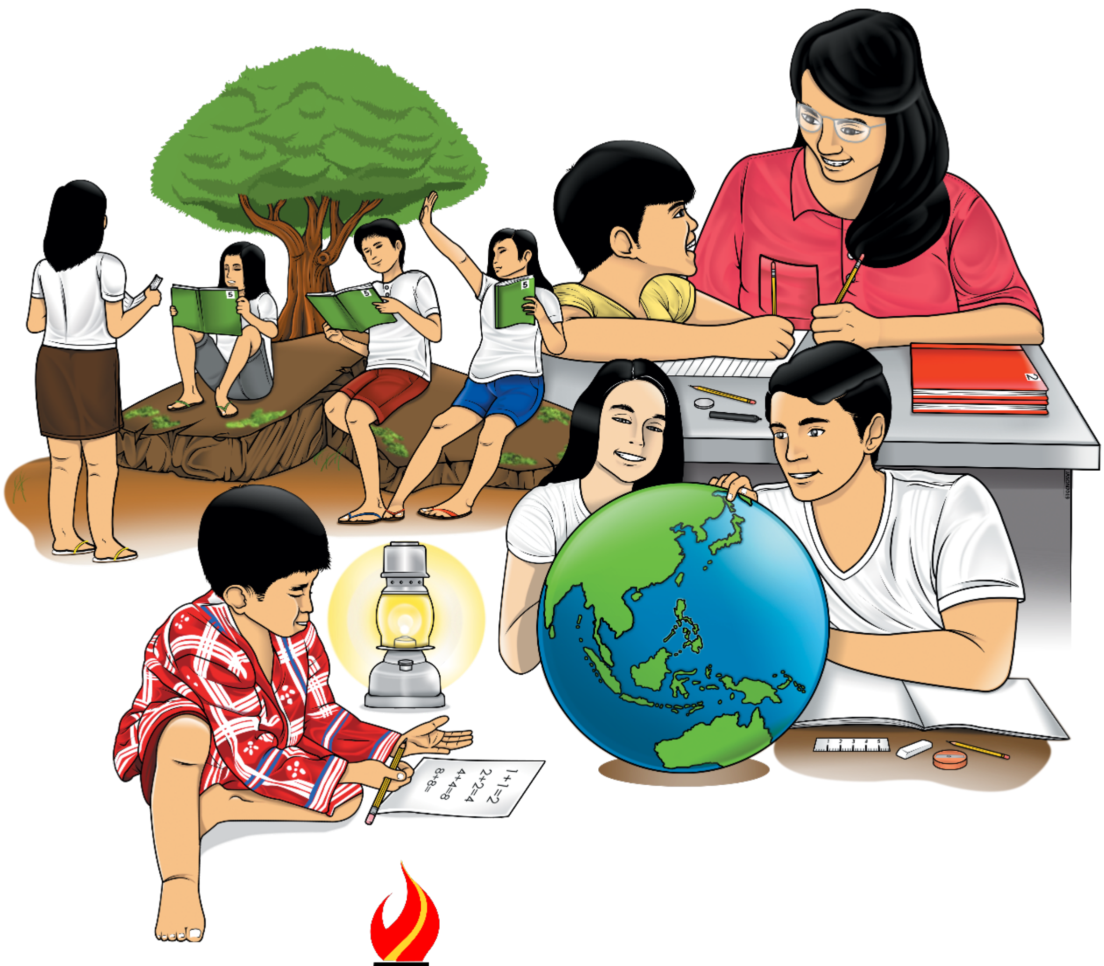
9

Mathematics
Quarter 2 – Module 9
Solving Equations Involving Radical Expressions



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Mathematics
Quarter 2 – Module 9:
Solving Equation Involving Radical Expressions

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Lesson 1 Radical Equations

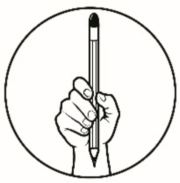
In the previous lessons, you have learned how to perform operations on radical expressions. This time, you will learn how to apply these concepts in solving radical equations.

What I Need to Know

LEARNING COMPETENCY:

The learners will be able to:

Solve equations involving radical expressions. **M9AL – IIi – j – 27**



What I Know

Directions: Choose the letter of the correct answer.

1. Solve for x in the equation $\sqrt{x-3} = 1$.
   a. $x = 4$ c. $x = 2$
   b. x =-4 d. There are no real solutions

2. Solve the radical equation: $\sqrt{3x} = 12$
   a. 12 c. 24
   b. 48 d. 36

3. Which of the following are value(s) of x that will satisfy the equation $\sqrt{5x+16} - x = 2$?
   a. $3 \ and \ 14$ c. $-1 \ and \ 5$
   b. 4 d. -3

4. Solve the equation: $\sqrt{3x-2} = 6$.
   a. 4 c. $\frac{38}{3}$
   b. $\frac{14}{3}$ d. 9

For numbers 5-8, refers to the choices below:

a. $x = 1$ b. $x = 33$ c.$x = 2$ d. $x = 20$

What is the solution of the equation?

