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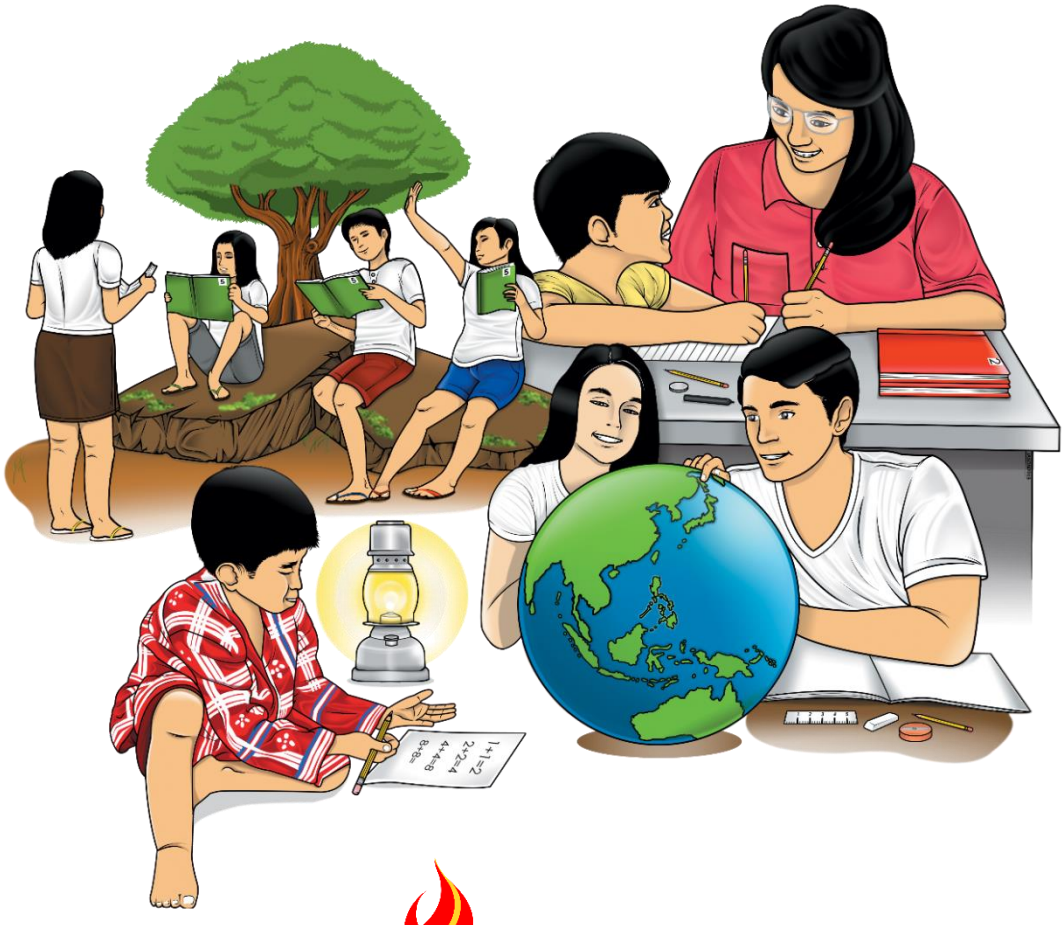
Mathematics

Quarter 1 – Module 9: Proving the Remainder and Factor Theorems



ALTERNATIVE DELIVERY MODE
ADM

Mathematics – Grade 10
Alternative Delivery Mode
Quarter 1 – Module 9: Proving the Remainder and Factor Theorems
First Edition, 2019

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writer's Name: Josefa H. Pugong
Editor's Name: Melchor B. Ticag; Jim D. Alberto
Reviewer's Name: Bryan A. Hidalgo; Heather G. Banagui; Laila B. Kiw-isen; Selalyn Maguilao
Management Team: May B. Eclar
Benedicta B. Gamatero
Carmel F. Meris
Marciana M. Aydinan
Ethielyn E. Taqued
Edgar H. Madlaing
Lydia I. Belingon

Printed in the Philippines by:

Department of Education – Cordillera Administrative Region

Office Address: Wangal, La Trinidad, Benguet
Telefax: (074) 422-4074
E-mail Address: car@deped.gov.ph

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Mathematics

Quarter 1 – Module 9: Proving the Remainder and Factor Theorems

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I need to Know

This module was designed and written with you in mind. It is here to help you understand better on how to prove the remainder and factor theorems. The scope of this module permits it to be used in many different learning situations. The lessons are done to follow the standard sequence of the course.

