

Mathematics

Quarter 1 – Module 21: Solving Problems Involving Polynomial Functions



Mathematics – Grade 10

Alternative Delivery Mode

Quarter 1 – Module 21: Solve problems involving polynomial functions

First Edition, 2019

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Published by the Department of Education

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Undersecretary:

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Printed in the Philippines by _____

Department of Education – Bureau of Learning Resources (DepEd-BLR)

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Mathematics

Quarter 1 – Module 21:

Solve problems involving polynomial functions

This instructional material was collaboratively developed and reviewed by educators from public and private schools, colleges, and or/universities. We encourage teachers and other education stakeholders to email their feedback, comments, and recommendations to the Department of Education at lfugao@deped.gov.ph

We value your feedback and recommendations.

Introductory Message

For the facilitator:

Hi. As the facilitator of this module, kindly orient the learner on how to go about in reading and answering this learning material. Please be patient and encourage the learner to complete this module. By the way, do not forget to remind the learner to use separate sheets in answering all of the activities found in this module.

For the learner:

Hello learner. I hope you are ready to progress in your Grade 10 Mathematics by accomplishing this learning module. This is designed to provide you with interactive tasks to further develop the desired learning competencies prescribed in the K-12 Curriculum of the Department of Education. In this module, you are expected to solve various problems involving polynomial functions.

Here is a guide on the parts of the learning modules which you need to understand as you progress in reading and analyzing its content.

ICON	LABEL	DETAIL
	What I need to know	This contains the learning objectives which you need to accomplish
	What I know	This assesses what you know about the lesson you are to tackle
	What's In	This connects the current lessons with the previous lessons
	What's New	This introduces the lesson through an activity
	What Is It	This contains a brief discussion of the lessons
	What's More	These are activities to check your understanding of the lesson
	What I have Learned	This summarizes the important ideas presented in the lesson
	What I Can Do	This is a real life application of what you have learned
	Assessment	This is a post assessment of what you have learned
	Additional Activities	This is an activity that will strengthen your knowledge about the lesson

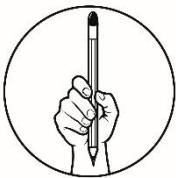


What I Need to Know

This module was designed and written with you in mind. It is here to help you solve problems involving polynomial functions applying the concepts learned in the previous modules. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course but the order in which you read and answer this module is dependent on your ability.

After going through this module, you are expected to:

1. Solve problems involving polynomial functions.



What I Know

Read each item carefully and choose the letter that corresponds to your answer. Write the letter of your answer in a separate sheet of paper.

1. What is the degree of the polynomial function $P(x) = 7x^3 + 6x^4 - 8x^6 + 6x + 11$?
A. 7 B. 6 C. 4 D. 5
2. If $P(x) = x^4 - 4x^2 + 3x + 2$, then $P(2) =$ _____.
A. 2 B. 8 C. 14 D. -10
3. Which of the following binomials is a factor of $a^3 + 5a^2 + 2a + 10$?
A. $(a - 2)$ B. $(a + 2)$ C. $(a + 5)$ D. $(a - 5)$
4. How many zeros does the polynomial function $(x+4)(x-3)(x+5)(x-2)$ have?
A. 4 B. less than 4 C. more than 4 D. at most 5
5. What are the zeros of the polynomial function $P(x) = (x + 6)(x - 4)$?
A. $[-6, -4]$ B. $[6, -4]$ C. $[-6, 4]$ D. $[6, 4]$
6. A cube has an edge that is x cm long. What is its capacity $C(x)$?
A. x^3 cu B. 4^3 cu C. c^2 cu D. x^2
7. What is the factored form of $x^3 + 2x^2 - 24x$?
A. $2x(x^2 - 4x + 6)$ B. $x(x - 4)(x + 6)$ C. $x^2(x - 2)(x - 1)$
8. What is the remainder when $x^4 + x^3 - 2$ is divided by $x - 2$?
A. 22 B. 27 C. 29 D. 31
9. What are the x -intercepts of $y = (x - 5)(x + 9)(x - 10)$?
A. 5, 9, 10 B. -5, 9, -10 C. 5, -9, 10 D. 5, 9, -10
10. What is the y -intercept of $y = (x - 3)(x + 1)(x - 1)$?
A. $y = -3$ B. $y = 3$ C. $y = 4$ D. $y = -4$

11. The length of a rectangular garden is $(x + 5)$ and the width is x . Which of the following represents the area of the garden?

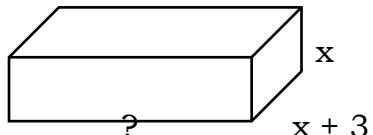
- A. $f(x) = x + 5x$ B. $f(x) = x^3 + 5$ C. $f(x) = x^2 + 5x$ D. $f(x) = x + 5x^2$

12. What is $B(x) + S(x)$ given that $B(x) = 7x^2 - 5x + 100$ and $S(x) = 20x^2 + 60x + 200$?

- A. $27x^2 + 55x + 300$ B. $27x^2 - 65x + 200$ C. $17x^2 + 45x + 300$

13. The volume of a box is $V(x) = 2x^3 + 7x^2 + 3x$. Which of the following expression represents the missing length?

- A. $x + 1$ C. $2x + 1$
B. $x + 2$ D. $2x + 3$



14. What is $f(3)$ if $f(x) = x^2 - 3x^3 + 2x^4 + 1$?

- A. 91 B. 10 C. 30 D. 3

15. The volume of a cube is 64 cm^3 . What is the length of its edge?

- A. 12cm B. 4cm C. 6cm D. 10cm

**Lesson
1**

Solve Problems Involving Polynomial Function

In your earlier module on polynomial, you learned how to graph solutions of polynomial functions of degree one (1) and two (2), the linear and quadratic functions respectively. In this module, you will focus on solving problems involving polynomial functions.

Review the previous module by doing the graphing activity below.



What's In

Given a polynomial function $f(x)$ you can draw its graph by constructing the table of values of x and $f(x)$ and plotting the points on the Cartesian plane.

Consider the polynomial function $f(x) = x^3 + x^2 - 4x - 2$.

Complete the table of values below and sketch the graph on the Cartesian plane using a graphing paper and answer the questions that follows.

x	-2	-1	0	1	2	3
y						

1. At what point does the graph intersect the x-axis?
2. How do we call such points?
3. At what point does the graph intersect the y-axis?
4. How do you call such point?
5. How many turning points does the graph have?
6. Identify the turning points.
7. Describe the graph.



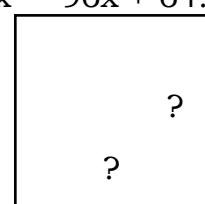
What's New

In the earlier modules you learned how to illustrate and graph polynomial functions. In this section, you will apply mathematical concepts on finding the measurements like area.

Solve for the unknown using the given information.

The area of a square garden is represented by $A(x) = 36x^2 - 96x + 64$.

- a. How long is one side?
- b. What is its perimeter?



Hint: Perimeter of a square = 4s

$$\text{Square of a square} = s^2$$

Solution: a. Area of the square = $36x^2 - 96x + 64$

$36x^2 - 96x + 64 = s^2$ Since the area is a perfect square trinomial, rewrite it as a square of a binomial

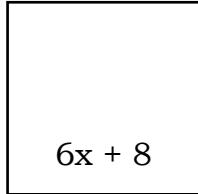
$$(6x + 8)^2 = s^2 \quad \text{Extract the root}$$

$$6x + 8 = s$$

The dimension of each side is $6x + 8$.

b. Square has four congruent sides.

$$P = 4s$$



$$P = 4(6x + 8) \quad P = 24x + 8$$

Did you find the correct dimension of the square? Do you remember the steps involved when solving problems? For better understanding, read the related concept below.



What is It

To solve word problems, you need to consider the following steps.

A – What is **ASKED**?

G – What are **GIVEN**?

O – What **OPERATION** to be used?

N – Make a **NUMBER SENTENCE or REPRESENT THE VARIABLES**

S – Write your **SOLUTION**

A – Final **ANSWER** in a complete sentence

Example 1: A cube has a capacity of 125cm^3 . What is the length of its edge?

Solution:

A – What is the length of its edge?

G – 125cm^3 – capacity of the cube

O – Multiplication & Extracting Roots

N – $125\text{cm}^3 = s \cdot s \cdot s$

S – $V = s \cdot s \cdot s$ Volume of a cube formula

$125\text{cm}^3 = s^3$ Substitute the given

$\sqrt[3]{125\text{cm}^3} = \sqrt[3]{s^3}$ Extract the cube root

$$5 \text{ cm} = s$$

A – The length of the edge of the cube is 5cm.

Example 2: A rectangle is $3x + 1$ units long and $2x - 3$ units wide. What is the area of the rectangle represented by $A(x)$?

A – What is the area of the rectangle?

G – $3x + 1$ length ; $2x - 3$ width

O – Multiplication

$$N - A(x) = (3x + 1) (2x - 3)$$

S – $A(x) = lw$ Formula in finding the area of a rectangle

$$A(x) = (3x + 1) (2x - 3)$$

$$A(x) = 6x^2 - 7x - 3$$

A – The area of the rectangle is $6x^2 - 7x - 3$.

You may also consider the following steps below in solving.

Example 3: Find the polynomial function with the following set of zeros; (3, -3, 1) that will represent the volume of the rectangular prism.

STEP 1 Write the given zeros of the polynomial function.

Given: $x = 3, x = -3, x = 1$

STEP 2 Know what is being asked for in the problem.

Unknown: Polynomial function of zeros 3, -3, 1

STEP 3 Solve the problem.

a. Write the factors from the roots.

$$P(x) = (x - 3) (x + 3) (x - 1)$$

$$P(x) = (x - 3) (x + 3) (x - 1) = 0 \quad \text{Equate to zero}$$

$$P(x) = (x^2 - 9) (x - 1)$$

$$P(x) = x^3 - x^2 - 9x + 9$$

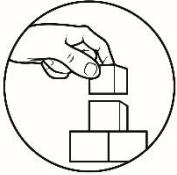
STEP 4 Review your answer.

STEP 5 Make a conclusion.

Final answer:

The volume of the rectangular prism is represented by $P(x) = x^3 - x^2 - 9x + 9$.

You have been provided illustrative examples on solving problems involving area, volume and zeros of polynomial function. Remember there are many ways and solution to every problem. Enjoy as you learn by performing the next activities.



What's More

Activity 1

1. Form the polynomial function with the following set of zeros -1 and 2 of multiplicity 3.

Solution: $P(x) = (x + 1)(x - 2)^3$

$$P(x) = (x + 1)(x^2 - 4x + 4)(x - 2)$$

$$P(x) = (x + 1)(x^3 - 6x^2 + 12x - 8)$$

$$P(x) = x^4 - 6x^3 + 12x^2 - 8x + x^3 - 6x^2 + 12x - 8$$

$$P(x) = x^4 - 6x^3 + x^3 + 12x^2 - 6x^2 - 8x + 12x - 8$$

$$P(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$$

The polynomial function is $P(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$.

Assessment 1

Let's check how well you were able to understand the given examples.

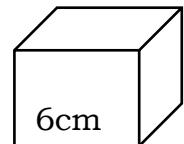
1. Find a third degree polynomial function with zeros 3 of multiplicity 2 and -4 of multiplicity 1.

Activity 2

The edge of a cube measures 6 cm. Give the polynomial function representing its volume.

Solution:

The volume of a cube can be solved by the formula: $V = s^3$



Let volume be $V(x) = s^3$

$$V(x) = (6\text{cm})^3 = 6\text{cm} \cdot 6\text{cm} \cdot 6\text{cm} = 216\text{cm}^3$$

Therefore, the polynomial function representing the volume is

$$V(x) = 216\text{cm}^3$$

Assessment 2

1. A cube has an edge of $x + 4$. What is the volume $P(x)$ of the cube?
2. The edge of a cube measures 7 cm. Give the polynomial function representing the volume.

Activity 3

A demographer predicts that the population, P , of a town t years from now can be modeled by the function $P(t) = 6t^4 - 5t^3 + 200t + 12000$. What will the population of the town be two years from now?

Solution:	$P(t) = 6t^4 - 5t^3 + 200t + 12000$	Given
	$P(2) = 6(2)^4 - 5(2)^3 + 200(2) + 12000$	Substitute 2
	$P(2) = 6(16) - 5(8) + 400 + 12000$	
	$P(2) = 96 - 40 + 12400$	
	$P(2) = 12,456$	

The population of the town after two years will be 12,456.

Assessment 3

Read and analyze carefully the problems. Write your answers with your solutions in a separate paper.

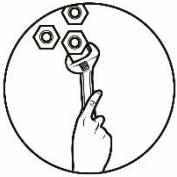
1. A farmer has a poultry farm whose area A is expressed by $8x^2 + 97x + 12$ m². What is the land area of the poultry farm if $x = 3$ m?
2. Annie went to the grocery and bought items which cost $C(x) = 5x^4 + 2x^3 + 4x + 18$ pesos p . If x is P4.00, how much did Annie pay?
3. A car manufacturer determines that its profit, P , in thousands of pesos, can be modeled by the function $P(x) = x^4 + 2x - 3$, where x represents the number of cars sold. What is the profit when $x = 200$?



What I Have Learned

Complete the steps in solving word problems. Use a separate sheet of paper as your answer sheet.

1. What is _____?
2. What are _____?
3. What _____ to be used?
4. Make a _____.
5. Write your _____
6. Final _____ in a complete sentence

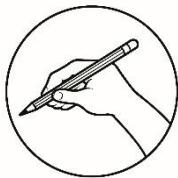


What I Can Do

Solve the given problem.

A company's profit in thousands of pesos is determined by the number of branches x it operates: $P(x) = -x(x - 10)(x - 30)$

- A. Classify the degree: _____
- B. Evaluate the polynomial function for $x = 10$. Interpret your answer in the context of the situation.
- C. Evaluate the polynomial function for $x = 20$. Interpret your answer in the context of the situation.
- D. Evaluate the polynomial function for $x = 35$. Interpret your answer in the context of the situation.
- E. If you are the manager of the company, how many branches should you maintain? Explain why?



Assessment

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

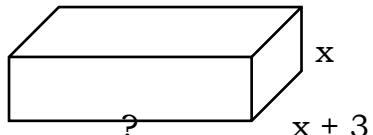
1. A cube has an edge that is x cm long. What is its capacity $C(y)$?
A. x^3 B. 4^3 C. c^2 D. y^2
2. What is the factored form of $x^3 + 2x^2 - 24x$?
b. $2x(x^2 - 4x + 6)$ B. $x(x - 4)(x + 6)$ C. $x^2(x - 2)(x - 1)$
3. What is the remainder when $x^4 + x^3 - 2$ is divided by $x - 2$?
B. 22 B. 27 C. 29 D. 31
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A. 5, 9, 10 B. -5, 9, -10 C. 5, -9, 10 D. 5, 9, -10
5. What is the y -intercept of $y = (x - 3)(x + 1)(x - 1)$?
A. $y = -3$ B. $y = 3$ C. $y = 4$ D. $y = -4$
6. The length of a rectangular garden is $(x + 5)$ and the width is x . Which of the following represents the area of the garden?
A. $f(x) = x + 5x$ B. $f(x) = x^3 + 5$ C. $f(x) = x^2 + 5x$ D. $f(x) = x + 5x^2$

7. What is $B(x) + S(x)$ given that $B(x) = 7x^2 - 5x + 100$ and $S(x) = 20x^2 + 60x + 200$?

A. $27x^2 + 55x + 300$ B. $27x^2 - 65x + 200$ C. $17x^2 + 45x + 300$

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- A. $x + 1$ C. $2x + 1$
B. $x + 2$ D. $2x + 3$



9. What is $f(3)$ if $f(x) = x^2 - 3x^3 + 2x^4 + 1$?

- A. 91 B. 10 C. 30 D. 3

10. The volume of a cube is 64 cm^3 . What is the length of its edge?

- A. 12cm B. 4cm C. 6cm D. 10cm

11. What is the degree of the polynomial function $P(x) = 7x^3 + 6x^4 - 8x^6 + 6x$?

- A. 7 B. 6 C. 4 D. 5

12. If $P(x) = y^4 - 4y^2 + 3y + 2$, then $P(2) = \underline{\hspace{2cm}}$.

- A. 2 B. 8 C. 14 D. -10

13. Which of the following binomials below is a factor of $a^3 + 5a^2 + 2a + 10$?

- A. $(a - 2)$ B. $(a + 2)$ C. $(a + 5)$ D. $(a - 5)$

14. How many zeros does the polynomial function $(x-8)(x+5)(x-1)(x+3)$ have?

- A. 4 B. less than 4 C. more than 4 D. at most 5

15. What are the zeros of the polynomial function $P(x) = (x + 6)(x - 4)$?

- A. -6, -4 B. 6, -4 C. -6, 4 D. 6, 4



Additional Activity

Read, analyze and solve the given problem. Use a separate sheet of paper as your answer sheet.

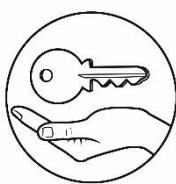
The cost C in pesos of removing x percent of pollutants from the swimming pool in Rational Resort is given by $C(x) = 50x^2 - 100x + 45000$.

Find: A. $C(50) = \underline{\hspace{2cm}}$

B. $C(70) = \underline{\hspace{2cm}}$

C. $C(90) = \underline{\hspace{2cm}}$

D. $C(99) = \underline{\hspace{2cm}}$



Answer Key

What I Know

- 1. B
- 2. B
- 3. C
- 4. A
- 5. C
- 6. A
- 7. B
- 8. A
- 9. C
- 10. B
- 11. C
- 12. A
- 13. C
- 14. A
- 15. B

- What I Have Learned**
- 1. Asked
 - 2. Given
 - 3. Operation
 - 4. Number Sentence
 - 5. Solution
 - 6. Answer
- Additional Activity**
- A. $C(50) = 165,000$
 - B. $C(70) = 283,000$
 - C. $C(90) = 441,000$
 - D. $C(99) = 525,150$

Assessment

- Assessment 3**
- $$x^3 + 12x^2 + 48x + 64$$
- Assessment 2**
- $$x^3 - 2x^2 - 15x + 36$$
- Assessment 1**
- $$1. A(3) = 375 \text{ sq. m}$$
- $$2. C(4) = P1442$$
- $$3. P(200) = 1,600,000,397$$
- What's More**
- 1. A
 - 2. B
 - 3. B
 - 4. C
 - 5. B
 - 6. C
 - 7. A
 - 8. C
 - 9. A
 - 10. B
 - 11. B
 - 12. C
 - 13. C
 - 14. A
 - 15. C

References

Callanta, Melvin M. et.al, Mathematics – Grade 10 Learner’s Module. Pasig City, REX Bookstore, Inc. 2015.

Capul, Erist A. , Hasmin T. Ignacio, Elsie M. Pacho, Suchi Christine Garcia-Rufino.2015, Makati City, Diwa Learning Systems, Inc.

Oronce, Orlando A. and Marilyn O. Mendoza, 2015. E-Math Worktext in Mathematics, Sampaloc Manila, Rex Bookstore

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