8

Mathematics

Quarter 4 – Module 5:

Determining the Conditions under which Lines and Segments are Parallel or Perpendicular

ALTERNATIVE DELIVERY MODE

ADM

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Mathematics

Quarter 4 – Module 5:

Determining the Conditions under which Lines and Segments are Parallel or Perpendicular

DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

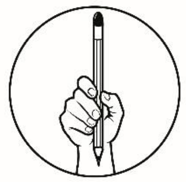
This module was designed and written with you in mind. It is here to help you master the skills in determining the conditions under which lines and segments are parallel or perpendicular. You are provided with varied activities to process the knowledge and skills learned and to deepen and transfer your understanding of the lesson. The scope of this module enables you to use it in many different learning situations. The lesson is arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains:

Lesson 1- Determining the Conditions under which Lines and Segments are Parallel or Perpendicular.

After going through this module, you are expected to:

1. Identify the properties of parallel lines, perpendicular lines, and quadrilaterals;
2. prove the conditions under which lines and segments are parallel or perpendicular;
3. use the properties of parallel lines, perpendicular lines, and quadrilaterals to find the measures of angles and sides involving quadrilaterals; and
4. apply properties of parallel and perpendicular lines to real life situations.



What I Know

PRE-ASESSMENT

Directions: Answer each of the following items accurately. Write the letter of the best answer on a separate sheet of paper. You may skip the module if you get 100% correct answers, otherwise, proceed.

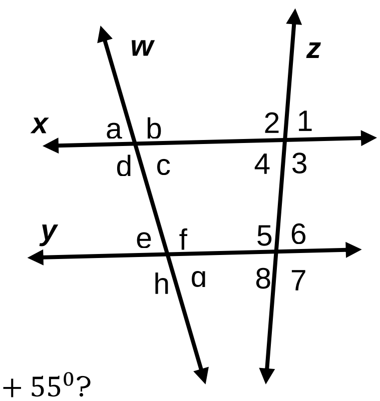
For items 1 - 2, refer to the characteristics below.

I. Lines are coplanar.
II. Lines are non-coplanar.
III. Lines do not intersect.
IV. Lines intersect and form right angles.

1. Which of the following statements above is/are ***NOT*** true about two parallel lines?
   A. I only B. I and III only C. III only D. II and IV only

2. Which is/are **true** about perpendicular lines?
   A. II only B. II and IV only C. I and IV only D. II and III only

For items 3 - 6, refer to the figure at the right.



3. If $x \parallel y$, which pair of angles are supplementary?
   A. $\angle 2$ and $\angle 7$ C. $\angle 2$ and $\angle 3$
   B. $\angle b$ and $\angle 4$ D. $\angle b$ and $\angle c$

4. If $x \parallel y$, which pair of angles are congruent?
   A. $\angle d$ and $\angle h$ C. $\angle c$ and $\angle f$
   B. $\angle f$ and $\angle 6$ D. $\angle 4$ and $\angle 5$

5. What is $m\angle 2$ if $x \parallel y$, $m\angle 3 = 3x - 15^0$ and $m\angle 5 = x + 55^0$?
   A. 75° B. 80° C. 85° D. 90°

6. If $m\angle f = m\angle 8$, which statement is true?
   A. $x \parallel y$ B. $x \perp y$ C. $w \parallel z$ D. $w \perp z$

7. Which of the following statements is true about trapezoid?
   A. A trapezoid has four right angles.
   B. A trapezoid has four congruent sides.
   C. A trapezoid has pairs of opposite sides parallel.
   D. A trapezoid has exactly one pair of opposite sides parallel.

8. If □MATH is a rhombus and $m\angle A = 88°$, *what is the measure of* $\angle H$?
   A. 45° B. 87° C. 88° D. 92°

9. Which of the following are properties of a rhombus?
    I. All sides are congruent.
    II. All angles are right angles.
    III. Diagonals are perpendicular to each other.
    IV. Diagonals are congruent.
    V. No sides are parallel.

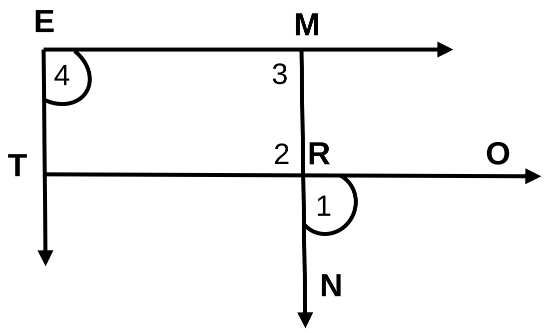
    A. I, II, III, IV, V
    B. I, III, IV
    C. I, II, III, IV
    D. I and III

10. Anna was asked by her teacher to find the perimeter of the floor of their classroom with a length of 18 m and a width of 12 m. Is Anna correct when she got a perimeter of 60 m?
    A. No, the perimeter of the floor of the classroom should be 216 m.
    B. Yes, since the floor of the classroom is a quadrilateral.
    C. Yes, since the floor of the classroom is a rectangle.
    D. No, the perimeter should be 30 m.

11. Given the parallelogram **FARM**, if $m\angle M = 42°$, what is the measure of $\angle F$?
    A. 138° B. 128° C. 48° D. 42°

For items 12 – 15, refer to the figure at the right and complete the proof on the table. Write the letter of the correct answer from the list of statements below.

A. $m\angle 4 + m\angle 3 = 180°$
B. $m\angle 1 = m\angle 4$
C. Vertical Angle Theorem
D. Interior Angles Same Side-Parallel Theorem
E. Substitution Property of Equality
F. Corresponding Angles-Parallel Theorem
G. Transitive Property of Equality



Given: $\overrightarrow{EM} \parallel \overrightarrow{TO}$ and $\angle 1 \cong \angle 4$

Prove: $\overrightarrow{MN} \parallel \overrightarrow{ET}$

Proof:

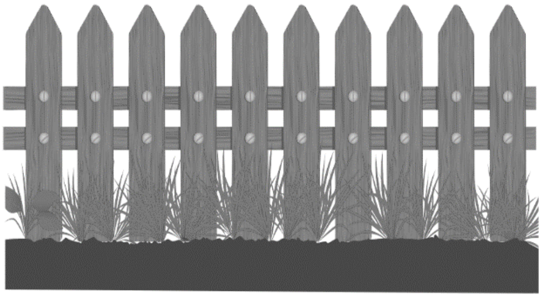
Statements	Reasons
1. $\overrightarrow{ME} \parallel \overrightarrow{OT}$	1. Given
2. $\angle 2$ *and* $\angle 3$ are supplementary angles	2. If two parallel lines are cut by a transversal, then same-side interior angles are supplementary (Interior Angles Same Side-Parallel Theorem).
3. $m\angle 2 + m\angle 3 = 180°$	3. Definition of supplementary angles
4. $\angle 1 \cong \angle 2$	4. ______(12)______________
5. $m\angle 1 + m\angle 3 = 180°$	5. Substitution Property of Equality
6. $\angle 1 \cong \angle 4$	6. Given
7. $m\angle 1 = m\angle 4$	7. Definition of Congruent Angles
8. _____(13)________	8. ________(14)______________
9. $\angle 4$ *and* $\angle 3$ are supplementary angles	9. Definition of supplementary angles
10. $\overrightarrow{MN} \parallel \overrightarrow{ET}$	10. ________(15)______________

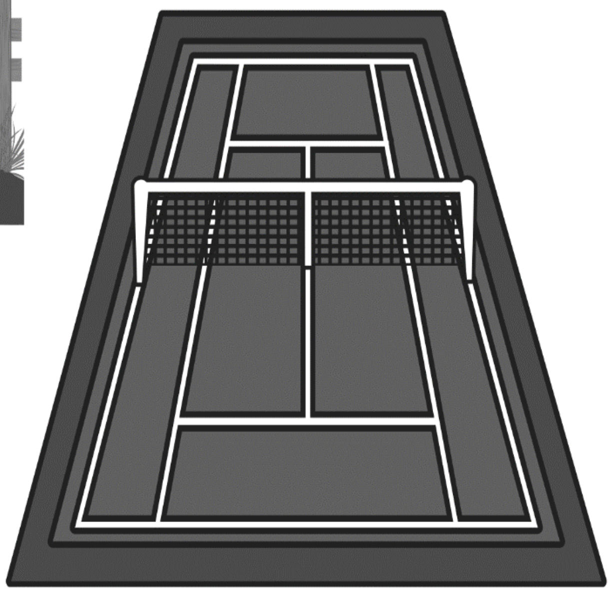
Lesson 1

Determining the Conditions under which Lines and Segments are Parallel or Perpendicular

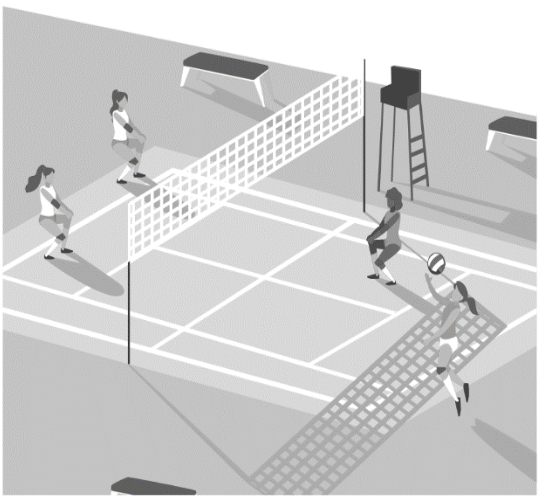
Do you like sports? Tennis, basketball, volleyball, badminton, and soccer are some of the sports that we Filipinos like playing or watching. Everybody loves sports but have you ever been curious of the design of each court? If you take a closer look at the courts, you will notice that they consist of lines: parallel lines and perpendicular lines. Parallel lines and perpendicular lines are evident in our surroundings, in our house, the perimeter fence, designs in our clothing, and many others. They play important roles in geometry and in real life.

Perimeter Fence





Tennis Court



Volleyball Court

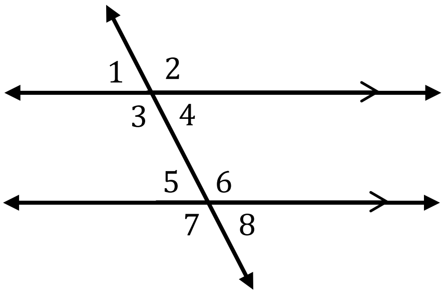
Ultimately, this lesson provides understanding of concepts in geometry that lead to prove the conditions under which lines and segments are parallel or perpendicular.

What's In

Activity: Supply Me!

You have learned in module 4 the different conditions that guarantee lines are parallel and the relationships between angles formed when two parallel lines are cut by a transversal. This time, you are encouraged to remember the angle pairs formed when parallel lines are cut by a transversal.

Directions: Use the figure below in answering each of the following questions. Supply the reason. The first item is done for you.



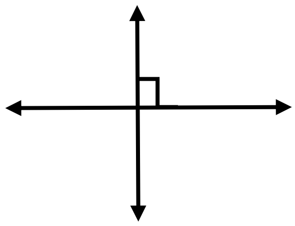
1. If the $m\angle 3 = 110°$, what is the $m\angle 6$? Answer: 110° Reason: Alternate interior angles are congruent.	4. If the $m\angle 7 = 75°$, what is the $m\angle 6$? Answer: ____________ Reason: ____________
2. If the $m\angle 3 = 70°$, what is the $m\angle 5$? Answer: ____________ Reason: ____________	5. If the $m\angle 8 = 115°$, what is the $m\angle 4$? Answer: ____________ Reason: ____________
3. If the $m\angle 2 = 105°$, what is the $m\angle 7$? Answer: ____________ Reason: ____________	6. If the $m\angle 7 = 95°$, what is the $m\angle 1$? Answer: ____________ Reason: ____________

What's New

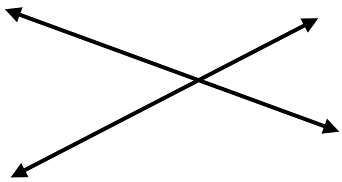
Activity 1: Am I Perpendicular or Not?

Directions: Given the figures below, determine whether the lines in each item are perpendicular or not. Write ***PL*** if they are perpendicular and ***PN*** if not.

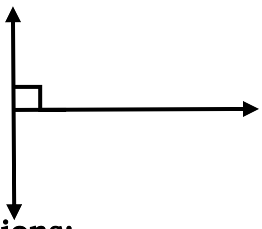
1.



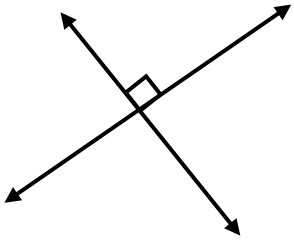
2.



3.



4.

