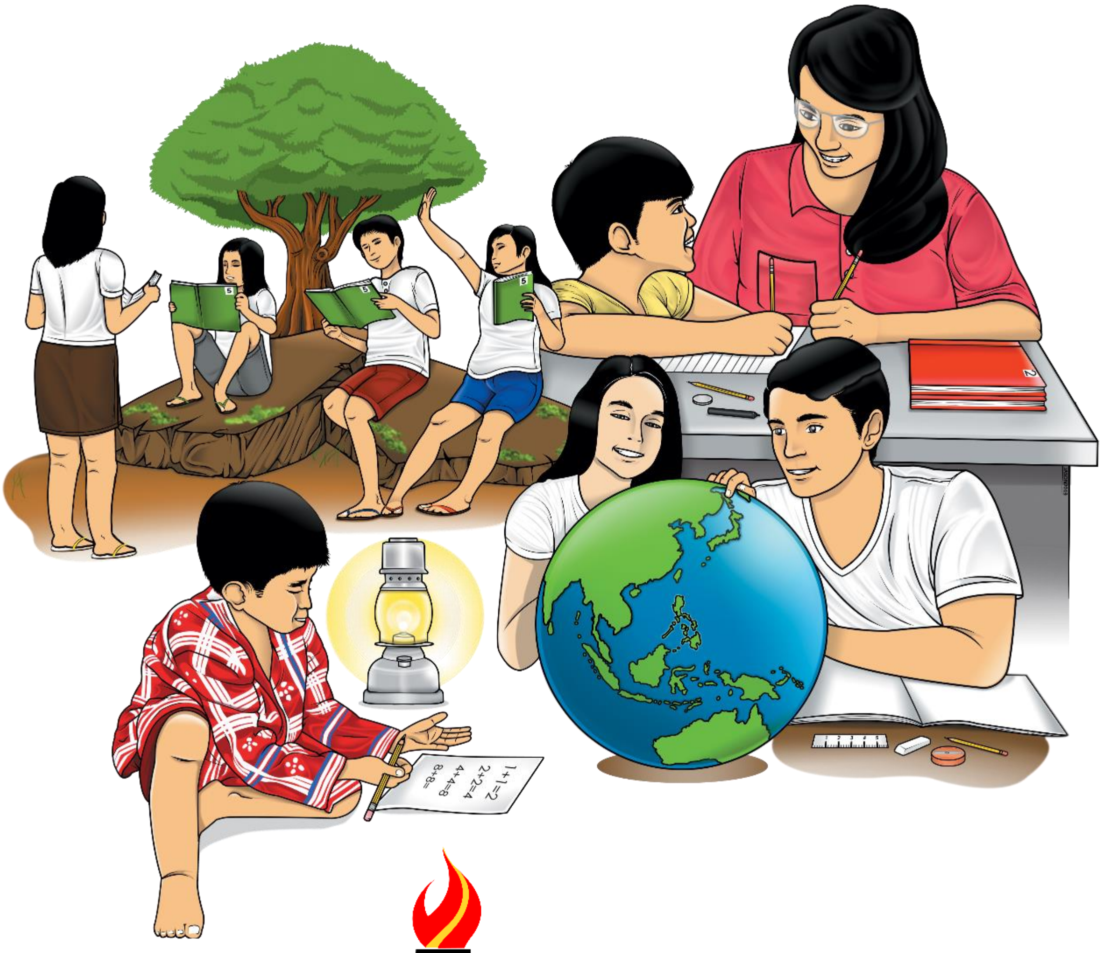
10

Mathematics

Quarter 3 – Module 26:

Linear Permutation of Distinguishable Objects



ALTERNATIVE DELIVERY MODE

ADM

CO_Q3_Mathematics 10_Module 26

Mathematics – Grade 10
Alternative Delivery Mode
Quarter 3 – Module 26: Linear Permutation of Distinguishable Objects
First Edition, 2020

Published by the Department of Education
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Undersecretary: Diosdado M. San Antonio
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Printed in the Philippines by ________________________

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Mathematics

Quarter 3 – Module 26:

Linear Permutation of Distinguishable Objects

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

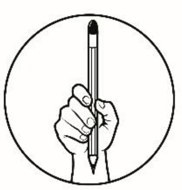
Thank you.