5. $\sqrt{3x+1} - 3 = 7$

6.$\sqrt{19-3x} - 1 = 3x$

7.$\sqrt{2x-15} = \sqrt{x+5}$

8. $\sqrt{x+2} + \sqrt{x-1} = 3$

9. If the given value of a solution is $n = 10$, what is the equation?
 a. $\sqrt{110-n} = n$ c. $x = \sqrt{8x}$
 b. $\sqrt{30-x} = x$ d. $\sqrt{4n} = n$

10. Which of the following value(s) of x satisfy the equation $\sqrt{6x+4} = 4$?
 a. $x = 2$ c. $x = 3$
 b. $x = 4$ d. *no solution*

11. Solve the equation: $\sqrt{x-1} = 3$
 a. $x = -10$ c. $x = -3$
 b. $x = 3$ d. $x = 1$

For numbers 12-15, refers to the choices below:
 a. $x = 9$ c. *no solution*
 b. $x = 21$ d. $x = 5\ ,\ x = 7$

Find the solution of the equation.

12. $-6 + \sqrt{x-5} = -2$

13. $\sqrt{x+72} = x$

14. $8\sqrt{x+6} - 7 = -79$

15. $x = 5 + \sqrt{2x-10}$

What's In

The following operations on radical expressions will be needed in solving equations involving radical equations:

1. Addition and Subtraction: Write all radicals in their simplest form then combine like radicals. (Like radicals have the same indices and radicand).
2. Multiplication: Apply the product rule for radicals ($\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$), then simplify. Apply distributive property when multiplying radicals with two or more terms.
3. Division: Write the quotient as fraction form and rationalize the denominator. We may also apply the quotient rule for radicals ($\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$), then simplify.

Let us review these operations by answering the exercises below.

Perform the indicated operations:

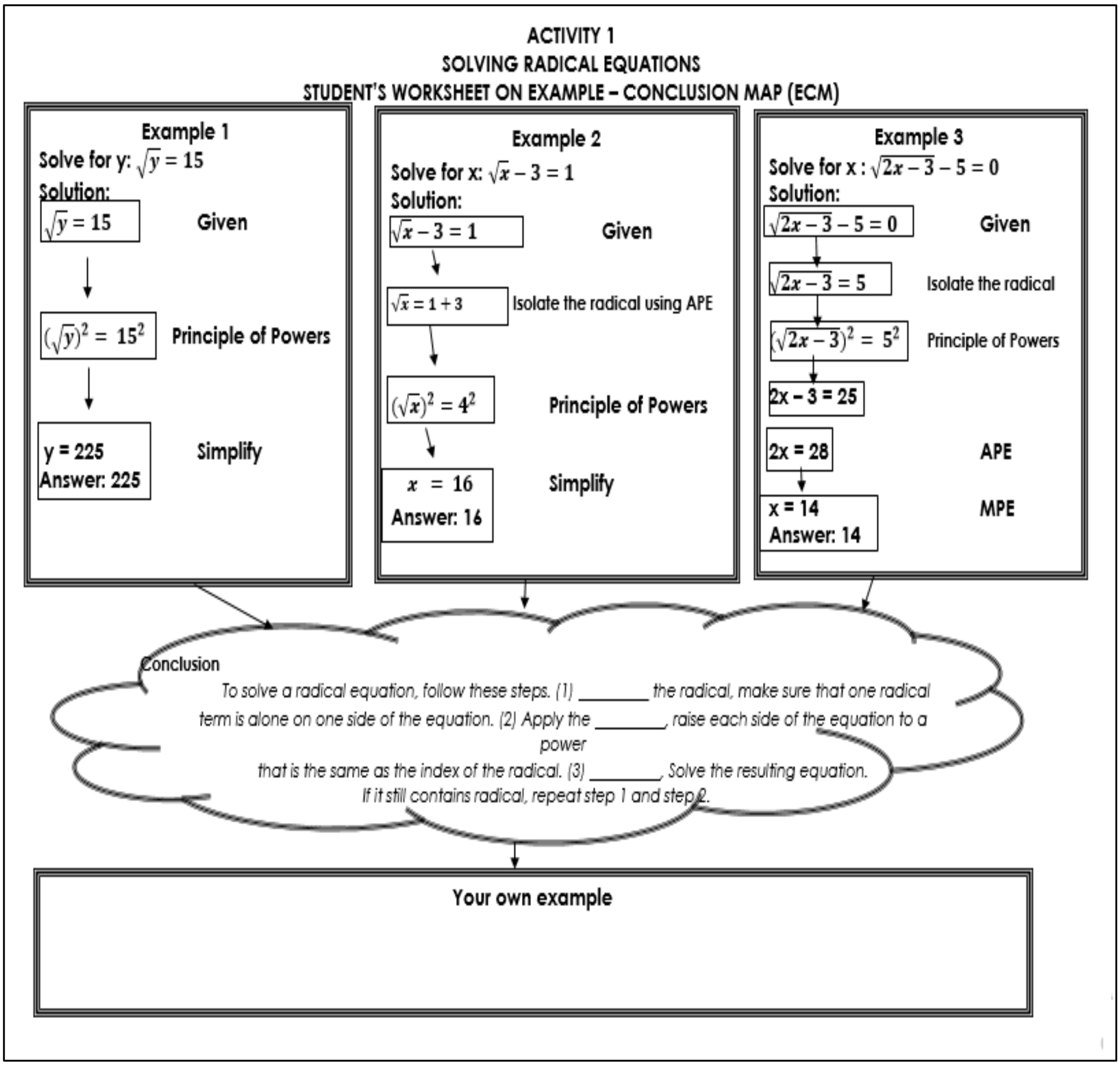
1. $8\sqrt{5} + \sqrt{5}$	*6.* $\sqrt{50} \cdot \sqrt{2m^5}$
2. $5\sqrt{3} - 3\sqrt{3}$	*7.* $\sqrt[4]{14} \div \sqrt[4]{2}$
3. $\sqrt{8} + \sqrt{20} - \sqrt{12}$	*8.* $\sqrt{21x^7} \div \sqrt{3x^2}$
4. $3\sqrt{20} - 2\sqrt{45}$	*9.* $\sqrt[4]{a}(\sqrt[4]{a^3} - 7)$
5. $\sqrt[3]{2a} \cdot \sqrt{2a}$	*10.* $7 \div (\sqrt{6} + \sqrt{5})$