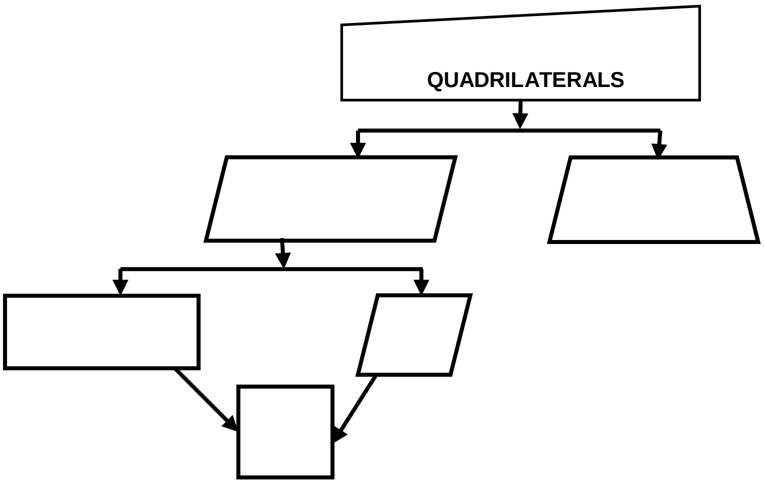


Questions:

1. What is your common observation in the figures given above?
2. What makes figures 1, 3 and 4 different from the second figure?
3. Which among the four figures show perpendicularity? You can check by using a protractor to measure the four figures.
4. Define perpendicular lines based on your observation in the figure above.
5. Cite at least five (5) real life examples that you can think perpendicular lines are used.

Activity 2: Name Me!

Directions: Below is the tree diagram that shows the relationship of some quadrilaterals. Complete the diagram. The first step is done for you.



Directions: Put a check on the characteristics applicable to the given quadrilaterals below.

Properties	**Parallelogram**	**Rectangle**	**Rhombus**	**Square**	**Trapezoid**
All sides are congruent.					
All angles are congruent.					
Diagonals are perpendicular to each other.					
Diagonals are congruent.					
Exactly one pair of parallel sides.					
Two pairs of opposite sides are congruent.					

Questions:

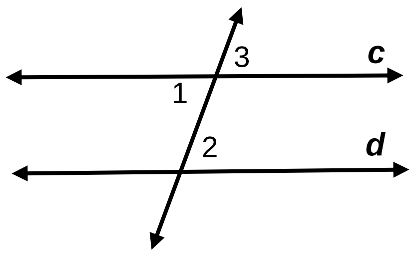
1. Did you find difficulty in answering the tasks above?
2. What can you say about the relationship of the quadrilaterals presented on the tree diagram and on the given table?
3. What do you think are the properties common to rectangle, square, and rhombus?
4. Why are parallelograms considered special quadrilaterals?
5. Compare and contrast the properties of parallelograms and list all properties that you found.

B. Complete Me!

Directions: Given the figures below, complete each table of proof.

1. Given: $\angle 1 \cong \angle 2$

   Prove: $c \parallel d$



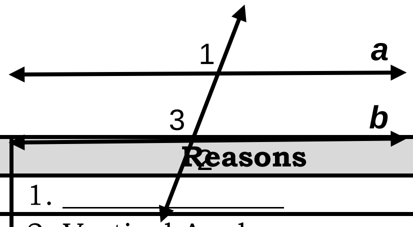
Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. ________________
2. $\angle 1 \cong \angle 3$	2. Vertical Angle Theorem
3. ______________	3. Transitive Property
4. $c \parallel d$	4. __________________

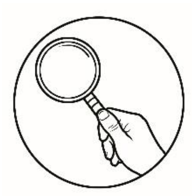
2. Given: $\angle 1 \cong \angle 2$

Prove: $a \parallel b$

Proof:



Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. ________________
2. $\angle 2 \cong \angle 3$.	2. Vertical Angles are congruent.
3. ______________	3. Transitive Property
4. $\boldsymbol{a \parallel b}$	4. ___________________



What is It

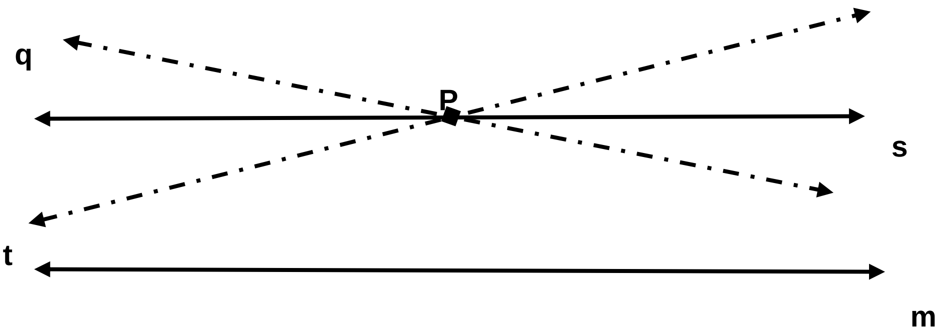
Are you already done answering the activities and questions provided above? Are you ready to check if your answers are correct or not? If so, let us now discover more about parallel lines, perpendicular lines, and quadrilaterals. Along the way, you can finally uncover the theorems and postulates on the conditions under which lines and segments are parallel including some properties of parallelogram.

Theorems on Conditions under which Lines and Segments are Parallel or Perpendicular

Recall that two lines are parallel if and only if they are coplanar and do not intersect. This concept is considered as a unique fundamental principle in geometry. Let us start with the Parallel Postulate.

Parallel Postulate

Given a line and a point not on the given line, there is exactly one line through the given point parallel to the given line.



Given, line $\boldsymbol{m}$ and point $\boldsymbol{P}$ not in $\boldsymbol{m}$. Only one line $s \parallel m$. Lines $\boldsymbol{t}$ and $\boldsymbol{q}$ are not parallel to $\boldsymbol{m}$ through $\boldsymbol{P}$.

Now, let us discuss further the theorems and postulates that prove that two lines are parallel or perpendicular. These theorems are converses of the theorems discussed in module 4.

Postulate and Theorems

Alternate Interior Angle-Parallel Postulate

If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the lines are parallel.

Alternate Exterior Angles-Parallel Theorem

If two lines are cut by a transversal and a pair of alternate exterior angles are congruent, then the lines are parallel.

Corresponding Angles-Parallel Theorem

If two lines are cut by a transversal and a pair of corresponding angles are congruent, then the lines are parallel.

Interior Angles Same Side-Parallel Theorem

If two lines are cut by a transversal so that the interior angles on the same side of the transversal are supplementary, then the lines are parallel.

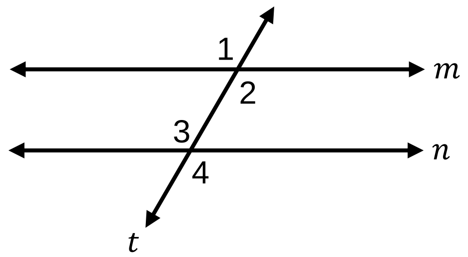
Perpendicular Same Line-Parallel Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

Example 1

Given: $\angle 1 \cong \angle 4$

Prove: $m \parallel n$



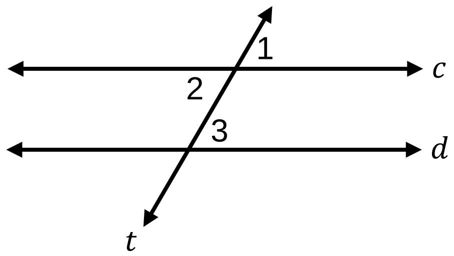
Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 4$	1. Given
2. $\angle 1 \cong \angle 2$	2. Vertical angles are congruent.
3. $\angle 2 \cong \angle 4$	3. Transitive Property of Congruence
4. $\angle 4 \cong \angle 3$	4. Vertical angles are congruent.
5. $\angle 2 \cong \angle 3$	5. Transitive Property of Congruence
6. $m \parallel n$	**6. Alternate Interior Angles-Parallel Postulate**

Example 2

Given: $\angle 2 \cong \angle 3$

Prove: $c \parallel d$



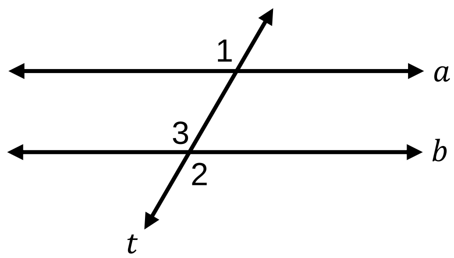
Proof:

Statements	Reasons
1. $\angle 2 \cong \angle 3$	1. Given
2. $\angle 1 \cong \angle 2$	2. Vertical angles are congruent.
3. $\angle 1 \cong \angle 3$	3. Transitive Property of Congruence
4. $c \parallel d$	**4. Corresponding Angles-Parallel Theorem**

Example 3

Given: $\angle 1 \cong \angle 2$

Prove: $a \parallel b$



Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 2 \cong \angle 3$	2. Vertical angles are congruent.
3. $\angle 1 \cong \angle 3$	3. Transitive Property of Congruence
4. $a \parallel b$	**4. Corresponding Angles-Parallel Theorem**

Here are two other theorems involving parallel lines.

The Three Parallel Lines Theorem

In a plane, if two lines are both parallel to a third line, then they are parallel

The Two Perpendicular Lines Theorem

If two coplanar lines are perpendicular to a third line, then they are parallel to each other.

The proofs of these theorems are left for you to prove in activity found in *What's More*.

Theorems on Perpendicular Lines

At this point, let us discuss and discover more about perpendicular lines.

Definition

Perpendicular Lines are defined as two lines that intersect to form right angles. Line segments and rays can also be perpendicular.

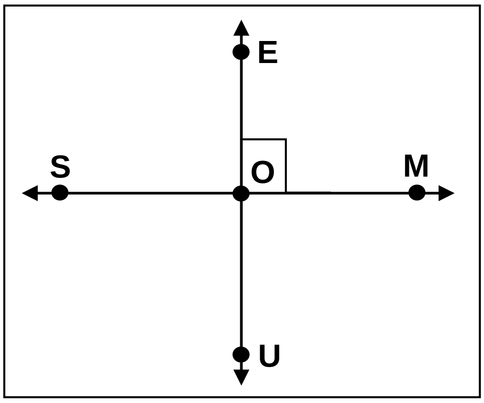
To prove that two lines are perpendicular, you must show that one of the following theorems is true.

Theorem 1

If two lines are perpendicular to each other, then they form right angles.

In the accompanying figure, $\overleftrightarrow{SM}$ and $\overleftrightarrow{EU}$ intersect at point O to form right angles. We can read it as " $\overleftrightarrow{SM}$ is perpendicular to $\overleftrightarrow{EU}$". In symbols, we write as $\overleftrightarrow{SM} \perp \overleftrightarrow{EU}$. Since, $m\angle MOE = 90°$, therefore $m\angle SOE = 90°$.

In the same manner, $m\angle SOU = 90°$ and $m\angle MOU = 90°$. Thus, perpendicular lines form four right angles.



If $\overleftrightarrow{EU}$ bisects $\overline{SM}$, then $\overleftrightarrow{EU}$ is called the ***perpendicular bisector*** of $\overline{SM}$.

Definition

A ***perpendicular bisector*** of a line segment is a line or a ray or another line segment that is perpendicular to the line segment and intersects it at its midpoint.

Theorem 2

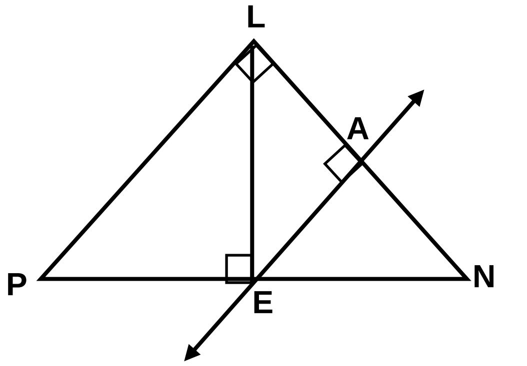
If the angles in a linear pair are congruent, then the lines containing their sides are perpendicular.

Refer to the figure above. Since $\angle SOE$ and $\angle MOE$ form a linear pair and are congruent, thus, $\overline{OE}$ and $\overline{SM}$ are perpendicular to each other. That is, $\angle SOE \cong \angle MOE$; and they are linear pair, $m\angle SOE + m\angle MOE = 180°$ therefore, $\overleftrightarrow{SM} \perp \overleftrightarrow{EU}$.

Applying the theorems and postulates on parallel lines and perpendicular lines to parallelograms, can you still determine which lines and segments are parallel or perpendicular? Let's take a look at the following examples.

EXAMPLE 4

Given: $\overline{PE} \cong \overline{EN}$, $\overline{LA} \cong \overline{AN}$



From the figure above, we can state the following: $\overline{PL} \perp \overline{LN}$, $\overline{AE} \perp \overline{LN}$, $\overline{PL} \parallel \overline{AE}$ since they are both perpendicular to the same line segment $\overline{LN}$. $\overline{PE} \perp \overline{LE}$, $\overline{EL}$ and $\overline{AE}$ are perpendicular bisectors of $\overline{PN}$ and $\overline{LN}$, respectively. Also, the measure of $\angle EAL$ is 90° and $m\angle PEL = 90°$ while, $\angle PEL$ and $\angle NEL$ are supplementary angles and at the same time adjacent, therefore they are linear pair.

Moreover, we can say that ***PEAL*** is a quadrilateral since it is a four-sided figure. Can you say that ▭***PEAL*** is a parallelogram?

This time let us discover the relationships of parallel and perpendicular lines in parallelograms.

Definition

A ***quadrilateral*** is a polygon with four sides. The symbol ▭ is used to indicate a quadrilateral. For example, ▭***PEAL***, this is read as "Quadrilateral ***PEAL***."

Quadrilaterals are classified according to the number of pairs of parallel sides.

1. Parallelogram – is a quadrilateral with two pairs of parallel sides. It is further classified as:
   a. Rhomboid – is a parallelogram with no right angle and any two consecutive sides are not congruent.

   b. Rectangle – is a parallelogram with four right angles.

   c. Rhombus – is a parallelogram with four congruent sides.
      - Square – is a rectangle with four congruent sides. It is both a rectangle and a rhombus because it satisfies the definition of a rectangle and a rhombus.

2. Trapezoid – is a quadrilateral with exactly one pair of parallel sides. If the non-parallel sides, called legs, are congruent, then the trapezoid is isosceles.

3. Trapezium – is a quadrilateral with no pair of parallel sides.

Given the definitions, we can now deduce the following properties.

Points to Remember

A quadrilateral is a parallelogram if:

- A pair of opposite sides are both parallel and congruent.
- Two pairs of opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.
- Opposite angles are congruent.

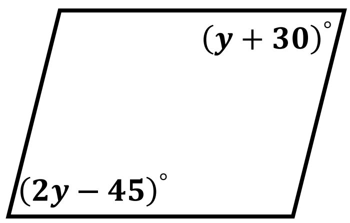
Let's take a look at this example.

EXAMPLE 5

1. Given the parallelogram at the right, find the value of y.

Solution:

In the given quadrilateral, we know that the two angles given are opposite angles and opposite angles are congruent in a parallelogram. Hence, we can formulate an equation out of this.



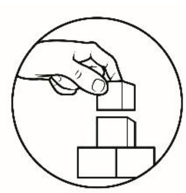
$(2y - 45°) = (y + 30°)$ — *Equate the two expressions.*

$(2y - y) - 45° = (y - y) + 30°$ — *Add (-y) to both sides of the equation, (Addition Property of Equality).*

$y + (-45° + 45°) = (30° + 45°)$ — *Add (+45), (Addition Property of Equality)*

$y = (30° + 45°)$ — *Simplify*

$y = 75°$ — *Result*

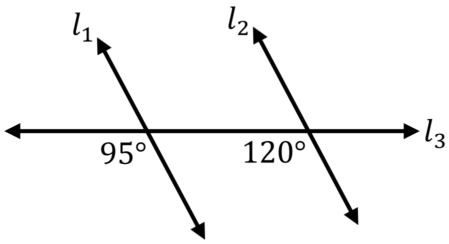


What's More

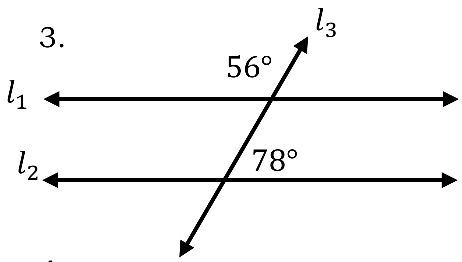
Activity 1: Justify Me!

A. Determine whether each pair of lines l_1 and l_2 are parallel or not and justify your answer.

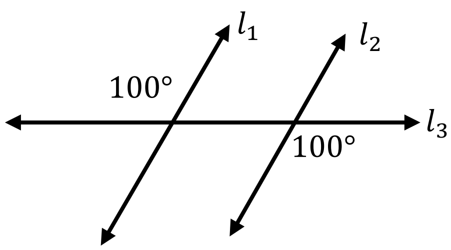
1.



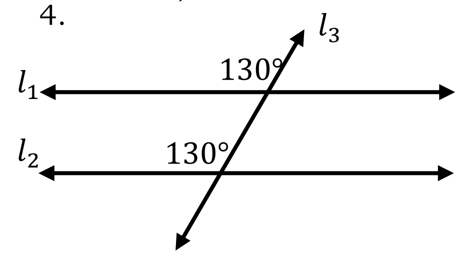
3.



2.

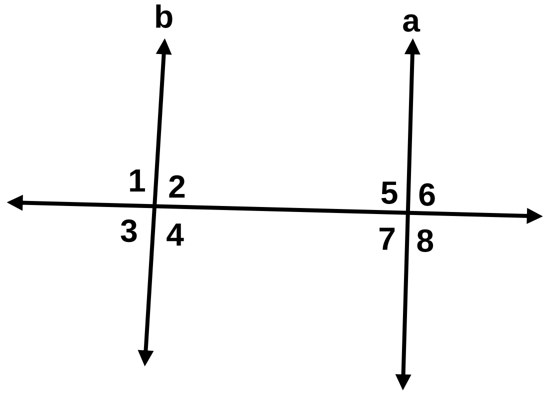


4.



B. Refer to the figure at the right, explain why $a \parallel b$. Justify your answer with a theorem or postulate. Choose your answers from the choices provided in the box.

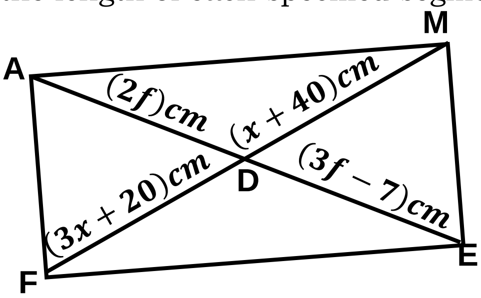
1. If $m\angle 1 = 50°$, then $m\angle 5 = 50°$
2. If $m\angle 7 = 78°$, then $m\angle 2 = 78°$
3. If $m\angle 4 = 120°$, then $m\angle 7 = 60°$
4. If $m\angle 3 = 99°$, then $m\angle 6 = 99°$
5. If $m\angle 6 = 85°$, then $m\angle 1 = 95°$



Corresponding Angles-Parallel Theorem	Law of Substitution
Interior Angles Same Side-Parallel Theorem	Reflexive Property
Alternate Interior Angles-Parallel Theorem	Supplementary Angles
The Three Parallel Lines Theorem	Vertical Angle Theorem
Alternate Exterior Angles-Parallel Theorem	Linear Pair Postulate
Transitive Property of Equality	Exterior Angles Same Side-Parallel Theorem

Activity 2: Think More!

Direction: Given the figure at the right, find the length of each specified segment.

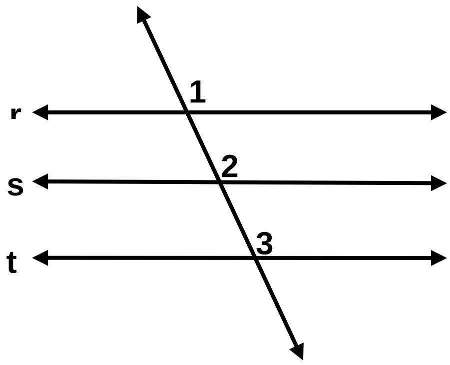


1. $\square$ *FAME* is a parallelogram.
   a. $\overline{AD}$
   b. $\overline{AE}$
   c. $\overline{FM}$
   d. $\overline{FD}$

2. In $\square$ *FAME*, if $m\angle AFE = 6a - 45°$ and $m\angle EMA = 4a + 15°$, what is $m\angle FEM$? (Show your solution).

Activity 3: Prove Me!

Given the figure below, complete each proof:



1. Given: $r \parallel t$, $s \parallel t$

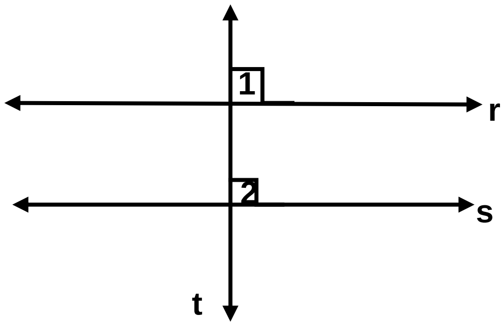
   Prove: $r \parallel s$

Proof:

Statements	Reasons
1. $r \parallel t$	1. ________________
2. $\angle 1 \cong \angle 3$	2. If two parallel lines are cut by transversal, then corresponding angles are congruent.
3. __________	3. Given
4. $\angle 3 \cong \angle 2$	4. ________________
5. __________	5. Transitive Property
6. $r \parallel s$	6. ___________________

2. Given: $r \parallel s$, $r \perp t$

Prove: $s \perp t$



Proof:

Statements	Reasons
1. $r \parallel s$, $r \perp t$	1. Given
2. $m\angle 1 = 90°$	2. ________________
3. ______________	3. Corresponding angles are congruent.
4. $m\angle 1 = m\angle 2$	4. _______________
5. $m\angle 2 = 90°$	5. Transitive Property of Equality
6. $s \perp t$	6. _________________

What I Have Learned

Directions: Tell whether each statement is true or false by checking the box that corresponds to your answer. If false, change the underlined word/s to make the statement true.

	True	**False**	
1. Parallel lines are coplanar lines that <u>do not intersect</u>.	☐	☐	_______
2. Two intersecting are <u>always</u> perpendicular.	☐	☐	_______
3. If two parallel lines are cut by a transversal and a pair of alternate interior angles are supplementary, then the transversal is <u>perpendicular</u> to the two parallel lines.	☐	☐	_______
4. Line 1 (l_1) is a transversal if it intersects two coplanar lines l_2 and l_3 at <u>the same</u> point/s.	☐	☐	_______
5. If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the lines are <u>parallel</u>.	☐	☐	_______

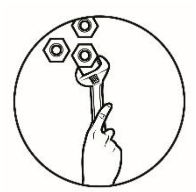
6. If a transversal is perpendicular to two parallel lines, then the interior angles on the same side of the transversal are <u>congruent</u>. ☐ ☐ ______

7. If a line is the perpendicular bisector of a line segment, then the line intersects the segment at <u>any point</u>. ☐ ☐ ______

8. If the angles in a linear pair are congruent, then the lines containing their sides are <u>perpendicular</u>. ☐ ☐ ______

9. If two coplanar lines are perpendicular to a third line, then the two coplanar lines are <u>perpendicular</u> to each other. ☐ ☐ ______

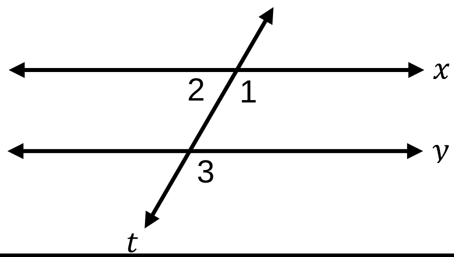
10. Given a line and a point not on the line, there is exactly <u>one</u> line through the point parallel to the given line. ☐ ☐ ______



What I Can Do

Directions: Refer to the figure below. Make a two – column proof to prove the following. The first part is done for you.

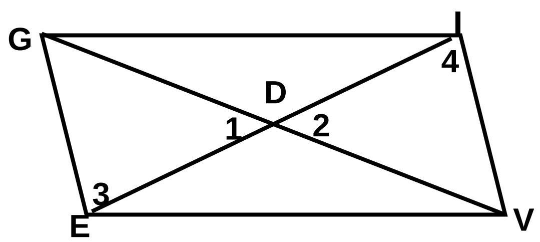
1. Given: $m\angle 1 \cong m\angle 3$.
   Prove: $x \parallel y$



Statements	Reasons
1. $m\angle 1 \cong m\angle 3$	
2. $m\angle 1 + m\angle 2 = 180°$	
3.	
4.	

2. Given: $\overline{GV}$ bisects $\overline{EI}$
   $\overline{EI}$ bisects $\overline{GV}$

Prove: *GIVE* is a parallelogram



Statements	Reasons
1. $\overline{GV}$ bisects $\overline{EI}$ $\overline{EI}$ bisects $\overline{GV}$	1. Given
2. $\overline{ED} \cong \overline{ID}$; $\overline{GD} \cong \overline{VD}$	2. Definition of segment bisector.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Assessment

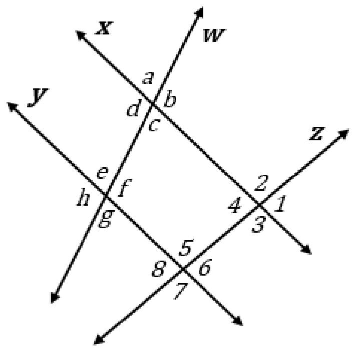
POST-ASSESSMENT

Directions: Read and answer each question carefully and write the letter of the correct answer on a separate sheet of paper.

1. The symbol used to indicate parallelism is _____.
   A. $\perp$ B. $\parallel$ C. $\cong$ D. $\leftrightarrow$

2. Lines that intersect to form right angles are said to be _____.
   A. coinciding B. intersecting C. parallel D. perpendicular

For items 3 - 7, refer to the figure at the right.



3. Which statement is true if $\angle f \cong \angle d$?
   A. $w \parallel z$ C. $x \parallel y$
   B. $w \perp z$ D. $x \perp y$

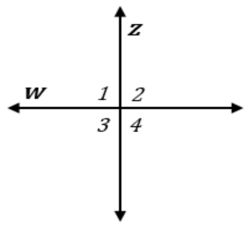
4. Which pair of angles are supplementary?
   A. $\angle 2$ *and* $\angle 3$ C. $\angle b$ *and* $\angle 4$
   B. $\angle 2$ *and* $\angle 7$ D. $\angle b$ *and* $\angle c$

5. If $x \parallel y$, which pair of angles are congruent?
   A. $\angle 1$ *and* $\angle 6$ B. $\angle 4$ *and* $\angle 5$ C. $\angle c$ *and* $\angle f$ D. $\angle d$ *and* $\angle g$

6. If $x \parallel y$, what is the value of x if $m\angle 3 = 3x + 16^o$ and $m\angle 5 = x + 66^o$?
   A. 25° B. 35° C. 50 D. 70°

7. If $x \parallel y$, $m\angle c = 13y + 9°$, $m\angle d = 6y$ and $m\angle 2 = 11y + 1°$, what is the measure of $\angle 8$?
A. 60° B. 80° C. 100° D. 120°

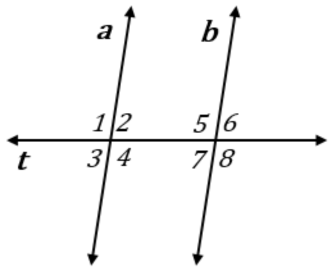
For items 8 - 9, refer to the figure at the right.



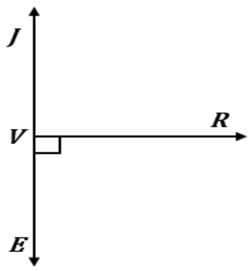
8. If $w \perp z$, all the following statements are true EXCEPT:
A. $m\angle 1 = 90°$
B. $m\angle 2 = 90°$
C. $m\angle 1 + m\angle 2 = 180°$
D. $m\angle 3 + m\angle 4 = 200°$

9. If $w \perp z$, $m\angle 1 = 20x + 2y$ and $m\angle 2 = 25x - 2y$, find the values of x and y.
A. $x = 2°,\ y = 4°$
B. $x = 3°,\ y = 5°$
C. $x = 4°,\ y = 5°$
D. $x = 4°,\ y = 7°$

10. In the figure at the right, which of the following guarantees that $a \parallel b$?
A. $\angle 1$ and $\angle 2$ are congruent.
B. $\angle 2$ and $\angle 5$ are supplementary.
C. $\angle 3$ and $\angle 7$ form a linear pair.
D. $\angle 4$ and $\angle 8$ are supplementary.

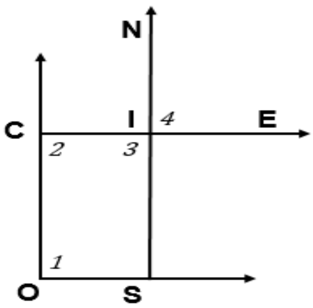


11. Given that $\overleftrightarrow{JE} \perp \overrightarrow{VR}$ at V, find the possible algebraic expressions to represent $\angle JVR$ and $\angle EVR$ such that the values for the variables x and y are 7 and 13, respectively.



A. $m\angle JVR = (8x - 14°)$, $m\angle EVR = (12y + 6°)$
B. $m\angle JVR = (9x - 25°)$, $m\angle EVR = (13y - 1°)$
C. $m\angle JVR = (12x + 6°)$, $m\angle EVR = (8y - 14°)$
D. $m\angle JVR = (13x - 1°)$, $m\angle EVR = (9y - 25°)$

For items 12-15, refer to the figure given below and complete the proof. Choose the letter of your answer from the box provided.



Given: $\overrightarrow{OC} \parallel \overrightarrow{SN}$ and $\angle 1 \cong \angle 4$

Prove: $\overrightarrow{CE} \parallel \overrightarrow{OS}$

Proof:

Statements	Reasons
$\overrightarrow{OC} \parallel \overrightarrow{SN}$	Given
1. $\angle 2$ *and* $\angle 3$ are same-side interior angles	1. Definition of Same-Side Interior Angles
2. $\angle 2$ *and* $\angle 3$ are supplementary angles	2. If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.
3. $m\angle 2 + m\angle 3 = 180°$	3. Definition of Supplementary Angles
4. $\angle 3$ *and* $\angle 4$ are vertical angles	4. Definition of Vertical Angles
5. $\angle 3 \cong \angle 4$	5. _______(12)____________
6. $m\angle 3 = m\angle 4$	6. Definition of Congruent Angles
7. $m\angle 2 + m\angle 4 = 180°$	7. Substitution Property of Equality
8. $\angle 1 \cong \angle 4$	8. Given
9. $m\angle 1 = m\angle 4$	9. Definition of Congruent Angles
10. _______(13)_____________	10. _______(14)____________
11. $\angle 1$ *and* $\angle 2$ are supplementary angles	11. Definition of Supplementary Angles
12. $\angle 1$ *and* $\angle 2$ are same-side interior angles	12. Definition of Same-Side Interior Angles
13. $\overrightarrow{CE} \parallel \overrightarrow{OS}$	13. ________(15)___________

A. $m\angle 1 = m\angle 4$

B. $m\angle 1 + m\angle 2 = 180°$

C. Vertical Angles Theorem

D. Transitive Property of Equality

E. Substitution Property of Equality

F. Corresponding Angles-Parallel Postulate

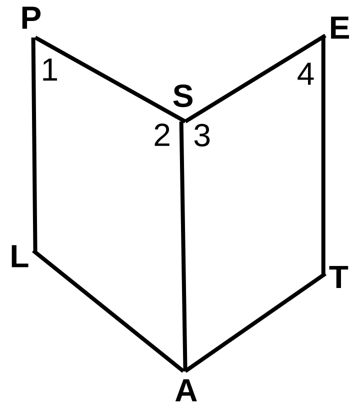
G. Interior Angles Same Side-Parallel Theorem

Additional Activities

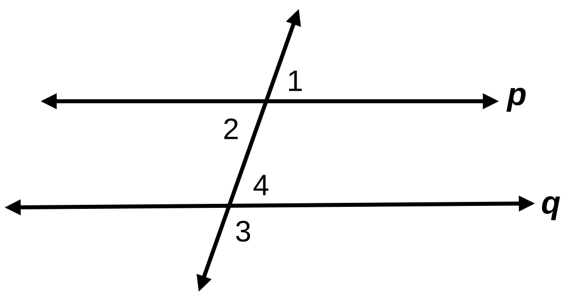
Construct Me!

Refer to the figure below and construct a two-column proof to prove the following given.

1. Given: $\angle 1$ and $\angle 2$ are supplementary angles.
   $\angle 3$ and $\angle 4$ are supplementary angles.
   Prove: $\overline{PL} \parallel \overline{ET}$



2. What is the value of x, for which $\boldsymbol{p} \parallel \boldsymbol{q}$



a. $m\angle 2 = 3x - 15°$, $m\angle 4 = 4x - 20°$
b. $m\angle 1 = 3x + 46°$, $m\angle 3 = 2x + 24°$
c. $m\angle 1 = 3x - 29°$, $m\angle 4 = 2$

Answer Key

What I Know

1. D	9. D
2. C	10. C
3. D	11. A
4. A	12. C
5. D	13. A
6. C	14. E
7. D	15. D
8. C	

What's In

2. 110°, interior angles on the same side of transversal are supplementary.
3. 105°, alternate exterior angles are congruent
4. 75°, vertical angles are congruent
5. 115°, corresponding angles are congruent
6. 85°, same-side exterior angles are supplementary

What's New

Activity A

1. PL 2. PN 3. PL 4.PL

Activity B

Parallelogram: Rectangle-Rhombus-Trapezoid-Square

Activity C

Properties	Para	Rect	Rho	Squa	Trap
All sides are congruent.			√	√	
All angles are congruent.		√		√	
Diagonals are perpendicular to each other.			√	√	
Diagonals are congruent.		√		√	
Exactly one pair of parallel sides.					√
Two pairs of opposite sides are congruent.	√	√	√	√	

What's More

Activity 1

A. 1. Not parallel
2. parallel
3. not parallel
4. parallel

B. 1. Corresponding Angles-Parallel Postulate
2. Alternate Interior Angles Theorem
3. Interior Angles Same Side-Parallel Theorem
4. Alternate Exterior Angles-Parallel Theorem
5. Exterior Angles Same Side-Parallel Theorem

Activity 2

1. a. 14 cm
   b. 28 cm
   c. 100 cm
   d. 50 cm
2. $m\angle FEM = 45°$

Activity 3

Statements	Reasons
1. $\boldsymbol{r \parallel t}$	1. Given
2. $\angle\mathbf{1} \cong \angle\mathbf{3}$	2.Corresponding Angles- Parallel Theorem
3. $\boldsymbol{s \parallel t}$	3. Given
4. $\angle\mathbf{3} \cong \angle\mathbf{2}$	4.Corresponding Angles- Parallel Theorem
5. $\angle\mathbf{1} \cong \angle\mathbf{2}$	5. Transitive Property
6. $\boldsymbol{r \parallel s}$	6 Corresponding Angles- Parallel Theorem

POST-ASSESSMENT

1. B	9. C
2. D	10. B
3. C	11. C
4. D	12. C
5. A	13. B
6. A	14. E
7. B	15. G
8. D	

What I Can Do

1.

Statements	Reasons
$\boldsymbol{m\angle 1 \cong m\angle 3}$	Given
$\boldsymbol{m\angle 1 + m\angle 2 = 180°}$	Definition of Linear pair.
$m\angle 1 + m\angle 3 = \mathbf{180°}$	Definition of Supplementary Angle
$x \parallel y$	Interior Angles Same-Side-Parallel Theorem

WHAT I HAVE LEARNED

1. True
2. False, sometimes
3. True
4. False, different
5. True
6. True
7. False, midpoint
8. True
9. False, parallel
10. True

What's New: Activity C

Properties	Paral	Rect	Rhombus	Square	Trapezoid
All sides are congruent.	√		√	√	
All angles are congruent.		√		√	
Diagonals are perpendicular to each other.			√		
Diagonals are congruent.	√	√	√	√	√
Exactly one pair of parallel sides.					√
Two pairs of opposite sides are congruent.	√	√	√	√	

What's More : Activity 3

Statements	Reasons
1. $r \parallel s$, $r \perp t$	1. Given
2. $m\angle 1 = 90°$	2. Definition of perpendicular lines
3. $\angle 1 \cong \angle 2$	3. Corresponding angles are congruent.
4. $m\angle 1 = m\angle 2$	4. Definition of Congruent angles
5. $m\angle 2 = 90°$	5. Transitive Property of Equality
6. $s \perp t$	6. Definition of perpendicular lines

What I Can Do

Statements	Reasons
1. $\overline{GV}$ bisects $\overline{EI}$ $\overline{EI}$ bisects $\overline{GV}$	1. Given
2. $\overline{ED} \cong \overline{ID}$; $\overline{GD} \cong \overline{VD}$	2. Definition of segment bisector.
3. $\angle 1 \cong \angle 2$	3. Vertical Angles are congruent.
4. $\Delta GDE \cong \Delta VDI$	4. SAS
5. $\angle 3 \cong \angle 4$, $\overline{EG} \cong \overline{IV}$	5. CPCTC
6. $\overline{EG} \parallel \overline{IV}$	6. Alternate Interior Angles-Parallel Postulate
7. GIVE is a parallelogram.	7. If a pair of opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram

Additional Activities

1. Proof

Statements	Reasons
1. $\angle 1$ *and* $\angle 2$ are supplementary angles.	1. Given
2. $\overline{PL} \parallel \overline{SA}$	2. Interior Angles Same Side-Parallel Theorem
3. $\angle 3$ *and* $\angle 4$ are supplementary angles.	3. Given
4. $\overline{ET} \parallel \overline{SA}$	4. Interior Angles Same Side-Parallel Theorem
5. $\overline{PL} \parallel \overline{ET}$	5. The Three Parallel Lines Theorem

2. a. x = 5 b. x = 22 c. x = 30

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