

STATISTICS AND PROBABILITY

Quarter 3: Module 4 Normal Probability Distribution



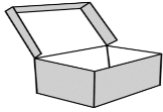
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What I Need to Know

Hello Grade 11 learners! In this module you will learn how to:

Illustrate a normal random variable and its characteristics **M11/12SP-IIIc-1**,

Identify regions under the normal curve corresponding to different standard normal values **M11/12SP-IIIc-3**,

Convert a normal random variable to a standard normal variable and vice-versa **M11/12SP-IIIc-4** and

Compute probabilities and percentiles using the standard normal table **M11/12SP-IIIc-d-1**.

You can say that you understood the lesson in this module if you can already:

1. Illustrate a normal random variable and its characteristics,
2. Identify regions under the normal curve corresponding to different standard normal values ,
3. Convert a normal random variable to a standard normal variable and vice-versa and
4. Compute probabilities and percentiles using the standard normal table .



What I Know

Read each item carefully. Encircle the letter that corresponds to the correct answer.

1. What do you call a continuous, symmetric, bell-shaped distribution of a variable where in the measures of central tendencies are equal?
 - A. Box-and-Whisker Plot
 - B. Histogram
 - C. Normal Probability Distribution
 - D. Stem-and-Leaf Plot
2. Which of the following is **NOT** a property of a normal distribution?
 - A. The total area is equal to 1.
 - B. The mean, median and mode are equal.
 - C. The area to the right and to the left of the mean is 0.5.
 - D. The tails of a normal curve are asymptotic relative to the vertical line.
3. Which of the following statement is TRUE about changing a random variable X to a standard score?
 - A. Subtract the mean from the value of X and divide the result by the standard deviation.
 - B. Subtract the value of X from the mean and divide the result by the standard deviation.

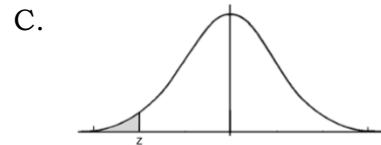
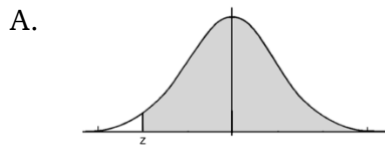


- C. Subtract the mean from the value of X and divide the result by the variance.
 D. Subtract the value of X from the mean and divide the result by the variance.
4. In a normal curve, the area from the mean minus 1 standard deviation to the left of mean plus 1 standard deviation to the right of mean is approximately _____.
- A. 0.50 B. 0.64 C. 0.95 D. 0.98
5. A set of test scores is considered normally distributed. Its mean is 80 and its standard deviation is 10. If these scores are converted to standard normal Z scores, what will be the value of the mode and median?
- A. 0 B. 1 C. 40 D. 80

For numbers 6-8, consider the given below.

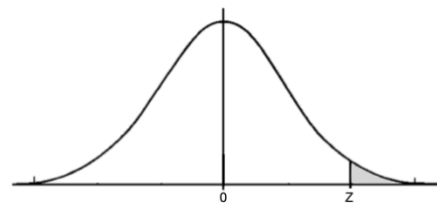
A random variable X is normally distributed with a mean of 42 and standard deviation of 5.

6. What is the value of the random variable X if the area to the left of Z is 0.0207?
- A. -2.04 B. 2.04 C. 31.8 D. 52.2
7. If the area to the right of Z is 0.9292, what will be the value of its random variable X ?
- A. -1.47 B. 1.47 C. 34.65 D. 49.35
8. Which of the following normal curves suits the description, to the left of Z is 0.8508?



9. Which of the following best describes the normal curve on the right?

- A. to the right of 0.0485
 B. to the left of 0.0485
 C. between 0 to Z is 0.0485
 D. to the right of 0.5



10. What is the probability of choosing a random variable from the area under a normal curve to the left of $Z = -2.34$?
- A. 0.0096 B. 0.0906 C. 0.9904 D. 0.9094

LESSON 1: Illustrating and Identifying Regions Under the Normal Curve

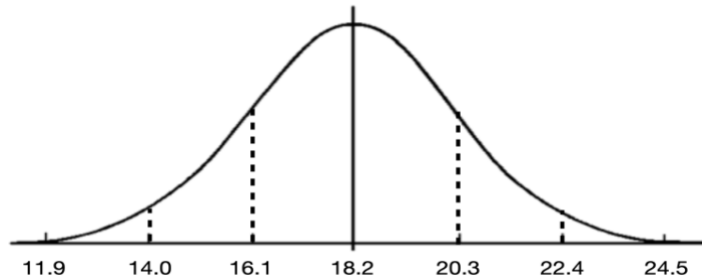


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What's In

A continuous random variable X is normally distributed with a mean of 18.2 and a standard deviation of 2.1. Find the following probabilities using the figure below.



