Senior High School

Department of Education
National Capital Region
SCHOOLS DIVISION OFFICE
MARIKINA CITY

GENERAL MATHEMATICS

Quarter 1: Module 2B

RATIONAL FUNCTIONS

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What I Need to Know

This module was designed and written with you in mind. This will help you master the **Rational Functions**. The scope of this module is to represent a rational function through its: (a) table of values, (b) graph, and (c) equation, find the domain and range of a rational function, determine the: (a) intercepts; (b) zeroes; and (c) asymptotes of rational functions, and (d) solve problems involving rational functions, equations, and inequalities.

This module on rational functions will help you learn and understand the content and how it is applicable to our daily life.

This module 2B is a continuation of Rational Function 2A and it is divided into 4 lessons, namely:

Lesson 1: A Rational Function through its:
- (a) table of values,
- (b) graph, and
- (c) equation

Lesson 2: Intercepts, Zeroes; and Asymptotes of Rational Functions
Lesson 3: The domain and range of a rational function
Lesson 4: Problems Involving Rational Functions, Equations, and Inequalities

After going through this module, you are expected to:

1. Represent a rational function through its:
   (a) table of values,
   (b) graph, and
   (c) equation,
2. Determine the: (a) intercepts; (b) zeroes; and (c) asymptotes of rational functions,
3. Find the domain and range of a rational function.and
4. Solve problems involving rational functions, equations, and inequalities.

What I Know

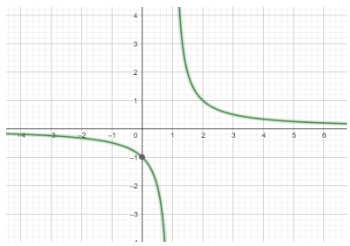
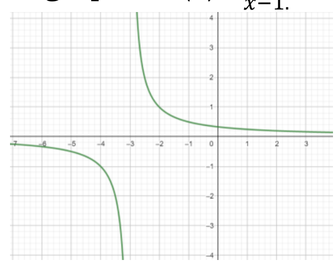
Write the letter of the correct answer on a clean sheet of paper.

1. Which of the following can be used to represent a function?
   A. Equation C. Table of Values
   B. Graph D. all of the above

2. Suppose I am driving along at 30 miles per hour. If I let x = number of hours traveled and y = distance traveled, then y is determined by x, so y is a function of x. Which of the following equations can be used to represent this function?

   A. y = 30x C. y = x – 30
   B. y = 30 + x D. None of these

3. Which of the following graphs represent the graph of $f(x)= \frac{1}{x-1.}$?

   A.
   
   C.
   
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   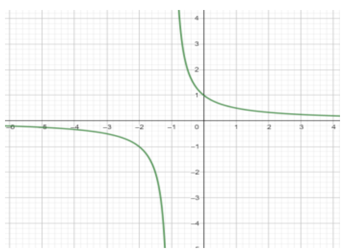
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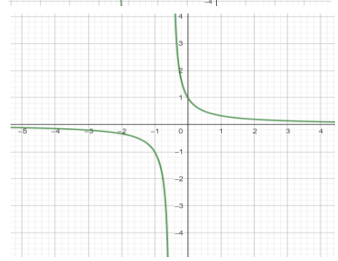
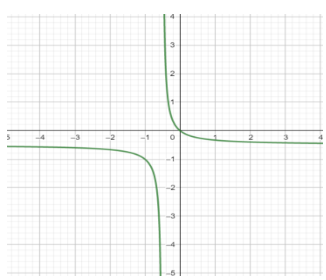
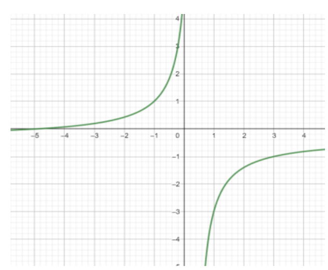
   

4. The graph of $f(x)= \frac{-x-5}{3}$ is ___________.

   A.
   
   C.
   
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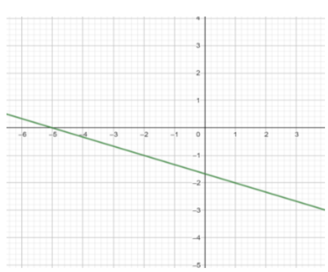
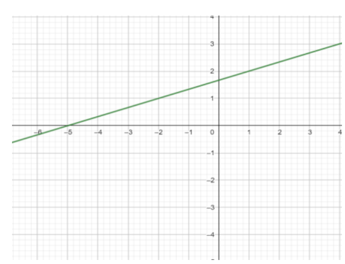
   
   D.
   

5. Given the rational function f(x)=$\frac{1}{x^2-4}$, what are the values that will make the denominator zero?
   A. -4 B. ± 4 C. -4 D. ± 2
6. The domain of the rational function f(x)=$\frac{x+1}{x-3}$ is____________________.
   A. $\{x/x \neq -1\}$ B. $\{x/x \neq -3\}$ C. $\{x/x \neq -1\}$ D. $\{x/x \neq 3\}$
7. The range of the rational function f(x)=$\frac{1}{x+2}$ is ___________________.
   A. $\{y/y \neq -2\}$ B. $\{y/y \neq -1\}$ C. $\{y/y \neq 0\}$ D. $\{y/y \neq 1\}$
8. The vertical asymptotes of the rational function f(x)=$\frac{1}{x^2-9}$ are at x=_____ and x=_____.
   A. -3 B) 3 C) -3 and 3 D) None
9. The rational function f(x)=$\frac{x^2-4x+4}{x+3}$ has an oblique asymptote at ______.
   A. y =x+7 B) y=x-7 C) y=x+3 D) y=x-3
10. A boy is swimming in a river with the rate of the current equal to 2 kph. If he can swim 5 km upstream in the same amount of time it would take him to swim 8 km downstream, what is his speed in still water?
   A. $\frac{22}{3}$ kph B. $\frac{25}{3}$ kph C. $\frac{29}{3}$ kph D. $\frac{26}{3}$ kph

Lesson 1: Rational Functions Through Its (a) Table of Values, (b) Graph, and (c) Equation

What's In

Construct a table of values for the following and draw their graph. Then, answer the questions below.

A. f(x) = 2x – 3

x	3	2	1	0
y				

B. f(x) = x^2

x	-1	0	1	2
y				

1. What properties of equality did you use in completing the table of values?
2. How did you graph/sketch the given functions above?
3. Can we use those steps in constructing the table of values and in graphing rational functions? Explain your answer.

What's New

Given that $f(x) = \frac{1}{x-3}$, the values of f(x) from the **left** of 3 are shown below:

x	0	1	2	2.5	2.9	2.99	2.999
f(x)	-0.333	-0.5	-1	-2	-10	-100	-1000

Similarly, computing the values of f(x) from the **right** of 3, we have the values below:

x	6	5	4	3.5	3.1	3.01	3.001
f(x)	0.333	0.5	1	2	10	100	1000

Answer the following questions briefly.

1. What observations have you noticed from the two table of values?
2. What happens to the function values as x goes near 3 from the right?
3. What happens to the function values as x goes near 3 from the left?

What is It

Here, you will learn on how to construct the table of values and the graph of rational functions.

Let us define once again a rational function.

A **rational function** is a function of the form $f(x)=\frac{p(x)}{q(x)}$; where p(x) and q(x) are polynomial functions and $q(x) \neq 0$.

A rational function can be represented through equation, table of values, or graph.

Construct a table of values and sketch the graph of the following rational functions:

a. $f(x) = \frac{1}{x}$

b. $g(x) = \frac{1}{x-2}$

c. $G(x) = \frac{x}{x+1}$

d. $F(x) = \frac{2}{(x-2)^2}$

SOLUTION

In dealing with a rational function, it is often helpful to find values for which the rational function is undefined. Later, you will find that the graph of the rational function approaches lines that contain these values for which the rational function is undefined hence the graph does not cross these lines.

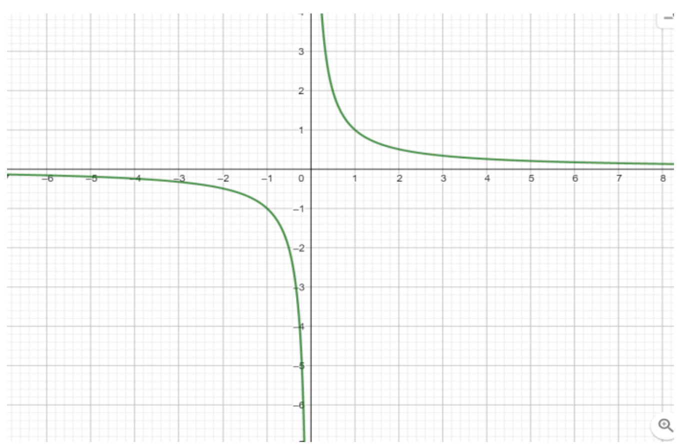
From the activity in What's new, if you know in which values the given rational function will be undefined, you can center your table of values around it, that is, you can choose values of x that move closer to this number, approaching from the left and the right.