LEARNING OBJECTIVES:

After going through this module, the learner should be able to:

1. prove the remainder and factor theorems,
2. find the remainder using synthetic division or the remainder theorem, and
3. solve word problems using the remainder and factor theorem

What I know

DIRECTION: Let us determine how much you already know about the remainder theorem and factor theorem. Read and understand each item, then choose the letter of your answer and write it on your answer sheet.

1) Which of the following binomials is a factor of the $P(x) = x^3 - 7x + 5$?
   a) $x - 1$ b) $x + 1$ c) $x + 2$ d) none of these

2) Is the first polynomial a factor of the second polynomial, $x - 1$; $x^2 + 2x + 5$?
   a) yes b) no c) uncertain d) invalid

3) Which is the missing factor in the equation $x^2 - 4 = (x - 2)$ (_____)?
   a) x -2 b) x+ 2 c) x + 4 d) x – 4

4) Which of the following is NOT a factor of $x^3 + 5x^2 - x - 5$?
   a) x + 1 b) x – 1 c) x – 5 d) x + 5

5) What are the factors of $x^2 - 2x - 24$?
   a) (x + 4) (x – 6) c) (x -12) (x+ 1)
   b) (x -8) (x + 3) d) (x + 12) (x – 12)

6) What is the remainder when $x^3 - 4x^2 + x + 8$ is divided by $x - 2$?
   a) 1 b) 2 c) 3 d) 4

7) When P(x) is divided by x – r and the remainder is equal to zero, it means _____.
   a) x – r is a factor of P(x) c) P(x) is the only factor of x – r
   b) P(x) is a factor of x – r d) x – r is the only factor of P(x)

8) Evaluate the polynomial $x^3 + x^2 + x + 3$ if the value of $x = -2$.
   a) 3 b) -3 c) 17 d) -17

9) Which polynomial gives a remainder of 0 when divided by $3x - 2$?
   a) $12x^2 + 15x - 18$ c) $12x^2 + 19x - 18$
   b) $12x^2 + 18x + 7$ d) $12x^2 + 8x + 7$

10) Which of the following is a factor of $2x^2 - 5x + 3$?
   a) x – 3 b) 2x + 3 c) x – 1 d) 3x + 2

11) What is the remainder if $x^2 - 7x - 4$ is divided by $x - 2$?
   a) -14 b) 5 c) 6 d) 7

12) Which statement is true?
   a) The quotient multiplied by the dividend plus the remainder equals the divisor.
   b) If $x^2 + 5x + 7$ is divided by $x + 2$, the remainder is 1.
   c) If the remainder is 0, then the dividend is a factor of the divisor.
   d) The remainder is a factor of the dividend if the quotient is 0.

13) Which is the remainder if $2x^3 - 7x^2 - 19x + 20$ is divided by $x - 5$?
a) -1 b) 0 c) 1 d) 2

14) $x - 3$ is a factor of $x^4 - 3x^3 - x + 3$.
a) false b) true c) uncertain d) invalid

15) Find another factor of $x^3 - 7x^2 + 4x - 28$ if $x - 7$ is a factor.
a) $x^2 + x + 4$ b) $x^2 - 2x - 4$ c) $x^2 - x - 4$ d) $x^2 + 4$

Lesson 1 The Remainder Theorem and the Factor Theorem

What's In

In the earlier lesson, you have learned how to divide polynomials using long division or by synthetic division. Read and understand the discussion below, then investigate.