- Find:
1. $P(X > 18.2)$
 2. $P(16.1 < X < 20.3)$
 3. $P(X > 20.3)$

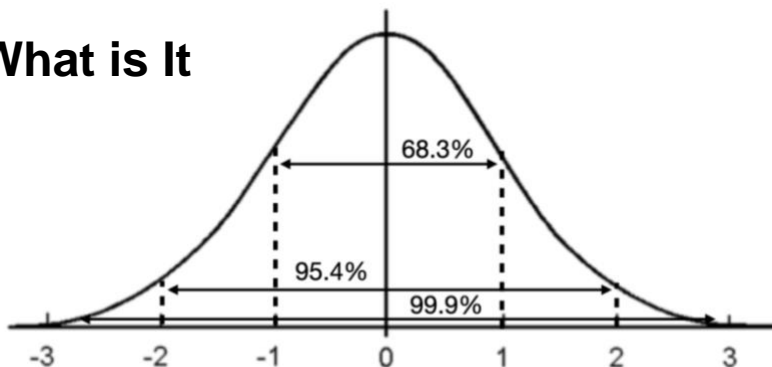


What's New

In the previous lessons, you have learned about the standard distribution table. In this lesson, you will be dealing with the areas under the normal curve and what does its shaded regions want to show and the properties underlying it. What do you think a normal curve wants to show if its shaded region is on the left side of 0? How about the right side of 0? What will it imply if the shaded region covers a part of the left side of the 0 up to its right side or vice-versa? You will also be dealing with normal random variables and how to illustrate them.



What is It



The illustration that you see above is an example of a normal distribution curve with an approximate area of 68.3% for the standard deviation $Z = -1$ and $Z = 1$,



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while the area within $Z = -2$ and $Z = 2$ has approximately 95.4% and for the area within $Z = -3$ and $Z = 3$ has 99.9%.

The random variable X that you will see in the next lesson can be shown in a normal equation that is called the normal random variable. It can be computed if the mean, standard deviation, and probability are given.

From this, you can make a statement that the probabilities of having a random variable Z between -1 and 1 is 0.683 , 0.954 between -2 and 2 then 0.997 probability between 0.3 and 3 . In probability notation, you can write these as:

$$P(-1 < Z < 1) = 0.683$$

$$P(-2 < Z < 2) = 0.954$$

$$P(-3 < Z < 3) = 0.997$$

On this point, let us enumerate the properties of a normal distribution.

- The total area under the standard normal curve is equal to 1.
- The standard normal distribution is symmetrical to the mean ($\mu = 0$).
- The area to the right and left of the mean is 0.5 ($\mu = 0$).
- The mean, the median and the mode are equal and located at the axis of symmetry.
- The horizontal line is asymptotic to the tails of a standard normal curve.
- At least three standard scores each to the left and to the right of the mean on the standard normal curve may be used before the tails appear to touch the horizontal line on both ends.
- The distance from one standard score to the next standard score is equal to 1 along the horizontal line.
- The area of the shaded region under the standard normal curve between two points a and b is equal to the probability that the standard random variable Z will fall into an interval a to b .

Now let us try identifying the region being described. Find the area and shade the region in a normal curve. (See table at the end of this module.)

1. $P(0 < Z < 1.25)$
2. $P(-2.43 < Z < 1.25)$
3. $P(Z < 1.25)$
4. $P(Z < -2.43)$
5. $P(1.25 < Z < 2.43)$

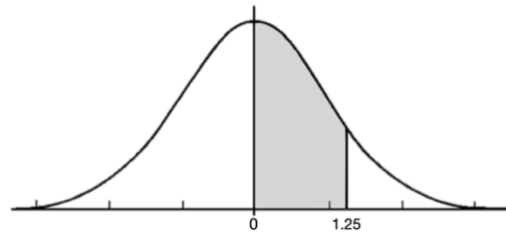
Answers:

1. $P(0 < Z < 1.25)$ = area under the normal curve between $Z = 0$ and $Z = 1.25$
= $A(Z = 1.25)$
= 0.3944

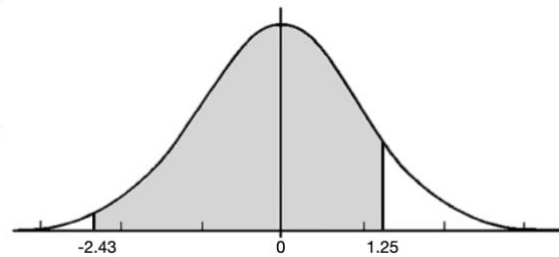
(Note: The value from the whole number up to the tenths place of the Z score will be looked at the vertical column and the value of the hundredths place of the Z score



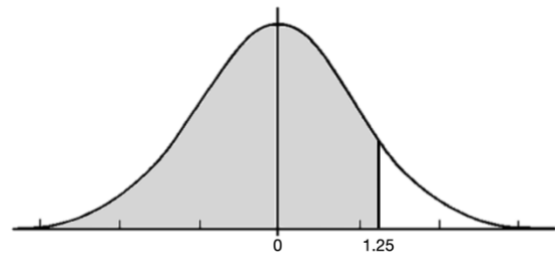
will be looked at the horizontal part of the table. The intersection of the two values is the area under the normal curve corresponding to the standard normal value $Z = 1.25$. The result is 0.3944.)