Solve the following:

11. *Maggie walks around a rectangular field. The field has a length of* $\sqrt{27}$ *meters and is* $2\sqrt{3}$*-meter wide. What is the total distance Maggie walks through the field?*
12. *The base of an isosceles triangle measures* $10\sqrt{2}$ *units while its height measures* $12\sqrt{2}$ *units. Express the area of the triangle as an exact value in simplest form.*

What's New

Mathematical problems that involve radical equations arise in a number of professions. We have rational equations that are used to calculate depreciation, inflation and interest rates. Engineers used radical equations for their measurements and calculations. Scientists use radical exponents for size comparison in research. Let us take a look at how these equations are solve



Solving Radical Equations

A radical equation is an equation that has variables in one or more radicands. Examples are $\sqrt{x} = 8$ and $2 = \sqrt[3]{m}$.To solve these kinds of radical equations, we need to apply the **Principle of Powers.** That is, $\boldsymbol{if a = b,\ then\ a^n = b^n}$.

Usually, radical equations are not as simple as the examples given above. Most of the time, the equation contains several terms on each side. To solve equations of this kind, the following steps must be followed.

1. Isolate the radical.
2. Apply the principle of powers.
3. Solve the equation. If there is still a radical, repeat 1 and 2.
4. Check for extraneous solution.

Examples

1. Solve $\sqrt{x} = 8$.

   Solution: $\sqrt{x} = 8$

   $(\sqrt{x})^2 = 8^2$ Principle of Powers

   $x = 64$

   Check: $\sqrt{x} = 8$

   $\sqrt{64} = 8$ Substitution

   $8 = 8$ Therefore, the solution/value of the variable is acceptable

2. Solve $2 = \sqrt[3]{m}$.

   Solution: $2 = \sqrt[3]{m}$

   $2^3 = (\sqrt[3]{m})^3$ Principle of Powers

   $8 = m$

   Check: $2 = \sqrt[3]{m}$

   $2 = \sqrt[3]{8}$ Substitution

   $2 = 2$ Therefore, the solution/value of the variable is acceptable

3. Solve $\sqrt{x+2} = 3$

   Solution: $\sqrt{x+2} = 3$

   $(\sqrt{x+2})^2 = 3^2$ Principle of Powers

   $x + 2 = 9$

   $x = 9 - 2$ Addition Property of Equality

   $x = 7$ Simplify

   Check: $\sqrt{x+2} = 3$

   $\sqrt{7+2} = 3$ Substitution

   $\sqrt{9} = 3$

   $3 = 3$ Therefore, the solution/value of the variable is acceptable

4. Solve $\sqrt{3x+2} = \sqrt{x+10}$

   Solution: $\sqrt{3x+2} = \sqrt{x+10}$

   $(\sqrt{3x+2})^2 = (\sqrt{x+10})^2$ Principle of Powers

   $3x + 2 = x + 10$

   $3x - x = 10 - 2$ APE

   $2x = 8$

   $\frac{2x}{2} = \frac{8}{2}$ MPE

   $x = 4$ Simplify

   Check: $\sqrt{3x+2} = \sqrt{x+10}$

   $\sqrt{3 \cdot 4 + 2} = \sqrt{4 + 10}$ Substitution

   $\sqrt{14} = \sqrt{14}$ Therefore, the solution/value of the variable is acceptable