For example, to graph the rational function

a. $f(x)=\frac{1}{x}$, the function will not be defined at x = 0. Therefore, you can choose x to take on the following values: -2, -1, $-\frac{1}{2}$, $-\frac{1}{4}$ and 2, 1, $\frac{1}{2}$, $\frac{1}{4}$.

x	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	2	1	$\frac{1}{2}$	$\frac{1}{4}$
y	$-\frac{1}{2}$	-1	-2	-4	undefined	$\frac{1}{2}$	1	2	4

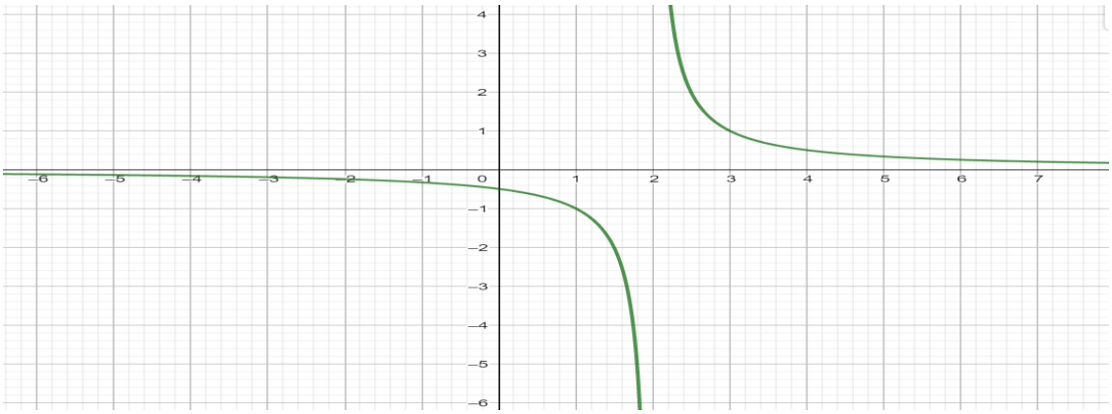
and can be determined and plotted in a Cartesian plane. With these points, the graph can be sketched.



b. The function $g(x) = \frac{1}{x-2}$, which can be represented by the equation $y = \frac{1}{x-2}$ is not defined when x=2. The graph of the line x=2 will not be crossed by the graph of $g(x) = \frac{1}{x-2}$. The table of values for the function is constructed and shown below.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	-0.14	-0.17	-0.20	-0.25	-0.33	-0.5	-1	undefined	1	0.5	0.33

Using these values, the points (-5,-0.14), (-4,-0.17), (-3,-0.20), (-2,-0.25), (-1,-0.33), (0,-0.5), (1,-1), (3,1), (4,0.5), and (5,0.33) can be determined and plotted in a Cartesian plane. With these points, the graph can be sketched.

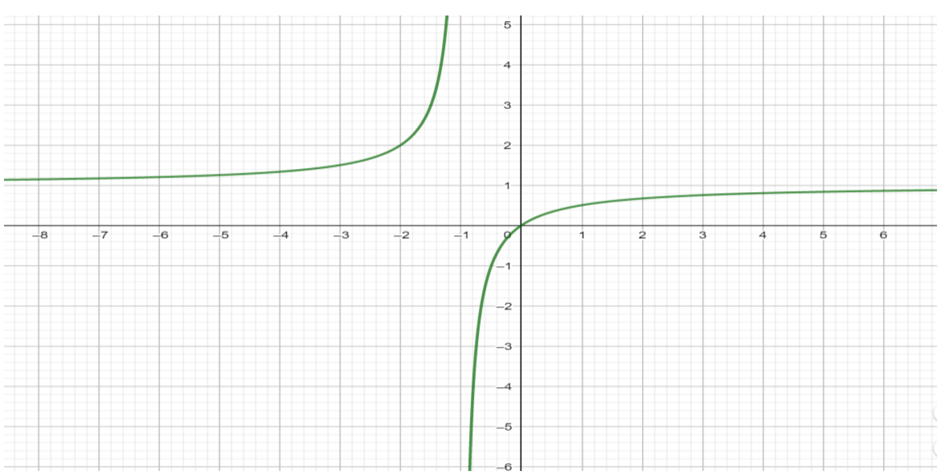


c. The function $G(x) = \frac{x}{x+1}$ which can be represented by the equation $y = \frac{x}{x+1}$ is not defined when x = -1. The graph of the line x= -1 will not be crossed by the graph of $G(x) = \frac{x}{x+1}$.

The table of values for the function is constructed and shown below.

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y	1.25	1.33	1.5	2	undefined	0	0.5	0.67	0.75	0.80	0.83

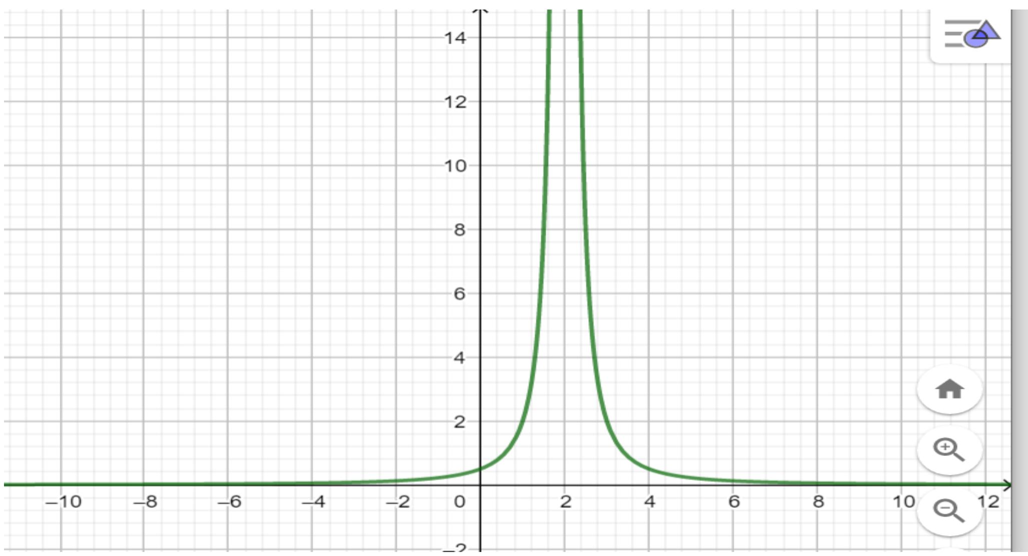
Using these values, the point (-5,1.25), (-4,1.33), (-3,1.5), (-2,2), (0,0), (1,0.5), (2,0.67), (3,0.75), (4,0.80), (5,0.83) can be determined and plotted in a Cartesian plane. With These points, the graph can be sketched.



d. The function $F(x) = \frac{2}{(x-2)^2}$ which can be represented by the equation $y = \frac{2}{(x-2)^2}$ is not defined when x = 2. Also, note that y can never be 0. Its value will never be negative, or y > 0, The graph of the line x =2 and y=0 will not be crossed by the graph of $F(x) = \frac{2}{(x-2)^2}$. The table of values for the functions is constructed and shown below.

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	0.04	0.06	0.08	0.125	0.22	0.50	2	undefined	2	0.50	0.22

Using these values, the points (-5,0.04),(-4,0.06), (-3,0.08), (2,0.125), (-1,0.22), (0,0.50), (1,2), (3,2), (4,0.50), (5,0.22) can be determined and plotted in a Cartesian plane. With these points, the graph can be sketched.



What's More

Represent each rational function by equation, table of values, and graph.

1. $f(x) = \frac{1}{2x}$
2. $f(x) = \frac{1}{x-1}$
3. $f(x) = \frac{2}{x+1}$
4. $f(x) = \frac{1}{x+2}$
5. $f(x) = \frac{1}{3x}$

What I Have Learned

Given the rational function $f(x)=\frac{x+1}{x-5}$, fill in the blanks with the value/s that will make the sentences true.

1. The value which will make the function undefined is x=___________.
2. The values that can be assigned to x are values to the right and left of ________.
3. What happens to the function values as x goes near 5 from the right?
4. What happens to the function values as x goes near 5 from the left?

What I Can Do

Make a table of values to represent the following rational function. (Answers may vary depending on the chosen x values).

1. $f(x) = \frac{1}{x+4}$
2. $g(x) = \frac{x+6}{x-8} + 5$
3. $s(t) = \frac{2}{t^2} + \frac{1}{t} + 1$

Rubrics for this activity

Score	Description
15 points	Complete solutions with correct answer
10 points	75% correct solutions with incorrect answer
5 points	50% correct solution with incorrect answer
No point earned	No output at all

Assessment

Write the letters of the correct answers on a clean sheet of paper.