1. Divide the polynomial function $f(x) = x^3 - 2x^2 + x - 3$ by $x - 2$ using synthetic division.

Solution:

2	1	-2	1	-3
		2	0	2
	1	0	1	-1

2. Since the divisor is $x - 2$, evaluate the above function at $x = \mathbf{2}$.

Solution: $f(x) = x^3 - 2x^2 + x - 3$
$f(2) = 2^3 - 2(2^2) + 2 - 3$ Substitute x by 2
$f(2) = 8 - 8 + 2 - 3$ Simplify
$f(2) = -1$ Perform the operations

Guide Question: What can you say about the results of the two separate solutions?

Notice:
- When the function was divided by $x - 2$, the remainder is -1.
- When the function was evaluated at $x = 2$, the result is -1.
- This leads us to the ***Remainder Theorem***.

What's New

Definition

REMAINDER THEOREM
If a polynomial ***P(x)*** is divided by ***x – a***, then the remainder is ***r = P(a)***.

Example:If $f(x) = x^3 - 2x^2 + x - 3$ divided by x – 2, then the remainder is $r = P(a)$.
Based from the solutions above, the remainder r = -1, and f(2) = -1.
Thus, r = f(2).

When a polynomial is divided by x – a, if the remainder is zero, we say that x – a is a factor of the polynomial. Through the ***remainder theorem***, we now know that the remainder is related to evaluation of the polynomial at the point x = a. We are then led to the ***factor theorem***.

Definition

FACTOR THEOREM
If *P(a) = 0, then x – a is a factor of P(x).*
Conversely, if x – a is a factor of P(x), then P(a) = 0.

What Is It

You have already learned the difference between the Remainder Theorem and the Factor Theorem. Now, you will learn how to use these theorems to solve problems.

Example 1. Find the remainder when $x^3 + 2x^2 - 5x + 2$ is divided by x + 3.

Solution: We can find the remainder in two methods: by synthetic division or by remainder theorem.

Using the Remainder Theorem:

$P(x) = x^3 + 2x^2 - 5x + 2$	At x = - 3
$P(-3) = (-3)^3 + 2(-3)^2 - 5(-3) + 2$	Substitute x by -3
$P(-3) = -27 + 2(9) + 15 + 2$	Simplify
$P(-3) = -27 + 18 + 15 + 2$	
$P(-3) = 8$	Perform the operations

Therefore, the remainder is **8**.

-3	1	2	-5	2
		-3	3	6
	1	-1	-2	**8**

We can check the answer using the Synthetic Division:

Example 2. Determine whether or not x + 2 is a factor of $x^3 - 2x^2 - 5x + 6$.

Solution: By definition of Factor Theorem, x + 2 is a factor of $x^3 - 2x^2 - 5x + 6$ if and only if the remainder is zero. Again, we can use two methods in finding the remainder.

Using the Remainder Theorem:

$P(x) = x^3 - 2x^2 - 5x + 6$	At x = - 2
$P(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$	Substitute x by -2
$P(-2) = -8 - 2(4) + 10 + 6$	Simplify
$P(-2) = -8 - 8 + 10 + 6$	
$P(-2) = 0$	Perform the operations

Since the remainder is zero, **x + 2 is a factor of $x^3 - 2x^2 - 5x + 6$**.

Using the Synthetic Division:

-2	1	-2	-5	6
		-2	8	-6
	1	-4	3	**0**

Note: It is not required to present the solution in two methods. Use the method that you are most comfortable with. However, if you feel uncertain with your answer, you may use the two methods.

Example 3. Show that x – 4 is a factor of $P(x) = x^3 - 6x^2 + 5x + 12$.

Solution: Since x – 4 is a factor of $P(x) = x^3 - 6x^2 + 5x + 12$, the remainder must be zero.

Using the Remainder Theorem:

$P(x) = x^3 - 6x^2 + 5x + 12$	At x = 4
$P(4) = (4)^3 - 6(4^2) + 5(4) + 12$	Substitute x by 4
$P(4) = 64 - 6(16) + 20 + 12$	Simplify
$P(4) = 64 - 96 + 20 + 12$	
$P(4) = 0$	Perform the indicated operations

The remainder is 0.