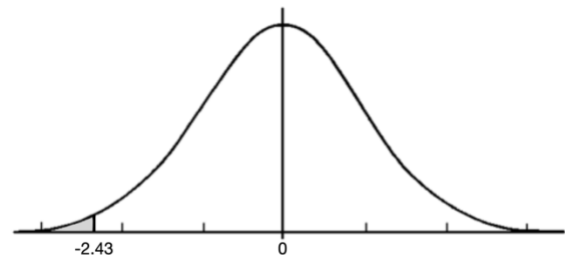
$$\begin{aligned}
 2. P(-2.43 < Z < 1.25) &= \text{area under the normal curve between } Z = -2.43 \text{ and } Z = 1.25 \\
 &= A(Z = -2.43) + A(Z = 1.25) \\
 &= 0.4925 + 0.3944 \\
 &= 0.8869
 \end{aligned}$$



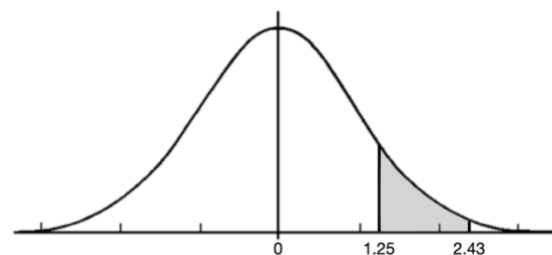
$$\begin{aligned}
 3. P(Z < 1.25) &= \text{area under the normal curve to the left of } Z = 1.25 \\
 &= 0.5 + A(Z = 1.25) \\
 &= 0.5 + 0.3944 \\
 &= 0.5944
 \end{aligned}$$



$$\begin{aligned}
 4. P(Z < -2.43) &= \text{area under the normal curve to the left of } Z = -2.43 \\
 &= 0.5 - A(Z = -2.43) \\
 &= 0.5 - 0.4925 \\
 &= 0.0075
 \end{aligned}$$



$$\begin{aligned}
 5. P(1.25 < Z < 2.43) &= \text{area under the normal curve between } Z = 1.25 \text{ and } Z = 2.43 \\
 &= A(Z = 2.43) - A(Z = 1.25) \\
 &= 0.4925 - 0.3944 \\
 &= 0.0981
 \end{aligned}$$

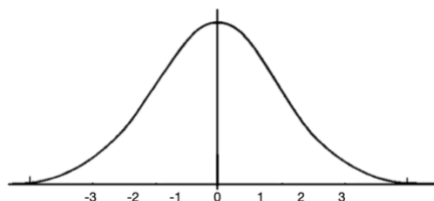




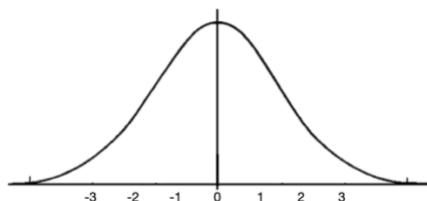
What's More

Identify the region being described by the following normal curves. Shade the region and find the area using the standard normal distribution table.

1. $P(0 < Z < 1.96)$



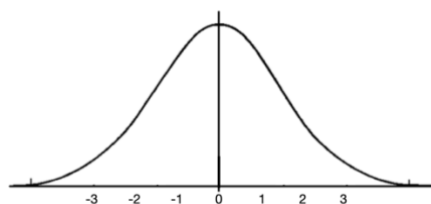
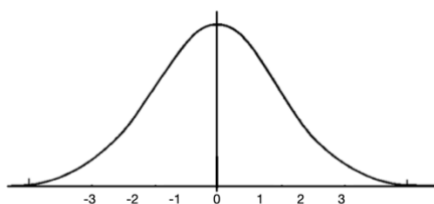
2. $P(-1.73 < Z < 0)$



3. $P(Z < -1.73)$

$P(Z < -1.73)$

4. $P(Z > 1.45)$



What I Have Learned

I learned that:

I discovered that:

I realized that:

I need help on:



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What I Can Do

PERFORMANCE TASK: A Math test has a mean of 80 and a standard deviation of 10.

Solve the problem using the adjoining **UPS Check** chart below.

<p><u>U</u>NDERSTAND <i>What is the problem all about?</i> State the given in the problem.</p>	<p><u>P</u>LAN <i>What information is needed to solve the problem?</i> Estimate a reasonable solution. Draw your model.</p>
<p><u>S</u>OLVE <i>What method is applicable to be used?</i> Show all your solutions in solving the problem.</p>	<p><u>C</u>HECK <i>Does my answer make sense? Is my estimate correct? Did I answer the question?</i> Justify your answer.</p>

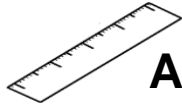
What percentage of students will lie between 65 and 85?

RUBRIC FOR THE ACTIVITY

	1	3	5
Computation	The answer and strategies used were incorrect.	The answer is incorrect, but the strategy used to solve is correct.	The answer and strategies used to solve are correct.
Explanation	No written explanation is included.	Explanation is included but not detailed or written in complete sentences.	Explanation is detailed and written in complete sentences.



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Assessment

Read each item carefully. Encircle the letter that corresponds to the correct answer.

1. What do you call a set of numerical values that is normally distributed and can be standardized by subtracting the population mean from the value of X and dividing the result the population standard deviation?

- A. Normal Random Variable
- B. Normal Distribution Table
- C. Standard Normal Variable
- D. Skewness

2. Which of the following is **NOT** a property of a normal distribution?

- A. The total area is equal to 1.
- B. The mean, median and mode are not equal.
- C. The area to the right and to the left of the mean is 0.5.
- D. The tails of a normal curve are asymptotic relative to the horizontal line.

3. In a normal curve, the area from the mean minus 1 standard deviation to the left of mean plus 1 standard deviation to the right of mean is approximately _____.