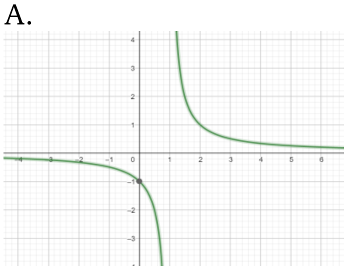
1. Which of the following can be used to represent a function?
   A. Equation C. Table of Values
   B. Graph D. all the above

2. Suppose I am driving along at 80 miles per hour. If I let x = number of hours traveled and y = distance traveled, then y is determined by x, so y is a function of x. Which of the following equations can be used to represent this function?
   A. y = 80x C. y = x – 80
   B. y = 80 + x D. None of these

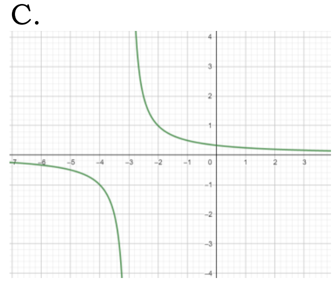
For item 3-4, determine the graph of each function.

3. $f(x)= \frac{1}{x+1.}$

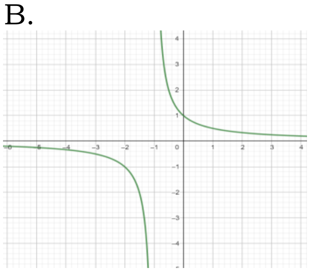
A.



C.



B.



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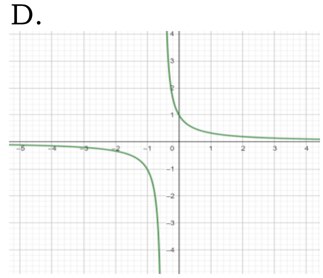
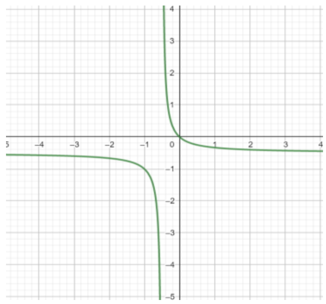
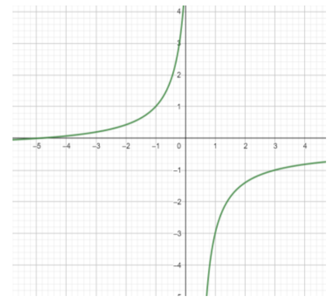


4. $f(x)=\frac{1}{2x+1}$

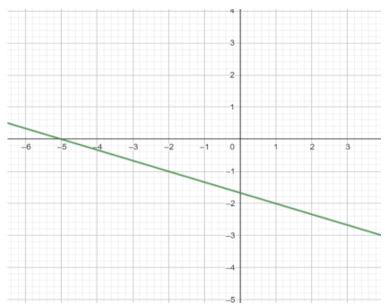
A.



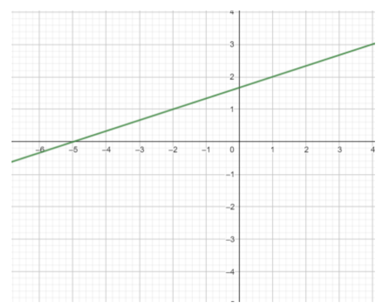
C.



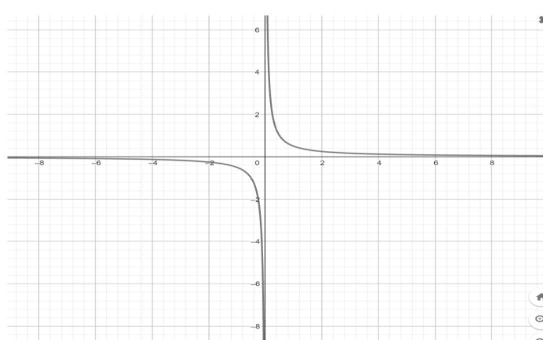
B.



D.



5. Which equation best represents the graph?



A. $f(x)=\frac{1}{2x}$

B. $f(x)=\frac{2}{x}+1$

C. $f(x)=\frac{1}{x}-1$

D. $f(x)=x+\frac{2}{x}-2$

Additional Activities

Given: $g(x) = \frac{6}{x-2}$

a. Construct a table of values.
b. Plot the points in the Cartesian plane and identify if the points form a smooth curve or a straight line

Lesson 2: Zeroes, Intercepts and Asymptotes of Rational Functions

What's In

1) Given the rational function $f(x)=\frac{3x^2}{x^2-1}$, what are the value(s) that will make the function undefined?
2) Using the rational function $f(x)=\frac{1}{x+1}$, complete the table below.

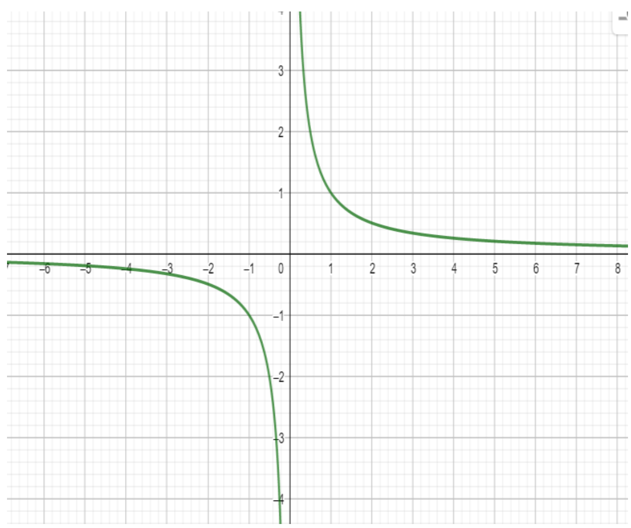
x	2	1	0	-0.5	-0.8
f(x)					

x	-2	-1.5	-1.25	-1.1	-1.01
f(x)					

3) Sketch the graph of the given rational function in #2.

What's New

In the previous lesson, we can construct table of values to graph the rational function $f(x)=\frac{1}{x}$, wherein you assign values of x to the right and left of the restricted value that will make the function undefined. After assigning values for x, you need to solve for the corresponding function values before you can plot the points and sketch the graph.



1. Is there an easier way to sketch the graph of a rational function aside from plotting points which takes a lot of work to do?

2. The graph seems to be getting closer to the y-axis, do the graphs of all rational functions just get closer to the y-axis but will never cross the y-axis?

3. The graph seems to be getting closer to the y-axis, do the graphs of all rational functions just get closer to the y-axis but will never cross the y-axis?

What is It

Aside from plotting points, there is another way to sketch the graph of a rational function, that is by getting the asymptotes, zeroes and intercepts of a rational function.

Let us look at the following definitions and illustrative examples.

In this lesson, you will use the following definitions:

Zeroes: These are the values of x which make the function equal to zero.

y-intercept: It refers to the function value when x=0.

x-intercept(s): It refers to the real numbered zeroes of a rational function.

Asymptote: It is a line that the graph gets close to but does not touch.

Vertical Asymptote: These are vertical lines x=a of a rational function f if the graph either increases or decreases without bound as the x values approaches from the right or left.

Horizontal Asymptote: It is a horizontal line y=b of the rational function f if f(x) gets closer to b as x increases or decreases without bound.

Oblique Asymptote: It is a line y=mx+b (disregarding the remainder) obtained after dividing the numerator by the denominator when the degree of the numerator is one.

Note:

Let n be the degree of the numerator and m be the degree of the denominator:

a. If n<m, the horizontal asymptote is y=0.

b. If n=m, the horizontal asymptote is at $y = \frac{a}{b}$, where a is the leading coefficient of the numerator and b is the leading coefficient of the denominator.

c. If n>m, there is no horizontal asymptote.

Examples:
Find the zeroes, intercepts and asymptotes of the following rational functions.

1. $f(x) = \frac{x^2-9}{x+3}$

Simplify the function if possible before getting the zeroes, intercepts, and asymptotes.

$= \frac{(x+3)(x-3)}{x+3}$

Factor the numerator and simplify.

$= x-3$

After simplifying, the rational function seems to be reduced to a linear function but not exactly a linear function since x≠-3.

To find the zero(s) of the function, set y=0 and solve for x.

f(x)=x-3 which can be written as

y = x - 3	
0 = x - 3	replace y by 0 and
x = 3	solve for x.