What I Need To Know

This module was designed and written with you in mind. It is intended to help you find the permutation of distinguishable objects. The scope of this module permits it to be used in many different learning situations. The lessons are arranged to follow the standard sequence of the course but the pacing in which you read and comprehend the contents and answer the exercises in this module will depend on your ability.

After going through this module, you are expected to be able to demonstrate understanding of key concepts of permutation of distinguishable objects. Specifically, you should be able to:

1. enumerate the different counting techniques;
2. find the number of outcomes of specified event;
3. find the permutation of distinguishable objects; and
4. solve word problems that involve permutation of distinguishable objects.



What I Know

Are you ready? You are tasked to answer the following questions before we proceed with our lesson. Do not worry, we only want to know how knowledgeable you are with the topics that we will be discussing in this module.

DIRECTION: Read and analyze each item carefully. Write the letter of the correct answer on the blank before each item number. You may answer here.

_____1. To which situation is counting techniques applicable?
A. Linda wants to know how many codes she can make for her padlock.
B. Rissa wants to jog around the playground together with her coach.
C. Mario wants to finish his assignment in his three subjects.
D. Luis wants to paint his basket in three ways.

_____2. In how many ways can one choose a crust and topping for a pizza order from two choices of crust and three choices of toppings?
A. 6 B. 5 C. 4 D. 3

_____3. In how many ways can the letters of the word MATH be arranged?
A. 4 B. 12 C. 24 D. 48

_____4. Which one is equivalent to 5!?
A. 5×1 C. $5 \times 4 \times 2$
B. 5×4 D. $5 \times 4 \times 3 \times 2 \times 1$

_____5. Which expression is equivalent to ${}_4P_4$?
A. 4×4 C. $4 \times 4 \times 3 \times 2 \times 1$
B. 3×4 D. $4 \times 3 \times 2 \times 1$

_____6. Which expression gives the number of ways of arranging five different books in a shelf?
A. $(5 - 1)!$ B. 5! C. 5×4 D. 5×5

_____7. Evaluate ${}_8P_3$.

A. 120 B. 336 C. 520 D. 5,040

_____8. A new model of a car is available in five exterior colors, four interior colors, and two interior styles. How many versions of car are available for order?

A. 20 B. 40 C. 80 D. 120

_____9. A bookshelf has *n* different Algebra books and three different Geometry books. If there are 5,040 ways to arrange the books on the shelf, how many Algebra books are on the bookshelf?

A. 4 B. 5 C. 6 D. 7

Use this situation for items 10 to 12. Find the number of different arrangements of the letters of the word HONESTY if:

_____10. all seven letters are used.

A. 300 B. 360 C. 3,600 D. 5,040

_____11. only two letters are taken at a time.

A. 30 B. 42 C. 64 D. 80

_____ 12. only four letters are taken at a time.

A. 360 B. 840 C. 1,260 D. 2,520

Use this situation for items 13 to 15. There are five boys and four girls to sit in a row. How many possible arrangements are there if:

_____ 13. the first seat is reserved for a boy?

A. 2,880 B. 10,080 C. 60,480 D. 201,600

_____ 14. the first and last seats are for girls?

A. 2,880 B. 10,080 C. 60,480 D. 201,600

_____ 15. the boys and girls sit alternately?

A. 2,880 B. 10,080 C. 40,320 D. 420,240

Lesson 1

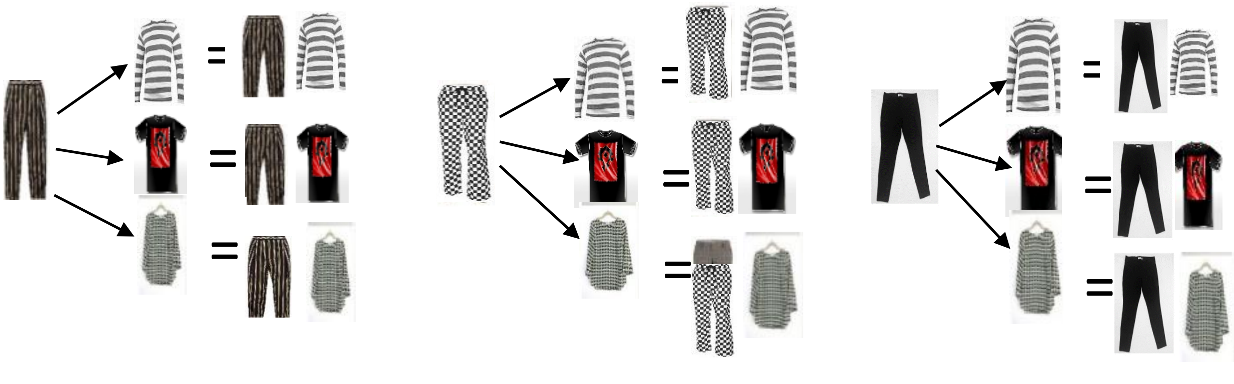
Permutation of Distinguishable Objects

What's In

At this point in time, I know you are already familiar with matching outfits, making passwords, plate numbers, menus, or other related sorts. If you notice, all of them come in group, and each member in the group has its position. Certainly, there are ways on how these members are arranged. Now, if you are asked, how many four-digit passwords can be formed from the digits 1,2,3? or how many ways can you match your three pants and three tops on hand? Can you do this systematically? Can you do it in less time and become accurate about it?

I'll help you recall how to deal with similar questions above by using this illustration. How many ways can you match the outfits presented below? You have three different pants and three different tops on hand.

Recalling our lessons in the previous grades, we simply match the three pants to the tops one by one and counting the results will give the desired number. We can use the ***tree diagram*** so that we can visualize the results.



From the illustration, we can see that there are nine different ways of matching the pants and the tops. Thus, there are nine resulting outfits one can make out of the three different pants and three different tops.

Now think, how about if there are more items to put together? Let's say there are eight pants and 10 tops. How would you imagine the outcome of your illustration using the tree diagram? It would be very taxing since we have to extend our illustration and probably would run out of space. Looking back at the results on the matching of three pants and three tops however, notice that we could simply determine the number of outcomes by multiplying the number of pants to the number of tops. That is: $3 \times 3 = 9$. There are nine ways of matching the three pants and three tops. Do you agree with the observation?

The manner of solving by multiplying the number of elements of two or more events to find the total number of outcomes for those events to occur is known as the **Fundamental Counting Principle (FCP)**. Others call it the multiplication principle. The **FCP** states that if one event has $\boldsymbol{m}$ possible outcomes and a second independent event has $\boldsymbol{n}$ possible outcomes, then there are $\boldsymbol{m \times n}$ total possible outcomes for the two events to occur together. Thus, the problem above where it involves three pants and three tops will have 3×3 which will give 9 as the number of outcomes or outfits.

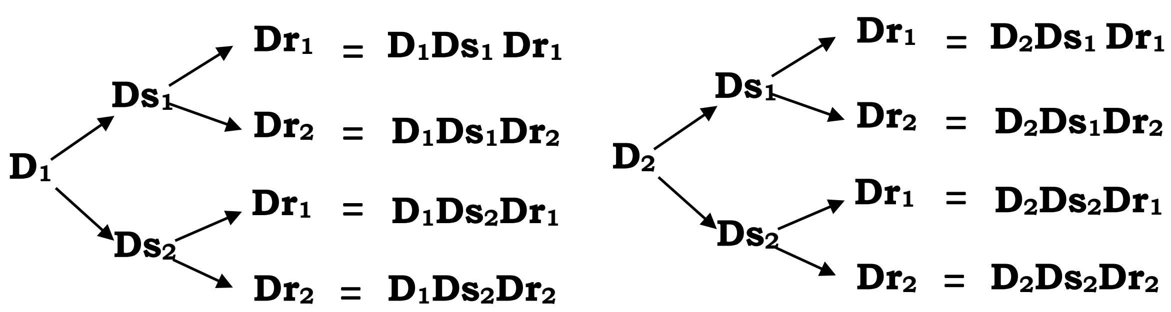
Another Example:

In how many ways can you order a menu composed of a main dish, a dessert, and a drink to be chosen from two different main dishes, two different desserts, and two different drinks?