5. Solve $\sqrt[3]{x+3}=2$.

Solution: $\sqrt[3]{x+3}=2$

$(\sqrt[3]{x+3})^3=2^3$ Principle of Powers

$x+3=8$

$x=8-3$ APE

$x=5$ Simplify

Check: $\sqrt[3]{x+3}=2$

$\sqrt[3]{5+3}=2$ Substitution

$\sqrt[3]{8}=2$

$2=2$ Therefore, the solution/value of the variable is acceptable

6. Solve $\sqrt{5a+1}+2=6$

Solution: $\sqrt{5a+1}+2=6$

$\sqrt{5a+1}=6-2$ APE

$(\sqrt{5a+1})^2=4^2$ Principle of Powers

$5a+1=16$

$5a=16-1$ APE

$5a=15$

$\frac{5a}{5}=\frac{15}{5}$ MPE

$a=3$ Simplify

Check: $\sqrt{5a+1}+2=6$

$\sqrt{5\cdot 3+1}+2=6$ Substitution

$\sqrt{16}+2=6$

$4+2=6$

$6=6$ Therefore, the solution/value of the variable is acceptable

7. Solve $\sqrt{7x+2}-2x=0$

Solution: $\sqrt{7x+2}-2x=0$

$\sqrt{7x+2}=2x$ APE

$(\sqrt{7x+2})^2=(2x)^2$ Principle of Powers

$7x+2=4x^2$

$0=4x^2-7x-2$ APE

$4x^2-7x-2=0$ Symmetric Property of Equality

$(4x+1)(x-2)=0$ Factoring

$4x+1=0\,;x-2=0$ Equate both factors to 0

$4x=-1 : x=2$ APE

$x=-\frac{1}{4}\,;x=2$ MPE

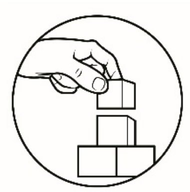
Check: By substitution

$x=-\frac{1}{4}$	$x=2$
$\sqrt{7x+2}-2x=0$	$\sqrt{7x+2}-2x=0$
$\sqrt{7(\frac{-1}{4})+2-2(\frac{-1}{4})}=0$	$\sqrt{7\cdot 2+2}-2\cdot 2=0$
$\sqrt{\frac{-7}{4}+2+\frac{2}{4}}=0$	$\sqrt{14+2}-4=0$
$\sqrt{\frac{-7+8}{4}+\frac{1}{2}}=0$	$\sqrt{16}-4=0$
$\sqrt{\frac{1}{4}+\frac{1}{2}}=0$	$4-4=0$

$\frac{1}{2}+\frac{1}{2}=0$ and $\qquad\qquad 0=0$

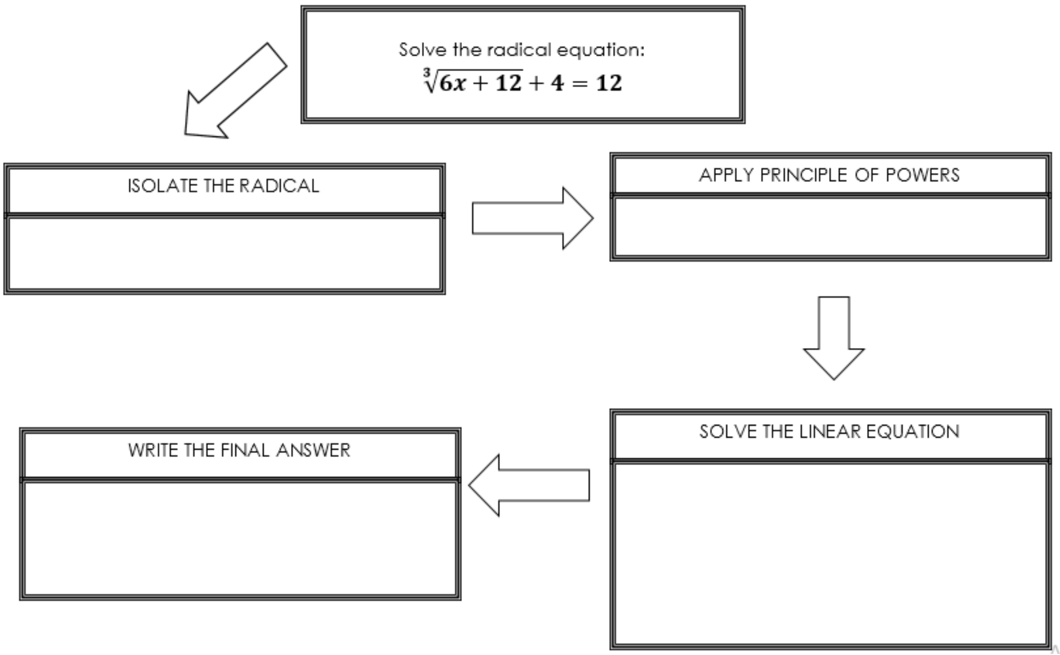
$1 \neq 0$

One value ($-\frac{1}{4}$)of x did not satisfy the equation. Therefore, it is ***not*** a solution. The solution of the given radical equation is just 2.



