

Module in Math 10

POLYNOMIAL FUNCTION



Photo Credit: Google.com.ph

Myra L. Delos Reyes
Master Teacher I
Maria Clara High School

Introduction

This module deals with polynomial functions of degree 3 and higher. You will find the zeros of polynomial function which are restricted to integral coefficients. You will be able to relate the zeros to the factor of the polynomial functions by applying some theorems. And you are also expected to sketch the graph of the polynomial functions.

To help you attain this goal, you will need to recall the definition of polynomial, terms and degrees of polynomials, and how to perform operations on polynomials and recall some factoring techniques.

"Math is like love – a simple idea but it can get complicated." ~ R. Drabek

LESSONS and COVERAGE

<i>Lesson 1</i>	-----	4 - 5
• Identifying Polynomials		
<i>Lesson 2</i>	-----	6 - 7
• Determining the Terms and Degrees of Polynomials		
<i>Lesson 3</i>	-----	8 - 11
• Performing the Fundamental Operations on Polynomials		
<i>Lesson 4</i>	-----	12 - 13
• Synthetic Division		
<i>Lesson 5</i>	-----	14 - 15
• The Remainder Theorem		
<i>Lesson 6</i>	-----	16 - 17
• Factoring Polynomials (A Recall)		
<i>Lesson 7</i>	-----	18 - 20
• The Factor Theorem		
<i>Lesson 8</i>	-----	21 - 22
• Finding the Roots/Zeros of Polynomial Equations/Functions		
<i>Lesson 9</i>	-----	23 - 25
• The Rational Root Theorem		
<i>Lesson 10</i>	-----	26 - 29
• Sketching the Graph of Polynomial Functions		



Lesson

Identifying Polynomials



What to Learn?

- To identify polynomials from a list of algebraic expressions.



What to Know?

A polynomial is an algebraic expression where the *exponent* of the variable is a *positive integer* or zero. There are *no variables* in the *denominator* or *under the radical sign*.

A polynomial function is written as

$$P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n$$

These are examples of

Polynomials

$$\frac{2}{3}x^3 + x^2 - 7x + 11$$

$$x^3 - 3$$

$$6x^4 - 5x^6 - x + 15x^2$$

$$9 + 5x - x^5$$

Polynomial Functions

$$f(x) = \frac{2}{3}x^3 + x^2 - 7x + 11$$

$$P(x) = x^3 - 3$$

$$g(x) = 6x^4 - 5x^6 - x + 15x^2$$

$$f(x) = 9 + 5x - x^5$$



What to Do?

Write *PAK* if the given expression is a polynomial and *GANERN* if not polynomial.
Write the answer on the space provided for.

_____ 1. $2x^2 - 5x^3 + x$

_____ 2. $x^3 - \frac{3}{x} - \frac{1}{4}$

_____ 3. $\frac{x^5}{5} + x - \frac{3}{4}$

_____ 4. $2x^{\frac{1}{3}} - 5x^2 + 22x - 4$

_____ 5. $x + 10 + x^3 - 6x^2$

_____ 6. $\sqrt{3}x^9 + 2x^2 - 3$

_____ 7. $2 + 5x - 3x^2$

_____ 8. $x^4 - x + 2x^2$

_____ 9. $5x^4 + x^2 - 2x + 7x^0$

_____ 10. $8x - x^2 + 1$



What to Enhance?

WHAT IS THE MESSAGE?

Directions: Cross – out the expressions which are not polynomials. Get the corresponding letters from the uncrossed expressions to form the hidden message.

$2x^2 - \frac{1}{x} - 7$ G	$x^{23} - x - 5$ M	$5x + 1$ A	$3 - x^{-7} + x$ N
$12 - x - 4x^3$ T	$\sqrt{3x^3 - x + 2}$ L	$x^5 - x^4 - 1$ H	$2x^5 - 3x$ M

$\sqrt[3]{16} - x^9 + 4x$ A	$6x^2 - x + \frac{2}{5}$ K	$\frac{6}{x^4} - 3x^2 + 1$ W	$x^5 - 3x - 12$ E
$x + 31 - x^7$ S	$x^{10} - x^{-3} + 16$ X	$\frac{4}{5}x + 3x^2 - 2$ Y	$-x^9 + 8x + 6$ O
$5\sqrt{x} - 2$ J	$5x^3 - 3x^2 + 6x$ U	$9x - 3x^6 - 5$ R	$\frac{3x}{4} + 3x^2 - 5$ L
$9x^{10} - 3x + 5$ I	$4 - 6x^{\frac{1}{2}} - 7x$ C	$x^{49} - x + 1$ F	$-2 - 2x + 2x^3$ E
$-x^4 - 2x - 2$ A	$x^2 - \frac{3}{x} - 3x^8$ B	$6x + 7x^5 - 9$ D	$x^6 - x^3 - x$ D
$5x^{-4} - x + 1$ Q	$-x^5 + x^2 + x - 3$ U	$9 - x^9 - x^{99}$ P	$3x + 1 - 2x^{\frac{2}{5}}$ V

Message: _____



What to Transfer?

Consider the area A of a circle with a variable r . Recall that $A = \pi r^2$. If for instance $r = \frac{1}{2}$, $A = \frac{\pi}{4}$; $r = 1$, $A = \pi$; $r = 2$, $A = 4\pi$; $r = 3$, $A = 9\pi$. In tabular form,

r	$\frac{1}{2}$	1	2	3
A	$\frac{\pi}{4}$	π	4π	9π

This means that there exists a relation between the radius and the area of the circle, which can be related to the idea of polynomial function. Since for every value of the radius there corresponds a unique value of the area of the circle, then the value of A depends upon the value of r . We can say that A is function of r and it can be denoted by $P(r)$.

Think of at least 5 formulas in Mathematics or in other subjects that you can relate to polynomials. List and write examples on how to use them.



Lesson

Determining the Terms and Degrees of Polynomials



What to Learn?

- To determine the degree and number of terms of a given polynomial expression.



What to Know?

A polynomial can be classified according to the number of terms in the expression.

- Monomial – a polynomial that consists of one term
- Binomial – a polynomial that consists of two terms
- Trinomial – a polynomial that consists of three terms
- Multinomial – a polynomial that consists of more than three terms

Examples:

Monomial	Binomial	Trinomial	Multinomial
$5x$ $-3m^5$ $15x^3y^2z$ -11	$7y - 3$ $3x + \frac{2}{5}$ $5a + 10b$	$m^2 - 6m + 9$ $3x^2 - 5x + \frac{1}{3}$ $1 + 2y^2 + y^4$	$3x^3 - 2x^2 + 5$ $5x^3y^3 + x^2y + 2xy^3$ $3a^3b - 2a^2b^3 - 5mn + 13$

The **degree** of a polynomial in one variable is determined by the highest exponent of the variable in the expression. If the given expression has more than one variable, get the sum of the exponents in each variable. The term with the highest exponent is known as the leading term.

Examples:

The expression $3x^3 - 2x^2 + 5$ is a polynomial in one variable and it has 3 terms (Trinomial) and since the highest exponent is 3, then the **degree** is **3** and the leading term is $3x^3$.

The expression $3a^3b - 2a^2b^3 - 5mn + 13$ is a polynomial in two variables and it has 4 terms (Multinomial) and since the highest exponent is 5, then the **degree** is **5** and the leading term is $-2a^2b^3$.