Example 4. Find k so that x + 5 is a factor of $P(x) = x^3 + 5x^2 - kx - 20$.

Solution: Since x + 5 is a factor of $P(x) = x^3 + 5x^2 - kx - 20$, the remainder is zero.

Using the Remainder Theorem:

$P(x) = x^3 + 5x^2 - kx - 20$	At x = -5
$P(-5) = (-5)^3 + 5(-5)^2 - k(-5) - 20$	Substitute x by -5
$0 = (-5)^3 + 5(-5)^2 - k(-5) - 20$	Change P(-5) by 0 (remainder is 0)
$0 = -125 + 125 + 5k - 20$	Simplify
$0 = 5k - 20$	
$5k = 20$	Solve for k
$k = 4$	Divide both sides by 5

So that x + 5 is a factor of $P(x) = x^3 + 5x^2 - kx - 20$, **k = 4**.

What's More

Exercise 1. Answer what is asked.

A. Use the Remainder Theorem or synthetic division to find each function value.

1. $f(x) = x^3 - 5x^2 - 7x + 4$	a) f(1)	b) f(-2)	c) $f\left(\frac{1}{2}\right)$
2. $g(x) = 2x^6 + 3x^4 - x^2 + 3$	a) g(2)	b) g(3)	c) g(-1)
3. $h(x) = 2x^3 - 7x + 3$	a) h(-3)	b) h(5)	c) h(-10)

B. Solve.

4. Determine if x – 3 is a factor of $P(x)$ where $P(x) = x^4 - 3x^3 - x + 3$.
5. Determine if x – 1 is a factor of $P(x)$ where $P(x) = x^{25} - 4$.
6. Find k so that x – 2 is a factor of $P(x) = x^3 - kx^2 - 4x + 20$.

Exercise 2. Answer what is asked.

1. Use the remainder theorem to find P(2) in $P(x) = x^4 + 4x^3 - x^2 - 16x - 4$.
2. Prove that y – 3 is a factor of $3y^3 - 7y^2 - 20$ using the remainder theorem.
3. Evaluate P(4) in $P(y) = 3y^3 - 7y^2 - 20$.
4. Use the factor theorem to determine whether x – 1 is a factor of $P(x) = x^2 + 2x + 5$.
5. Use the remainder theorem to find the remainder R in $(3x^2 + 5x^3 - 8) \div (x - 4)$.

What I have learned

1. How will you determine that x –r is a factor of P(x)?
2. In the remainder theorem, what will you substitute in the polynomial expression?
3. What are the two ways on how to find the remainder when P(x) is divided by x – r?

What I can do

A. Fill the blanks to complete the statement. Write your answer in the separate sheet of pad paper.

1. x – r is a factor of P(x) if and only if the remainder R when P(x) ÷ (x – r) is _____.
2. By the Remainder Theorem, R = 0 if and only if _____.
3. Thus, x – r is a factor of P(x) if and only if _____.

B. Solve this problem correctly: The volume of a rectangular solid is $(x^3 + 3x^2 + 2x)$ cubic cm, and its height is (x+1) cm. What is the area of the base?

Assessment

DIRECTION: Let us determine how much you have learned from this module. Read and understand each item, then choose the letter of your answer and write it on your answer sheet.

1. What is the remainder if $x^2 – 7x – 4$ is divided by x – 2?
   a) 4 b) -14 c) 2 d) -2

2. Which of the following statements is true?
   a) The quotient multiplied by the dividend plus the remainder equals the divisor.
   b) If $x^2 + 5x + 7$ is divided by x + 2, the remainder is not 1.
   c) If the remainder is 0, then the dividend is a factor of the divisor.
   d) The quotient is a factor of the dividend if the remainder is 0.

