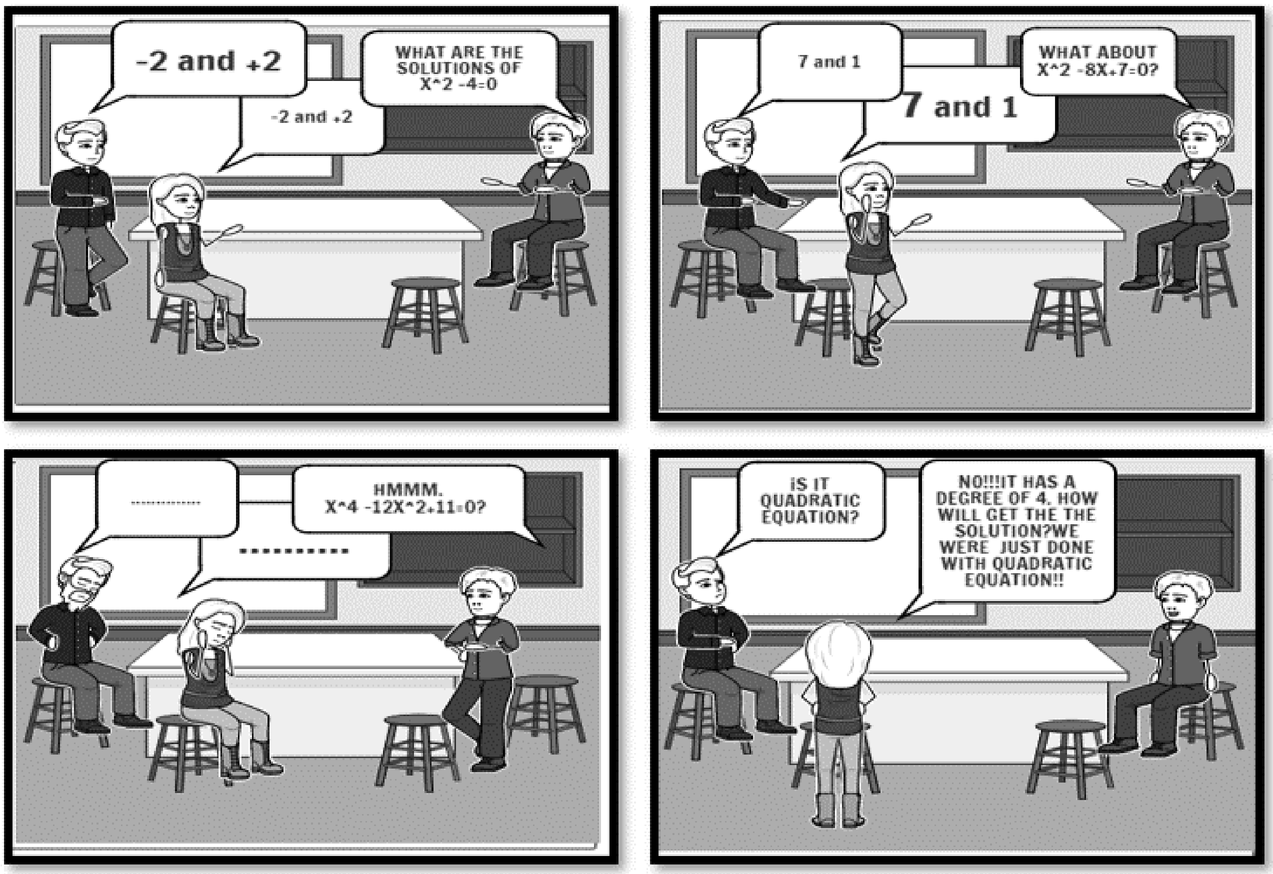
Using any method of finding the zeroes of quadratic equation, solve for the following:

1. $x^2 - 9 = 0$
2. $x^2 - 5x + 6 = 0$
3. $6x^2 + 7x - 15 = 0$
4. $2x^2 + 3x - 5 = 0$
5. $3x^2 + 7x + 4 = 0$

WHAT'S NEW

Communication and collaboration

HIGHER DEGREE!