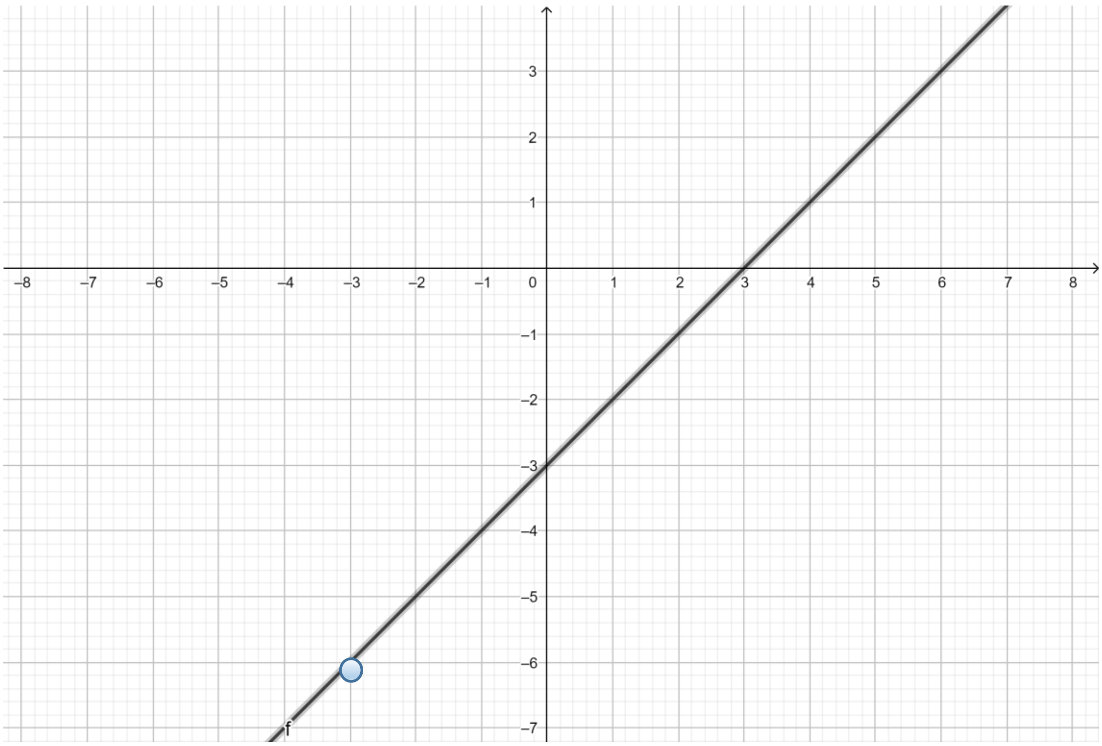
x=3 x = 3 is the zero of the function and at the same time the x-intercept because 3 is a real number.

To find the y-intercept, set x=0

f(x)=x-3 which can be written as

y = x - 3	
y = 0 - 3	replace x by 0 and
y = - 3	simplify.

The y-intercept is at y=-3.
Since the function is reduced to a seemingly "linear function" but not exactly linear because it has a restriction at x = -3 in the denominator, after simplifying there will be no asymptotes and the graph has a hole at (-3,-6).



2. $f(x) = \frac{x+3}{x^2-9}$

$= \frac{x+3}{(x+3)(x-3)}$

$= \frac{1}{x-3}$; $x \neq -3$, $x \neq 3$

Simplify if possible before getting the zeroes, intercepts, and asymptotes.

Factor the expressions that can be factored and simplify.

To find the zero(s) of the function, set y=0 and solve for x.

$f(x)=\frac{1}{x-3}$
can be written as

$y = \frac{1}{x-3}$

$0 = \frac{1}{x-3}$ Replace y by 0 and solve for x.

$(x-3)(0) = \left(\frac{1}{x-3}\right)(x-3)$ Multiply both sides by x-3.

$0=1$ False

There aThere are no zeroes as well as x-intercepts since the solution resulted to a false statement.

To find the y-intercept, set x=0.

$f(x)=\frac{1}{x-3}$

can be written as

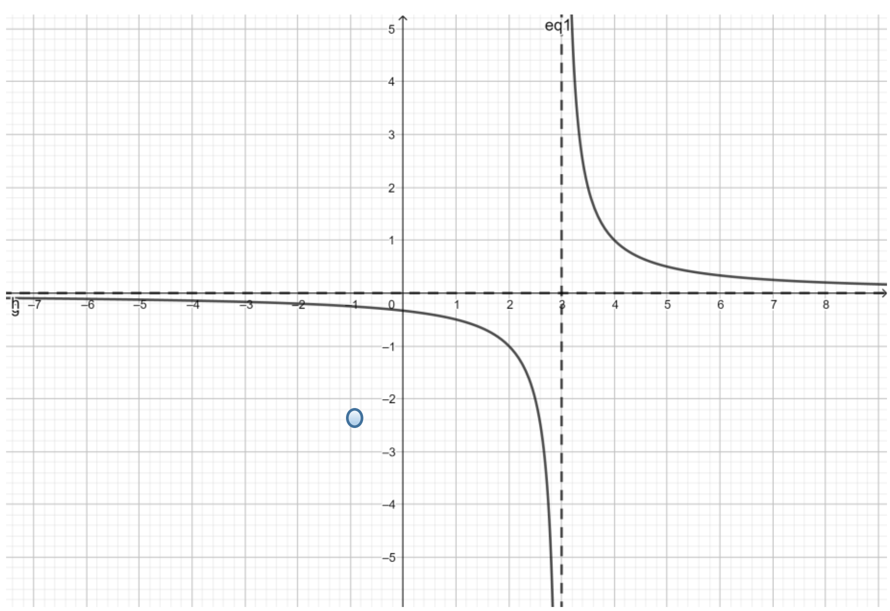
$y = \frac{1}{x-3}$

$y = \frac{1}{0-3}$ Replace x by 0 and simplify.

$y = -\frac{1}{3}$

The The y-intercept is at $y=-\frac{1}{3}$.

From the original function $f(x)=\frac{x+3}{x^2-9}$, the vertical asymptotes will pass at the values that will make the denominator equal to zero, that is supposedly when x=-3 and x=3 but after simplifying, x+3 in the numerator and x+3 in the denominator cancels out, which means x+3 or x=-3 is no longer a vertical asymptote but instead a hole on the graph at $(-3,-\frac{1}{6})$. Therefore, the equation of the vertical asymptote is x=3 (vertical line drawn in dashes) since the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is at y=0 (horizontal line drawn in dashes).



3. $F(x) = \frac{x^2+6x+9}{x^2-9}$

Simplify if possible before getting the zeroes, intercepts and asymptotes.

$f(x) = \frac{(x+3)(x+3)}{(x+3)(x-3)}$

Factor out expressions both in the numerator and in the denominator that can be factored out and simplify.

$f(x)= \frac{x+3}{x-3}; x\neq-3,\ x\neq3$

To find the zero(s) of the function, set y=0 and solve for x.

$f(x)=\frac{x+3}{x-3}$ can be written as

$y = \frac{x+3}{x-3}$	
$0 = \frac{x+3}{x-3}$	Replace y by 0 and solve for x.
$(x-3)(0) = \left(\frac{x+3}{x-3}\right)(x-3)$	Multiply both sides by x-3.
$0 = x+3$ $x = -3$	Simplify and solve for x.

x=-3 is the zero of the function and at the same time the x-intercept because -3 is a real number.

To find the y-intercept, set x=0.

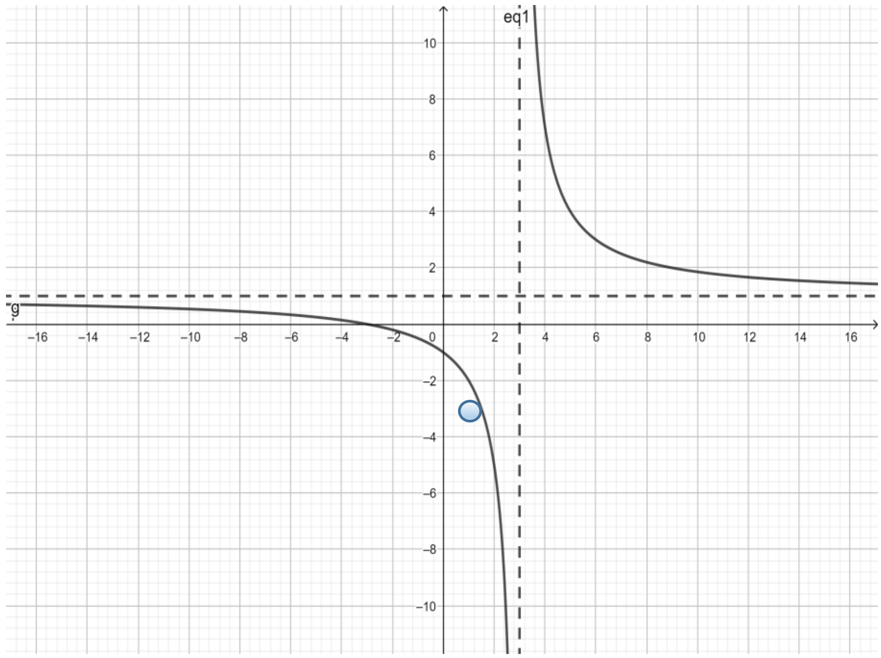
$f(x)=\frac{x+3}{x-3}$ can be written as

$y = \frac{x+3}{x-3}$	
$0 = \frac{0+3}{0-3}$	Replace x by 0 and solve for y.
$y = -1$	

The y-The y-intercept is at y=-1.