What to Do?

Complete the table by classifying the given polynomial according to the number of terms and give the degree and leading term of each polynomial based from the highest exponent.

Polynomial	No. of Terms	Classification	Highest Exponent	Degree	Leading Term
1. $14ax$					
2. $m^3 - m^4 + 7$					
3. $\frac{1}{4}x + 8$					
4. $13 - n$					
5. $8x^8 + 6x^9 - x + x^3 - 1$					
6. $ab^4 - 2a + 3$					
7. $5x^5 + 8x^3 - x - 4$					

8. -15					
9. $2x^7 + 4x^5 - x^3 - 3x + 9$					
10. $y^5 - \frac{1}{3}y^{15}$					



What to Enhance?

A famous quotation by Plato is given in the boxes. To find this, shade the box that does not illustrate a true statement. The remaining boxes give the quotation.

1. LETTERS	2. NUMBERS	3. ARE	4. OF
In the monomial $10x^5$, 10 is the leading term	In the expression $5x^3$, 3 is the exponent or the degree	$\frac{3x^2y}{4}$ is a monomial	$\frac{ab^3}{2}$ is a binomial
5. THE	6. THIS	7. HIGHEST	8. DEGREE
In $2x - 5$, x is the literal coefficient	$\frac{x-2y}{3} + 1$ is a multinomial	$7 - 4x^3 + 2x$ is a trinomial	In the polynomial $4x - 5x^4$, $-5x^4$ is the leading term
9. FORM	10. OF	11. WISDOM	12. KNOWLEDGE
The leading coefficient in the trinomial $x^2 + 3x - 2$ is 2	$-\frac{12a}{5} + b$ is a binomial	14 is not a polynomial	$3 + 4x^5 - 3x^3 + x^2$ is a multinomial

Quote:



Challenge Yourself!

“What is so special about the number 4?”

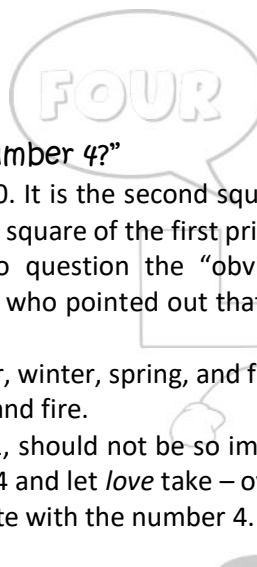
The number 4 is considered as polynomial of degree 0. It is the second square number, though the first which is $1 = 1^2$ is hardly ever interesting. It is also the square of the first prime number.

Until before the 1900's, scientists did not seem to question the “obvious” situation that everything we see is three dimensional. It was Albert Einstein who pointed out that a fourth dimension exists – time.

The number of seasons in some countries is 4: summer, winter, spring, and fall. The ancients also believed in the four fundamental elements: water, wind, soil, and fire.

Surely, let us not forget $1 - 4 - 3$. The first, which is 1, should not be so important as not to be selfish. To think 3 alone is to be foolishly blind. But to think of 4 and let love take – over is the best of all.

Think of other real – life situation that you can associate with the number 4.





Lesson

Performing the Fundamental Operations on Polynomials



What to Learn?

- To add, subtract, multiply, and divide polynomials.



What to Know?

- To add polynomials, simply combine similar terms and to subtract polynomials get the additive inverse of the subtrahend then follow the rules in addition.

Examples:

$$1. (2x^4 + 4x^2 + 5) + (x^3 - 5x^2 + 6x) \quad 2. (7x^3 - 2x^2 + 4x - 7) - (3x^3 + x^2 - 2x + 8)$$

$$\begin{array}{r} 2x^4 \quad \quad + 4x^2 \quad \quad + 5 \\ \quad \quad x^3 - 5x^2 + 6x \\ \hline = 2x^4 + x^3 - x^2 + 6x + 5 \end{array}$$

$$\begin{array}{r} 7x^3 - 2x^2 + 4x - 7 \\ -3x^3 - x^2 + 2x - 8 \\ \hline = 4x^3 - 3x^2 + 6x - 15 \end{array}$$

- To multiply polynomial by another polynomial apply the Distributive Property of Multiplication, then simplify.

Examples:

$$1. -7x^3(5x^2 - 4x + 10) = -7x^3(5x^2) - 7x^3(-4x) - 7x^3(10) \\ = -35x^5 + 28x^4 - 70x^3$$

$$2. (3x^2 - 5x)(6x^2 - 5x + 2) \\ = 3x^2(6x^2) + 3x^2(-5x) + 3x^2(2) - 5x(6x^2) - 5x(-5x) - 5x(2) \\ = 18x^4 - 15x^3 + 6x^2 - 30x^3 + 25x^2 - 10x \\ = 18x^4 - 45x^3 + 31x^2 - 10x$$

- To divide polynomial by monomial, divide each term of polynomial by monomial, then apply the law of exponents for quotient.

Example:

$$(81x^6 - 6x^4 - 18x^3 + 9x^2) \div (3x^2) = \frac{81x^6}{3x^2} - \frac{6x^4}{3x^2} - \frac{18x^3}{3x^2} + \frac{9x^2}{3x^2} \\ = 27x^4 - 2x^2 - 6x + 3$$

- To divide polynomial by other polynomial using long division the process is similar to that of whole numbers.

Example:

$$(6x^2 + 5x^3 - 17x + 25) \div (x + 3)$$

1. Arrange the terms in descending order both in divisor and dividend

$$x + 3\sqrt{5x^3 + 6x^2 - 17x + 25}$$

2. Divide the first term of the dividend by the first term of the divisor to get the first term of the quotient

$$x + 3\sqrt{5x^3 + 6x^2 - 17x + 25} \quad \begin{array}{r} 5x^2 \end{array}$$

3. Multiply the divisor by the first term of the quotient and subtract the product from the dividend

$$x + 3\sqrt{5x^3 + 6x^2 - 17x + 25} \quad \begin{array}{r} 5x^2 \\ 5x^3 + 15x^2 \\ -9x^2 \end{array}$$

4. Bring down the next term, thus obtaining a new dividend

$$x + 3\sqrt{5x^3 + 6x^2 - 17x + 25} \quad \begin{array}{r} 5x^2 \\ 5x^3 + 15x^2 \\ -9x^2 - 17x \end{array}$$

5. Continue the process until the remainder is zero or if the remainder is not equal to 0, write the remainder as the numerator of the fraction with the divisor as its denominator, and add this fraction to the partial quotient

$$x + 3\sqrt{5x^3 + 6x^2 - 17x + 25} \quad \begin{array}{r} 5x^2 - 9x + 10 + \frac{-5}{x+3} \\ 5x^3 + 15x^2 \\ -9x^2 - 17x \\ -9x^2 - 27x \\ 10x + 25 \\ 10x + 30 \\ -5 \end{array}$$



What to Do?

Do what is asked.

1. Find the sum of $x^3 + 10x^2 - 23x + 14$ and $-3x^3 + 6x^2 - 4x + 8$
2. Add $-2x^4 + 7x^2 + 3x - 8$ and $3x^4 - 5x^3 + 2x - 3$
3. Take $x^3 - 2x^2 + 5x - 4$ from $4x^3 - 2x^2 - 8x + 5$

4. The difference of $5a - 9a^2 + 10a^5 - 3$ and $4a^3 - 2a^5 + 19a + 9$
5. Multiply $(3m^4 - 13m^3 - 11m^2 + 22m - 14)$ by $(m + 2)$
6. Find the product of $(2x^2 - 5xy - 6y^2)(x^2 - 5x + 1)$
7. When dividing $8a^4b^5 - 64a^4b^4 + 16a^3b^5 - 32a^5b^6$ by $4a^3b^3$ the result is
8. Divide $10p^4 - 5p^3 - 5p + 12$ by $2p + 3$ using long division
9. $(3y^4 - 13y^3 - 11y^2 + 22y - 4) \div (y^2 - 5y + 1)$
10. *The product of $(3x^3 + 5)$ and $(4x^3 - 1)$ added to the difference between $(4x^3 + 5x^2 - 9x + 3)$ and $(9x^3 + 16x^2 - 19x - 3)$ the result is



What to Enhance?

CROSS POLYNOMIAL PUZZLE

1				2		
			3			4
		5				
	6				7	
8						
			9			

Perform the indicated operation.

Across:

1. $(5x^2 - 8x + 6) + (-6x^2 - 9)$
2. $(4x^2 - 5) - (4 + 2x^2)$
3. $(2x - 1)(3x)$
5. $(x + 4)^2$
6. $(x - 7)(x + 2)$
7. $(x - 5)(x + 5)$
8. $(2x + 1)(x + 5)$
9. $(42x^5 - 12x^3 + 6x) \div (6x)$

Down:

1. $(3x^2 + 4x) - (4x^2 - 6x + 5)$
2. $(x^2 + 2x + 8) + (x^2 - 5x + 8)$
3. $2(3x + 7)(x - 1)$
4. $(4x^3 - 3x + 3x^2 - 6) + (2x^3 - 3x^2 + 5x - 19)$
5. $(3x^2 - 8x + 1) - (2x^2 - 3x - 4)$
6. $(x + 6)(x + 5)$
7. $(2x^3 - 11x^2 + 17x - 5) \div (2x - 5)$



Challenge Yourself!

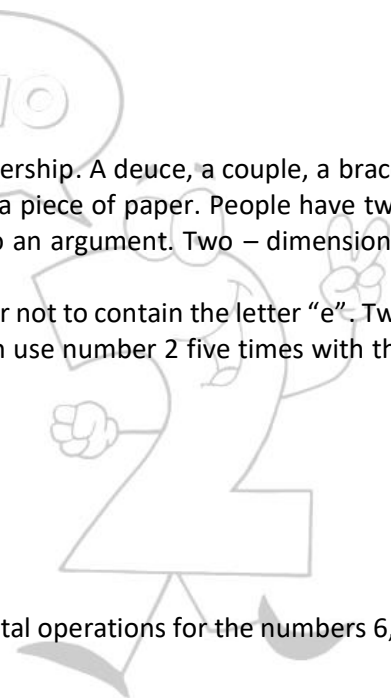
FIVE 2's

Many say, "Good things come in pairs". Two symbolizes partnership. A deuce, a couple, a brace, or a duo. There are two blades on a pair of scissors and two sides to a piece of paper. People have two hands and so do some clock. There are two genders and two sides to an argument. Two – dimensional means that something has length and width, but no depth.

Two is the only even prime number and the only prime number not to contain the letter "e". Two has a very special property because $2 + 2 = 4$ and $2 \times 2 = 4$. You can use number 2 five times with the fundamental operations:

$$\begin{aligned} 2 + 2 - 2 - 2 \div 2 &= 1 \\ 2 + 2 + 2 - 2 - 2 &= 2 \\ 2 + 2 - 2 + 2 \div 2 &= 3 \\ 2 \cdot 2 \cdot 2 - 2 - 2 &= 4 \\ 2 + 2 + 2 - 2 \div 2 &= 5 \end{aligned}$$

Make your own equation using five 2's showing the fundamental operations for the numbers 6, 7, 8, 9, and 10 as the result.





Lesson

Synthetic Division



What to Learn?

- To illustrate the process of synthetic division.
- To find the quotient and the remainder when $P(x)$ is divided by $x - k$ using synthetic division.



What to Know?

When you do long division, you are using the four operations. But in synthetic division, you can only use multiplication and addition. The process of synthetic division for polynomials greatly simplifies division of polynomials in x , where the divisor is in the form of $x - k$.

Example:

Divide $P(x) = x^4 - 10x^2 - 2x + 4$ by $(x + 3)$ using synthetic division.

Steps:

- ❖ Arrange the coefficients of the polynomial in descending powers of x , placing 0s for the missing terms
- ❖ Get the constant divisor in $x - k$
- ❖ Bring down the leading coefficient
- ❖ Multiply the leading coefficient to the constant divisor and write the product beneath the second number, then get their sum
- ❖ Repeat the process until the last number is obtained
- ❖ The entries in the third row give the coefficients of the quotient and the last number is the remainder
- ❖ The exponent is one degree less than the degree of the polynomial

Synthetic Division

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -10 & -2 & 4 \\ & & -3 & +9 & 3 & -3 \\ \hline & 1 & -3 & -1 & 1 & 1 \end{array} \quad \begin{array}{l} \text{Divide } x^4 - 10x^2 - 2x + 4 \text{ by } x + 3 \\ \\ \\ \\ \text{Remainder} \end{array}$$

$$\frac{x^4 - 10x^2 - 2x + 4}{x + 3} = x^3 - 3x^2 - x + 1 + \frac{1}{x + 3}$$



What to Do?

A. Choose the correct synthetic division for $(x^4 - 2x^2 - x - 6) \div (x - 2)$.

a.
$$\begin{array}{r|rrrr} 2 & 1 & -2 & -1 & -6 \\ & & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & -8 \end{array}$$

b.
$$\begin{array}{r|rrrr} 2 & 1 & 0 & -2 & -1 & -6 \\ & & 2 & 4 & 4 & 6 \\ \hline & 1 & 2 & 2 & 3 & 0 \end{array}$$

B. Find the quotient and the remainder using synthetic division.

1. $f(x) = 3x^3 - x^2 + 2x - 4;$	$x + 3$	_____
2. $g(x) = x^5 - 3x^4 + 2x^2 + x - 2;$	$x + 1$	_____
3. $h(x) = x^3 - 2x^2 - 3x - 1;$	$x - 1$	_____
4. $j(x) = 2x^5 - 2x^2 + x - 4;$	$x + 2$	_____
5. $m(x) = x^3 + 16x^2 + 9x - 90;$	$x + 3$	_____

6. $n(x) = x^4 + x^3 - 13x^2 - 25x - 12$; $x + 2$
7. $P(x) = 4x^3 + 2x^2 + x + 7$; $x + 1$
8. $Q(x) = x^3 - 3x^2 + x + 7$; $x - 2$
9. $*R(x) = x^4 + 2x^3 + 6x^2 - x - 10$; $x^2 - x + 2$
10. $*S(x) = 6x^5 + 23x^4 - 39x^3 - 13x^2 + 33x - 10$; $2x^2 + x - 1$



What to Enhance?

TRIVIA: The commonly used division symbol “/” is called a “solidus” or “virgule”, while an “obelus” appears like a hyphen with dots above and below it. What do you call the other division notation which is a horizontal line that is sometimes referred to as a “fractional bar”?

Directions: To find the answer to the question above, determine the quotient and remainder using synthetic division. Match column A with column B. Place the letter that represents the quotient and the remainder in the space that corresponds to the number representing the problems.

A		B	
1.	$P(x) = 3x^3 - 5x + 4$; $x + 2$	L	$x^3 - x^2 + 2x + 4$ R. 7
2.	$P(x) = 2x^5 - 2x^3 + 4x^2 - 1$; $x + 1$	M	$2x^4 - 2x^3 + 4x - 4$ R. -5
3.	$P(x) = x^4 - 3x^3 + 4x^2 - 1$; $x - 2$	C	$2x^2 - 5x + 17$ R. -54
4.	$P(x) = x^4 - x^2 - x - 1$; $x - 1$	V	$3x^3 + x^2 + 4x + 7$ R. 13
5.	$P(x) = 3x^4 - 5x^3 + 2x^2 - x - 1$; $x - 2$	N	$3x^2 - 6x + 7$ R. -10
6.	$P(x) = 2x^3 + x^2 + 2x - 3$; $x + 3$	I	$x^3 + x^2 - 1$ R. -2
7.	$P(x) = x^4 - 2x^3 - 3x + 3$; $x - 3$	U	$x^3 + x^2 + 3x + 6$ R. 21

5	4	1	6	7	3	7	2



Challenge Yourself!

The REMAINDER 3 and 6!

Choose any prime number greater than 3. Let's say 17

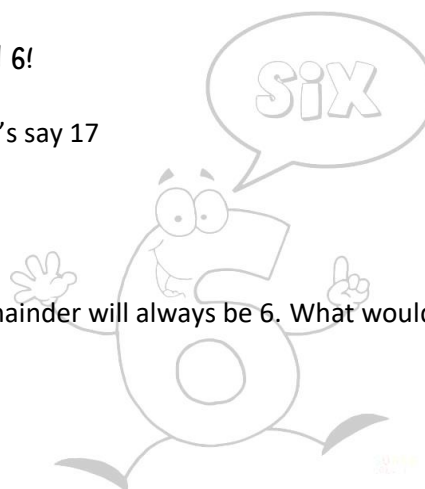
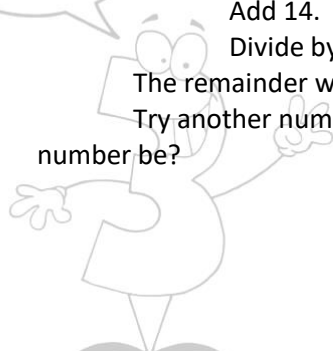
Square that number. $(17)^2 = 289$

Add 14. $289 + 14 = 303$

Divide by 12. $303 \div 12 = 25 \text{ r}3$

The remainder will always be 3!

Try another number to add instead of 14, and the remainder will always be 6. What would that number be?





Lesson

The Remainder Theorem



What to Learn?

- To use Synthetic Division and the Remainder Theorem in finding the value of a polynomial function.



What to Know?

In finding the values of polynomial functions, it is a convenient way to use synthetic division and the Remainder Theorem at the same time.

Example:

- Find the remainder when $(5x^2 - 2x + 1)$ is divided by $(x + 2)$.
- Evaluate $P(x) = 5x^2 - 2x + 1$ when $x = -2$.

1. Using synthetic division

$$\begin{array}{r|rrr} & 5 & -2 & 1 \\ & & -10 & 24 \\ \hline & 5 & -12 & 25 \end{array} \quad -2$$

The remainder is 25. Therefore, $P(-2) = 25$.

$$2. P(-2) = 5(-2)^2 - 2(-2) + 1$$

$$P(-2) = 5(4) + 4 + 1$$

$$P(-2) = 20 + 4 + 1$$

$$P(-2) = \boxed{25}$$

The Remainder Theorem states that when the polynomial $P(x)$ is divided by $x - k$, the remainder is $P(k)$.



What to Do?

Work with a partner to find the value of the polynomial function, one will use the Synthetic Division and the other will use the Remainder Theorem, compare your answer.

- $P(x) = x^3 + 8x^2 + 13x - 10$; $k = -3$
- $P(x) = 3x^4 - 2x^3 - 2$; $k = 2$
- $P(x) = -3x^4 + 5x^3 - 2x - 2$; $k = -1$
- $P(x) = x^4 + 2x^2 + 2x - 2$; $k = 1$
- $P(x) = 2x^3 + 3x - 3$; $k = -2$
- $P(x) = x^3 - 2x^2 - 5x + 3$; $k = -2$
- $P(x) = 3x^4 - 2x^2 + 2x - 1$; $k = 2$
- $P(x) = 2x^4 - 3x^2 + 2$; $k = 1$
- $P(x) = x^3 + 3x^2 + 2x - 1$; $k = -1$
- $P(x) = 2x^3 - 2x^2 - x + 4$; $k = 3$



What to Enhance?

MESSAGE UNDER THE TABLE!

Find the values of the given polynomial function and then place the syllable or word with its corresponding answer in the table to decode the message.

k	-3	-2	-1	
$f(x)$	$x^3 - 7x - 6$	$4x^4 + 9x^3 + 3x^2 + 4$	$x^5 - 2x^3 + x^2 + 2$	
$f(k)$				
Message				

0	1	2	3	4
$x^{10} - 3x^5 + 19$	$2x^4 - 2x^2 + x - 2$	$3x^3 - x^2 + 3x + 3$	$3x^3 - 5x^2 + x - 2$	$x^3 - 3x^2 - x - 3$

$$f(k) = -12 \text{ [Life]}$$

$$f(k) = 8 \text{ [with]}$$

$$f(k) = 9 \text{ [less]}$$

$$f(k) = -1 \text{ [be]}$$

$$f(k) = 19 \text{ [will]}$$

$$f(k) = 37 \text{ [ing]}$$

$$f(k) = 4 \text{ [out]}$$

$$f(k) = 29 \text{ [mean]}$$

$$f(k) = 0 \text{ [Mathematics]}$$



What to Transfer?

“How the Early Egyptians Solved Equations?”

The Egyptians, one of the first civilizations to use Mathematics, they created symbols to represent numbers. A rod represented number one, a heel bone stood for ten, a snare for 100, a lotus flower for 1000, a bent finger for 10,000, a fish for 100,000, and a kneeling figure for 1,000,000.

Many problems appearing on Egyptians papyri (1800 B.C.) were solved by the method of false position. A value was assigned to the unknown. When this value was checked and the given conditions not satisfied, the value was changed by a simple proportion. For example, consider “Aha, its whole, its quarter, it makes 30.” We have $x + \frac{x}{4} = 30$.

Assume any value of x , say $x = 4$. Then $x + \frac{x}{4}$ becomes $4 + \frac{4}{4} = 5$. Since the desired answer was 30, 5 should be multiplied by 6 to obtain it. The value of x that we assumed is 4 multiplied by 6, $x = 4 \cdot 6$, $x = 24$.

Applying the polynomial function here, we have $f(x) = x + \frac{x}{4}$, and $f(x) = 30$, find the value of x . If $x = 24$, then $f(24) = 24 + \frac{24}{4}$, thus $f(24) = 30$.

Supposed you were an Egyptian, think of a simple everyday problem that you can apply evaluating of polynomial function similar to the given example then write it in your journal. State your reflection in the identified problem.



Lesson

Factoring Polynomials (A Recall)



What to Learn?

- Find the factors of polynomials.



What to Know?

To factor a polynomial means to write it as a product of two or more polynomials.

Some factoring techniques are:

- Largest Common Factor:

Example: Factor $10x^4 + 15x^3 - 25x$

Get the common monomial factor, that is $5x$

The factors are $5x(2x^3 + 3x^2 - 5)$

- Perfect Square Trinomial: For all x and y

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

Examples: Factor

- $x^2 + 8x + 16 = (x + 4)^2$
- $9a^2 - 30ab + 25b^2 = (3a - 5b)^2$
- $4y^4 + 12y^2 + 9 = (2y^2 + 3)^2$

- Difference of Two Squares: For all x and y

$$x^2 - y^2 = (x + y)(x - y)$$

Examples: Factor

- $a^2 - 100b^2 = (a + 10b)(a - 10b)$
- $49y^2 - 1 = (7y + 1)(7y - 1)$
- $9x^6 - 36 = 9(x^6 - 4) = 9(x^3 + 2)(x^3 - 2)$

- Sum or Difference of Two Cubes: For all x and y

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Examples: Factor

- $x^3 + 125 = (x + 5)(x^2 - 5x + 25)$
- $8a^3 - 27b^3 = (2a - 3b)(4a^2 + 6ab + 9b^2)$
- $64y^6 + 1 = (4y^2 + 1)(16y^4 - 4y^2 + 1)$

- Quadratic Trinomial: For all x and y

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

Examples: Factor

- $x^2 + 5x + 6 = (x + 3)(x + 2)$
- $x^2 - 5x + 6 = (x - 3)(x - 2)$
- $x^2 + 5x - 6 = (x + 6)(x - 1)$
- $x^2 - 5x - 6 = (x - 6)(x + 1)$



What to Do?

Factor completely:

1. $16a^3b^2 - 2a^2b^2 + 8ab^2$ _____
2. $5x^4 - 125x^2$ _____
3. $x^2 - 10xy + 25y^2$ _____
4. $49x^2 + 56x + 16$ _____
5. $16m^4 - 1$ _____
6. $3x^2 - 12y^2$ _____
7. $y^3 - 216$ _____
8. $27x^9 + 64$ _____
9. $x^2 - x - 56$ _____
10. $5x^2 + 9x - 2$ _____



What to Enhance?

"MATH HUGOT"

Boy: Miss, I heard that you are good in Algebra?
 Girl: Hmmm...and so?
 Boy: Well, can you please substitute my "eX" without asking "Y"?

Girl: Do you know that, to get a functional relationship you can only match with one, you cannot cheat into matching to two or three others? "Parang tayo lang..."

Boy: Really! "Paanong parang tayo lang"?

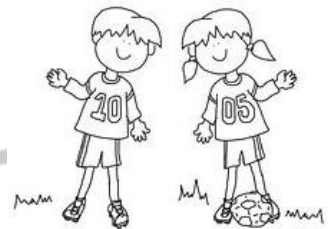
Girl: To know what I mean, you must answer this problem first.

$$\frac{\frac{2a^2M+6aM+18M}{a^3-27}}{\frac{4a+2}{2a^2e-5ae-3e}} + \frac{\frac{3u^2v-3u^2}{uv^2-u}}{\frac{3}{v+1}} = \frac{F}{t} \left(\frac{\frac{a^4-a^2}{3a^2-a-2}}{\frac{a^2+a}{3at^2e+2t^2e}} \right)$$

Boy: But I'm not as good as you in Math!

Girl: That's why we will ask the class to help you with my message.

- What do you think is the girl's message? Help the boy to reveal it by factoring each polynomial.



The Factor Theorem



Lesson



What to Learn?

- To determine if a binomial is a factor of a given polynomial function.
- To find the factors of polynomial functions.



What to Know?

When you divide 28 by 7, you will get a quotient of 4 and a remainder of 0. If there is no remainder what does it tell us? Yes, 4 and 7 are factors of 28, since 7 exactly divides 28.

A similar condition exists in polynomials when divided by a binomial $(x - k)$.

Example:

Find the remainder if $P(x) = x^3 - 7x + 6$ is divided by $x - 2$

Solution: Using the Remainder Theorem

$$P(2) = 2^3 - 7(2) + 6$$

$$P(2) = 8 - 14 + 6$$

$$P(2) = 0$$

Since the remainder is 0, it means that the binomial exactly divides the polynomial function, so we can say that the binomial $x - 2$ is a factor of the given polynomial $P(x) = x^3 - 7x + 6$. This illustrates the Factor Theorem.

The Factor Theorem states that if polynomial function $P(x)$ is divided by $(x - k)$ and $P(k) = 0$, then $x - k$ is a factor of $P(x)$.

More Examples:

1. Determine whether $(x + 4)$ is a factor of $f(x) = x^3 + 7x^2 + 2x - 40$.

Solution: Using the Remainder Theorem

$$f(-4) = (-4)^3 + 7(-4)^2 + 2(-4) - 40$$

$$f(-4) = -64 + 7(16) - 8 - 40$$

$$f(-4) = -112 + 112$$

$$f(-4) = 0$$

By the Factor Theorem, since $f(-4) = 0$, $(x + 4)$ is a factor of $f(x) = x^3 + 7x^2 + 2x - 40$

2. Is $(x + 2)$ a factor of $P(x) = x^3 - 5x^2 - 2x + 24$? If yes, find the other factors.

Solution: Using Synthetic Division

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -2 & 24 \\ & & -2 & 14 & -24 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$

Since the remainder is 0, therefore $x + 2$ is a factor of $P(x)$. To find the other factors of $P(x)$, get the factors of the quotient. The coefficients of the quotient are 1 -7 12, the equivalent polynomial function is $Q(x) = x^2 - 7x + 12$. Thus,

$$x^2 - 7x + 12 = 0$$

$$(x - 4)(x - 3) = 0$$

Therefore, the other factors are $(x - 4)$ and $(x - 3)$.



What to Do?

A. Divide the class into 5 groups. Use the Factor Theorem to determine which of the following binomials is/ are the factor/s of the given polynomial.

Group I

1. $f(x) = x^3 - 2x^2 - x + 2$
 - a. $x - 2$
 - b. $x + 2$
 - c. $x - 3$
 - d. $x + 3$

Group II

2. $f(x) = x^4 - 8x^3 + 2x^2 + 5$
 - a. $x - 1$
 - b. $x + 1$
 - c. $x - 3$
 - d. $x + 3$

Group III

3. $f(x) = 3x^2 + x^3 - 11x + 2$
 - a. $x - 1$
 - b. $x - 2$
 - c. $x + 1$
 - d. $x + 2$

Group IV

4. $f(x) = 2x^4 + 9x^3 + 15x^2 + 11x + 3$
 - a. $x - 1$
 - b. $x + 1$
 - c. $x - 2$
 - d. $x - 3$

Group V

5. $f(x) = x^5 - 2x^3 - x + 14$
 - a. $x + 1$
 - b. $x - 2$
 - c. $x + 2$
 - d. $x - 3$

B. Do what is asked.

1. For what value of t is $x^3 + 2tx^2 + x - 6$ exactly divisible by $x + 2$?
2. Find the value of k so that $x - 3$ is a factor of $kx^3 - 7x - 6 = 0$.
3. The polynomials $x^3 + mx^2 - x + n$ and $x^3 + nx^2 - 5x + 3m$ have a common factor of $x + 2$. Find m and n .
4. Find the value of q if $x + 1$ is a factor of $3x^4 - 15x^3 - qx^2 + qx + 10$.
5. Use the Factor Theorem to show that $x - 1$ is a factor of $(x - 1)^3 + (x - 1)^2 + (1 - x)$.



What to Enhance?

"FEAR OF NUMBERS"

Most students fear numbers. An individual might fear all kinds of numbers, especially complex mathematical computations. Just a thought of solving a difficult arithmetic equation in school or doing day – to – day life could cause intense panic in the person. Having been the subject of ridicule, being spanked, scolded, bullied for not having done well in Math exam could also trigger to fear numbers. Failing or doing poorly in Math at school could lead to permanent fear of numbers. What do you call the fear of numbers?

To answer the question, find the factors of each polynomial function and place the letters in the respective boxes below.

- B** $f(x) = x^3 + 4x^2 + x - 6$
I $f(x) = x^3 - x^2 - 10x - 8$
A $f(x) = 2x^4 - x^3 - 14x^2 + 19x - 6$
O $f(x) = x^3 - x^2 - 8x + 12$
P $f(x) = x^3 + 6x^2 + 5x - 12$
M $f(x) = x^4 - x^3 - 7x^2 + x + 6$
H $f(x) = x^4 + 7x^3 + 9x^2 - 7x - 10$
T $f(x) = x^4 - 9x^2 + 4x + 12$
R $f(x) = 4x^3 + 16x^2 - x - 4$

$(x+3)$	$(2x+1)$	$(x+2)$	$(x+3)$	$(x+5)$	$(x+1)$	$(x+3)$	$(x+4)$	$(x+5)$	$(x+3)$	$(x+3)$	$(x+2)$	$(x+3)$
$(x-2)$	$(2x-1)$	$(x+1)$	$(x+1)$	$(x+2)$	$(x-1)$	$(x-2)$	$(x+3)$	$(x+2)$	$(x-2)$	$(x+2)$	$(x+1)$	$(x-2)$
$(x-1)$	$(x+4)$	$(x-4)$	$(x-2)$	$(x+1)$	$(x+2)$	$(x-2)$	$(x-1)$	$(x+1)$	$(x-2)$	$(x-1)$	$(x-4)$	$(x-1)$
$(2x-1)$			$(x-2)$	$(x-1)$	$(x-3)$			$(x-1)$				$(2x-1)$



Challenge Yourself!

AMAZING NUMBER

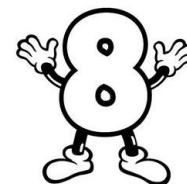
The number 12,345,679 is an amazing number. Notice that 8 is missing in this number. When this number is multiplied by 9, the product is all 1. When multiplied by 18, the product is all 2.

$$12,345,679 \times 9 = 111,111,111$$

$$12,345,679 \times 18 = 222,222,222$$

Find other factors to be multiplied to 12,345,679 so that the product will be all 3, all 4, all 5, and up to the product of all 9.

Finding the Integral Roots/Zeros of Polynomial Equations/Functions



Lesson



What to Learn?

- Use the Factor Theorem to find the integral roots/zeros of a polynomial equation/function of degree greater than 2.



What to Know?

Zeros of the functions are the values of x that make the polynomial equal to zero.

With the Factor Theorem, you can get the zeros of the polynomial functions by the help of Synthetic Division if and only if the remainder is zero.

Study the table below:

Polynomial	Degree	Factors	Zeros
1. $P(x) = x - 3$	1	$(x - 3)$	3
2. $P(x) = x^2 - x - 12$	2	$(x - 4)(x + 3)$	4, -3
3. $P(x) = x^3 - 3x^2 - x + 3$	3	$(x - 1)(x - 1)(x - 3)$	-1, 1, 3

What conclusions can you get from the above table? What is the relationship of the degrees of the polynomial functions to their zeros?

The fundamental theorem of Algebra states that "Every polynomial function $P(x) = 0$ of degree n has exactly n zeros."

Example:

Find the zeros of $P(x) = x^4 + x^3 - 7x^2 - x + 6$

Solution:

Find the factors of the constant term (6)

The factors are $\pm 6, \pm 1, \pm 3, \pm 2$

Select a factor of 6 and use it as constant divisor and if the remainder becomes zero, then the selected factor is a zero of the given polynomial function.

$$\begin{array}{r|rrrrr}
 1 & 1 & 1 & -7 & -1 & 6 \\
 & & 1 & 2 & -5 & -6 \\
 \hline
 & 1 & 2 & -5 & -6 & 0
 \end{array}$$

Since the remainder is 0, therefore 1 is a zero of $P(x)$

Using the depressed equation $(x^3 + 2x^2 - 5x - 6)$ as dividend, find the other zeros using the same process.

$$\begin{array}{r|rrrr}
 -1 & 1 & 2 & -5 & -6 \\
 & & -1 & -1 & 6 \\
 \hline
 & 1 & 1 & -6 & 0
 \end{array}$$

The remainder is 0, then -1 is another zero of $P(x)$

Repeat the process until all zeros are obtained.

$$\begin{array}{r|rrr}
 2 & 1 & 1 & -6 \\
 & & 2 & 6 \\
 \hline
 & 1 & 3 & 0
 \end{array}$$

The remainder is 0, then 2 is the other zero of $P(x)$

$$\begin{array}{r|rr} -3 & 1 & 3 \\ & & -3 \\ \hline & 1 & 0 \end{array}$$

Therefore, the four zeros of $P(x) = x^4 + x^3 - 7x^2 - x + 6$ are 1, -1, 2, -3



What to Do?

Complete the table.

Polynomial Functions	Factors	Zeros
1. $f(x) = x^3 - 3x^2 + 3x - 1$		
2. $f(x) = x^3 - 7x + 6$		
3. $f(x) = x^4 - 13x^2 + 36$		
4. $f(x) = x^3 - 5x^2 - 2x + 24$		
5. $f(x) = x^4 + 4x^3 - x^2 - 16x - 12$		
6. $f(x) = x^3 - 9x^2 + 26x - 24$		
7. $f(x) = x^3 + x^2 - 17x + 15$		
8. $f(x) = x^4 - 2x^3 - 7x^2 + 8x + 12$		
*9. $f(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$		
*10. $f(x) = x^6 - 5x^5 - 16x^4 + 10x^3 + 29x^2 - 5x - 14$		



What to Enhance?

Did You Know?

The first use of the symbol which we recognize today as the notation for zero “0” is omicron, the first letter of the Greek word for nothing namely “ouden”. The name zero derives ultimately from the Arabic sifr which means nothing or empty. Zero was originally called as “_____”.

Find the zeros of the given polynomial functions. Write the letters corresponding to the zeros of the functions in the boxes below to decode the answer.

R $f(x) = x^3 + 5x^2 + 2x - 8$

P $f(x) = x^4 + x^3 - 7x^2 - x + 6$

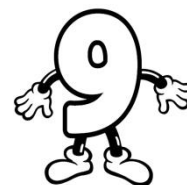
I $f(x) = x^3 + 7x^2 + 2x - 40$

H $f(x) = x^3 + x^2 - 25x - 25$

C $f(x) = x^4 - 7x^3 + 11x^2 + 7x - 12$

E $f(x) = x^4 - 2x^3 - 7x^2 + 8x + 12$

-1, 1, 3, 4	-5, -4, 2	-3, -1, 1, 2	-5, -1, 5	-2, -1, 2, 3	-4, -2, 1



Lesson

The Rational Root Theorem



What to Learn?

- Find the rational roots/zeros of the given polynomial equation/function if there are any using the Rational Root Theorem.



What to Know?

The Rational Root Theorem

Let $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0 = 0$, where $a_n \neq 0$ and a_i is an integer for all $i, 0 \leq i \leq n$, be a polynomial equation of degree n . If $\frac{p}{q}$, in lowest terms, is a rational root of the equation, then p is a factor of a_0 and q is a factor of a_n .



Study the table below:

Polynomial Equation	Factors of the Leading Coefficients (q)	Factors of the Constant Term (p)	Possible Rational Roots ($\frac{p}{q}$)	Rational Roots
1. $x^3 - 5x^2 + 8x - 4 = 0$	± 1	$\pm 1, \pm 2, \pm 4$	$\pm 1, \pm 2, \pm 4$	1, 2, -2
2. $3x^3 + 8x^2 - 15x + 4 = 0$	$\pm 1, \pm 3$	$\pm 1, \pm 2, \pm 4$	$\pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$	-4, $1\frac{1}{3}$
3. $12x^4 + 8x^3 - 7x^2 - 2x + 1 = 0$	$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$	± 1	$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{4}, \pm \frac{1}{6}, \pm \frac{1}{12}$	-1, $-\frac{1}{2}, \frac{1}{2}, \frac{1}{3}$

You may observe that the rational roots of a polynomial equation are related to the factors of the constant term and the factors of the leading coefficient.

The process of finding the rational roots/zeros of a polynomial equation/function is similar in finding the integral zeros of the function in Lesson 8. The only difference lies in the possible roots to be tested. The Rational Root Theorem is used when the leading coefficient of the polynomial a_n is neither zero nor one.

Examples:

- Find the rational zeros of $f(x) = 2x^4 - 11x^3 + 11x^2 + 15x - 9$

Solution:

By the Rational Root Theorem, if $\frac{p}{q}$ is a root of $f(x)$, then p must be a factor of -9 (constant term) and q must be a factor of 2 (the leading coefficient).

$p = -9$ The factors of -9 are $\pm 1, \pm 3, \pm 9$

$q = 2$ The factors of 2 are $\pm 1, \pm 2$

$\frac{p}{q} = \left\{ \pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2} \right\}$

The degree is 4, so there must be 4 zeros.

$$\begin{array}{r|rrrrrr} 3 & 2 & -11 & 11 & 15 & -9 & \\ & & 6 & -15 & -12 & 9 & \end{array} \quad x - 3 \text{ is a factor of } f(x), x = 3 \text{ is a zero of } f(x)$$

$$\begin{array}{r|rrrrr} 3 & 2 & -5 & -4 & 3 & 0 \\ & & 6 & 3 & -3 & \end{array} \quad x - 3 \text{ is a factor of } f(x), x = 3 \text{ is a zero of } f(x)$$

$$\begin{array}{r|rrrr} -1 & 2 & 1 & -1 & 0 \\ & & -2 & 1 & \end{array} \quad x + 1 \text{ is a factor of } f(x), x = -1 \text{ is a zero of } f(x)$$

$$\begin{array}{r|rrr} \frac{1}{2} & 2 & -1 & 0 \\ & & 1 & \\ \hline & 2 & 0 & \end{array} \quad 2x - 1 \text{ is a factor of } f(x), x = \frac{1}{2} \text{ is a zero of } f(x)$$

Thus, the four zeros of $f(x)$ are $3, 3, -1, \frac{1}{2}$.

2. Find the polynomial function $P(x)$ whose zeros are $\frac{1}{2}, 1, -\frac{1}{2}$.

Solution:

Write the given zeros in factored form, then solve.

$$\begin{array}{lll} x = \frac{1}{2} & x = 1 & x = -\frac{1}{2} \\ 2x - 1 = 0 & x - 1 = 0 & 2x + 1 = 0 \end{array}$$

$$\begin{aligned} \text{Thus, } P(x) &= (2x - 1)(x - 1)(2x + 1) \\ P(x) &= (4x^2 - 1)(x - 1) \\ P(x) &= 4x^3 - 4x^2 - x + 1 \end{aligned}$$



What to Do?

Use the Rational Root Theorem to find all the rational roots of the following polynomial equations.

1. $2x^3 - 3x^2 - 5x + 6 = 0$ _____
2. $4x^4 - 17x^2 + 4 = 0$ _____
3. $3x^3 - 4x^2 - 17x + 6 = 0$ _____
4. $x^3 - 8x^2 + 19x - 12 = 0$ _____
5. $3x^3 - 2x^2 - 7x - 2 = 0$ _____
6. $2x^4 + 9x^3 + 15x^2 + 11x + 3 = 0$ _____
7. $5x^4 - 14x^3 - 8x^2 + 14x + 3 = 0$ _____
8. $8x^3 - 6x^2 - 11x + 3 = 0$ _____

9. $6x^3 + 11x^2 - 3x - 2 = 0$ _____

10. $9x^4 - 6x^3 - 23x^2 - 4x + 4 = 0$ _____



What to Enhance?

Complete the table.

$P(x)$	Possible Rational Roots	Factored Form	Zeros
$2x^4 - 5x^3 - 5x^2 + 5x + 3$	1.	2.	3.
4.	5.	$(x + 2)(x - 1)(3x + 1)$	6.
7.	8.	9.	$-1, 3, -\frac{1}{2}, \frac{2}{3}$
$6x^4 - 7x^3 - 9x^2 + 7x + 3$	10.	11.	12.
13.	14.	$(x - 1)(x + 4)(2x + 1)(3x - 1)(x - 2)$	15.

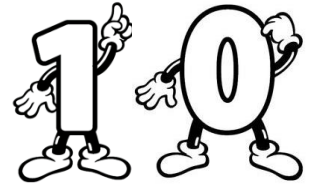


What to Transfer?

A Gift Box

You are making a project in your T.L.E. class on how to make a rectangular gift box. You are given the expression $(3x^3 + 8x^2 - 45x - 50)$ cubic units for the volume of the box. Suppose you want the height of your box to be $(x + 1)$ units, how will you find an expression for the length and the width of your desired box? Show a mathematical procedure in determining the dimensions of your box. Then use the computed values of the length, width, and height in making a gift box. Your final output will be graded based on creativity and accuracy of your work.

Sketching the Graph of Polynomial Functions



Lesson



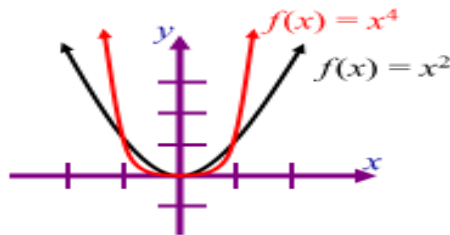
What to Learn?

- To sketch the graph of polynomial function of degree greater than 2.

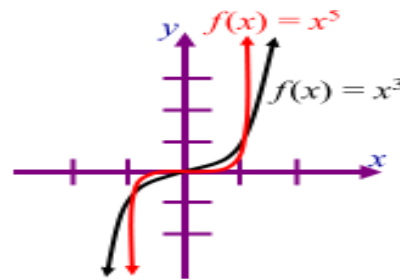


What to Know?

- Graphs of polynomial functions are continuous. That is, they have no breaks, holes, or gaps. Polynomial functions are also smooth with rounded turns. Graphs with points or cusps are not graphs of polynomial functions.
- Polynomial functions of the form $f(x) = x^n, n \geq 1$ are called Power Functions.



If n is even, their graphs resemble the graph of $f(x) = x^2$.



If n is odd, their graphs resemble the graph of $f(x) = x^3$.

- Moreover, the greater the value of n , the flatter the graph near the origin.
- Another important characteristic of a polynomial function is the end behavior. The end behavior is the behavior of the graph of the function to the far left or far right. The end behavior of a polynomial function depends on the degree of the function and the sign of its leading coefficient. The leading coefficient is the coefficient of the highest degree of the variable. The graphs below illustrate the four types of end behavior for non - constant polynomial functions.

The Leading Coefficient Test

As x increases or decreases without bound, the graph of the polynomial function $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$ ($a_n \neq 0$) eventually rises or falls. In particular,

- For n is odd: $a_n > 0$

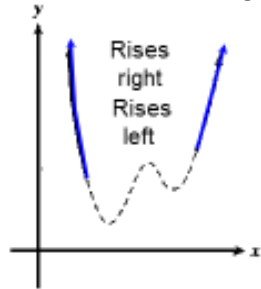
If the leading coefficient is positive, the graph falls to the left and rises to the right.

- For n is odd: $a_n < 0$

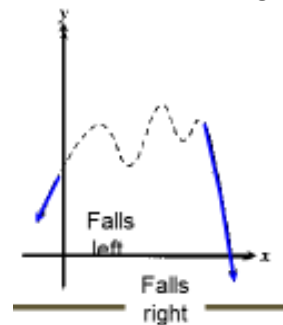
If the leading coefficient is negative, the graph rises to the left and falls to the right.



3. For n is even: $a_n > 0$
 If the leading coefficient is positive, the graph rises to the left and to the right.



4. For n is even: $a_n < 0$
 If the leading coefficient is negative, the graph falls to the left and to the right.

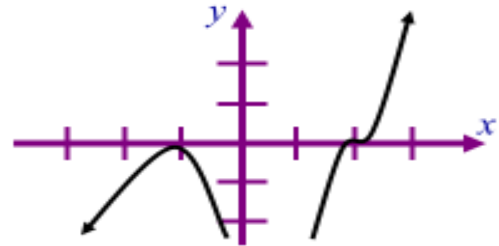


- For repeated zeros, if k is the largest integer for which $(x - a)^k$ is a factor of $f(x)$ and $k > 1$. Then a is the repeated zero of multiplicity k .
 1. If k is *odd* the graph of $f(x)$ crosses the x -axis at $(a, 0)$.
 2. If k is *even* the graph of $f(x)$ touches, but does not cross through, the x -axis at $(a, 0)$.

Examples:

1. Determine the multiplicity of the zeros of $f(x) = (x - 2)^3(x + 1)^4$, then sketch the graph.

Zero	Multiplicity	Behavior
2	3 (odd)	Crosses x -axis at $(2, 0)$
-1	4 (even)	Touches x -axis at $(-1, 0)$



2. Find all the real zeros and turning points of the graph of $f(x) = x^4 - x^3 - 2x^2$.

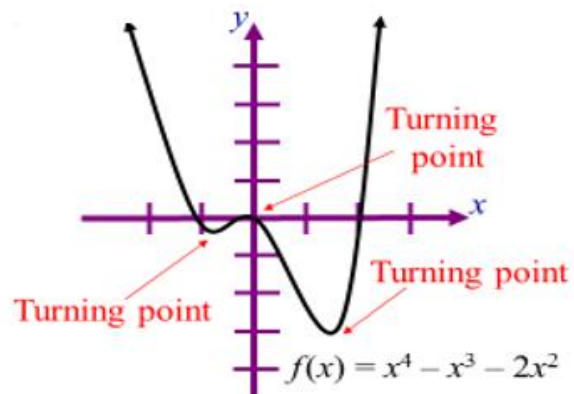
Factor completely:

$$f(x) = x^4 - x^3 - 2x^2 = x^2(x + 1)(x - 2)$$

The real zeros are $x = 0$ of multiplicity 2, $x = -1$, $x = 2$.

These corresponds to the x -intercepts $(0, 0)$, $(-1, 0)$, $(2, 0)$.

The graph shows that there are three turning points. Since the degree is four, this is the maximum number of turning point possible.



- Here are the steps in sketching the graph of polynomial functions:
 1. Use the Leading Coefficient Test to determine the graphs' end behavior.
 2. Find the x -intercepts by setting $P(x) = 0$ and solve the resulting polynomial equation. If there is an x -intercept at a as a result of $(x - a)^k$ in the complete factored form of $P(x)$, then
 - If k is even, the graph touches the x -axis at $(a, 0)$ and turns around.
 - If k is odd, the graph crosses the x -axis at $(a, 0)$.
 3. Find the y -intercept by equating x to 0 and solve $P(0)$.
 4. Use the fact that the maximum number of turning points of the graph is $n - 1$ to check whether the graph is drawn correctly.



What to Do?

Sketch the graph of the polynomial function given the following conditions:

1. the factors of $P(x)$ are $(x + 1)^2$, $(x - 1)$, $(x - 5)$
2. $f(x) = x^4 - x^3 - 10x^2 - 8x$
3. the zeros of a polynomial function are $-2, 1, -3, 2$
4. $f(x) = (3x - 2)(2x + 1)(x - 3)$
5. $P(x) = 2x^5 - x^4 - 10x^3 + 5x^2 + 8x - 4$



What to Enhance?

Determine the following for each given polynomial function, then sketch the graph.

- a. Leading term
 - b. End behavior
 - c. x -intercepts
 - d. Multiplicity of roots
 - e. y -intercept
 - f. Number of turning points
1. $f(x) = (x - 5)(x + 3)(x + 2)$
 2. $f(x) = (x + 2)^3(x - 3)^2$
 3. $f(x) = (x^2 - 4)(x - 3)(-x + 2)$

4. $f(x) = (x - 2)(x - 4)(2x + 1)$

5. $f(x) = x^4 - 13x^2 + 36$



What to Transfer?

Graphs in Real – world!

URGENT!

MLDR Company will open the latest amusement park in Caloocan City. In this regard, we are needing Grade 10 students who have skills in architectural designing. You may submit your blueprint or design through email. Use polynomials to describe the curves of the different amusement rides. The designer who shows creativity, uniqueness, and accuracy would be accepted.

Submit your design now!

References

Cliparts. <https://www.google.com.ph>

Daet, Marciana et. al (2006). *Scoring High in Math: Workbook in Algebra*, Gabay Eskwela Publishing House

Lerida – M Eclevia Razon (2005). *Math Builders*, JO – ES Publishing House, INC.

Malini Lingarajaprabhu (2010). *malini – math.blogspot.com*

Mrs Strong Math, Math Trivia. <https://sites.google.com>

Oronce & Mendoza (2007). *e – math IV (Advanced Algebra and Trigonometry)*. Rex Book Store

Pascual, Ferdinand et. al (2001). *Worktext in Mathematics IV: Simplified Concepts & Structures*, Innovative Educational Materials, INC.