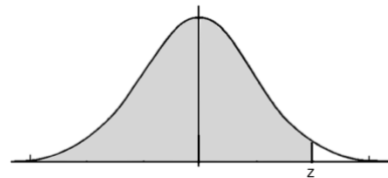
- A. 0.50
- B. 0.64
- C. 0.95
- D. 0.98

4. A set of test scores is considered normally distributed. Its mean is 90 and its standard deviation is 5. If these scores are converted to standard normal Z scores, what will be the value of the mode and median?

- A. 0
- B. 1
- C. 45
- D. 90

5. Which of the following best describes the normal curve on the right?

- A. to the right of 0.5967
- B. to the left of 0.5967
- C. between 0 to Z is 0.5967
- D. to the right of 0.5



Additional Activities

Suppose that your aunt is pregnant and due in 100 days. The probability density distribution function for having a child is approximately normal with a mean of 100 and a standard deviation of 8. You are in a vacation and need to return in 85 days so that you will be home for your aunt's delivery of her baby. If your mother asked you to returned home immediately so that there is a 99% chance that you will make it back for the birth, when must you return home?





What's In

When do we use the symbol μ for the mean and σ for the standard deviation?



What's New

The average score in a Math test is 80 with a standard deviation of 10. What is the standard score of Cardi and Rita whose grades are 97 and 86 respectively?



What is It

Using a formula, you can convert raw scores to standard scores and a normal random variable X is in its standardized form if its value is expressed as the number of standard deviations that can be found either to the left or to the right of its mean. Also, the normal random variable X can be standardized by getting the difference between the value of X and population mean and dividing the result by the population standard deviation.

Below is the formula to convert raw scores to standard scores:

$$Z = \frac{X - \mu}{\sigma}$$

where: Z is the standard score

μ is the population mean

X is the raw score

σ is the population standard deviation

By using this formula, you can identify the comparison of a normal variable X and a standard normal variable Z .

To solve the problem above regarding the standard scores for Cardio and Rita, we will have:



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Cardo

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{97 - 80}{10}$$
$$Z = 1.7$$

Rita

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{86 - 80}{10}$$
$$Z = 0.6$$

Therefore, the standard scores of Cardo and Rita are 1.7 and 0.6 respectively.

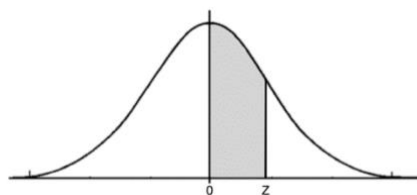
Let us have an example where in the random variable X is normally distributed with a mean of 40 and standard deviation of 3. Find the values of the random variable X and Z given the areas of the intervals under the standardized normal curve.

1. between 0 and Z is 0.3944
2. between 0 and Z is 0.2967
3. to the left of Z is 0.0207
4. to the right of Z is 0.0485
5. to the right of Z is 0.9292
6. to the left of Z is 0.9861

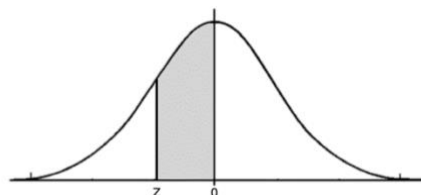
Answers:

1. between 0 and Z is 0.3944

The area from 0 to Z is 0.3944, therefore, if you are going to look for this at the standard normal distribution table, you will have either $Z = +1.25$ or $Z = -1.25$. To illustrate this you will have:



$$Z = \frac{X - \mu}{\sigma}$$
$$1.25 = \frac{X - 40}{3}$$
$$X = 43.75$$



$$Z = \frac{X - \mu}{\sigma}$$
$$-1.25 = \frac{X - 40}{3}$$
$$X = 36.35$$

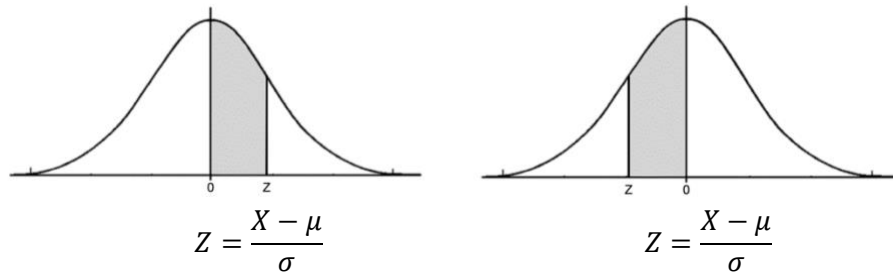
Therefore the value of the random variable X is 43.75 or 36.35.

2. between 0 and Z is 0.2967



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The area from 0 to Z is 0.2967, therefore, if you are going to look for this at the standard normal distribution table, you will have either $Z = +0.83$ or $Z = -0.83$. To illustrate this you will have:



$$0.83 = \frac{X - 40}{3}$$

$$X = 42.49$$

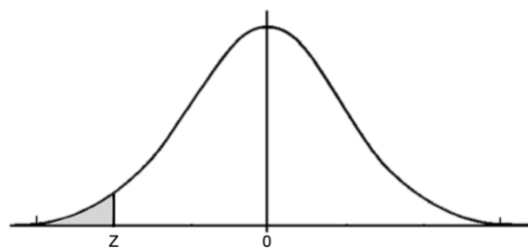
$$-0.83 = \frac{X - 40}{3}$$

$$X = 37.51$$

Therefore the value of the random variable X is 42.49 or 37.51.