Since the degree of the numerator is the same as the degree in the denominator, the horizontal asymptote is the ratio of the leading coefficient of the numerator and the denominator, which is at y=-1 (horizontal line drawn in dashes).

The vertical asymptotes pass at the values of x that make the denominator zero and those are supposedly at x=-3 and x=3, but after simplifying, the common factor x+3 cancels each other out, which means x=-3 is no longer a vertical asymptote but instead a hole on the graph at (-3,0) and therefore x=3 is the only vertical asymptote of the graph (vertical line drawn in dashes).



4. $f(x) = \frac{x^2+3x+2}{x+3}$

Simplify if possible before getting the zeroes, intercepts and asymptotes.

$f(x) = \frac{(x+2)(x+1)}{x+3}$

Factor out expressions in the numerator and in the denominator that can be factored out and simplify.

Since there are no common factors both in the numerator and in the denominator, the function can no longer be simplified, we will solve for the zeroes, intercepts and asymptotes using the original function.

To find the zero(s) of the function, set y=0 and solve for x.

$f(x)=\frac{x^2+3x+2}{x+3}$ can be written as

$$y = \frac{x^2+3x+2}{x+3}$$

$$0 = \frac{x^2+3x+2}{x+3}$$

Replace y by 0 and solve for x.

$$(x+3)(0)=\left(\frac{x^2+3x+2}{x+3}\right)(x+3)$$

Multiply both sides by x+3

$$0=x^2+3x+2$$

Solve the resulting quadratic equation either by factoring or by quadratic formula. By factoring

$$0=x^2+3x+2$$

$$0 = (x+2)(x+1)$$

$$X + 2 = 0, x + 1 = 0$$

$$x=-2 \quad ; \quad x=-1$$

The zeroes of the function are at x=-2 and x=-1 which at the same time the x-intercepts because -2 and -1 are both real numbers.

To find the y-intercept, set x=0.

$f(x)=\frac{x^2+3x+2}{x+3}$

can be written as

$$y=\frac{x^2+3x+2}{x+3}$$

$$y=\frac{0^2+3(0)+2}{0+3}$$ Replace x by 0 and simplify.

$$y=\frac{2}{3}$$

The y-in the y-intercept is at $y=\frac{2}{3}$.

The vertical asymptote passes at x=-3 (the values that makes the denominator zero). It is the vertical line drawn in dashes on the graph.

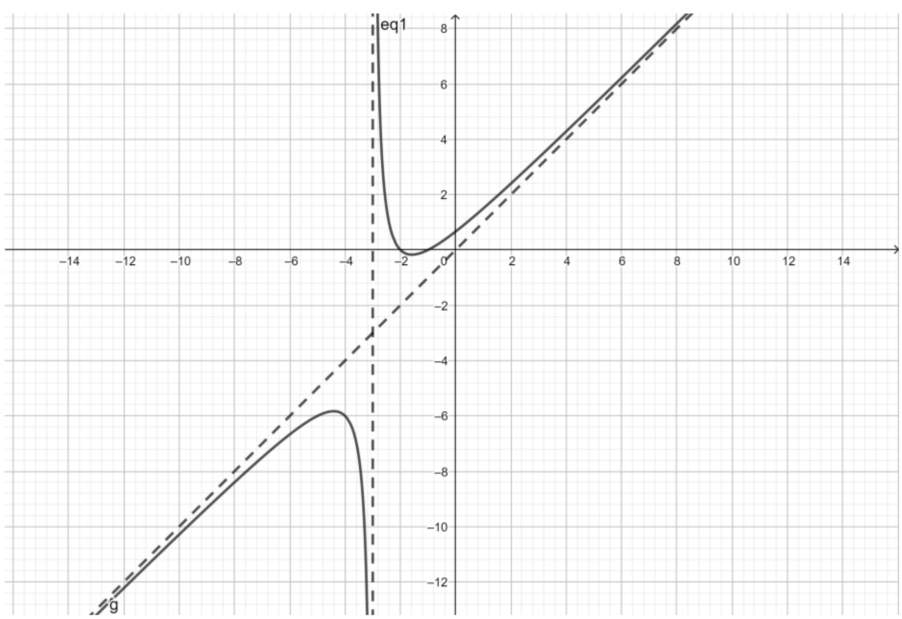
There is no horizontal asymptote because the degree of the numerator is one degree higher than the degree in the denominator; instead there will be an oblique asymptote.

To find the oblique asymptote, divide the numerator by the denominator either by long division or by synthetic division. The oblique asymptote will be the quotient disregarding the remainder.

-3	1	3	2
		-3	0
	1	0	2

$\frac{x^2+3x+2}{x+3}=x+\frac{2}{x+3}$

The oblique asymptote is at y=x (slant line drawn in dashes).



What's More

Activity 1: Find the zeroes, intercepts, and asymptotes of the following rational functions.

1) $f(x)=\frac{x+1}{2x^2-2}$

3) $f(x)=\frac{x+2}{x^2+4x+4}$

5) $f(x)=\frac{x^2+4}{x+1}$

2) $f(x)=\frac{2x^2-2}{x+1}$

4) $f(x)=\frac{x^2-x-6}{x^2+4x+4}$

What I Have Learned

Fill in the blanks. Write the term/s or expression/s that will complete the sentence.

1) The rational function $f(x)=\frac{1}{x+5}$ will have a vertical asymptote at __________ and a horizontal asymptote at _______________.
2) The zeroes of the rational function $f(x)=\frac{x^2+6x+5}{x+6}$ are__________.
3) The rational function $f(x)=\frac{x^2+6x+5}{x+5}$ will have an oblique asymptote at y=_____________________.
4) The y-intercept of the rational function $f(x)=\frac{1}{x^2-1}$ is at ________.
5) The rational function $f(x)=\frac{x^2+6x+5}{x+6}$ will have a vertical asymptote at _____________, a horizontal asymptote at _______________ and an oblique asymptote at ________________.

What I Can Do

Find the zeroes, intercepts, and asymptotes of the following rational functions.

1) $f(x)=\frac{x+2}{2x^2-8}$

3) $f(x)=\frac{x-2}{x^2-4x+4}$

5) $f(x)=\frac{x^2-9}{x+1}$

2) $f(x)=\frac{2x^2-2}{2x+2}$

4) $f(x)=\frac{x^2-x-6}{x^2-6x+9}$

Rubrics for Scoring

Score	Description
15 points	Complete solutions with correct answer
10 points	75% correct solutions with incorrect answer
5 points	50% correct solutions with incorrect answer
No point earned	No output at all

Assessment

Write the letter of the correct answer on a clean sheet of paper.

1. It is a line that the graph gets close to but does not touch.
   A. Asymptotes C. y-intercepts
   B. x-intercepts D. Zeroes

2. The vertical asymptotes of the rational function $f(x)=\frac{1}{x^2-9}$ are at x=_____ and x=_____.
   A. -3 B) 3 C) -3 and 3 D) None

3. The y-intercept of the rational function $f(x)=\frac{2x^3+3x-10}{x+5}$ is at y=____.
   A. -2 B. -1 C. 1 D. 2

4. The rational function $f(x)=\frac{x^2-4x+4}{x+1}$ has an oblique asymptote at ______.
   A. y =x+5 B) y=x-5 C) y=x+3 D) y=x-3

5. The rational function $f(x)=\frac{x^2-1}{x+1}$ has a horizontal asymptote at y=____.
   A. -1 B. 1 C. -1 and 1 D. None of these

Additional Activities

Write the letter of the correct answer on a clean sheet of paper.

1. The zeroes of the function $f(x)=\frac{x^2-5x+6}{x^2+2x-24}$ are ___________.
   A. -3 and -2 B. -3 and 2 C. -2 and 3 D. 2 and 3

2. The vertical asymptotes of the rational function $f(x)=\frac{1}{x^2-49}$ are at x=_____ and x=_____.
   A. -7 B. 7 C. -7 and 7 D. None

3. The y-intercept of the rational function $f(x)=\frac{x+3}{x+5}$ is at y=____.
   A. $\frac{3}{5}$ B. $-\frac{3}{5}$ C. $\frac{5}{3}$ D. $-\frac{5}{3}$

4. The rational function $f(x)=\frac{x^2+3x-4}{x+2}$ has an oblique asymptote at ______.
   A. y =x+1 B. y=x-1 C. y=x+2 D. y=x-2

5. The rational function $f(x)=\frac{x^2-16}{x+4}$ has a horizontal asymptote at y=______.
   A. -1 B. 1 C. -1 and 1 D. None

Lesson 3: Domain and Range of a Rational Function

What's In

Solve for the values of x and answer the questions that follow.

