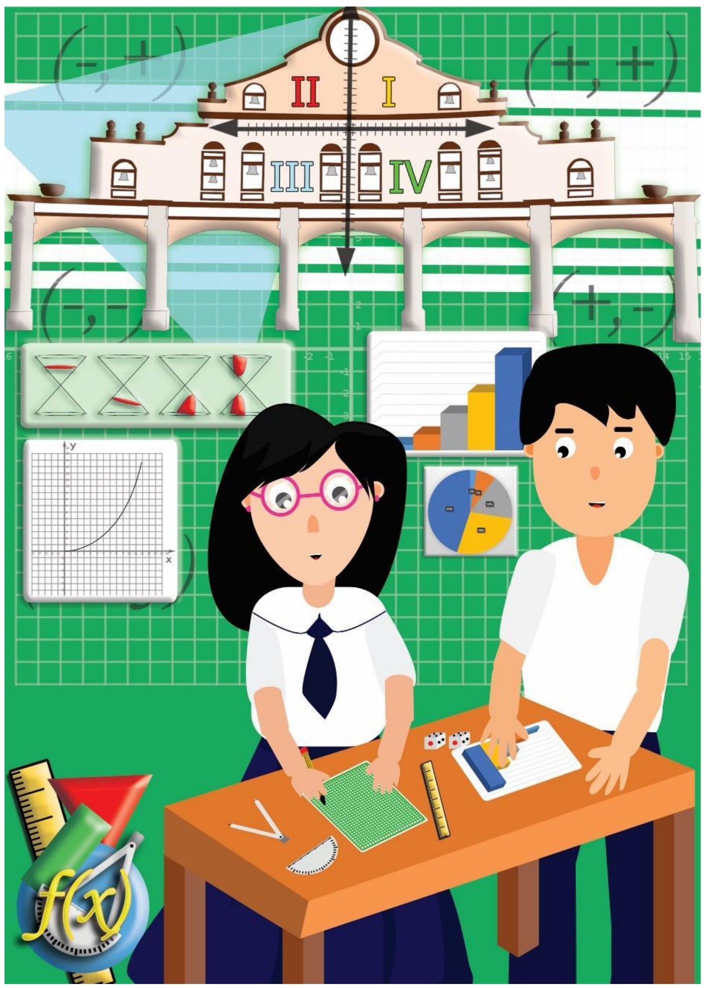
Senior High School

Department of Education
National Capital Region
SCHOOLS DIVISION OFFICE
MARIKINA CITY

MATHEMATICS

Quarter 1: Module 6
LOGARITHMIC FUNCTIONS



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What I Need to Know

This module was designed and written with you in mind. It is here to help you master the Logarithmic Functions. The scope of this module is to represent real-life situations using logarithmic functions, distinguishes logarithmic function, logarithmic equation, and logarithmic inequality, solve logarithmic equations and logarithmic inequalities.

Introduction to Logarithms
Logarithmic Functions, Equations, Inequalities and Basic Logarithmic Properties of Logarithms
Laws of Logarithms and Logarithmic Equations
Logarithmic Inequality

After going through this module, you are expected to:

1. define logarithm
2. change exponential form to logarithmic form and vice versa.
3. evaluate logarithms
4. simplify logarithmic expressions using the properties.
5. represent real life situation using logarithmic functions.
6. distinguishes logarithmic function, logarithmic equation, and logarithmic Inequality.
7. apply laws of logarithms and solve logarithmic equation.
8. solve logarithmic inequality.

What I Know

Write the letter of the correct answer on a clean sheet of paper.

1. Change $5^y = 15$ to logarithm.
   A. $5 = y$ B. $15 = y$ C. $y = 15$ D. $y = 5$
2. Evaluate 125
   A. -3 B. $-\frac{1}{3}$ C. $\frac{1}{3}$ D. 3

3. How long will it take for a deposit of ₱250 000.00 to grow to ₱320 000 at r% annual interest rate compounded quarterly? Express the time as a function f(r).

   A) $f(r) = \frac{loglog\left(\frac{320\,000}{250\,000}\right)}{4loglog\left(1+\frac{r}{4}\right)}$ C) $f(r) = \frac{loglog\left(\frac{250\,000}{320\,000}\right)}{4loglog\left(1+\frac{r}{4}\right)}$

   B) $f(r) = \frac{loglog\left(\frac{320\,000}{250\,000}\right)}{4loglog\left(1-\frac{r}{4}\right)}$ D) $f(r) = \frac{loglog\left(\frac{250\,000}{320\,000}\right)}{4loglog\left(1-\frac{r}{4}\right)}$

4. The following represents a logarithmic equation EXCEPT:

   A. $\log_x 2 = 4$ B. $\log x^2 = 2$ C. $\log_3 (3x - 2) = 2$ D. $\log_2 x > 3$

5. Which of the following is an example of logarithmic function?

   A. $y = 3\log_4 x$ B. $\log_x 2 = 4$ C. $\log_2 x > 3$ D. $\log x^2 = 2$

6. $x\log_2(x) - 2 > 0$ is an example of ______________________?

   A. logarithmic equation C. Logarithmic inequality

   B. logarithmic function D. Neither

7. What is the value of $\ln e^3$?

   A. 0 B. 1 C. 2 D. 3

8. Expand $\log_2 3x$

   A. 2log3x B. $\log 3x^2$ C. $\log_2 3 + \log_2 x$ D. $\log_2 \frac{3}{x}$

9. Solve log(x+3)+log(x)=1

   A. −5, 2 B. 2 C. 7 D. 10

10. Determine the solution set for $\log_2(2x + 3) > \log_2(x + 6)$.

    A. (2, +∞) B. (-∞, 3) C. (3, +∞) D. (-∞, 2)

LESSON 1: Introduction to Logarithms

What's In

Answer the following questions.

1. To which exponent must 2 be raised to get 8?
2. 5 to what power gives 625?
3. To which exponent must 3 be raised to get 81?

What's New

Answer the following questions briefly:

Think about this!

To which exponent must 2 be raised to get 10?