3. to the left of Z is 0.0207

The area from 0 to Z is 0.0207. Since this area is less than 0.5, you can state that the value of Z is negative. To get its value, find the area from 0 to Z by subtracting 0.0207 from 0.5. The answer will give you an area of 0.4793 which is equivalent to 2.04 from the standard normal distribution table and since Z is located at the left side of 0, $Z = -2.04$.



$$Z = \frac{X - \mu}{\sigma}$$

$$-2.04 = \frac{X - 40}{3}$$

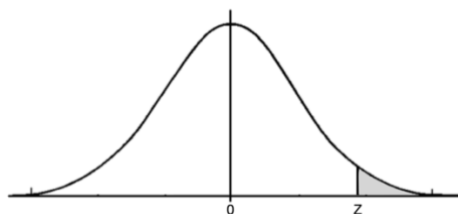
$$X = 33.88$$

Therefore the value of the random variable X is 33.88.

4. to the right of Z is 0.0485

The area from 0 to Z is 0.00485. Since the area is less than 0.5 but located at the right of Z, you can state that Z is positive. To get its value, find the area from 0 to Z by subtracting 0.0485 from 0.5. The answer will give you an area of 0.4515 which is equivalent to 1.66 from the standard normal distribution table. Therefore, $Z = 1.66$.





$$Z = \frac{X - \mu}{\sigma}$$

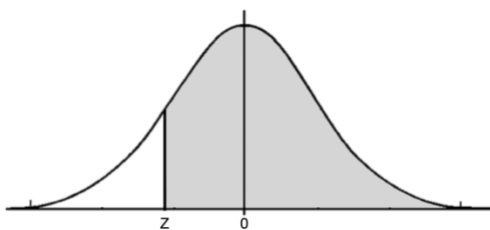
$$1.66 = \frac{X - 40}{3}$$

$$X = 44.98$$

Therefore the value of the random variable X is 44.98.

5. to the right of Z is 0.9292

The area from 0 to Z is 0.9292. Since the area is greater than 0.5 but located at the right of Z , you can state that Z is negative since the boundary of area will cover the left part of 0. To get its value, find the area from 0 to Z by subtracting 0.5 from 0.9292. The answer will give you an area of 0.4292 which is equivalent to 1.47 from the standard normal distribution table. Therefore, $Z = -1.47$.



$$Z = \frac{X - \mu}{\sigma}$$

$$-1.47 = \frac{X - 40}{3}$$

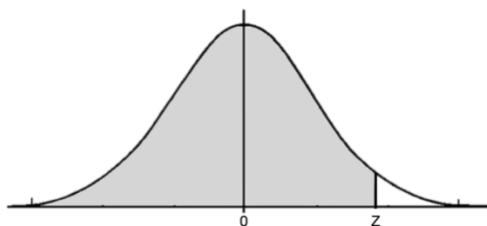
$$X = 35.59$$

Therefore the value of the random variable X is 35.59.

6. to the left of Z is 0.9861

The area from 0 to Z is 0.9861. Since the area is greater than 0.5 but located at the left of Z , you can state that Z is positive since the boundary of the area will end at the right part of 0. To get its value, find the area from 0 to Z by subtracting 0.5 from 0.9861. The answer will give you an area of 0.4861 which is equivalent to 1.47 from the standard normal distribution table. Therefore, $Z = 2.20$.





$$Z = \frac{X - \mu}{\sigma}$$

$$2.20 = \frac{X - 40}{3}$$

$$X = 46.60$$

Therefore the value of the random variable X is 46.60.

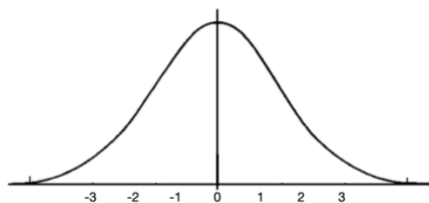
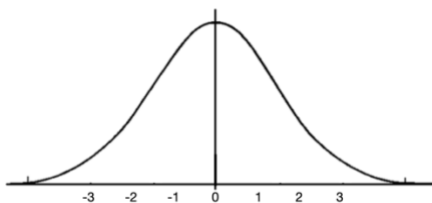


What's More

A. Shade the normal curve and find the required area which lies for each of the following.

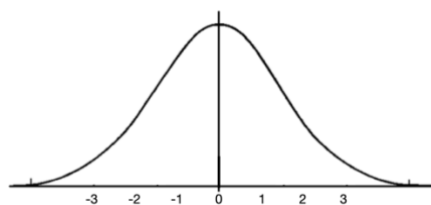
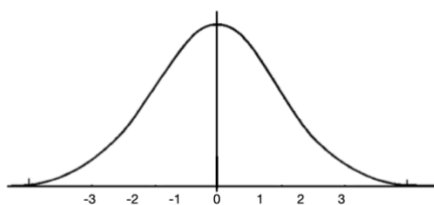
1. between $Z = 0$ and $Z = 2.23$

2. between $Z = -2.66$ and $Z = -0.42$



3. to the right of $Z = 0.70$

4. to the left of $Z = -1.06$



B. Read and analyze the statement below then answer the questions that follow.

The scores of grade 11 students in their midterm examination for General Mathematics has a mean of 32 and a standard deviation of 5. Find the z-scores corresponding to each of the following:

1. 37

2. 22

3. 33

4. 28



What I Have Learned

I learned that:



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I discovered that:
I realized that:
I need help on:



What I Can Do

PERFORMANCE TASK: The scores of the whole grade 11 in a Math test is normally distributed with a mean of 60 and a standard deviation of 8.