1) $x + 1 = 2x - 5$
2) $x^2 - 81 = 0$
3) $x^2 - 19x + 90 = 0$
4) $(x)(x-3)(x+2) = 0$
5) $x(x-2)(x+5)(x+1) = 0$

Guide Questions:

1. Which of the following illustrates an example of a linear equation?
2. How do we solve a linear equation?
3. Which of the following illustrates a quadratic equation?
4. What are the ways to solve a quadratic equation?
5. Is it possible for an equation to have 2 or more solutions? Why?

What's New

Consider the statement:

> All real numbers have a corresponding reciprocal (multiplicative inverse).

All real numbers have a corresponding reciprocal (multiplicative inverse) which can be represented by the rational function $f(x) = \frac{1}{x}; x \neq 0$ which means x can take any value that will not make the denominator equal to zero, which makes the function undefined. This set of possible values of x is called the **DOMAIN of the function.**

1) How about the corresponding values of y, is it the same as its domain?
2) If it is not the same as its domain, is zero included so that the corresponding values of y will be the set of all real numbers?
3) What are the ways to determine the possible values of y?

What is It

The function $f(x) = \frac{1}{x}$ can be written as $y = \frac{1}{x}$, if x can take any values except zero, how about the corresponding y values or function values $f(x)$? Can it also take all values except zero?

Consider the table as x takes a very large number to the right.

x	1	2	5	10	20	40	50	100	1000
y=f(x)	1	0.5	0.2	0.1	0.05	0.025	0.02	0.01	0.001

It seems that the function values y=f(x) is getting closer to zero or almost zero. How about as x takes a very small negative number to the left?

x	-1	-2	-5	-10	-20	-40	-50	-100	-1000
f(x)	-1	-0.5	-0.2	-0.1	-0.05	-0.025	-0.02	-0.01	-0.001

It also seems that the function values y=f(x) is getting closer to zero or almost zero.

Is zero included, so that the corresponding y values or function values can take all values or simply the set of all real numbers($\mathbb{R}$)?

When y = 0, then $y = \frac{1}{x}$ becomes $0 = \frac{1}{x}$ replacing y by 0.

$x(0) = \left(\frac{1}{x}\right)x$ Multiply both sides by x to get rid of the denominator x

$0=1$ This is a false statement. Which means 0 is not included in the set of y values or functions.

Since $x \neq 0$, what happens to the function values as the values of x get closer to the right or left of 0?

X	0.5	0.25	0.2	0.1	0.01	0.001	0.0001	0.00001
f(x)	2	4	5	10	100	10000	10000	100000

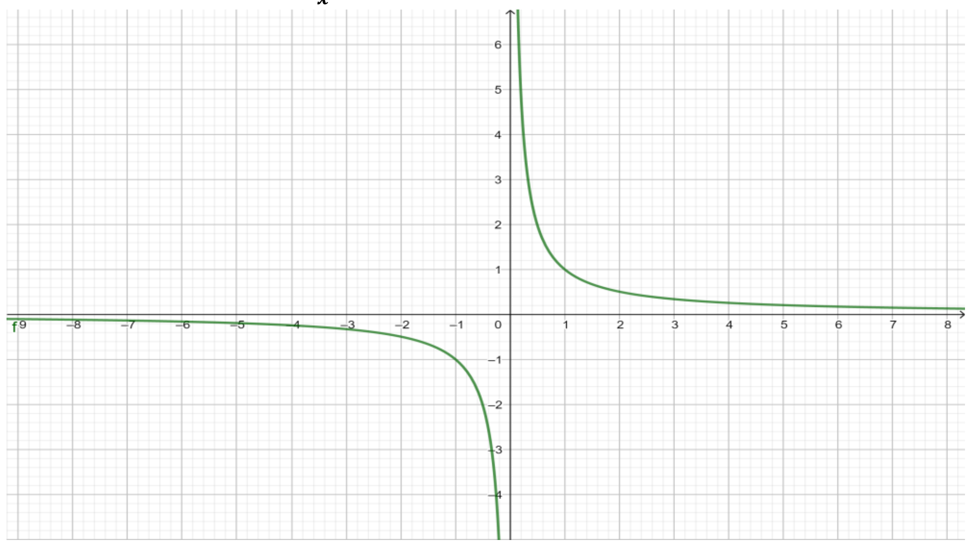
The function values gets larger up to $+\infty$.

X	-0.5	-0.25	-0.2	-0.1	-0.01	-0.001	-0.0001	-0.00001
f(x)	-2	-4	-5	-10	-100	-10000	-10000	-100000

The function values get smaller up to $-\infty$.

Therefore, the set of all possible y values or function values is the set of all real numbers except 0 which is called the **RANGE of the function.**

From the graph of $f(x) = \frac{1}{x}$



It is clearly seen that the graph never crosses x=0 (y-axis) and y=0 (x-axis), which means 0 is not included both in the domain and range of the function $f(x) = \frac{1}{x}$.

What you will learn here is about getting the domain and range of rational functions.

Definition: Domain: The set of all values of x for which the function is defined. Range: The set of all function values (output values) that the function takes.

To find the domain of the rational function:
Set the denominator to zero, to determine the values that will make the function undefined which must be excluded.

For rational functions, you can determine the range by inspecting the graph.

Examples:

1) Find the domain and range of the rational function $f(x) = \frac{x}{x+1}$.

To find the domain of the function:

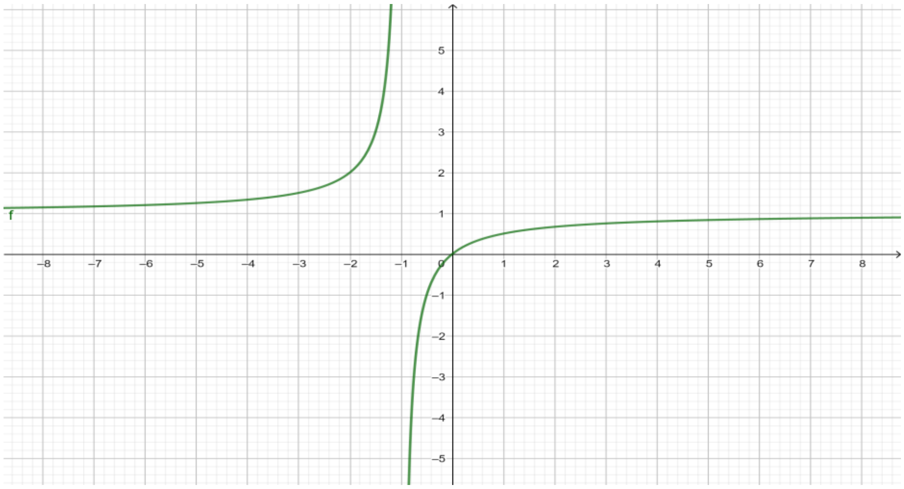
Set the denominator to zero to determine the values that will make the denominator zero which must be excluded in the domain.

x+1= 0	Set the denominator to zero.
x =-1	Add -1 to both sides to solve for the value of x (addition property of equality).

Therefore, the domain is $\{x/x \neq -1\}$ or $(-\infty, -1) \cup (-1, +\infty)$.

To find the range of the function:

We need to examine the graph of $f(x) = \frac{x}{x+1}$, which can be sketched either by plotting points or by getting the intercepts, zeroes and asymptotes.



The graph has a horizontal asymptote at y=1 and it is clearly seen that the graph only gets closer to the line y=1 but never crosses the line y=1. Therefore, the range:$\{y/y \neq 1\}$or $(-\infty, 1) \cup (1, +\infty)$.