3. Which is the remainder if $2x^3 + 4x^2 – x + 7$ is divided by $x – 2$?
   a) 35 b) 36 c) 37 d) 38

4. Is $x – 3$ a factor of $x^4 – 3x^3 – x + 3$?
   a) false b) true c) uncertain d) invalid
5. Find another factor of $x^3 – 7x^2 + 4x – 28$ if $x – 7$ is a factor.
   a) $x^2 + x + 4$ b) $x^2 – 2x – 4$ c) $x^2 – x – 4$ d) $x^2 + 4$

6. Determine which of the following binomials is a factor of $P(x) = x^3 - 7x + 5$.
   a) $x – 1$ b) $x + 1$ c) $x + 2$ d) none of these

7. Is the first polynomial a factor of the second polynomial, $x – 1$; $x^2 + 2x + 5$?
   a) yes b) no c) uncertain d) invalid

8. Which is the missing factor in the equation $x^2 – 4 = (x – 2)$ (_____)?
   a) $x – 2$ b) $x + 2$ c) $x + 4$ d) $x – 4$

9. Which of the following is a factor of $x^3 + 5x^2 – x – 5$?
   a) $x + 1$ b) $x + 2$ c) $x – 5$ d) none of these

10. What are the factors of $x^2 – 2x – 24$?
    a) $(x + 4)(x – 6)$ b) $(x - 8)(x + 3)$ c) $(x - 12)(x + 1)$ d) $(x + 12)(x – 12)$

11. What is the remainder when $x^3 – 4x^2 + x + 8$ is divided by $x – 2$?
    a) 1 b) 2 c) 3 d) 4

12. When P(x) is divided by $x – r$ and the remainder is equal to 0, it means _____.
    a) $x – r$ is a factor of P(x)
    b) P(x) is the only factor of $x – r$
    c. P(x) is a factor $x – r$
    d) $x - r$ is the only factor of P(x)

13. Evaluate the polynomial $x^3 + x^2 + x + 3$ at the given value of $x = -2$
    a) 3 b) -3 c) 17 d) -17

14. Which polynomial gives a remainder of zero when divided by $3x – 2$?
    a) $12x^2 + 15x – 18$
    b) $12x^2 + 18x + 7$
    c) $12x^2 + 19x – 18$
    d) $12x^2 + 8x + 7$

15. Which of the following is a factor of $2x^2 – 5x + 3$?
    a) $x – 3$ b) $2x + 3$ c) $x – 1$ d) $3x + 2$

Additional Activity

DIRECTION. Perform each given task.

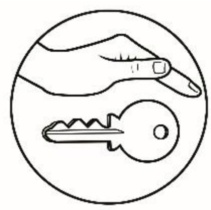
Task A. Let $P(x) = x^3 - 64$

a. Find P(4).
b. Find the remainder when P(x) is divided by x – 4.

Task B. Let $P(x) = x^3 - 64$

a. Find P(-4).
b. Find the remainder when P(x) is divided by (x + 4).

Task C. Compare your answers. What do you observe?



Answer Key

What I Know

1. d	4. c	7. a	10. c	13. b
2. b	5. a	8. b	11. a	14. b
3. b	6. b	9. c	12. b	15. d

What's More

Exercise 1

(A)

1) a = - 7 b = - 10 $c = -\frac{5}{8}$

2) a = 175 b = 1695 c = 7

3) a = -30 b = 218 c = -1927

(B)

4) x – 3 is a factor

5) x – 1 is not a factor

6) k = 5

Exercise 2

1) 8 3) Not

2) Not 4) 360

3) 60

What I Can Do

(A) 1) Zero 2) (x – a) is a factor of P(x) 3) the remainder of P(x) ÷ x – r is zero

(B) $x^2 + 2x$

Assessment

1. b	4. b	7. b	10. a	13. b
2. d	5. d	8. b	11. b	14. c
3. c	6. d	9. a	12. a	15. c

Additional Activity

Task A

a) 0

b) 0

Task B

a) -128

b) -128

Task C

The remainder and the value of the polynomial at r taken from x – r are equal.

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For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph: