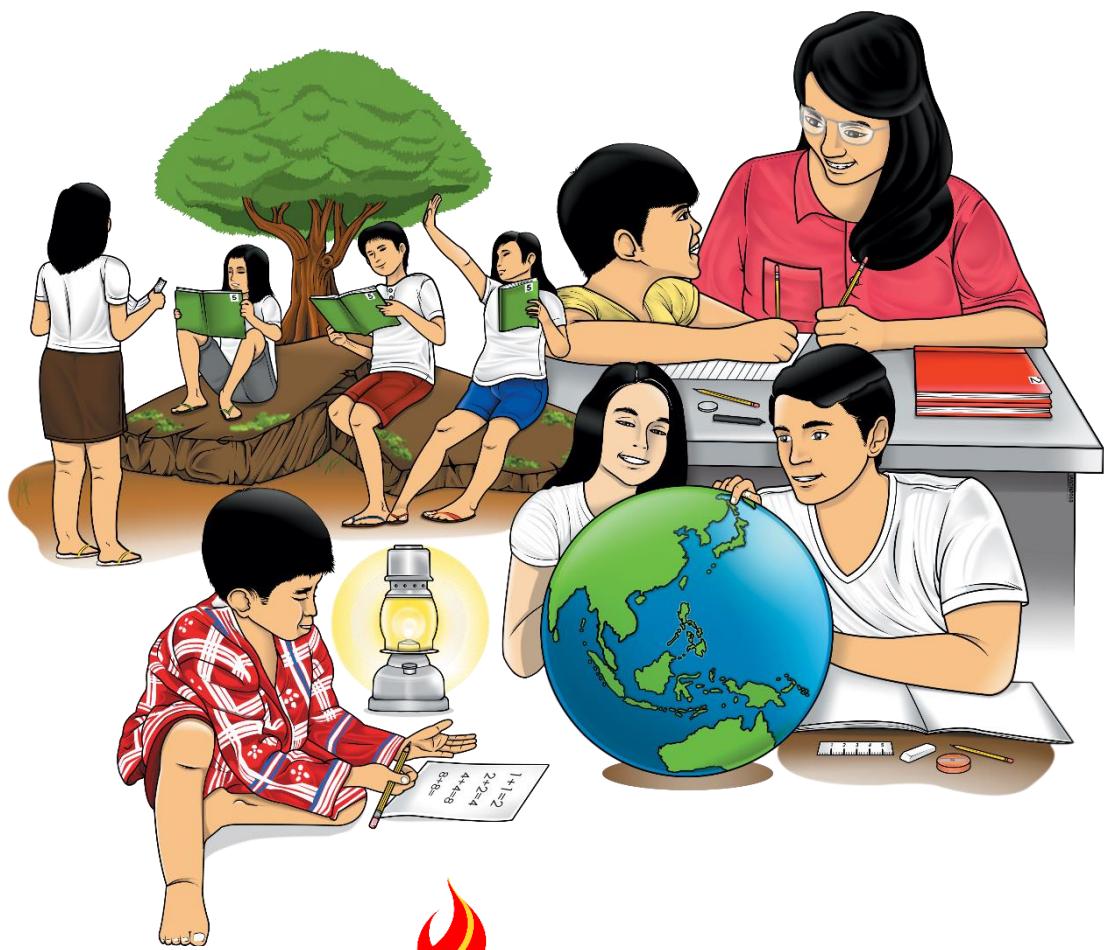


Mathematics

Quarter 3 – Module 27: Permutation of Identical Objects and Circular Permutation



Mathematics – Grade 10

Alternative Delivery Mode

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First Edition, 2020

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Mathematics

**Quarter 3 – Module 27:
Permutation of Identical Objects
and Circular Permutation**

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

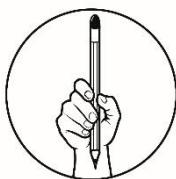


What I Need To Know

This module was designed and written with you in mind. It is here to help you solve problems involving permutation of identical objects and circular permutation. The scope of this module permits it to be used in many different learning situations. The lessons are arranged to follow the standard sequence of the course but the pacing in which you read and comprehend the contents and answer the exercises in this module will depend on your ability.