What's More

ACTIVITY 2
SOLVING RADICAL EQUATIONS
STUDENT'S WORKSHEET ON MATH-BREAKER MAP (MBM STRUCTURED)



A. Read each item carefully and determine whether the statement is true or false.
   1. $\sqrt{5}+x=x^2$ is an example of a radical equation.
   2. Radical equations may not always have real solutions.
   3. The value x = -4 is a solution to $\sqrt{5-x}=x+7$.
   4. The $\sqrt{x+1}=2$ has two solutions.
   5. The $\sqrt[3]{3x+1}=-2$ has a positive solution.

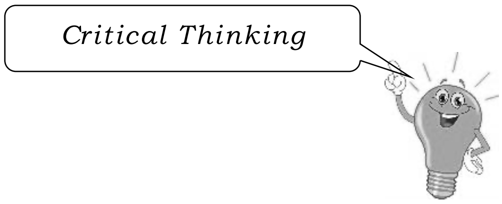
B. Find the value(s) of x in the following radical equations:
   1. $\sqrt{x}-3=5$
   2. $\sqrt{x+8}=3$
   3. $\sqrt[3]{2x+3}+5=0$
   4. $\sqrt{3x-1}-\sqrt{x+5}=0$
   5. $x=\sqrt{x-4}+4$
   6. $x+4=\sqrt{x+10}$

What I Have Learned

The solution of a radical equation is a value that will satisfy the given equation. To solve a radical equation, we follow these steps:

1. Isolate the radical on one side of the equation.
2. Apply the Principle of Powers.
3. If radical remains, repeat steps 1 and 2.
4. Check if the obtained value or values satisfy the given equation.

A. Analyze each item and identify on which part, the solution started to get wrong. Write down the letter only.



1. $\sqrt{20-n} = n$

   $\left(\sqrt{20-n}\right)^2 = n^2$ **(A)**

   $20 - n = n^2$ **(B)**

   $0 = n^2 - n - 20$ **(C)**

   $n^2 - n - 20 = 0$

   $(n-5)(n+4) = 0$

   $n = 5 \text{ or } n = -4$

2. $\sqrt{x} = -3$

   $\left(\sqrt{x}\right)^2 = (-3)^2$ **(A)**

   $x = 9$ **(B)**

   The solution to the radical equation is 9. **(C)**

3. $\sqrt{x+3} - 1 = 7$

   $\sqrt{x+3} = 7 - 1$ **(A)**

   $\sqrt{x+3} = 6$

   $\left(\sqrt{x+3}\right)^2 = 6^2$ **(B)**

   $x + 3 = 36$

   $x = 33$ **(C)**

4. $\sqrt{x} = \sqrt{x+2}$

   $\left(\sqrt{x}\right)^2 = \left(\sqrt{x+2}\right)^2$ **(A)**

   $x^2 = x + 2$ **(B)**

   $x^2 - x - 2 = 0$ **(C)**

   $(x-2)(x+1) = 0$

   $x = 2 \text{ or } x = -1$

5. $\sqrt[3]{2x} = -4$

$\left(\sqrt[3]{2x}\right)^3 = (-4)^3$ **(A)**

$\sqrt{2x} = -4$ **(B)**

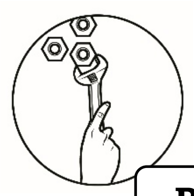
$(\sqrt{2x})^2 = (-4)^2$

$2x = 16$

$x = 4$ **(C)**

B. Solve for the value(s) of x.

1. $\sqrt{2x} = 6$
2. $3\sqrt{4x} = 36$
3. $\sqrt{x-7} = 3$
4. $\sqrt{x} + 13 = 20$
5. $\sqrt{5x+11} - 1 = x$
6. $\sqrt[3]{x} - 1 = 2$
7. $\sqrt{25 - x^2} = 4$
8. $1 + \sqrt{2x+3} = 6$
9. $\sqrt[3]{x^2 + 28} = 4$
10. $\sqrt{2x-2} = x - 1$



What I Can Do

PROBLEM – BASED LEARNING WORKSHEET

HERON'S FORMULA

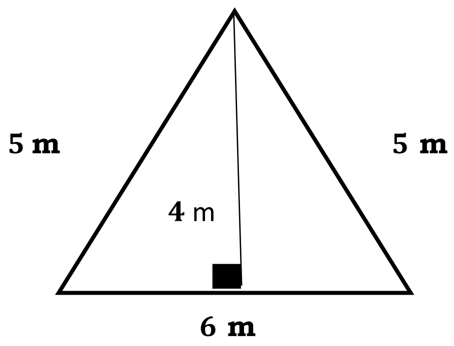
The area (A) of a triangle is given by the formula $A = \frac{1}{2}bh$, where b is the length of the base and h is the height. However, we can still compute for the area of a triangle using the lengths of its three sides. The Greek mathematician Heron proved this by developing a formula that uses the *semiperimeter* ($\frac{1}{2}$the perimeter) of the given triangle.