1. Were you able to get the exponent at which 2 must be raised to get 10 easily? Why?
2. Does the exponent at which 2 must be raised an integer?
3. If the exponent is not an integer, what is your closest estimate for the exponent at which 2 must be raised to get 10? Why?

What is It

From the activity in What's In, the question "To which exponent must 2 be raised to get 10? can be represented in exponential form as $2^y = 10$, where y represents the unknown exponent.

We can estimate that the exponent (y) is between 3 and 4 because $2^3 = 8$ and $2^4 = 16$. To get a more accurate estimate we will apply logarithms.

A real number y is called the **logarithm of x, with base a** if $a^y = x$.

In notation, we say that: $y = x$ if and only if $a^y = x$.

In the example, this number y is the exponent at which the base (2) should be raised to get a result of 10.

Therefore, we can write $2^y = 10$ in logarithm as $y = 10$ which is approximately equal to 3.32 using a scientific calculator.

To evaluate logarithms, the following basic properties of logarithms are useful.

BASIC PROPERTIES OF LOGARITHMS

Definition:

Let b and x be real numbers such that b>0 and b≠ 1, the basic properties of logarithms are the following:

1. $1 = 0$
   Since b>0, 1 means, a number (b) to what power gives 1 and that happens only when the exponent (power) is zero. That is why $1 = 0$
2. $b^x = x$
   b^x in exponential means, a number (b) to what power gives b^x and that happens only when the exponent (power) of b is x. That is why $b^x = x$.
3. If x>0, then $b^x = x$
   To verify this property, suppose $b^x = y$, which can be written as $y = x$ and this is true when x=y. That is why $b^x = x$.

Examples:

Evaluate the following logarithms.

1. $\log_5 125$
   Solution: $\log_5 5^3 = 3$ (using Property 2)
2. $\log_2 1$
   Solution: $\log_2 1 = 0$ (Using Property 1)
3. $\log_7 343$
   Solution: $\log_7 7^3 = 3$ (Using Property 2)
4. $3^{\log_3 4}$
   Solution: $3^{\log_3 4} = 4$ (Using Property 3)
5. $\frac{1}{2}$
   Solution: suppose $\frac{1}{2} = x$

$4^x = \frac{1}{2}$	Change the logarithm to exponential form.
$2^{2x} = 2^{-1}$	Solve the resulting exponential equation by making the bases equal.
2x = -1	Apply the property: If $a^n = a^m$, then n = m
$\frac{2x}{2} = -\frac{1}{2}$	Divide both sides by 2 to solve for x.
x = $-\frac{1}{2}$	

6. Log 1000

Solution:

If the base is not indicated in the logarithm, it is called a common logarithm whose base is 10. Log 1000 = 3 becomes $log\, log\, 10^3 = 3$ (property 2)

7. $\frac{1}{25}$

Solution:

$\frac{1}{25}$ becomes $5^{-2} = -2$ (property 2)

8. $\frac{4}{9}$

Solution:

$\frac{4}{9}$ becomes $\left(\frac{3}{2}\right)^{-2}$ = -2 (property 2)

9. ln 1

Solution:

The logarithm ln refers to the natural logarithm whose base is the natural number e. ln 1 = 0 (property 1) because $e^0 = 1$

10. $e^{ln\, 2x+1}$

Solution:

$e^{ln\, 2x+1} = 2x + 1$ (property 3)

Note that:

A real number y is called the **logarithm of x, with base** a if $a^y = x$.In notation, we say that: $y = x$ if and only if $a^y = x$. Thus, the function x which can be written as x is called a Logarithmic function.

There are real life situations which can be solved using logarithmic functions.

A. Compound Interest

The times it takes for an investment to grow at a given compound interest rate (r) is given by the formula: $t = \frac{log\left(\frac{A}{P}\right)}{nlog\left(1+\frac{r}{n}\right)}$

Where:

P: principal amount

A: accumulated amount

r: interest rate

n: number of times a year the interest is paid

t: number of time intervals that have passed

Example:

How long will it take for a deposit of ₱10 000.00 to grow to ₱35 000.00 at x% annual interest rate compounded monthly. Express the time (t) as a function f(x).

Solution:

P= 10 000

A=35 000

r= x interest rate

n=12 (compounded monthly)

Substitute these given data in the problem into the formula:

$t = \frac{log\left(\frac{A}{P}\right)}{nlog\left(1+\frac{r}{n}\right)}$	Formula
$t = \frac{log\left(\frac{35\ 000}{10\ 000}\right)}{12log\left(1+\frac{x}{12}\right)}$	Replace the given data into the formula.
$f(x) = \frac{log\left(\frac{35\ 000}{10\ 000}\right)}{12log\left(1+\frac{x}{12}\right)}$	Replace t as f(x) to express the time (t) as a function f(x).

B. The time it takes for the property to depreciate (decrease) in value at a given rate of decrease is given by the formula $t = \frac{log\left(\frac{y}{a}\right)}{log(1-r)}$

Where:

y: value after the rate of decrease at the number of time intervals that have passed.

a: initial value

r: rate of decrease

t: number of time intervals that have passed

Example:

In how many years will a television set depreciates from ₱51 990.00 to ₱29 000.00 if the value decreases by m % annually. Express the time (t) in terms of a function f(m).

Solution:

y=29 000

a=51 990

r= m%

Substitute these given data in the problem into the formula:

$t = \frac{log\left(\frac{29\ 000}{51\ 990}\right)}{log(1-m)}$	
$t = \frac{log\left(\frac{29\ 000}{51\ 990}\right)}{log(1-m)}$	Replace the given data into the formula.
$f(m) = \frac{log\left(\frac{29\ 000}{51\ 990}\right)}{log(1-m)}$	Replace t by f(m) to express the time t as a function f(m).

C. The time it takes for the property to appreciate (increase) in value at a given rate of increase is given by the formula $t = \frac{loglog\left(\frac{y}{a}\right)}{log(1+r)}$

Where:
y: value after rate of increase at the number of time intervals that have passed.
a: initial value
r: rate of increase
t: number of time intervals that have passed
Example:

In how many years will a set of gold jewelries to appreciate from ₱500 000.00 to ₱900 000.00 if the value increases by m% annually. Express the time (t) in terms of a function f(m).