Solve the problem using the adjoining **UPS Check** chart below.

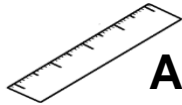
<p><u>U</u>NDERSTAND <i>What is the problem all about?</i> State the given in the problem.</p>	<p><u>P</u>LAN <i>What information is needed to solve the problem?</i> Estimate a reasonable solution. Draw your model.</p>
<p><u>S</u>OLVE <i>What method is applicable to be used?</i> Show all your solutions in solving the problem.</p>	<p><u>C</u>HECK <i>Does my answer make sense? Is my estimate correct? Did I answer the question?</i> Justify your answer.</p>

If there were 250 students who took the test, about how many students got a score higher than 64?



RUBRIC FOR THE ACTIVITY

	1	3	5
Computation	The answer and strategies used were incorrect.	The answer is incorrect, but the strategy used to solve is correct.	The answer and strategies used to solve are correct.
Explanation	No written explanation is included.	Explanation is included but not detailed or written in complete sentences.	Explanation is detailed and written in complete sentences.



Assessment

Read each item carefully. Encircle the letter that corresponds to the correct answer.

1. Which of the following statement is TRUE about changing a random variable X to a standard score?

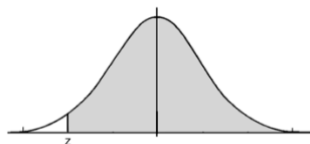
A. Subtract the value of X from the mean and divide the result by the variance.
 B. Subtract the mean from the value of X and divide the result by the variance.
 C. Subtract the value of X from the mean and divide the result by the standard deviation.
 D. Subtract the mean from the value of X and divide the result by the standard deviation.

For numbers 2-4, consider the given below.

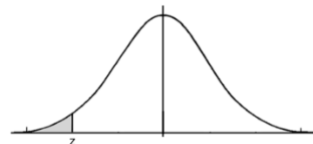
A random variable X is normally distributed with a mean of 50 and standard deviation of 3.

2. What is the value of the random variable X if the area to the left of Z is 0.0207?
 A. -2.04 B. 2.04 C. 43.88 D. 60.2
3. If the area to the right of Z is 0.9292, what will be the value of its random variable X ?
 A. -1.47 B. 1.47 C. 45.59 D. 54.41
4. Which of the following normal curves suits the description, to the right of Z is 0.8508?

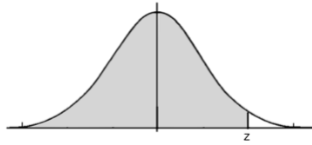
A.



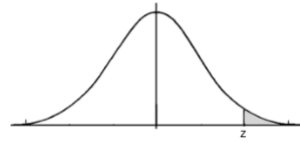
C.



B.



D.



5. In a given normal distribution, the sample mean is 20.5 with a standard deviation of 5.4. Find its corresponding standard score of 18.

A. -0.46

B. -7.12

C. 7.12

D. 0.46



Additional Activities

You want to be admitted in a university who only accepts the top 5% of the total examinees on their entrance exam. The result of this year's entrance exam follow a normal distribution with a mean of 285 and a standard deviation of 12. What must be at least your score so that you can still be admitted in that university?



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LESSON 3: COMPUTING PROBABILITIES AND PERCENTILES USING THE STANDARD NORMAL TABLE



What's In

Find the raw score of a standard score of 2.52 that is normally distributed with a mean of 42 and a standard deviation of 3.



What's New

A local bank located at the heart of a busy city wants to improve their service to their customers during lunch break. They have observed the flow of traffic inside the bank and the data observed are as follows:

- i. The customers spent an average time of 15.2 minutes.
- ii. The computed standard deviation of time spent by the customer is 1.5 minutes.

How will you illustrate the given problem? If the time is normally distributed, what is the probability that a customer chosen at random will consume a time more than 15 minutes?



What is It

In answering the problem given above, you need to recall your knowledge about standard score and the use of standard normal distribution.

To answer this, you need to convert 15 minutes to a standard score.

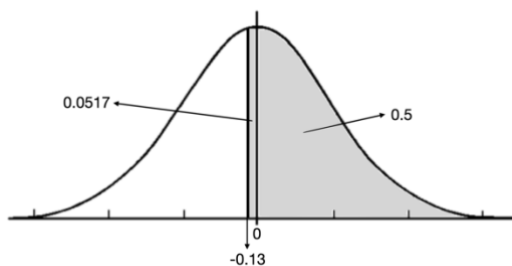
$$Z = \frac{X - \mu}{\sigma} = \frac{15 - 15.2}{1.5} = -0.1\bar{3}$$

To find the probability we will use the table for the standard normal distribution. (See table at the end of this module.)

To use the table, the computed standard normal value Z is rounded off to the nearest hundredths. The value from the whole number up to the tenths place of



the Z score will be looked at the vertical column and the value of the hundredths place of the Z score will be looked at the horizontal part of the table. The intersection of the two values is the area under the normal curve corresponding to the standard normal value $Z = -0.13$. The result is 0.0517. To illustrate this area you will have:



$$P(Z > -0.13) = 0.0517 + 0.5 = 0.5517$$

Therefore, 0.5517 is the probability that a bank customer will finish his transaction in more than 15 minutes.