2. Determine the domain and range of the rational function

$$f(x)=\frac{x+1}{x^2-9}.$$

To find the domain of the function:

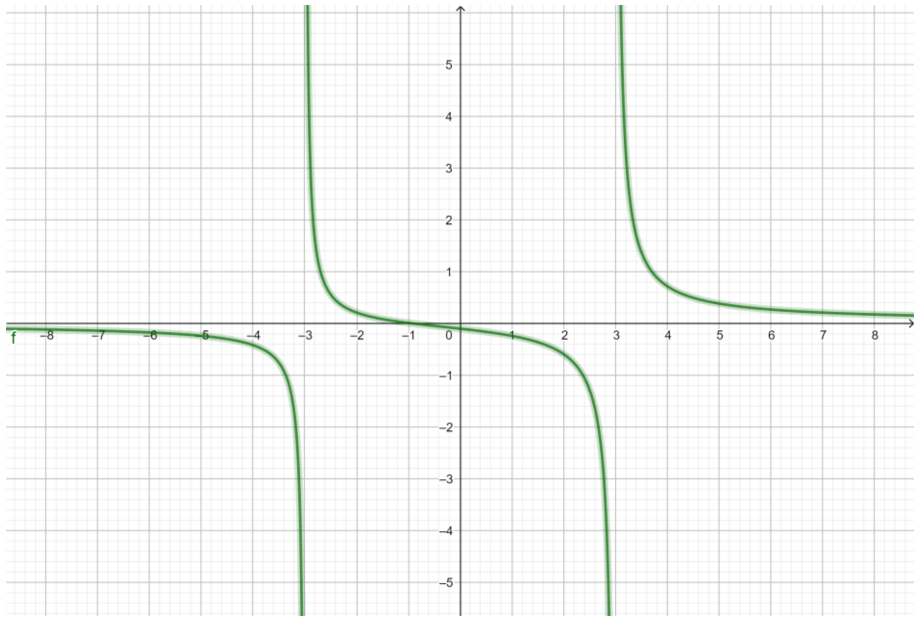
Set the denominator to zero to determine the values that will make the denominator zero which must be excluded in the domain.

$x^2 - 9 = 0$		Set the denominator to zero.
$(x+3)(x-3) = 0$		Factor out the left-hand side of the equation.
$x + 3 = 0$	$x - 3 = 0$	Set each factor to zero.
$x = -3$	$x = 3$	Solve for the values of x.

Therefore, the domain is $\left\{\frac{x}{x} \neq -3,\ x \neq 3\right\}$or $(-\infty, -3) \cup (-3,3) \cup (3, +\infty)$.

To find the range of the function:

We need to examine the graph of the rational function $f(x) = \frac{x+1}{x^2-9}$, which can be sketch either by plotting points or by getting the asymptotes, intercepts and zeroes of a rational function.



It is clearly seen from the graph that the function values take the set of all values of y since the graph continuously extends without bounds above and below the x-axis. Therefore, the range will be $(-\infty, +\infty)$.

What's More

Find the domain and range of the following rational functions in interval notation.

1) $f(x)=\frac{5}{x+3}$

2) $f(x)=\frac{x+1}{x-1}$

3.) $f(x)=\frac{x^2-9}{x-3}$

4) $f(x)=\frac{x+1}{x^2-4}$

What I Have Learned

Fill in the blanks. Write the statement/s or expression/s on the blank to complete each sentence.

1. The domain of the function is the set of all values of x that will not make the ____________________ equal to zero.
2. To find the domain of the function, solve for the values that will make the denominator equal to ______________.
3. The range of the function is the set of all _______________values.
4. The only real number with no multiplicative inverse is __________.

What I Can Do

Problem Set: Find the domain and range of the following rational functions in interval notation.

1) $f(x)=\frac{5}{2x+3}$ 2) $f(x)=\frac{x+1}{2x-1}$ 3) $f(x)=\frac{x^2-49}{x-7}$

Rubrics for Scoring

Score	Description
15 points	Complete solutions with correct answer
10 points	75% correct solutions with incorrect answer
5 points	50% correct solution with incorrect answer
No point earned	No output at all

Assessment

Write the letter of the correct answer on a clean sheet of paper.

1. Given the rational function $f(x)=\frac{1}{x^2-4}$, what are the values that will make the denominator zero?

A. -4 B. ± 4 C. -4 D. ± 2

2. The domain of the rational function $f(x)=\frac{x+1}{x-3}$ is____________________.

A. $\{x/x \neq -1\}$ B. $\{x/x \neq -3\}$ C. $\{x/x \neq -1\}$ D. $\{x/x \neq 3\}$

3. The range of the rational function $f(x)=\frac{1}{x+2}$ is ____________________.

A. $\{y/y \neq -2\}$ B. $\{y/y \neq -1\}$ C. $\{y/y \neq 0\}$ D. $\{y/y \neq 1\}$

4. What are the restricted values in the rational function $f(x)=\frac{x+2}{x^2-7x+12}$?

A. 3 and 4 C. -3 and 4
B. -4 and 3 D. -4 and -3

5. What is the domain of the rational function $f(x)=\frac{x+1}{x^2+1}$?

A) $(-\infty,-1) \cup (-1,+\infty)$ C) $(-1,1)$
B) $(-\infty,+\infty)$ D) $(-\infty,1) \cup (1,+\infty)$

Additional Activities

Write the letter of the correct answer on a clean sheet of paper.

1. What are the values that will make $f(x) = \frac{x+1}{x^2-1}$ undefined?

   A. -1 B. 1 C. ± 1 D. 0

2. The function $f(x) = \frac{2x}{x^2-x-6}$ is defined if ______________.

   A. $x \neq -2, x \neq 3$ C. $x \neq -2, x \neq -3$
   B. $x \neq -3, x \neq 2$ D. $x \neq 2, x \neq 3$

3. The domain of the rational function $f(x) = \frac{x+1}{x+3}$ is____________________.

   A. $\left\{\frac{x}{x} \neq -1\right\}$ B. $\left\{\frac{x}{x} \neq -3\right\}$ C. $\left\{\frac{x}{x} \neq -1\right\}$ D. $\left\{\frac{x}{x} \neq 3\right\}$

4. The range of the rational function $f(x) = \frac{4x}{2x+4}$ is ____________________.

   A. $\left\{\frac{y}{y} \neq 2\right\}$ B. $\left\{\frac{y}{y} \neq -1\right\}$ C. $\left\{\frac{y}{y} \neq -2\right\}$ D. $\left\{\frac{y}{y} \neq 1\right\}$

5. What are the restricted values in the rational function $f(x) = \frac{x+2}{x^2-4x-12}$?

   A. 2 and 6 C. -6 and 2
   B. -2 and 6 D. -2 and -6

Lesson 4: Problems Involving Rational Functions, Equations and Inequalities

What's In

Solve the following equations.

1. $\frac{x}{2} - \frac{x}{5} = 6$ Answer:________________________

2. $\frac{x-1}{2} = \frac{x+2}{4}$ Answer: ________________________

3. $\frac{5}{x} - \frac{3}{4x} = 1$ Answer: _______________________

4. $\frac{x+4}{2x-10} = \frac{8}{7}$ Answer: _______________________

5. $\frac{2x+1}{5} - \frac{x-1}{3} = \frac{1}{2}$ Answer: _______________________

Guide Questions:
1. How do we solve rational equations?
2. How do we find the zeros of rational function?
3. Can you give an example of applications of rational functions in real-life situations?

What's New

Not all types of real-life problems can be represented by linear, quadratic, or any other polynomials function. Rational functions are of great importance in solving some number-related problems, motion, and work problems. A systematic approach in solving problems should be observed to have a smooth and simplified procedure.

What is It

In this lesson, you will learn all about solving word problems involving rational.

A rational equation has a big role and it is useful for representing real life situations. Let's us study the following illustrative examples for you to appreciate its application in real-life situations.

Example 1: **Number Problem**

Problem 1: The difference of a whole number and its reciprocal is $\frac{63}{8}$, what is the number?

Step 1. Represent the unknown in the problem.

Let x = the whole number

$\frac{1}{x}$ = the reciprocal of the number

Step 2. Form the equation that describes the problem.

$$x - \frac{1}{x} = \frac{63}{8}$$

Step 3. Solve the equation.

$x - \frac{1}{x} = \frac{63}{8}$ Multiply by the LCD (8x).

$8x\left(x - \frac{1}{x}\right) = \left(\frac{63}{8}\right)8x$

$8x^2 - 8 = 63x$ Arrange in the form of $ax^2 + bx + c = 0$.

$8x^2 - 63x - 8 = 0$ Find the factors.

$(8x + 1)(x - 8) = 0$

$8x + 1 = 0$ $\quad$ $x - 8 = 0$ Solve for x.

$x = -\frac{1}{8}$ $\quad$ $x = 8$

Since the missing number is a whole number, - $\frac{1}{8}$ cannot be accepted as the answer. Thus, the whole number in the problem is 8.

Problem 2: When five times the square of a certain integer is divided by four more than three times the same number, the result is -10. What is the integer?