Solution:
y = 900 000
a = 500 000
r = m%
Substitute these given data in the problem into the formula:

$$t = \frac{loglog\left(\frac{y}{a}\right)}{log(1+r)}$$

$$t = \frac{loglog\left(\frac{900\,000}{500\,000}\right)}{log(1+m)}$$ Replace the given data into the formula.

$$f(m) = \frac{loglog\left(\frac{900\,000}{500\,000}\right)}{log(1+m)}$$ Replace t by f(m) to express the time t as a function f(m)

What's More

A. Change the following to Logarithmic form
1. $a^b = c$
2. $5^3 = 125$
3. $3^x = 5$

B. Change the following to exponential form.
1. $w = z$
2. $7 = x$
3. $121 = 2$

C. Evaluate the following
1. 7
2. $\frac{4}{25}$
3. 512
4. $\frac{1}{8}$
5. $\frac{1}{2}$
6. $e^{ln\,x+3}$
7. 5^x
8. $10^{loglog\,2x-5}$
9. $\frac{1}{3}$
10. $\frac{1}{81}$

D. Express the following situations as a function f(n).

1. A car is worth ₱1 345 000.00, after how many years will it cost ₱650 000.00 if the value depreciates by n% a year. Express the time (t) as a function f(n).
2. How long will it take ₱50 000.00 to quadruple, if it is invested in a savings account that pays n% annual interest rate compounded semi-annually. Express the time (t) in terms of a function f(n).
3. How many years will it take for a piece of land worth P1 000 000.00 to double its value. If its value increases by n% yearly. Express the time (t) in terms of a function f(n).

What I Have Learned

Fill in the blanks to make the given statement true.
Complete the sentence below.

1. Real-life situations can be modeled using logarithmic functions such as ____________, ________________and ________________.

2. The time it takes for an investment to grow at a given compound interest rate is given by ______________.

3. The time it takes for the property to appreciate/increase to a desired value at a given rate of increase is given by ____________.

4. The time it takes for the property to depreciate/decrease at a given rate of decrease is given by ___________.

What I Can Do

A. Change the following to Logarithmic form

1. $x^y = z$
2. $13^2 = 169$
3. $3^n = 5$

B. Change the following to exponential form.

1. $p = s$
2. $7 = m$
3. $110 = n$

C. Evaluate the following

1. 3
2. $\frac{7}{9}$
3. 256
4. $\frac{1}{5}$
5. $\frac{1}{2}$
6. $e^{2ln\,x}$
7. 5^{3x}
8. $10^{3loglog\,2x-5}$
9. $\frac{1}{3}$
10. $\frac{1}{243}$

D. Express the following situations as a function f(a).

1. A motorcycle is worth ₱345 000.00, after how many years will it cost ₱150 000.00 if it depreciates in value at a% a year. Express the time (t) as a function f(a).
2. How long will it take for an investment of ₱500 000.00 to double, if it is invested in a savings account that pays a% annual interest rate compounded semi-annually. Express the time (t) in terms of a function f(a).
3. How long will it take for an antique collection worth ₱590 000.0 to ₱1 000 000.00 if its value increases by a% yearly. Express the time (t) in terms of a function f(a).

Rubrics for Scoring

Score	Description
15 points	Complete solutions with correct answers
10 points	75% correct solutions with incorrect answer.
5 points	50% correct solution with incorrect answer.
No point earned	No output at all

Assessment

Write the letter of the correct answer on a clean sheet of paper.

1. Change $3^x = 5$ to logarithm.

   A. $5 = x$ B. $3 = x$ C. $x = 3$ D. $5 = 3$

2. How many years will it take for a motorcycle worth P120 000.00 to depreciate to P89 990.00 if its value decreases by r % yearly? Express the time (t) as a function f(r).

   A. $f(r) = \frac{loglog\left(\frac{89\,990}{120\,000}\right)}{loglog\,(1-r)}$ C. $f(r) = \frac{loglog\left(\frac{120\,000}{89\,990}\right)}{loglog\,(1-r)}$

   B. $f(r) = \frac{loglog\left(\frac{89\,990}{120\,000}\right)}{loglog\,(1+r)}$ D. $f(r) = \frac{loglog\left(\frac{120\,000}{89\,990}\right)}{loglog\,(1+r)}$

3. How long will it take for a deposit of ₱25 000.00 to grow to ₱42 000 at an r% annual interest rate compounded quarterly? Express the time as a function f(r).

   A) $f(r) = \frac{loglog\,(\frac{42\,000}{25\,000})}{4loglog\left(1-\frac{r}{4}\right)}$ C) $f(r) = \frac{loglog\,(\frac{25\,000}{42\,000})}{4loglog\left(1-\frac{r}{4}\right)}$

   B) $f(r) = \frac{loglog\,(\frac{42\,000}{25\,000})}{4loglog\left(1+\frac{r}{4}\right)}$ D) $f(r) = \frac{loglog\,(\frac{25\,000}{42\,000})}{4loglog\left(1+\frac{r}{4}\right)}$

4. How long will it take ₱25 000.00 to quadruple, if it is invested in a savings account that pays w% annual interest rate compounded semi-annually. Express the time in terms of a function p(w).

   A) $p(w) = \frac{loglog\,(\frac{100\,000}{25\,000})}{2loglog\left(1-\frac{w}{2}\right)}$ C) $p(w) = \frac{loglog\,(\frac{25\,000}{100\,000})}{2loglog\left(1-\frac{w}{2}\right)}$

   B) $p(w) = \frac{loglog\,(\frac{100\,000}{25\,000})}{2loglog\left(1+\frac{w}{2}\right)}$ D) $p(w) = \frac{loglog\,(\frac{25\,000}{100\,000})}{2loglog\left(1+\frac{w}{2}\right)}$

5. Juan Luna's painting costs 46 million pesos. How long will it take for the painting to cost 100 million pesos if its price increases by p% annually? Express the time (t) as a function f (p).