For you to find the probability that a continuous random variable X lies on the interval from a to b , you need to do the following steps:

- Convert the value of the continuous random variable X to its standardized random variable Z .
- Use the standard normal distribution table.



What's More

Read and analyze each statement below then answer the question that follows.

1. Alex has an online job and his earnings for a week was randomly selected. If the mean of his salary for a week is Php240.00 and has a standard deviation of Php60.00, what is the probability that he will make less than Php255.00?

2. Studies shows that the average annual summer job pays for high school graduates is normally distributed with a mean of Php60,000.00 and a standard deviation of Php15,000.00.

A. Find the probability of randomly selecting a student graduate earning less than Php45, 000.00 annually.

B. Find the probability of randomly selecting a high school graduate that makes more than Php80, 000.00 a year.



What I Have Learned



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I learned that:
I discovered that:
I realized that:
I need help on:



What I Can Do

The admission office of Pamantasan ng Lungsod ng Marikina found out that the IQ of the incoming 1000 Grade 11 students for School Year 2020-2021 is normally distributed with a mean of 110 and has a standard deviation of 8.

According to Resing and Blok (2002), IQ can be interpreted as follows:

IQ Score	Interpretation
> 130	very gifted
121 - 130	gifted
111 - 120	above average intelligence
91 - 110	average intelligence
81 - 90	below average intelligence
71 - 80	cognitively impaired

The university is planning to create different programs where all their students can benefit.

Assuming that you are the President of the Mathematics Club and you are asked by the University Dean to give some recommendations in helping the school decide on what school programs that they need to provide so that it will cater all the students' need based from their IQ scores.

In order for you to give your recommendations to the University Dean, you need to answer the following questions:

1. How many students have an average IQ or lower?
2. If the university plans to grant a scholarship worth Php 10,000.00 per semester to all the gifted and very gifted students, how much budget do they need for this school year?
3. If one of your recommendations will be requiring all the students who have an IQ scores of 87 and lower are needed to attend a remedial program, how many are they?
4. If a Grade 11 student will be selected at random to be a student assistant, what is the probability of selecting a student with an IQ between 111 and 130?

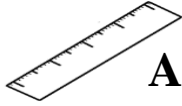


5. The University Dean is planning to offer additional Math enriching programs to the Top 20% based from the result of their IQ scores. What should be the IQ score (at least minimum) to be qualified for the said program?

OUTPUT: Submit a formal letter addressed to the University Dean stating your recommendations. Attached a complete solution/explanation as a basis. Include illustrations if needed.

	1	2	3	4
Organize and use data to support claims or conclusions.	Represent data in tables and/or various graphical displays (bar graphs, pictographs and/or pie charts to reveal patterns.	Construct, analyze, and/or interpret graphical displays of data (charts, graphs, and/or tables) to identify relationships in data.	Analyze data using tools, technologies, and/or models in order to make valid and reliable scientific claims or determine an optimal design solution.	Using collected data to enhance or change existing model/claim to support claim or conclusion
Utilization of visual representations of data and their limitations.	Correctly sorting data or proper visual representation(s) of data	Correctly sorting data and proper visual representation(s) of data	Consider limitations of data analysis such as measurement of error and sample selection.	Using qualitative data and creating quantitative data. Extrapolate proposed process or system to optimize it relative to criteria for success
Analyze data to make sense of phenomena or determine an optimal design solution.	Analyze and interpret data to make sense of phenomena, using logical reasoning, mathematics, and/or computation. Compare and contrast data collected by different groups in order to discuss similarities and differences in their findings.	Analyze and interpret data to provide evidence for phenomena. Apply concepts of statistics and probability to analyze and characterize data. Define an optimal operational range for a proposed object, tool, process or system that best meets criteria for success.	Apply concepts of statistics and probability to scientific and engineering questions and problems. Evaluate the impact of new data on a working explanation or model or a proposed process or system.	Comparing and evaluating data from multiple sources in order to evaluate the impact, validity, and reliability of the data





Assessment

Read each item carefully. Encircle the letter that corresponds to the correct answer.

1. What is the probability of choosing a random variable from the area under a normal curve to the left of $Z = 1.50$?
A. 0.9223 B. 0.9332 C. 0.9393 D. 0.9932
2. Let z be a normal random variable with a mean of 0 and a standard deviation of 1. Determine $P(z \leq 1.4)$.
A. 0.9192 B. 0 C. 0.0808 D. 0.4192
3. A normal random variable has a mean of 35 and a standard deviation of 4. Find $P(30 \leq x \leq 37)$.
A. 0.2971 B. 0.5859 C. 0.6915 D. 0.1056
4. Given that X is normally distributed random variable with a mean of 50 and a standard deviation of 2, find the probability that X is less than 54.
A. 0.9500 B. 0.4772 C. 0.9772 D. 0.0228
5. What is the probability that X is found between 47 and 54 if it has a mean of 50 and a standard deviation of 2?
A. 0.4104 B. 0.0896 C. 0.9104 D. 0.5896



Additional Activities

Read and analyze the problem below. Show illustrations if necessary.

The customers in a milk tea shop follows a normal distribution having a mean of 45 and a standard deviation of 8. What is the probability that the number of customers will be less than 42?



SUMMATIVE TEST

Read each item carefully. Encircle the letter that corresponds to the correct answer.