1Comic generated using storyboard

The last given equation is $x^4 - 12x + 11 = 0$.
Obviously, it is not a second-degree equation and that is their problem.
Since it is not a quadratic equation, how will they find the solution?
What are the values of x?

WHAT IS IT

Communication, Critical Thinking, and Collaboration

Some equations which are obviously not quadratic in form may be solved using the same methods as in a quadratic equation after performing a few substitutions or simplifying the whole equation.

Example 1: Transform the following equations to quadratic.

a. $x^4 - 7x^2 + 12 = 0$
b. $(x^2 - 6x)^2 - 2(x^2-6x) - 35 = 0$
c. $\frac{2}{y+1} + \frac{1}{y-1} = 1$

Solution

a. The equation $x^4 - 7x^2 + 12 = 0$ can be transformed to quadratic using the law of exponent. We can see that,

$$x^4 - 7x^2 + 12 = 0 \qquad \Rightarrow \qquad (x^2)^2 - 7(x^2) + 12 = 0$$

Then, we make a substitution as follows: Let $m = x^2$

$$(x^2)^2 - 7(x^2) + 12 = 0 \qquad \Rightarrow \qquad m^2 - 7m + 12 = 0$$

b. In the given equation $(x^2 - 6x)^2 - 2(x^2-6x) - 35 = 0$, we can clearly see that $(x^2 - 6x)$ is the expression that we need to set into another variable.

Thus, let $r = x^2 - 6x$, then:

$$(x^2 - 6x)^2 - 2(x^2-6x) - 35 = 0 \qquad \Rightarrow \qquad r^2 - 2r - 35 = 0$$

c. To transform $\frac{2}{y+1} + \frac{1}{y-1} = 1$ to quadratic equation, multiply both sides of the equation by the least common denominator.

$$(\boldsymbol{y}+\mathbf{1})(\boldsymbol{y}-\mathbf{1})\left(\frac{2}{y+1} + \frac{1}{y-1}\right) = (1)(\boldsymbol{y}+\mathbf{1})(\boldsymbol{y}-\mathbf{1})$$
$$2(y - 1) + (y + 1) = (y + 1)(y - 1)$$
$$2y - 2 + y + 1 = y^2 - 1$$
$$0 = y^2 - 3y$$

Thus, $\frac{2}{y+1} + \frac{1}{y-1} = 1 \quad \Rightarrow \quad y^2 - 3y = 0$

Example 2: Given a rational algebraic equation, $x + \frac{8}{x-2} = 1 + \frac{4x}{x-2}$, what are the values of x?

Solution:

$$(x-2)\left(x + \frac{8}{x-2}\right) = \left(1 + \frac{4x}{x-2}\right)(x-2) \quad \textit{Multiply both side by the LCD}$$
$$x(x-2) + 8 = (x-2) + 4x$$
$$x^2 - 2x + 8 = x - 2 + 4x$$
$$x^2 - 2x - x - 4x + 8 + 2 = 0$$
$$x^2 - 7x + 10 = 0 \qquad \textit{This is now in quadratic form}$$

You may use factoring to find the solution.

$$x^2 - 7x + 10 = 0$$
$$(x-5)(x-2) = 0$$

Hence, $x = 5 \text{ } or \text{ } x = 2$
But $x = 2$ is an extraneous solution. Why?

What is an extraneous root or solution?
It is a root of a transformed equation which is not a root of the original equation since it was excluded from the domain of the original equation.
https://www.varsitytutors.com/

Example 3: Solve: $x^4 - 12x^2 + 11 = 0$

Solution: The given equation can be written in the form of

$$(x^2)^2 - 12(x^2) + 11 = 0.$$

Using the quadratic formula,

$$x^2 = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(11)}}{2(1)}$$
$$= \frac{12 \pm \sqrt{144 - 44}}{2} = \frac{12 \pm \sqrt{100}}{2} = \frac{12 \pm 10}{2}$$
$$x^2 = 11,\ 1$$

Therefore, $x = \pm\sqrt{11},\ \ \pm 1$

Example 4: Solve: $(x^2 + 2x)^2 - 2(x^2 + 2x) + 3 = 0$

Solution: This example will show the method of solving a complicated equation by using a new variable and substituting it for an expression in the given equation.

So, in this problem, let $m = x^2 + 2x$
Substituting m to the original equation, we have:

$$m^2 - 2m + 3 = 0$$
$$(m+1)(m-3) = 0$$
$$m + 1 = 0 \quad or \quad m - 3 = 0$$
$$m = -1 \qquad\qquad m = 3$$

Substituting the values of m to $m = x^2 + 2x$, we have:

$$x^2 + 2x = 3 \quad \text{or} \quad x^2 + 2x = -1$$
$$x^2 + 2x - 3 = 0 \qquad x^2 + 2x + 1 = 0$$
$$(x-1)(x+3) = 0 \qquad (x+1)(x+1) = 0$$
$$(x-1) = 0 \ \boldsymbol{or}\ (x+3) = 0 \qquad (x+1) = 0 \quad \text{or} \quad (x+1) = 0$$
$$x = 1 \ \boldsymbol{or}\ x = -3 \qquad \textit{These will be both} \ \ x = -1$$

Since we are looking for 4 solutions, the solutions are: $\boldsymbol{x = 1, -3, -1, -1}$

WHAT'S MORE

Transform the following equations into quadratic form and solve for the unknown variable.

$y^4 - 15y^2 = 16$

Quadratic form:____________
Solutions:__________________

$\frac{x-4}{x-5} - \frac{2x-1}{x+4} = \frac{1-2x}{x^2-x-20}$

Quadratic form:____________
Solutions:__________________

$(x^2+1)^2 + 3(x^2+1) - 10 = 0$

Quadratic form:____________
Solutions:__________________

WHAT I HAVE LEARNED

In order to determine whether a certain equation can be solved using the same method as in quadratic equations, you may do or consider any of the following:

1. The equation can be written in the form am² + bm + c = where m is an algebraic expression.
2. For complicated equations, you can make it simple by replacing a new variable for an expression in the equation,
3. For the solutions of a rational algebraic equation, you must simplify first the expression on both sides (includes the removal of the denominator). This will make you see clearly that the resulting equation is in quadratic form.

WHAT I CAN DO

Solve the following equations:

1. $\frac{4}{x} - \frac{5}{x+3} = 1$
2. $\frac{x^2+1}{x^2-3} + \frac{2x^2-}{x^2-3} = 4$
3. $(r^2+2)^2 + (r^2+2) - 6 = 0$
4. $x^4 - 5x^2 + 4 = 0$
5. $x^4 - 3x^2 + 2 = 0$

ASSESSMENT

Write the letter of the correct answer on your answer sheet. If your answer is not among the choices, write E together with your final answer.

1. Which of the following rational algebraic equations could be transformed to quadratic form?
   A. $\frac{1}{r} = \frac{6}{5r} + 1$
   B. $\frac{h-2}{h+3} - 1 = \frac{3}{h+2}$
   C. $\frac{1}{6d^2} = \frac{1}{2d} + \frac{7}{6d^2}$
   D. $\frac{q+5}{q^2+q} = \frac{1}{q^2+q} - \frac{q-6}{q+1}$

2. Which of the following is the quadratic transformation of the equation: $\frac{2x-1}{x+1} = \frac{x+1}{x-1}$?
   A. $x^2 - 5x = 0$
   B. $x^2 + 5x = 0$
   C. $x^2 - 5x + 1 = 0$
   D. $x^2 + 5x + 1 = 0$

3. Referring to the equation in item #2, which of the following are the solutions?
   A. $0, -5$
   B. $0, 5$
   C. $\frac{3 \pm \sqrt{6}}{3}$
   D. $\frac{-3 \pm \sqrt{6}}{3}$

4. The rational equation $1 + \frac{x^2-5x-24}{3x} = \frac{x-6}{3x}$ is reducible into quadratic form. Which of the following are the solutions of this equation?
   A. $3, 6$
   B. $-3, -6$
   C. $-3, 6$
   D. $3, -6$

5. Which of the following are the solutions of $b(b-5) - 9 = 0$?
   A. $\frac{5 \pm \sqrt{61}}{2}$
   B. $\frac{-5 \pm \sqrt{61}}{2}$
   C. $5, 9$
   D. $-5, -9$

6. What are the solutions of the equation: $(x+3)^2 + 2(x+3) - 8 = 0$
   A. $-7, -1$
   B. $7, 1$
   C. $3, 8$
   D. $-3, -8$

7. Which of the following are the solutions of the 4th degree equation $x^4 - 8x^2 + 12 = 0$?
   A. $\pm 6, \pm 2$
   B. $\pm\sqrt{6}, \pm 2$
   C. $\pm 6, \pm\sqrt{2}$
   D. $\pm\sqrt{6}, \pm\sqrt{2}$

8. What are the solutions of $x^4 - 10x^2 + 9 = 0$?
   A. $1,3,1,3$
   B. $-1,-3,1,3$
   C. $1,-3,1,-3$
   D. $1,3,-1,-3$

9. Which of the following rational equations is transformable to a quadratic equation?
   A. $\frac{v+1}{2} - \frac{v+2}{4} = 7$
   B. $\frac{2}{r} + \frac{3}{r+1} = 5$
   C. $\frac{2b-1}{3} + \frac{1}{2} = \frac{3b}{4}$
   D. $\frac{3}{z-2} + \frac{4}{z+2} = \frac{7}{z}$

10. What are the solutions of the equation $(y^2 - 8y + 8)^2 - 6(y^2 - 8y + 8) + 8 = 0$?
    A. $4 \pm 2\sqrt{3}, 4 \pm \sqrt{10}$
    B. $-4 \pm 2\sqrt{3}, 4 \pm \sqrt{10}$
    C. $-4 \pm 2\sqrt{3}, -4 \pm \sqrt{10}$
    D. $-4 \pm 2\sqrt{3}, 4 \pm \sqrt{10}$

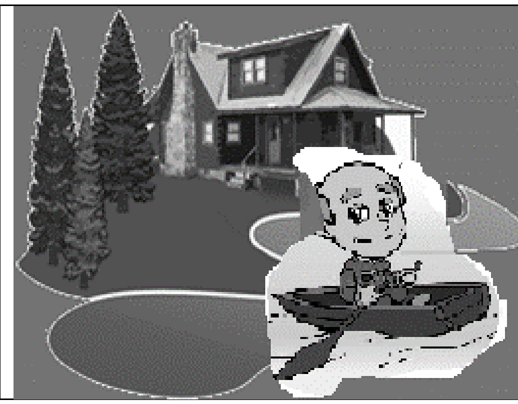
ADDITIONAL ACTIVITIES

Communication, Critical Thinking, Creativity

Finding the Average Speed! Read and study the given scenario. Then answer the questions that follow.

Mang Kevin is a vendor who lives 7km upstream from the town. When the current is 4 km/hr, he can paddle his boat downstream to reach the town and come back in 2 hours.

1. Make a mathematical model representing the scenario.
2. What is the average paddling speed in still water?



In this lesson, we have found that some equations can be made look like quadratic equations especially those with higher degree and rational algebraic equations. Consequently, this can be solved using the same techniques as the quadratic equations. Think and reflect! What are the other concepts related to these techniques that can be used particularly in real life situations?

E-Search

You may also check the following links for your reference and further learnings on solving equations in quadratic form:

- https://www.khanacademy.org/math/in-in-grade-10 ncert/x573d8ce20721c073:quadratic-equations-advanced/x573d8ce20721c073:solving-quadratic-equations-advanced/e/equations-reducible-to-quadratic-equations
- https://www.youtube.com/results?search_query=equations+reducible+to+quadratic+form+worksheet

REFERENCES

Dugopolski, Mark.2006.*Elementary and Intermediate Algebra 2nd edition.*MCGraw-Hill.New York City

Morgan, F. M., & Paige, B. L. (n.d.). *Algebra 2.* pp 98-102 .America: Henry Holt and Company.

Oronce, O. A., Santos, G. C., & Ona, M. I. (n.d.). *Interactive Mathematics III (Concepts,Structures, adn Metods for High School.* pp 250-256Manila: Rex Book Store.

http://tutorial.math.lamar.edu/Classes/Alg/ReducibleToQuadratic.aspx

https://www.jing.fm/iclip/u2q8a9o0a9r5e6e6_cabin-clipart-river-house-colorado-spruce/

https://www.dreamstime.com/illustration/old-man-sea.html

https://www.freepik.com/free-vector/woman-with-long-hair-teaching-online_7707557.htm

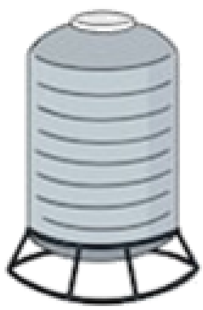
https://www.freepik.com/free-vector/kids-having-online-lessons_7560046.htm

https://www.freepik.com/free-vector/illustration-with-kids-taking-lessons-online-design_7574030.htm

PROBLEM – BASED WORKSHEET

The Water Station

In a water refilling station, the time that a pipe takes to fill a tank is 10 minutes more than the time that another pipe takes to fill the same tank. If the two pipes are opened at the same time, they can fill the tank in 12 minutes.



1. If x represents the time for the first pipe to fill the tank, how will you represent the time for the second pipe to fill the same tank?
2. What equation describes the situation?
3. How many minutes does each pipe take to fill the tank?

ANSWER KEY

WHAT'S IN
1. ± 3
2. 3,2
3. $\frac{-7\pm\sqrt{409}}{12}$
4. $1, -\frac{5}{2}$
5. $-1, -\frac{4}{3}$

WHAT I CAN DO
1. $-6,2$
2. ± 13
3. $-2, 1, \frac{-1\pm\sqrt{-11}}{2}$
4. $\pm\sqrt{5}, \pm\sqrt{-1}$
5. $\pm 2, \pm 1$

WHAT'S MORE
1. Quadratic Form: $(y^2)^2 - 5y^2 - 16 = 0$
   Solutions: $\pm 4, \pm i$
2. Quadratic Form: $x^2 - 13x - 22 = 0$
   Solutions: 11,2
3. Quadratic Form: Letting $y = x^2 + 1$,
   we have $y^2 + 3y - 10 = 0$
   Solutions: $\pm 6i, \pm 1$

ADDITIONAL ACTIVITIES

1. $\frac{7}{x-4} + \frac{7}{x+4} = 3$

2. $\frac{7+\sqrt{113}}{2}$ kph $\approx$ 8.815 kph

WHAT I KNOW
1. B
2. B
3. B
4. B
5. D
6. A
7. B
8. A
9. B
10. C

ASSESSMENT
1. D
2. A
3. B
4. C
5. A
6. A
7. D
8. B
9. B
10. A

PISA-BASED WORKSHEET

Solutions:

1. If x = time for the pipe to fill the one tank, then
x + 10 = time for the pipe to fill the second tank

2. $\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$

3. To solve for the value/s of x

$$12x\,(x+10)\left(\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}\right)12x\,(x+10)$$
$$12(x+10) + 12x = x\,(x+10)$$
$$12x + 120 + 12x = x^2 + 10x$$
$$0 = x^2 - 14x - 120$$

By Factoring:
$$(x - 20)(x + 6) = 0$$
$$x - 20 = 0 \quad \text{or} \quad x + 6 = 0$$
$$x = 20 \qquad\qquad x = -6$$

Since -6 is not a valid solution, 20 minutes is the answer. Thus, it takes 20 minutes for the pipe to fill one tank and 30 minutes to fill the other tank.