   A. $f(p) = \frac{loglog\,(\frac{100\,000\,000}{46\,000\,000})}{loglog\,(1-p)}$ C. $f(p) = \frac{loglog\,(\frac{46\,000\,000}{100\,000\,000})}{loglog\,(1-p)}$

   B. $f(p) = \frac{loglog\,(\frac{100\,000\,000}{46\,000\,000})}{loglog\,(1+p)}$ D. $f(p) = \frac{loglog\,(\frac{46\,000\,000}{100\,000\,000})}{loglog\,(1+p)}$

Additional Activities

Express the following situations as a function f(a).

1. How long will it take for a deposit of ₱250 000.00 to grow to ₱420 000 at an a% annual interest rate compounded quarterly? Express the time as a function f(a).
2. How many years will it take for the price of an e-bike to depreciate from ₱179 990.00 to ₱139 500.00 if the value decreases by a% annually? Express the time in terms of a function f(a).
3. How long will it take ₱250 000.00 to quadruple, if it is invested in a savings account that pays a% annual interest rate compounded semi-annually. Express the time in terms of a function f(a).

LESSON 2. Distinguishing Logarithmic Functions, Logarithmic Equations, and Logarithmic Inequalities

What's In

Rewrite the following exponential equations in logarithmic form	Rewrite the following logarithmic equations in exponential form
$4^3 = 64$ Answer:	$\log x = y$ Answer:
$5^0 = 1$ Answer:	$\log_5 25 = 2$ Answer:
$6^x = 36$ Answer:	$\ln 8 = x$ Answer:
$3^{-3} = \frac{1}{27}$ Answer:	$\log_{\sqrt{6}} 6 = 2$ Answer:

Guide Questions:
1. How did you rewrite exponential equation into logarithmic form?
2. How did you rewrite logarithm equation into exponential equation?
3. What is a common logarithm?
4. What is a natural logarithm?

What's New

Given in the table below are examples of Logarithmic functions, logarithmic equations, and logarithmic inequalities.

Logarithmic Functions	Logarithmic Equations	Logarithmic Inequalities
$y = x$	$x = 3 - 2x$	$x < 4$
$f(x) = log\ log\ \frac{x}{3}$	$(x^2 - 3x + 1) = 3$	$5 - 2x > x$
$ln\ x = y$	$(1\ -\ 3x)$	$(x^2 - x + 2)$

Answer the following questions based on observation from the given examples in the table.
1. What is a logarithmic function?
2. What is logarithmic equation?
3. What is logarithmic inequality?

What is It

Logarithmic Function: A function of the form $f(x) = \log_b x$ ($b > 0$, $b \neq 1$)

Examples: $y = \log_3(2x+5)$
$f(x) = \log_2 x$
$g(x) = 3\log_3 x$

Logarithmic Equation: An equation involving logarithms.

Examples: $\log_x 3 = 9$
$\log_2 x = 16$
$\log x + \log (x - 6) = 1$

Logarithmic Inequality: An inequality involving logarithms.

Examples: $\log_4 x > 4$
$\log(x+3)<2$
$x\log_3 x>-5$

What's More

A. Determine if the following is a logarithmic function, a logarithmic equation, or a logarithmic inequality.

1. $\log_4 (3x+1) < \log_4 x + 1$
2. $\ln x = 5$
3. $g(x) = \log x^2$
4. $\log x^2 = 8$
5. $y = \log_3 20$

What I Have Learned

Complete the statement/s. Write your answer on a clean sheet of paper.

1. ________________ is a function of the form $f(x) = \log_b x$ $(b > 0, b \neq 1)$
2. ________________ an equation involving logarithms.
3. ________________ an inequality involving logarithms.

What I Can Do

Determine whether the following is a logarithmic function, logarithmic equation, or a logarithmic inequality.

1. $(2x + 3)$
2. $(x + 5) = 3$
3. $(2x - 3) = (x + 5)$
4. $3x > 3$
5. x
6. $2x$
7. $(3x + 1) < (x + 3)$
8. $(x - 3) = 6$
9. $log\ log\ x = y$
10. $f(x)= \ln x$

Assessment

Write the letter of the correct answer on a clean sheet of paper.

1. Which of the following is an example of a logarithmic inequality?
   A. $y = 3\log_4 x$ B. $\log_x 2 = 4$ C. $\log_2 x > 3$ D. $\log x^2 = 2$
2. Which of the following is not a logarithmic function?
   A. $y = 3\log_4 x$ B. $\log_x 2 = f(x)$ C. $\log_2 x > y$ D. $\log x^2 = g(x)$
3. The following is logarithmic equation EXCEPT:
   A. $\log_x 2 = 4$ B. $\log x^2 = 2$ C. $\log_3 (3x - 2) = 2$ D. $\log_2 x > 3$
4. Which of the following is an example of logarithmic function?
   A. $y = 3\log_4 x$ B. $\log_x 2 = 4$ C. $\log_2 x > 3$ D. $\log x^2 = 2$

For item number 5 – 10. Determine whether the given is a logarithmic function, equation, and inequality.

5. $\log x^2 = 2+x$
   A. logarithmic equation C. Logarithmic inequality
   B. logarithmic function D. Neither
6. $(2x + 3) + 3x < 10$
   A. logarithmic equation C. Logarithmic inequality
   B. logarithmic function D. Neither
7. $g(x) = \log_2 x$:
   A. logarithmic equation C. Logarithmic inequality
   B. logarithmic function D. Neither
8. $x\log_2(x) - 2 > 0$:
   A. logarithmic equation C. Logarithmic inequality
   B. logarithmic function D. Neither
9. $\log(5x) = 2\log 4$:
   A. logarithmic equation C. Logarithmic inequality
   B. logarithmic function D. Neither
10. $\log x^3 = 8$:
    A. logarithmic equation C. Logarithmic inequality
    B. logarithmic function D. Neither

Additional Activities

Determine whether the following is a logarithmic function, logarithmic equation, or a logarithmic inequality.

1. $\log_9 \frac{1}{9} = x$
2. $\log_6 216 > 1 - 2x$
3. $2x + 5 = x$
4. $f(x) = x$
5. $y = \ln e^{(3x)}$

LESSON 3. Laws of Logarithms and Solving Logarithmic Equations

What's In

Solve the following exponential equations:

1. $3^{2x} = 81$
2. $4^{x+1} = 8^x$
3. $2^{x-3} = 32$
4. $2^x = 5$

Guide Questions:

1. How did you solve for the value of x for item numbers 1-3?
2. Can we solve item number 4 using concepts of exponential function?
3. How do we solve logarithmic equation?