Step 1. Represent the unknown in the problem.
Let x = integer
Five times the square of the integer is $5x^2$.
Four more than thrice the number is 3x + 4.
Step 2. Form the equation that describes the problem $\frac{5x^2}{3x+4} = -10$
Step 3. Solve the equation.

$$\frac{5x^2}{3x+4} = -10$$
$$5x^2 = -10\,(3x + 4)$$
$$5x^2 = -30x - 40$$
$$5x^2+30x+40 = 0$$
$$\frac{1}{5}\;[5x^2 + 30x + 40 = 0\,]\;\frac{1}{5}$$
$$x^2 + 6x + 8 = 0$$
$$(x+2)\,(x + 4)= 0$$
$$x + 2 = 0 \qquad x + 4 = 0$$
$$x = -2 \qquad x = -4$$

Step 4. Check if the computed values satisfy the given condition,

If x = -2,
then $\frac{5(-2)^2}{3(-2)+4} = -10$
$\frac{20}{-2} = -10$

If x = -4
then $\frac{5(-4)^2}{3(-4)+4} = -10$
$\frac{80}{-2} = -10$

Both solutions satisfy the given condition. Thus, the number may be -2 or -4.
Example 2: **Motion Problem**
Motion problems are usually modeled by rational equations. When solving motion problems, we should recall that distance is equal to the product of the rate and time. In symbol, we have

$$\boldsymbol{d = rt}$$

where: d = distance r = rate t = time
Transforming the formula, we also have

$\mathbf{r} = \frac{d}{t}$ and $\mathbf{t} = \frac{d}{r}$

Problem 3: A sprinter runs 9 km/hr faster than the other sprinter. When the faster sprinter travelled a distance of 93 km, the other one has gone 66 km. What is the speed of both runners?

Step 1. Represent the unknown in the problem.

Note that time is expressed as t = $\frac{d}{r}$.

Let x = rate of the sprinter

X + 9 = rate of the second sprinter

	distance	rate	time
1st sprinter	66	x	$\frac{66}{x}$
2nd sprinter	93	x + 9	$\frac{93}{x+9}$

Step 2. Form the equation that describes the problem.

Since the time spent by both sprinters is equal, we have the equation

$$\frac{66}{x} = \frac{93}{x+9}$$

Step 3. Solve the equation.

$$\frac{66}{x} = \frac{93}{x+9}$$

$$(x)(x+9)\ \frac{66}{x} = \frac{93}{x+9}(x)\ (x+9)$$

$$66\ (x+9) = 93x$$

$$66x + 594 = 93x$$

$$93x - 66x = 594$$

$$27x = 594$$

$$x = 22$$

The rate of the first sprinter is 22 km/hr.

Step 4. Check if the computed value satisfies the given condition.

	distance	rate	Time
1st sprinter	66	22	$\frac{66}{22} = 3$
2nd sprinter	93	22 + 9 = 31	$\frac{93}{22+9} = 3$

Example 3: **Work Problems**

Work problems are also modeled by rational equations. To solve work problems, keep in mind that if someone can finish a work in 5hrs, then he can finish $\frac{1}{5}$ of the total work in an hour.

Another important fact to remember is that, if two or more people will be working on a certain task, the time that they will spend working together is definitely shorter than any of the individual work.

Problem 4:
Lanie can finish her Mathematics portfolio in 3 hours while Nelia can finish the same job in 5 hours. If their teacher asks them to work together in finishing the Mathematics portfolio, how long will it take them to finish the work?

Step 1. Represent the unknown in the problem.

Nelia can finish the work in 5hours:

She can do $\frac{1}{5}$ of the work in an hour.

Lanie can finish the work in 3hours:

She can do $\frac{1}{3}$ of the work in an hour.

Nelia and Lanie can finish the work together in x hours.

They can finish $\frac{1}{x}$ portion of the work in an hour.

Step 2. Form the equation that describes the problem.

$$\frac{1}{5} + \frac{1}{3} = \frac{1}{x}$$

Step 3. Solve the equation.

$$\begin{aligned}
\frac{1}{5} + \frac{1}{3} &= \frac{1}{x} \\
15x\ [\frac{1}{5} + \frac{1}{3} &= \frac{1}{x}]\ 15x \\
3x + 5x &= 15 \\
8x &= 15 \\
x &= \frac{15}{8} \text{ or } 1\frac{7}{8}
\end{aligned}$$

Lanie and Nelia can finish the work together in $1\frac{7}{8}$ hours.

What's More

Solve the following problems. Present a complete and systematic solution.

1. Myra takes 2 hours to plant 500 flower bulbs. Francis takes 3 hours to plant 450 flower bulbs. Working together, how long should it take them to plant 1500 bulbs?

2. Jamie, Pria and Paul can paint a room together in 2 hours. If Pria does the job alone she can paint the room in 5 hours. If Paul works alone, he can paint the room in 6 hours. If Jamie works alone, how long will it take her to paint the room?

3. One pipe can fill a pool 5 hours faster than a second pipe. If both pipes are open, the pool can be filled in 6 hours. If only the slower pipe is open, how long would it take to fill the pool?

What I Have Learned

Complete the statement/s below and write your answer on the space provided.
1.Real life problems like number-related, work, and motion problems can be represented and solved by ________________________________.
2. To solve problems involving rational functions, follow these steps:

a. _______________ the given information.
b. _______________ the unknown quantity by a variable.
c. Form the equation representing the relationship among the _______________.
d. _______________.
e. _______________.

What I Can Do

Write the working equations which best represent the following problems.
1. The secretary of the council can type a communication letter in 15 minutes while the assistant secretary can finish the same in 30 minutes. Working together, how long should it take them to type 60 communication letters?

2. A number more than 12 divided by 24 more than twice a number is equal to $\frac{1}{2}$.
3. Twice the sum of a number and its reciprocal is same as $\frac{13}{3}$.
4. A certain boat traveled 32 kilometers along with the current and 16 kilometers against the current. In the same amount of time, find the rate of the boat if the rate of the current is 2 kph.
5. A student can finish a project 2 hours earlier than the other student. How long will it take them to finish the project together?

Rubrics for Scoring

Score	Description
15 points	Complete solutions with correct answer
10 points	75% correct solutions with incorrect answer
5 points	50% correct solution with incorrect answer
No point earned	No output at all

Assessment

Write the letters of the correct answers on a clean sheet of paper.

1. Jeffrey can finish a job in x hours. If Bryan can finish the same job three times faster than Jeffrey and the two of them can finish the job in five hours if they work together, solve for x.
   A. $\frac{20}{3}$ hours B. $\frac{20}{6}$ hours C. $\frac{22}{3}$ hours D. $\frac{22}{4}$ hours

2. Working together, it takes Jom, Jass, and Jack two hours to finish a typing job. When Jack works alone, he can finish the job in eight hours. When Jass works alone, she can finish the job in three hours. How long would it take Jom to finish the job on his own?
   A. 20 hours C. 24 hours
   B. 22 hours D. 23 hours

3. Belen walks 4.8 km to school and returns home on a bike. She averages 2 km per hour faster when cycling than when walking. If it takes her two hours to complete the roundtrip, what is her walking speed?
   A. 5 kph B. 4 kph C. 3 kph D. 6 kph

4. A boy is swimming in a river with the rate of the current equal to 2 kph. If he can swim 5 km upstream in the same amount of time it would take him to swim 8 km downstream, what is his speed in still water?
   A. $\frac{22}{3}$ kph B. $\frac{25}{3}$ kph C. $\frac{29}{3}$ kph D. $\frac{26}{3}$ kph

5. A box with a square base is to have a volume of 54 cubic meters. Let x be the length of the side of the square base and h be the height of the box. What is the measure of a side of the square base if the height should be twice the measure of a side of the square base?
   A. 2 meters C. 3 meters
   B. 6 meters D. 9 meters

Additional Activities

1. The denominator exceeds its numerator by 4. If 6 is added to the numerator and 2 is subtracted from the denominator, the resulting fraction equals to 5. Find the fraction.
   A. $\frac{1}{3}$ B. $-\frac{1}{3}$ C. $\frac{9}{5}$ D. $\frac{5}{9}$

2. A helicopter can fly at 90 kph in still air. If it can fly 220 kilometers with a tailwind in the same time that it can fly 180 kilometers against a headwind, what is the speed of the wind?
   A. 5 kph B. 7 kph C. 9 kph D. 11 kph

3. The ratio of men and women at Rio Olympics 2018 was 4 to 3. If there were 12 000 more men than women in attendance, then how many men and how many women were in attendance?
   A. 48 000 men and 36 000 women
   B. 12 000 men and 24 000 women
   C. 24 000 men and 12 000 women
   D. 36 000 men and 48 000 women

4. Amy and Alex play tennis almost every weekend. So far, Amy has won 12 out of 20 matches. How many matches will Amy have to win in a row to improve her winning percentage to 75%?
   A. 10 B. 12 C. 14 D. 15
5. Val can paint a room in six hours. Working with Vincent, the two of them can paint the room in two hours. How long would it take Vincent to paint the room alone?
   A. 2 hours C. 5 hours
   B. 4 hours D. 3 hours

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