If a triangle has sides whose lengths are a, b and c and s is $\frac{1}{2}$ the sum of a, b and c, the area (A) of the triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

LET'S ANALYZE

1. Who developed the formula?
2. What do you mean by semiperimeter?
3. Give the two formulas used to find the area of a triangle.
4. Analyn has a triangular lot whose dimensions are shown below. What is the lot's perimeter? Semiperimeter?



5. Using the Heron's formula, compute for the area of Analyn's lot. Check your answer using the other formula.
6. An equilateral triangle has an area of $9\sqrt{3}$ units. How long is each side of the triangle?

Assessment

Read the questions carefully. Write the letter of the correct answer on your answer sheet.

1. A ________ is an equation with one unknown in one or more of the radicals.
   a. quadratic equation
   b. linear equation
   c. radical equation
   d. none of these

2. What value(s) of ***y*** satisfy the equation $\sqrt{5y+1} = \sqrt{3y+1}$?
   a. 0
   b. $\frac{9}{4}$
   c. $0, \frac{9}{4}$
   d. no solution

3. The value of x in the equation $\sqrt[3]{x+1} = 3$
   a. 23 b. 26 c. 27 d. 29

(4-5 Refer to the given problem below)
The square root of the two consecutive odd integers is equal to 6.

4. What is the equation in terms of y?
   a. $\sqrt{y} + \sqrt{y+2} = 6$
   b. $\sqrt{y + (y+2)} = 6$
   c. $\sqrt{y + \sqrt{y+2}} = 6$
   d. $\sqrt{y + (y+2)} = \sqrt{6}$

5. Find the larger integer.
   a. 17 b. 10 c. 6 d. 19

(6-8 Solve and check)

6. $2\sqrt{x-3} = 12$
   a. 36 b. 30 c. 39 d. 35

7. $\sqrt{5x+4} + 6 = 4$
   a. 0 b. 2 c. 4 d. no solution

8. $\sqrt{x-2} = \sqrt{3x-14}$
   a. 2 b. 4 c. 6 d. no solution

9. What is the solution of $\sqrt{6x-4} = 3$
   a. $\frac{4}{3}$ b. $-\frac{7}{6}$ c. $\frac{13}{6}$ d. $-\frac{13}{6}$

10. The square root of the sum of two consecutive integers is 7. Find the two integers.

    a. 3 and 4
    b. 11 and 12
    c. 17 and 18
    d. 24 and 25

Additional Activities

Critical Thinking& Collaboration

Who is this Mathematician?
This Welsh Mathematician introduced the equal sign in 1557.

To find out:

1. Find the solution to each radical equation.
2. Write the letter that corresponds to the solution in the decoder below.

B	**C**	**D**	**E**
$\sqrt{x} = 7$	$12 = \sqrt{a + 108}$	$\sqrt{2y + 9} = 1$	$\sqrt{5n + 9} + 4 = 7$

O	**R**	**T**
$\sqrt{10 - 3x} = \sqrt{2x + 20}$	$\sqrt[4]{x + 8} = \sqrt[4]{3x}$	$4 = \sqrt{x + 7}$

DECODER:

____ 4	____ -2	____ 49	____ 0	____ 4	____ 9	
____ 4	____ 0	____ 36	____ -2	____ 4	____ -4	____ 0

E-Search

To further explore the concept learned today and if it's possible to connect the internet, you may visit the following links:

https://www.youtube.com/watch?v=0gicD4STzpg
https://www.youtube.com/watch?v=TrJUOKLKlsU
https://www.youtube.com/watch?v=w79rpKCKlFw

Answer Key

WHAT I KNOW

1 A
2 B
3 B
4 C
5 B
6 A
7 D
8 C
9 A
10 A
11 D
12 B
13 A
14 C
15 D

WHAT'S IN

1. $9\sqrt{5}$
2. $2\sqrt{3}$
3. $2\sqrt{2} + 2\sqrt{5} - 2\sqrt{3}$
4. 0
5. $\sqrt[6]{2^5a^5}$
6. $10m^2\sqrt{m}$
7. $\sqrt[4]{7}$
8. $x^2\sqrt{7x}$
9. $a - 7\sqrt[4]{a}$
10. $7(\sqrt{6} - \sqrt{5})$
11. $10\sqrt{3}$ yards
12. 120sq. units

WHAT'S MORE

A.

1. False
2. True
3. True
4. False
5. False

B.

1. x = 64
2. x = 1
3. x = -64
4. x = 3
5. x = 5, 4
6. x = -1

WHAT I CAN DO

A.

1. C, it should be $0 = n^2 + n - 20$
2. C, there is no real solution.
3. A, it should be $\sqrt{x+3} = 7 + 1$
4. B, it should be $x = x + 2$
5. B, it should only be $2x = -64$

B.

1. x =18
2. x =36
3. x =16
4. x =49
5. x =5
6. x =27
7. x = ±3
8. x =11
9. x = ±6
10. x =1, 3

ASSESSMENT

1. C
2. A
3. B
4. B
5. D
6. C
7. D
8. C
9. C
10. D

ADDITIONAL ACTIVITIES

ROBERT RECORDE

PROBLEM – BASED LEARNING WORKSHEET

1. Heron, a Greek Mathematician
2. Half of the perimeter
3. $A = \frac{1}{2}bh$ and
   $A = \sqrt{s(s-a)(s-b)(s-c)}$
4. $P = 16$ m; $s = 8$ m
5. $A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{8(8-5)(8-5)(8-6)}$

$A = \sqrt{8(3)(3)(2)}$

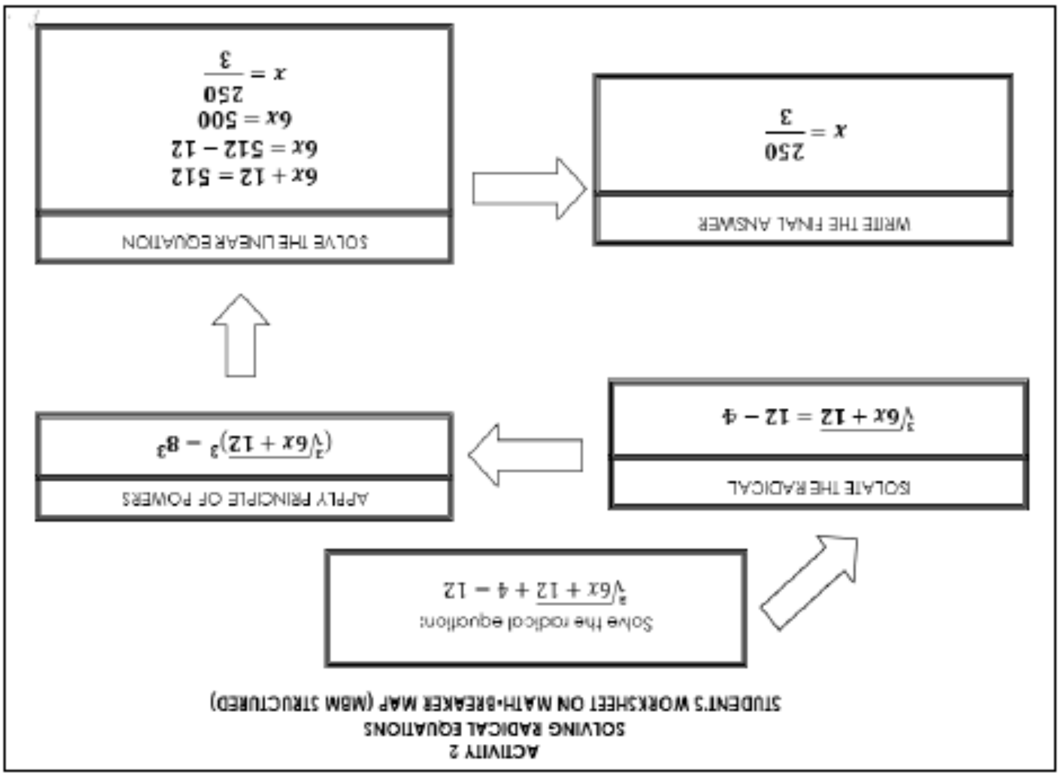
$A = 12$ sq. m

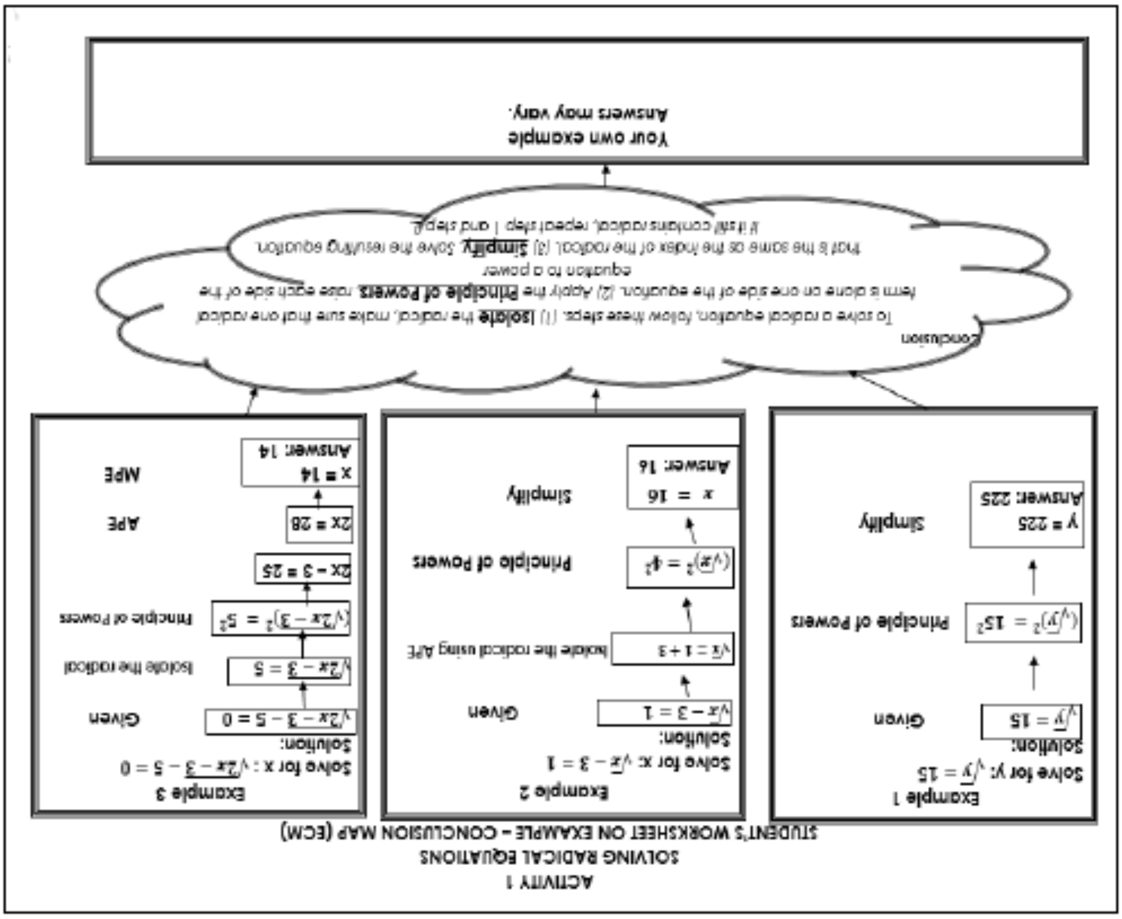
$A = \frac{1}{2}bh$

$A = \frac{1}{2}(6)(4)$

$A = 12$ sq. m

6. 6 units


ACTIVITY 2
SOLVING RADICAL EQUATIONS
STUDENT'S WORKSHEET ON MATH-BREAKER MAP (MBM STRUCTURED)
Solve the radical equation:
$\sqrt[3]{6x+12}+4=12$
ISOLATE THE RADICAL
$\sqrt[3]{6x+12}=12-4$
APPLY PRINCIPLE OF POWERS
$(\sqrt[3]{6x+12})^3=8^3$