Solution:

a. Using the **tree diagram**

Let's denote the main dishes as D, desserts as Ds, and drinks as Dr.



b. Using **FCP**

$2 \times 2 \times 2 = 8$ different ways

Activity 1
Find what is asked in each problem. You may use the tree diagram or the FCP to answer the problem. Use a separate sheet for your computations and answers.

1. Find the total number of outcomes of each given situations below.
   a) tossing a coin three times
   b) answering two multiple questions with four choices each

2. How many 4-digit codes can be formed from the digits 1,3,5,7, and 9 if repetition of digits is not allowed?

What's New

If you notice from the situation in Activity 1, each of the outcomes or results follow certain arrangements which are orderly. In this section, we shall encounter more situations that involve orderly arrangement of objects. The term **permutation** is used to indicate ordered arrangements of objects.

Take a look at these examples:

1. How many permutations are there in the letters of the word **LOVE**?

   The problem simply means we are to find how many ways can the letters L, O, V, and E be arranged. It involves orderly arrangements. Since there are four letters involved, then it is the same as filling up four spaces to form the different arrangements. The problem involves four events since there are four spaces. Thus, we can use any of the four letters at the first space, any of the three remaining letters in the next space since we already used one letter in the first space, then any of the two remaining letters in the third space since two letters were already used, and finally the remaining letter in the fourth space.

   Since there are four events or four spaces to fill up, then we will use the **FCP** in answering this problem.

   We will then have $4 \times 3 \times 2 \times 1 = 24$ permutations for the letters of the word **LOVE**.

2. In how many ways can three boys and two girls be arranged in a row with five seats?

This is also an orderly arrangement that involves five objects or seats where the three boys and two girls will sit.

So, we have $5 \times 4 \times 3 \times 2 \times 1 = 120$.
There are 120 ways of arranging the three boys and two girls.

Activity 2

Match each problem on the left to its corresponding answer on the right. Write the letter of the correct answer on the space before each number.

Problem	Number of Ways
____ 1. In how many ways can six books, no two of which are the same, be arranged in a shelf?	A) 12 B) 24
____ 2. How many permutations are there for the letters of the word **ALIVE?**	C) 120
____ 3. A code is composed of two letters followed by three digits, using M, C, 1, 2, and 3. How many codes can be formed if repetition is not allowed?	D) 600 E) 720
____ 4. In how many ways can four bikes, no two of which are the same, be displayed in a window?	F) 800

What Is It

How did you find Activity 2? How did you answer the problems? What did you notice about their solutions?

Let's look at the last item in the previous activity. The number of ways 4 bikes where no two of which are the same, be displayed in a window is computed as $4 \times 3 \times 2 \times 1$. That means there are 24 ways to display the bikes. What can you say about its computation? Notice that the factors in the computation are consecutive counting numbers arranged in decreasing order from four down to one (1).

So, if there are five bikes to arrange where no two bikes are identical, then we have 5 × 4 × 3 × 2 × 1. If there are six bikes, then the computation is 6 × 5 × 4 × 3 × 2 × 1. Notice how the factors in the computation decrease down to one (1). Thus, if there are ***n*** bikes to be displayed the computation would then be: $\boldsymbol{n\,(n-1)(n-2)(n-3)\ \ldots\ (3)(2)(1)}$. To write this in a shorter way, it can actually be expressed as a factorial (symbol: !). Therefore 6! = 6 × 5 × 4 × 3 × 2 × 1 .

Definition:

The Factorial Notation:

The **factorial notation** (symbol:!) is the process of **multiplying consecutive decreasing whole numbers** from an identified number down to one.

$n! = n\,(n-1)(n-2)(n-3)\ \ldots\ (3)(2)(1)$

Thus, 8! = 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 40,320

Another example: In how many ways can seven different books be arranged in a shelf? This also involves orderly arrangements or permutation of seven books. So, we have 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 that will give us the total number of arrangements which is 5,040.