What's New

Use the properties to find the value of the following	Write your answer here and state the property is used.
1. $\log_4 4^3$	
2. $\ln e^{-3}$	
3. $\log_6 1$	
4. $e^{\ln 2}$	
5. $\log 10^{x+2}$	

What is It

Let us consider the following Laws of Logarithms which are useful in solving logarithmic equations.

Laws of Logarithms

Let b > 0, b 1 and let n R. For u > 0, v > 0, then

1. $\log_b (uv) = \log_b u + \log_b v$

2. $= \log_b u - \log_b v$

3. $\log_b u^n = n \log_b u$

4. $\log_b u =$

These laws of logarithms can be proven following the laws of exponents since logarithms are exponents.

1. Logarithm of a product: The logarithm of a product (uv) is equal to the sum of the logarithms of its factors $(u + v)$.
   $(uv) = u + v$
   Proof:
   Let: $x = u$ and v

$x = u \iff b^x = u$	Definition of logarithm
$y = v \iff b^y = v$	Definition of logarithm
(u)(v)=$(b^x)(b^y)$	Multiplication property of equality
uv=b^{x+y}	Law of exponent $(a^m)(a^n) = a^{m+n}$
$(uv) = x + y$	Definition of logarithm
$(uv) = u + v$	Substitution

2. Logarithm of a quotient: The logarithm of a quotient $\left(\frac{u}{v}\right)$ is equal to the logarithm of the numerator u) minus the logarithm of the denominator (v).
   $\left(\frac{u}{v}\right) = u - v$
   Proof:
   Let: $x = u$ and v

$x = u \Leftrightarrow b^x = u$	Definition of logarithm
$y = v \Leftrightarrow b^y = v$	Definition of logarithm
$\left(\frac{u}{v}\right)=\left(\frac{b^x}{b^y}\right)$	Division property of equality
$\frac{u}{v}=b^{x-y}$	Law of exponent $(a^m) \div (a^n) = a^{m-n}$
$\left(\frac{u}{v}\right) = x - y$	Definition of logarithm
$\left(\frac{u}{v}\right) = u - v$	Substitution

3. Logarithm of a power: The logarithm of a power u^n is equal to the product of the exponent (n) and the logarithm of u with the base b.
   $u^n = nu$
   Proof:
   Let: $u = x$

$b^x = u$	Definition of logarithm
$b^{nx} = u^n$	Raise both sides by the exponent n
$u^n = nx$	Definition of logarithm
$u^n = (u)\, n$	Replace x by u
$u^n = n(u)$	Commutative property of multiplication

4. Change of base formula for logarithm
   $u = \frac{u}{b}$
   Proof:
   Let: $u = y$

$b^y = u$	Definition of logarithm
$b^y = u$	Take the logarithm on both sides of the equation using the same base.
$b = u$	Apply logarithm of a product on the left-hand side of the equation.
$y = \frac{u}{b}$	Solve for y by dividing both sides of the equation by b .
$u = \frac{u}{b}$	Replace y by u .

Example 1. Use the properties of logarithms to expand each expression.

a. log($2x^3$)

Solution:

log ($2x^3$) = log 2 + log x^3	Logarithm of a product
$log\ log\ (2x^3)$ = log 2 + 3logx	Logarithm of a power $log\ log\ x^3 = 3\ log\ log\ x$

b. $\left(\frac{2}{x}\right)^3$ Solution:

$\left(\frac{2}{x}\right)^3 = 3 \cdot \left(\frac{2}{x}\right)$	Logarithm of a power
$\left(\frac{2}{x}\right)^3 = 3 \cdot (2 - x)$	Logarithm of a quotient
$\left(\frac{2}{x}\right)^3 = 32 - 3x$	Distributive property of multiplication.

c. ln[(2)(x+2)]

Solution:

ln[(2)(x+2)] = ln2 + ln(x+2) Logarithm of a product

Example 2. Use the properties of logarithm to condense the expressions as a single logarithm.

a. log 3 + log 4

Solution:

log 3 + log 4 = log (3)(4) Logarithm of a product

= log 12 Simplify

b. 3lnx – 2lny

Solution:

$\ln x^3 - \ln y^2$	Logarithm of a power 3lnx=$\ln x^3$ and 2lny = $\ln y^2$
$\ln \frac{x^3}{y^2}$	Logarithm of a quotient

c. $\log_2 (x^3) - 4\log_2 x$

Solution:

$\log_2 (x^3) - 4\log_2 x = \log_2 x^3 - \log_2 x^4$	Logarithm of a power $4x = x^4$
$=\left(\frac{x^3}{x^4}\right)$	Logarithm of a quotient
$=\left(\frac{1}{x}\right)$	Simplify
$=x^{-1} = -x$	Rewrite $\frac{1}{x}$ to $x^{-1}$and apply logarithm of a power to get $-x$

Aside from the Laws of Logarithms, the property of logarithmic equations is also helpful in solving logarithmic equations.

Property of logarithmic Equations

If b>1, then the logarithmic function y = $\log_b$ x is increasing for all x. If 0<b<1, then the logarithmic function y = $\log_b$ x is decreasing for all x. This means that $\log_b$ u = $\log_b$ v if and only if u = v.

Steps in Solving Logarithmic Equations

1. Isolate the logarithmic term on one side of the equation.
2. Change the logarithm into an exponential form.
3. Solve for the unknown variable by applying either the properties of logarithm and/or laws of logarithm.
4. Check for extraneous solution.

Example 3: Solve each equation.

1. $16 = x - 2$

Solution:

There is no need to isolate the logarithmic term on one of the equation, since 16 is already isolated on the left-hand side of the equation

x-2 = 4	Simplify 16 = 4
x – 2 + 2 = 4 + 2	Add 2 to both sides of the equation to solve for x and simplify.
x = 6	

Alternative solution:

2^{x-2}=16	Change $16 = x - 2$ to exponential form.
2^{x-2}=2^4	Change 16 to $2^4$to make the bases the same (base 2)
x-2 =	Apply the rule: If $a^m = a^n$, then m = n
x – 2 + 2 = 4 + 2	Add 2 to both sides of the equation and simplify to solve for x.
x = 6	

To check if x = 6 is a solution of $16 = x - 2$

$16 = 6 - 2$	Replace x by 6 and simplify.
$16 = 4$	$16 = 4$ which is equal to the right-hand side of the equation.
4 = 4	True statement

Therefore, x = 6 is a solution of $16 = x - 2$.

2. $\log_3 5x = \log_3 15$

There is no need to isolate the logarithmic term on one side of the equation because $5x$ is already isolated on the left-hand side of the equation.

Solution:

$\log_3 5x = \log_3 15$	One-to-one property
$5x = 15$	Divide both sides by 5 and simplify to solve for x.
$x=3$	

To check if $x = 3$ is a solution of $\log_3 5x = \log_3 15$

$\log_3 5(3) = \log_3 15$	Replace x by 3 and simplify.
$\log_3 15 = \log_3 15$	One to one property of logarithm.
$15 = 15$	True statement

Therefore $x = 3$ is a solution of $\log_3 5x = \log_3 15$.

3. $\log_5 \frac{1}{125} = 3x - 2$

Solution:

$5^{3x-2} = \frac{1}{125}$	Change to exponential equation.
$5^{3x-2} = 5^{-3}$	$\frac{1}{125} = \frac{1}{5^3}$ □□□□□ □□□ □□ □□□□□□□□ □□ 5
$3x - 2 = -3$	Apply the rule: If $a^m = a^n$ then m = n
$3x - 2 + 2 = -3 + 2$	Add 2 to both sides of the equation to solve for x and simplify.
$3x = -1$	Divide both sides by 3 and simplify.
$x = -\frac{1}{3}$	

Alternative solution:

$-3 = 3x - 2$	Evaluate $\log_5 \frac{1}{125}$ *which is equal to* -3
$-3 + 2 = 3x - 2 + 2$	Add 2 to both sides of the equation and simplify.
$-1 = 3x$ or $3x = -1$	Commutative property of equality
$\frac{3x}{3} =$	Divide both sides of the equation by 3 and simplify to solve for x.
$x = \frac{-1}{3}$	

To check if $x = \frac{-1}{3}$ is a solution of $\log_5 \frac{1}{125} = 3x - 2$.

$\log_5 \frac{1}{125} = 3\left(-\frac{1}{3}\right) - 2$	Replace x by $-\frac{1}{3}$ and simplify.
$\log_5 \frac{1}{125} = -3$	Evaluate $\log_5 \frac{1}{125}$ which is equal to -3
$-3 = -3$	True statement

Therefore x = $-\frac{1}{3}$ is a solution of $\log_5 \frac{1}{125} = 3x - 2$.

4. $\log_x 64 = 2$
Solution:

$x^2 = 64$	Change to exponential equation.
$x = \pm\sqrt{64}$	Get the $\pm$ square root of 64 because there are two values of x which when squared is 64.
x = + 8 and -8	

From the definition of logarithmic function, $f(x) = \log_b x$ (b > 0, b ≠ 1)
The base x in the logarithmic equation $\log_x 64 = 2$ must be (x>0, x≠ 1). The solution x=-8 is an extraneous solution.

To check if x=8 is a solution of $\log_x 64 = 2$

64 = 2	Replace x by 8 and simplify.
2 = 2 True statement	Evaluate 64 is equal to 2.

Thus, x = 8 is a solution of $\log_x 64 = 2$.

5. log x = 3
Solution:

log x = 3	Change to exponential equation. If the base is not indicated in the logarithm log x, understood the base is 10 and it is called a common logarithm.
$x = 10^3$	Simplify 10^3
x = 1000	

To check if x=1000 is a solution of log x = 3.

Log 1000 = 3	Replace x by 1000 and simplify.
3 = 3 True statement	Evaluate Log 1000 which is equal to 3.

Thus, x = 1000 is a solution of log x = 3.

6. $m + (m - 2) = 3$

$m(m-2) = 3$	Logarithm of a product
$m(m-2) = 2^3$	Change the logarithm to exponential form.
$m^2 - 2m = 8$	Multiply
$m^2 - 2m + (-8) = 8 + (-8)$	Set the equation to zero by adding -8 to both sides of the equation and simplify.

$m^2 - 2m - 8 = 0$	Solve the resulting quadratic equation either by the quadratic formula or by factoring (if it is factorable)
(m-4)(m+2) = 0	Factor the left-hand side of the equation.
m – 4 = 0 or m + 2 = 0	Set each factor to zero and solve for m.
m = 4 or m = - 2	

When m=-2 it will result to a logarithm of a negative number to the base 2 which is undefined. Therefore m = -2 is an extraneous solution.

To check if m=4 is a solution of $m + (m - 2) = 3$

$4 + (4 - 2) = 3$	Replace m by 4 and simplify.
$4 + 2 = 3$	Evaluate each logarithm by applying the properties of logarithm. $4 = 2^2 = 2$ and $2 = 1$
2 + 1 = 3	
3 = 3 True statement	The left hand side of the equation is equal to the right hand of the equation which are both equal 3.

Thus, m = 4 is a solution of $m + (m - 2) = 3$.

What's More

A. Use the properties of logarithms to expand each expression:

1. $\log_b x^2 \sqrt{y}$
2. $\log \left(\frac{x^3}{y^4}\right)$

B. Use the properties of logarithms to condense the expression as a single logarithm.

1. $\frac{1}{3}\log x - 2\log y$
2. $\log_5 x^3 + \log_5 x$

C. Solve each equation.
1. $\log_x 9 = 2$
2. $\log_3 243 = 4x - 1$

What I Have Learned

Fill in the Blank:

1. The $\log_b(uv)$ = ______________________________
2. The $\log_b(u/v)$ = ______________________________
3. The $\log_b u^n$ = ______________________________
4. The $\log_b u$ = ______________________________
5. The value of x in $\log_4 256 = x^2 - 3x$ is ______________________________.

What I Can Do

1. Solve for x in each of the following logarithmic equations.

   a. $\log_{25} 125 = 6 - 4x$

   b. $2^x = 7$

2. Give your own 2 logarithmic equations and solve for x.

Rubrics for Scoring

Score	Description
15 points	Complete solutions with correct answers
10 points	75% correct solutions with incorrect answer.
5 points	50% correct solution with incorrect answer.
No point earned	No output at all

Assessment

Write the letter of the correct answer on a clean sheet of paper.

1. Expand log(xy^2) in terms of the logarithms of the factors.

A. logx + log y B. 2logx+logy C. logx + logy^2 D. $\frac{logx}{2logy}$

2. Simplify: log 2 + log 4

A. log 2+4 B. 2log4 C. log6 D. log 8

3. Solve $\log_x 8 = -\frac{1}{2}$.

A. -64 B. -16 C. $\frac{1}{64}$ D. 4

4. Solve log(x+3)+log(x)=1

A. −5, 2 B. 2 C. 7 D. 10

5. 3lnx – 2lny: Use the properties of logarithm to condense the expressions as a single logarithm.

A. $\ln\frac{x^3}{y^2}$ B. 6lnxy C. $\ln x^3 + \ln y^2$ D. $\ln\frac{y^3}{x^2}$

6. Expand $\log_2 3x$

A. 2log3x B. log $3x^2$ C. $\log_2 3 + \log_2 2$ D. $\log_2 \frac{3}{x}$

7. Solve for x: $\log_3 x = 3$

A. 1 B. 3 C. 9 D. 27

8. What is $\log_3 (x+1) = 1$

A. 1 B. 2 C. 3 D. 4

9. Solve for x: $\log_5 x = 5$

A. 1 B. 5 C. 25 D. 125

10. What is ln x = -1

A. e B. e^{-1} C. 0 D. 1

Additional Activities

Use the Change – of – Base Formula and compute the approximate value.

1. $\log_5 12$ (change to the base 10)

2. $\log_4 \frac{1}{8}$ (change to base 2)

3. $5^x = 120$

4. $\log_7 343 = x$

5. $4^x = 112$

Lesson 4. Solving Logarithmic Inequalitites

What's In

Evaluate each of the following:

1. $\log_2 16$ Answer: ____________________

2. $\log_5 1$ Answer: ____________________

3. $\log_3 \frac{1}{729}$ Answer: ____________________

4. $\log_6 36$ Answer: ____________________

5. $\log_{25} 5$ Answer: ____________________

Guide Questions:
1. What are the properties of logarithms?
2. What are the laws of logarithms?

What's New

Solve for x:	Solve for x:
$3^{x+1} > 27$	$5^{2x-1} \leq 125$
$3^{x+1} > 3^3$	$5^{2x-1} \leq 5^3$
$x + 1 > 3$	$2x - 1 \leq 3$
$x > 3 - 1$	$2x \leq 4$
$x > 2$	$x \leq 2$

The table above showed how to solve exponential inequalities. Do you think solving exponential inequality is the same with solving logarithmic inequality? Discuss your answer.

What is It

Property of Logarithmic Inequalities

If $0 < b < 1$, then $x_1 < x_2$ if and only if $\log_b x_1 > \log_b x_2$.

If $b > 1$, then $x_1 < x_2$ if and only if $\log_b x_1 < \log_b x_2$.

Aside from using the properties above, logarithmic inequalities can be solved using the following:

- $x > y$ is same as $x > b^y$ or
- $x < y$ is same as $x < b^y$
- If $x = y$ then, $x = y$

To solve logarithmic inequalities:

1. Convert the logarithm to exponential form,
2. Solve the resulting inequality,
3. Exclude values that will result to log 0 and logarithm of a negative number and
4. Check if the solution set satisfies the logarithmic inequality.

Example 1: $3^x > 3$

Solution:

$3^x > 3$	can be written as $x > \log_3 3$
$x > \log_3 3$	Apply property 2 since the base b>1 Simplify $3 = 1$
$x > 1$	

To check if the solution $x > 1$ satisfies the inequality $3^x > 3$, take a test value in the solution set and solve if the test value satisfies the inequality.

Suppose we take $x = 2$

$3^2 > 3$	Replace x by 3 and simplify.
$9 > 3$ True statement	

Thus, $x > 1$ is solution of the inequality $3^x > 3$.

From the solution x > 1
Thus, the solution set of the inequality is $\{x \,/\, x > 1\}$ or in interval notation (1, +∞).

Example 2: Solve $\log_2(4x-2) > \log_2(x-3)$
Solution:

$\log_2(4x-2) > \log_2(x-3)$	Given
$4x-2 > x-3$	Apply property 2 since the base b > 1
$3x > -1$	Simplify by combining similar terms
$x > -\frac{1}{3}$	Divide both sides by 3 to solve for x.

But the solution of $\log_2(4x-2)$ is (2, +∞) and the solution of $\log_2(x-3)$ is (3, +∞). To determine the solution of $\log_2(4x-2) > \log_2(x-3)$ we need to get the intersection of the three solution sets $\left(-\frac{1}{3}, +\infty\right)$, (2, +∞), and (3, +∞).
Thus, the solution is (3, +∞)

Example 3: Solve $\log_2(2x+1) < 3$
Solution:

$2x + 1 < 2^3$	Change to exponential form
$2x + 1 < 8$	Simplify 2^3 as 8
$2x + 1 + (-1) < 8 + (-1)$	Add -1 to both sides of the inequality and simplify.
$2x < 7$	Divide both sides of the inequality by 2 and simplify.
$x < \frac{7}{2}$	

From the solution set $x < \frac{7}{2}$, we need to exclude values that will result to log 0 and logarithm of a negative number and that happens when $2x + 1 \leq 0$, whose solution set is $x \leq -\frac{1}{2}$.
To determine the solution set, we need to exclude the solution set $x \leq -\frac{1}{2}$ from the computed solution set $x < \frac{7}{2}$.
Thus, the solution is $\left\{x \,/ -\frac{1}{2} < x < \frac{7}{2}\right\}$ or in interval notation $\left(-\frac{1}{2}, \frac{7}{2}\right)$.