SOLVE THE LINEAR EQUATION
$6x+12=512$
$6x=512-12$
$6x=500$
$x=\frac{250}{3}$
WRITE THE FINAL ANSWER
$x=\frac{250}{3}$


ACTIVITY 1
SOLVING RADICAL EQUATIONS
STUDENT'S WORKSHEET ON EXAMPLE – CONCLUSION MAP (ECM)
Example 1
Solve for y: $\sqrt{y}=15$
Solution:
$\sqrt{y}=15$
Given
$(\sqrt{y})^2=15^2$
Principle of Powers
y = 225
Answer: 225
Simplify
Example 2
Solve for x: $\sqrt{x}-3=1$
Solution:
$\sqrt{x}-3=1$
Given
$\sqrt{x}=1+3$
Isolate the radical using APE
$(\sqrt{x})^2=4^2$
Principle of Powers
$x=16$
Answer: 16
Simplify
Example 3
Solve for x : $\sqrt{2x-3}-5=0$
Solution:
$\sqrt{2x-3}-5=0$
Given
$\sqrt{2x-3}=5$
Isolate the radical
$(\sqrt{2x-3})^2=5^2$
Principle of Powers
2x – 3 = 25
2x = 28
APE
x = 14
Answer: 14
MPE
Conclusion
To solve a radical equation, follow these steps. (1) Isolate the radical, make sure that one radical term is alone on one side of the equation. (2) Apply the Principle of Powers, raise each side of the equation to a power
that is the same as the index of the radical. (3) Simplify. Solve the resulting equation.
If it still contains radical, repeat step 1 and step 2.
Your own example
Answers may vary.

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