1. What do you call the random variable X in the normal equation?
A. Box-and-Whisker Plot C. Normal Probability Distribution
B. Histogram D. Normal Random Variable
2. Which of the following is **NOT** a property of a normal distribution?
A. The total area is equal to 1.
B. The mean, median and mode are equal.
C. The area to the right and to the left of the mean is 0.5.
D. The distance from one standard score to the next standard score is equal to 0.5 along the horizontal line.
3. Which of the following statement is TRUE about changing a random variable X to a standard score?
A. Subtract the value of X from the mean and divide the result by the standard deviation.
B. Subtract the mean from the value of X and divide the result by the standard deviation.
C. Subtract the value of X from the mean and divide the result by the variance.
D. Subtract the mean from the value of X and divide the result by the variance.
4. In a normal curve, the area from the to the right and left of the mean is _____.
A. 0.50 B. 0.64 C. 0.95 D. 0.98
5. A set of test scores is considered normally distributed. Its mean is 90 and its standard deviation is 10. If these scores are converted to standard normal Z scores, what will be the value of the mode and median?
A. 0 B. 1 C. 45 D. 90

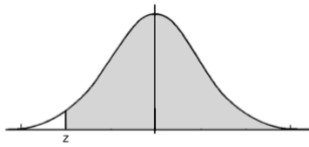
For numbers 6-8, consider the given below.

A random variable X is normally distributed with a mean of 45 and standard deviation of 3.

6. What is the value of the random variable X if the area to the left of Z is 0.1255?
A. -2.04 B. 2.04 C. -41.56 D. 41.56
7. If the area to the right of Z is 0.9292, what will be the value of its random variable X ?
A. -1.47 B. 1.47 C. -49.41 D. 49.41
8. Which of the following normal curves suits the description, to the right of Z is 0.8508?



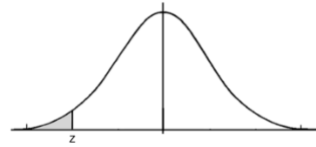
A.



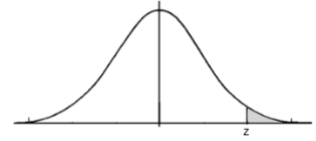
B.



C.

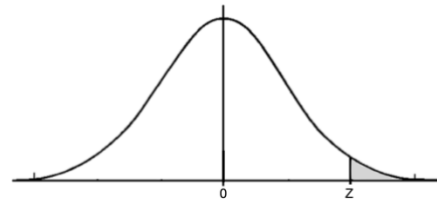


D.



9. Which of the following best describes the normal curve on the right?

- A. to the left of 0.0160
- B. to the right of 0.0160
- C. between 0 to Z is 0.0160
- D. to the right of 0.5



10. What is the probability of choosing a random variable from the area under a normal curve to the left of $Z = 1.91$?

- A. 0.1479
- B. 0.1497
- C. 0.4719
- D. 0.4791



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STANDARD NORMAL TABLE (Z)

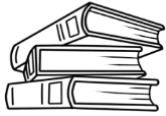
Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

Source: https://www.google.com/search?q=standard+normal+distribution+table&safe=active&rlz=1C5CHFA_enPH923PH924&source=lnms&tbn=isch&sa=X&ved=2ahUKEwii4y1udbsAhUnxosBHcrrB6cQ_AUoAXoECAQQAaw&biw=1440&bih=821#imgref=Rol221NpHD47DM



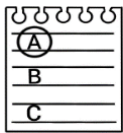
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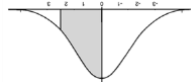
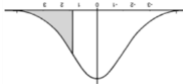
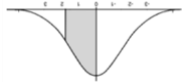
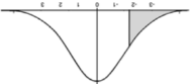
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Calaca, Ninia I. & et al. (2016). *Statistics and Probability*. Quezon City Philippines. Vibal Group, Inc.



Answer Key

LESSON 2:	
What's In	
You will use μ for the mean and σ if you are talking about an entire population that is normally distributed.	
What's More	
A.	
	1.
	3.
B.	
1. 1	
2. -2	
3. 0.2	
4. -0.8	
	2.
	4.
What I Have Learned	
Answer varies.	
What I Can Do	
30.85%	
Assessment	
1. D	
2. A	
3. A	
4. A	
5. A	
Additional Activities	
305	
LESSON 3:	
What's In	
49.56	
What's More	
1. 0.6	
2. A. 15.87%	
2. B. 9.18%	
What I Have Learned	
Answer varies.	
What I Can Do	
Answer varies.	
Assessment	
1. B	
2. A	
3. B	
4. C	
5. C	
SUMMATIVE TEST	
1. D	
2. D	
3. B	
4. A	
5. D	
6. D	
7. D	
8. D	
9. B	
10. C	

What I Know

1. C
6. C

LESSON 1:

What's In

1. 0.5

What's More

A.

1.

3.

What I Have Learned

Answer varies.

What I Can Do

63.1%

Assessment

1. C

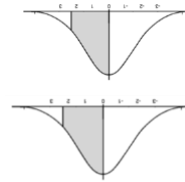
2. B

3. A

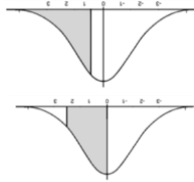
4. D

5. C

Additional Activities
You must return in 81 days.



4.



2.

3. 0.1585

2. 0.683

3. A
8. B

4. A
9. A

5. D
10. A



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