Example 4: $\log_4 9 > 2\log_4 x$
Solution:

$\log_4 9 > \log_4 x^2$	Apply logarithm of a power $2\log_4 x = \log_4 x^2$
$9 > x^2$ or $x^2 < 9$	Apply one-to-one property of logarithm
$x^2 - 9 < 0$ critical values are $x = 3$ or $x = -3$	Add -9 to both sides of the inequality and solve the resulting quadratic inequality using inspection of signs.
Since x > 0, hence, 0 < x < 3 or (0,3)	

What's More

Solve the following inequalities:

1. $(\frac{1}{3})^x < \frac{1}{9}$

2. $\log_3 (3x - 2) < 2$

3. $\log_2 x + 3 \geq \log_2 (1 - x)$

What I Have Learned

Fill in the Blank:

1. Write $5^x > 25$ in logarithmic inequality form: ______________________.
2. The solution set for $5^x > 25$ is ________________________________.
3. If $0 < b < 1$, then $x_1 < x_2$ if and only if ___________________________.
4. If $b > 1$, then $x_1 < x_2$ if and only if ______________________________.
5. The solution of $\log_2 x + \log_2 (x+2) \leq \log_2(x + 6)$ is ____________________.

What I Can Do

Solve the following inequalities:

1. $4^{x+2} > 3$

2. $\log_3 (x - 1)^2 > 2$

Rubrics for Scoring

Score	Description
15 points	Complete solutions with correct answers
10 points	75% correct solutions with incorrect answer.
5 points	50% correct solution with incorrect answer.
No point earned	No output at all

Assessment

Write the letter of the correct answer on clean sheet of paper.

1. Find the solution to $4^x > 8$.
   A. A. $(2, +\infty)$ B. $(-\infty, 3)$ C. $(3, +\infty)$ D. $(-\infty, 2)$
2. Solve for $(\frac{1}{5})^{2x} \geq \frac{1}{25}$.
   A. $(1, +\infty)$ B. $(-\infty, 1)$ C. $(2, +\infty)$ D. $(-\infty, 2)$
3. Solve for x: $\log_4(2x) = \log_4 8$
   A. 1 B. 2 C. 3 D. 4
4. Solve $\log_3 (2x - 2) > 1$
   A. $x > 3$ B. $x < -3$ C. $x > 5$ D. $x < -5$
5. Solve for x in the inequality $\log_3 x \geq 2$.
   A. $x \geq 3$ B. $x \geq 6$ C. $x \geq 9$ D. $x \geq 27$
6. Find the solution to the given exponential inequality $2^x > 2$.
   A. $(1, +\infty)$ B. $(-\infty, 1)$ C. $(2, +\infty)$ D. $(-\infty, 2)$
7. Which of the is the solution of $(\frac{1}{3})^{2x} \geq \frac{1}{9}$?
   A. $(1, +\infty)$ B. $(-\infty, 1)$ C. $(2, +\infty)$ D. $(-\infty, 2)$
8. Solve $\log_5 (x+3) > 1$.
   A. $(1, +\infty)$ B. $(-\infty, 1)$ C. $(2, +\infty)$ D. $(-\infty, 2)$
9. Determine the solution set for $\log_2(2x + 3) > \log_2(x + 6)$.
   A. $(2, +\infty)$ B. $(-\infty, 3)$ C. $(3, +\infty)$ D. $(-\infty, 2)$
10. Solve for $\log_2 (2x+1) > \log_2(x+3)$.
    A. $(2, +\infty)$ B. $(-\infty, 3)$ C. $(3, +\infty)$ D. $(-\infty, 2)$

Additional Activities

1. Solve for x : $\log_3 x + \log_3 6 \geq 2$

2. Find the solution set of $\log_3 x - 3\log_3 2 \leq 1$.

SUMMATIVE TEST

Write the letter of the correct answer on a clean sheet of paper.

1. Change $3^m = 15$ to logarithm.
   A. $3 = m$ B. $15 = m$ C. $y = 15$ D. $y = 3$
2. What is the value of $\ln e^5$?
   A. 2 B. 3 C. 4 D. 5
3. Evaluate 729
   A. -3 B. $-\frac{1}{3}$ C. $\frac{1}{3}$ D. 3
4. Which of the following is an example of logarithmic function?
   A. $y = 3\log_4 (x+5)$ B. $\log_x 8 = 4$ C. $\log_2 x > 9$ D. $\log x^2 < 16$
5. How long will it take for a deposit of ₱250 000.00 to grow to ₱320 000 at r% annual interest rate compounded quarterly? Express the time as a function f(r).
   A. $f(r) = \frac{loglog\left(\frac{320\,000}{250\,000}\right)}{4loglog\left(1+\frac{r}{4}\right)}$ C. $f(r) = \frac{loglog\left(\frac{250\,000}{320\,000}\right)}{4loglog\left(1+\frac{r}{4}\right)}$
   B. $f(r) = \frac{loglog\left(\frac{320\,000}{250\,000}\right)}{4loglog\left(1-\frac{r}{4}\right)}$ D. $f(r) = \frac{loglog\left(\frac{250\,000}{320\,000}\right)}{4loglog\left(1-\frac{r}{4}\right)}$
6. The following represents a logarithmic equation EXCEPT:
   A. $\log_x 2 = 4$ B. $\log x^2 = 2$ C. $\ln(3x - 2) = 2$ D. $\log_3 (x+3) < 9$
7. $x\log_2(2x + 5 < 0$ is an example of ________________________?
   A. logarithmic equation C. Logarithmic inequality
   B. logarithmic function D. Neither
8. Determine the solution set for $\log_2(x + 5) > \log_2(2x + 3)$.
   A. $(\frac{3}{2}, 2)$ B. $(-\frac{3}{2}, 2)$ C. $(-2, \frac{3}{2})$ D. $(-2, -\frac{3}{2})$
9. Expand $\log_2 5x$
   A. 2log3x B. $\log 3x^2 - 5$ C. $\log_2 5 + \log_2 x$ D. $\log_2 \frac{3}{x}$
10. Solve log(4x)+log(x)=1
    A. $\pm\frac{1}{2}$ B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 2

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