Arranging seven books on a shelf shows permutation or orderly arrangements of seven objects in seven spaces. Permutation also has its notation. The permutation notation of this case of seven books in seven spaces is ${}_7P_7$ which is the same as 7! and is equal to 5,040.

The Permutation of *n* Objects taken *n* at a time:

The number of permutations or different arrangements of ***n*** different objects taken ***n*** at a time in a row is ***nPn*** which is equal to ***n!***.

There are cases, however, that not all the objects are taken all at the same time. If we consider the problem on arranging seven books, let's say the space available is good for five books only. Thus, we are to arrange five books at a time from the given seven books. In how many ways can we do this? The first space is occupied by any of the seven books, the second space is occupied by any of the remaining 6 books, the third space is occupied by any of the remaining 5 books, the fourth space is occupied by the remaining 4 books, and fifth space is occupied by any of the remaining 3 books. Since

5 spaces were filled up, the total number of arranging those 7 different books taken 5 at a time can be computed using FCP, thus 7 × 6 × 5 × 4 × 3 which gives 2,520 distinct arrangements. This computation is equivalent to a permutation notation $_7P_5$ and is computed as:

$$_7P_5 = \frac{7!}{(7-5)!}$$

$$_7P_5 = \frac{7x6x5x4x3x2x1}{2!} = \frac{7x6x5x4x3x2x1}{2x1}$$

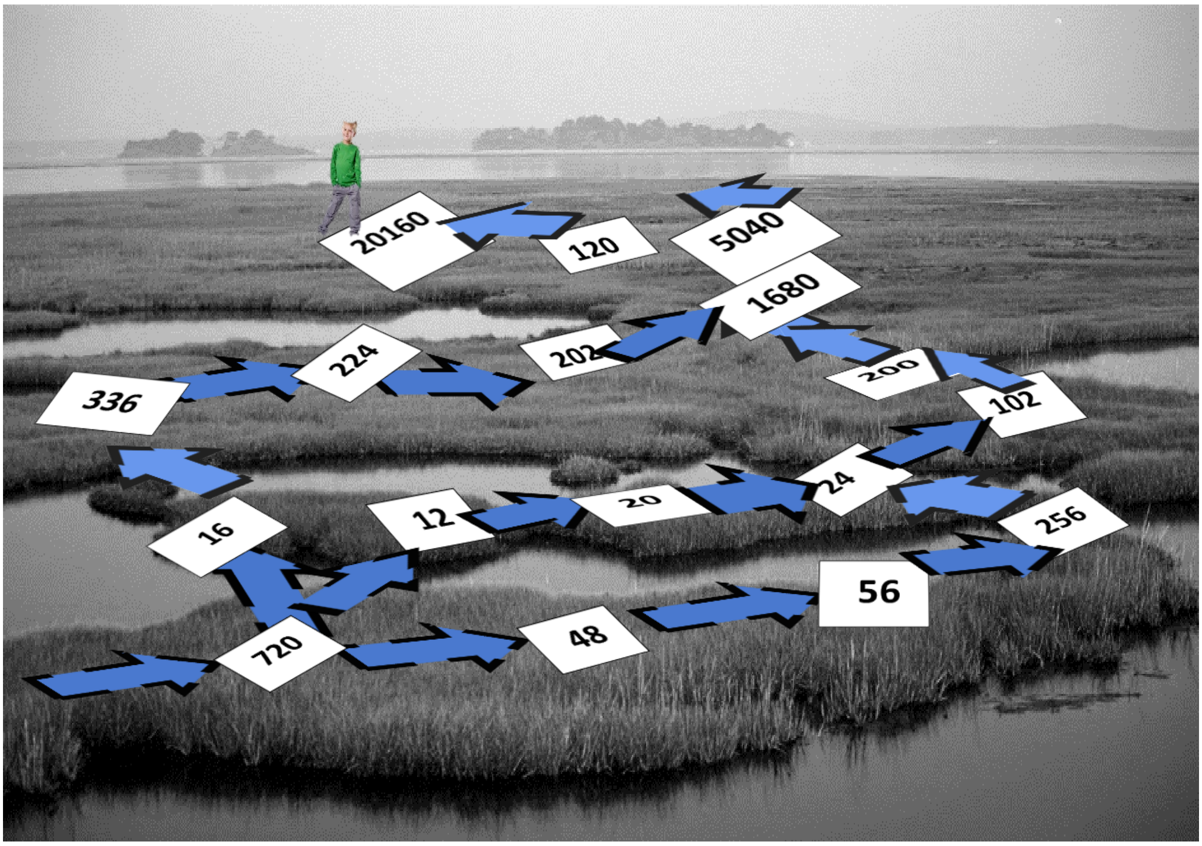
$$_7P_5 = \frac{5{,}040}{2} = 2{,}520$$

There are 2,520 ways of arranging the 7 different books taken 5 at a time in a shelf.

Activity 3: Trace the path of the boy going to the other side of the swamp. The answer to each question below will give a clue to his path.

1. 6! 2. 2!3! 3. $\frac{5!}{3!}$ 4. $_4P_4$ 5. $_8P_4$

6. How many permutations are there in the letters of the word **HOPEFUL**?
7. How many four-digit codes can be made out of the digits 0, 2, 4, 6, and 8 if repetition of digits is not allowed?
8. In how many ways can 4 distinct red cars and 4 distinct black cars be parked in a row of 6 – car garage?



What's More

Going further, let's see other situations where permutation is applied. Let's consider the following examples below, then compare them with the previous situations discussed.

1. How many 5-digit numbers can be formed, with no digit repetition, using the digits 1, 2, 3, 4, 5, if:

a) the first digit is a 5?

If we will put spaces for the five digits, then this will be the representation.

5
_ _ _ _ _ In this is the case, the digit 5 is put in the first space as the first digit, and since repetition of digits is not allowed, then the remaining four digits 1, 2, 3, and 4 are permuted for the remaining four spaces. The notation for the computation is $1x_4P_4$, where 1 is the number of digit 5 and $_4P_4$ is the permutation of the remaining four digits 1, 2, 3, ad 4. Thus,

$$1x_4P_4 = 1 \text{ x } 4! = 1 \text{ x } 24 = 24.$$

There are 24 five-digit numbers that can be formed from the digits 1, 2, 3, 4, and 5, the first digit of which is a 5, as shown below.

54321	54213	53124	52431	52314	51324
54312	54231	53142	52413	52341	51342
54123	53421	53214	52143	51432	51234
54132	53412	53241	52134	51423	51243

b) the first and last digits are even?

E   E
_ _ _ _ _ If the first and last digits are even, then we are left with three spaces to fill. Since there are two even digits from the given digits 1, 2, 3, 4, and 5, then 2 and 4 are permuted for first and last spaces, $(_2P_2)$ as per given condition. Then the remaining three digits which are all odd will be permuted for the second, third, and fourth spaces, $(_3P_3)$.

The notation for the computation is ${}_2P_2$ x ${}_3P_3$, where ${}_2P_2$ is the permutation of the two even digits for the first and fifth spaces, while ${}_3P_3$ is the permutation of the three odd digits for the second, third, and fourth spaces. Thus,

$${}_2P_2 \times {}_3P_3 = 2! \times 3! = 2 \times 6 = 12.$$

There are 12 5-digit numbers that can be formed from the digits 1, 2, 3, 4, and 5, the first and last digits of which are even, as shown.

21354 21534	23154 23514	25134 25314	41352 41532	43152 43512	45132 45312

2. In how many ways can four different Algebra books and three different Geometry books be arranged in a shelf if:

a) the shelf is good for five books only?

Again, considering the spaces for the books we have: _ _ _ _ _.

The space is good for five books only but there are seven books to be arranged.

Solution: $${}_7P_5 = \frac{7!}{(7-5)!} = 2{,}520$$

There are 2,520 various ways to arrange the seven books in a shelf that could hold five books.

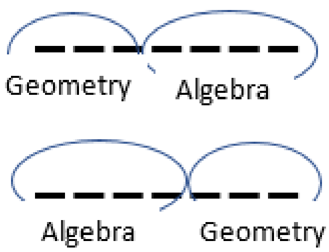
b) two particular books are grouped together?

_ _ _ _ _ _ _ If two particular books are together, then we can treat them as one but are permuted, ${}_2P_2$. The two particular books that are together, counted as one, and the remaining five books will be permuted, ${}_6P_6$. As if we are arranging six books in the shelf.

Solution: $${}_6P_6 \times {}_2P_2 = 6! \times 2! = 1{,}440$$

There are 1,440 distinct arrangements where two particular books are together.

c) books of the same kind/area are grouped together?



The encircled spaces represent books of the same kind. Notice that there are three different spaces on the first group and four on the second. Further, these groupings can still be interchanged, which means that the two groups can be permuted as ${}_2P_2$, while the Geometry books can also be permuted as ${}_3P_3$, and so with the Algebra books as ${}_4P_4$.

Solution: ${}_4P_4 \times {}_3P_3 \times {}_2P_2 = 4! \times 3! \times 2! = 288$

There are 288 arrangements where books of the same kind are put or grouped together.

I hope that you have understood all the discussions. Brace yourself with the activities ahead.

Activity 4

Answer each given situation below. Show complete computations, then box all final answers.

1. There are 12 athletes joining the Baguio Marathon Event. How many ways can the first, second, and third placers be chosen?

2. Five boys and four girls are to arrange themselves to form a line. How many arrangements can there be if:

   (a) the boys stand together, and the girls stand together?

   (b) the boys and the girls stand alternately?

3. There are 10 participants to a barangay meeting to talk about safety measures to be implemented during the pandemic. In how many ways can these 10 barangay participants arrange themselves in a row if three of them should stay together?

4. A couple with their three children are to pose for a family picture. In how ways can they be arranged in a row if:

   (a) they can take any position?

   (b) the mother and the father stay at both ends?

   (c) In how many ways can they be arranged in two rows if the parents will stay at the front row?

What I Have Learned

Let's see how much you learned from the previous discussions.

1. Do what is asked for in each diagram below. Be sure to include important points or concepts in your description.

TREE DIAGRAM	FUNDAMENTAL PRINCIPLE OF COUNTING
Description:	Description:
Illustration:	Illustration:

2. Write a solution for each of the following situations using permutation notation. No need to write the final in standard form/simplified answer.

(a) In how many ways can five students be seated in a row of five chairs?

(b) In how many ways can five students be seated in a row of three chairs?

(c) In how many ways can five students be seated in a row of two 2 chairs?

(d) In how many ways can five students be seated on a chair?

What I Can Do

Let's see how fun it is to play with arrangements. A more systematic way of finding the answers to some queries here is possibly by permutation.

Instructions: Read the premise on the first circle, then do what is asked in the second and third column circles. Show computations beside the circles and final answers must be written inside the circles.

1\. I am **GREAT.** Use each of my letters to find what is asked.

I am of three letters and I am always despised in homes.

In how many ways can you arrange my letters?

I am of four letters and I flow from your eyes.

In how many ways can you arrange my letters with my first letter stuck to its place?

.

I am an emotion which is one letter short of DANGER.

Who am I?

How many ways can you arrange my letters without feeling the same?

Assessment

DIRECTIONs: Let us determine how much you have learned from this module. Read and analyze each item carefully. Write the letter of the correct answer on the blank provided for.

______ 1. If one event can occur in m ways and a second thing can occur in n ways, then which expression gives the number of ways both events can occur?

A) $m!\ n!$ B) $m \times n$ C) $m!$ D) $n!$

______ 2. Which expression is equivalent to $n!$?

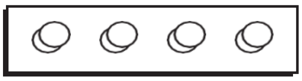
A) $n(n-1)(n-2)\ldots 2 \times 1$ C) $n(n-1)$

B) $n - (n - 1)$ D) $(n - 1)(n - 2)$

______ 3. It is referred to as an arrangement of things in definite order or distinct arrangement of distinguishable objects.

A) factorial B) tree diagram C) permutation D) listing

_______ 4. A coat hanger has four knobs, and each knob is to be painted by a distinct color. If six different colors of paint are available, how many ways can the knobs be painted?



A) 24 B) 360 C) 720 D) 1,296

_______ 5. Evaluate ${}_7P_5$.

A) 42 B) 120 C) 2,520 D) 5,040

______ 6. A new model of a car is available in six exterior colors, three interior colors, and two interior styles. How many versions of car are available for order?

A) 2 B) 6 C) 36 D) 720

______ 7. A multiple-choice test contains five questions, and each question has four options. In how many ways can the five questions be answered?

A) 20 B) 25 C) 625 D)1,024

______ 8. A bookshelf has five different Algebra books and n different Geometry books. If there are 40,320 ways to arrange the books on the shelf, how many Geometry books are on the bookshelf?

A) 6 B) 5 C) 4 D) 3

______ 9. How many ways can the letters from the word FLEA be arranged such that each "word" starts with a consonant and ends with a vowel?

A) 8 B) 12 C) 24 D) 27

Use this situation for items 10 to 14. How many three-digit numbers can be formed using the digits 5, 7, 8 and 2 if:

______ 10. the digits cannot be repeated?

A) 24 B) 64 C) 256 D) 512

______ 11. repetition of digits is not allowed, and the number is less than 700?

A) 48 B) 24 C) 12 D) 6

______ 12. 5 is the first digit and repetition is not allowed?

A) 6 B) 8 C) 10 D) 12

______ 13. 5 is the first digit, 2 is the second digit, and repetition of digits is allowed?

A) 6 B) 8 C) 4 D) 2

______ 14. 5 is the first digit, 2 is the last digit, and repetition of digits is not allowed?

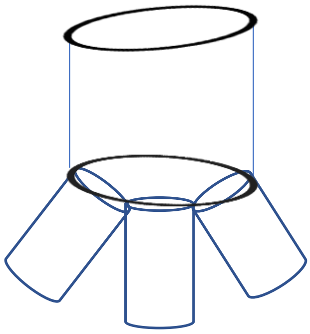
A) 16 B) 8 C) 4 D) 2

______ 15. How many permutations are there in the letters of the word HELPING if ING remain in that order?

A) 1,000 B) 720 C) 380 D) 120

Additional Activity

You are asked to drop three different coins into the figure below with one tube connected to three smaller tubes. In how many ways can the coins drop to the ground with only one coin passing through the tubes? Illustrate your answers by making a diagram below the figure.



Solution:

Answer:

Your diagram here:

Answer Key

Pre-test:

1. A	2. A	3. C
4. D	5. D	6. B
7. B	8. B	9. A
10. D	11. B	12. B
13. D	14. C	15. A

Activity 1:

1. 60 ways 2. 120 codes

Activity 2:

1. E 2. C 3. A 4. B

ACTIVITY 3:

1. 720	2. 12	3. 20
4. 24	5. 1680	6. 5 040
7. 120	8. 20 160	

Activity 4:

1. $_{12}P_3 = \frac{12!}{(12-3)!} = 1,320$ 2. a) $_5P_5\ _4P_4\ _2P_2 = 5!4!2! = 5,760$ b) $_5P_5\ _4P_4 = 5!4! = 2,880$

3. $_8P_8\ _3P_3 = 8!3! = 241,920$ 4. a) $_5P_5 = 5! = 120$

b) $_2P_2\ _3P_3 = 2!3! = 12$ c) $_2P_2\ _3P_3 = 2!3! = 12$

WHAT I HAVE LEARNED:

Tree Diagram Description : A diagram that shows the relationships between different pieces of information using lines that are connected and that have different branches. The end of each branch is one outcome.

Illustration on the branching out will do.

FCP Description: if one event has ***m*** possible outcomes and a second independent event has ***n*** possible outcomes, then there are ***m*** × ***n*** total possible outcomes for the two events together.

Illustration: ***4* × *5*** if there are 4 choices for one event and 5 choices on the other.

WHAT I CAN DO:

1. RAT 3! = 6 arrangements
   TEAR 3! = 6 arrangements
2. ANGER 6! = 720 arrangements

ASSESSMENT:

1. B 2. A 3. C 4. B 5. C

6. C 7. D 8. D 9. A 10. A

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https://www.analyzemath.com/statistics/permutations_combinations.html

For inquiries or feedback, please write or call:

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