After going through this module, you are expected to be able to demonstrate understanding of key concepts of permutation. Specifically, you should be able to:

- 1) illustrate the permutation of identical objects and circular permutation; and
- 2) solve problems involving permutation of identical objects and circular permutation.



What I Know

Are you ready? You are tasked to answer the following questions before we proceed with our lesson. Do not worry, we only want to know how knowledgeable you are with the topics that we will be discussing in this module.

DIRECTION: Read and analyze each item carefully. Write the letter of the correct answer on the blank before the item number.

- ___ 1. Which concept is involved when the letters of the word “LOVE” taken 4 at a time equals 24?
A. Combination B. FPC C. Permutation D. Probability

- ___ 2. In how many ways can the letters of the word “SWIMMING” be arranged?
A. 10,800 B. 10,080 C. 1,860 D. 1,680

- ___ 3. There are three identical green flags, three identical white flags, and two identical red flags. Using all eight flags, how many signals can be made?
A. 560 B. 1,120 C. 6,720 D. 20,160

- ___ 4. How many six-digit numbers can be made using the digits 9, 4, 4, 3, 4, and 9?
A. 720 B. 360 C. 120 D. 60

- ___ 5. A department store sells two same jackets, two same shirts, two same ties, and four same pairs of pants. How many different suits consisting of jacket, shirt, tie, and pants are possible?
A. 25,200 B. 19,120 C. 18,900 D. 4,200

- ___ 6. How many different ten-digit numerals can be written using the digits 1, 3, 3, 4, 4, 5, 5, 6, 6, and 9?
A. 226 B. 2,268 C. 22,860 D. 226,800

- ___ 7. Find the number of distinguishable permutations of the letters of the word “PANAGBENGA”.
- A. 3,628,800 B. 518,400 C. 302,400 D. 151,200
- ___ 8. In how many ways can three identical Mathematics magazines, two identical Statistics magazines, and four identical Science magazines be put in a stack?
- A. 362,880 B. 40,320 C. 1,728 D. 1,260
- ___ 9. In how many ways can nine people be seated around a circular table?
- A. 362,880 B. 40,320 C. 5,040 D. 720
- ___ 10. How many ways can 12 people be seated around a circular table if two insist to sit next to each other?
- A. 7,257,600 B. 3,628,800 C. 725,760 D. 362,800
- ___ 11. Five different keys are to be placed on a circular key chain. How many different arrangements are there?
- A. 4 B. 5 C. 12 D. 24
- ___ 12. Seven different beads will be put together to form a bracelet with a lock. How many ways can the beads be arranged?
- A. 5,040 B. 2,520 C. 720 D. 360
- ___ 13. How many ways can eight unique beads be arranged on a chain without a clasp?
- A. 40,320 B. 20,160 C. 5,040 D. 2,520
- ___ 14. In how many ways can six people be seated in a round table?
- A. 24 B. 120 C. 720 D. 5,040
- ___ 15. Ten people are to be seated around a table. One of them is to be seated close to the door. How many arrangements are possible?
- A. 479,001,600 B. 39,916,800 C. 3,628,800 D. 362,880

Lesson 1

Permutation of Identical Objects and Circular Permutation



What's In

In the previous module, you have learned that a permutation is an ordering or an arrangement of certain objects. Oftentimes, permutation problems will ask you in how many ways can you pick and arrange a certain number of objects from a larger set of objects. Let's review our previous lesson on permutation of distinguishable objects by studying the following examples:

Example 1

In how many ways can you arrange five (5) people to be seated in a row?

Solution

The diagram illustrates the five seats. Each person can be arranged in different ways.



In how many ways can the person be seated?

Why?

1st
person

5 ways

The first person can be seated in any one of the five slots. Each slot is different from the others.

2nd
person

4 ways

Since the first person is seated, the second person may occupy any of the four vacant seats.

3rd
person

3 ways

The third person may occupy any of the three vacant seats.

4 th person	2 ways	The fourth person may occupy any of the two vacant seats.
5 th person	1 way	The fifth person occupies the last remaining seat.

We do the computation ${}_5P_5 = 5 \times 4 \times 3 \times 2 \times 1 = 5!$ to know the number of ways five people can be seated in a row. After doing the computation, we conclude that there are 120 ways to arrange five people in a row.

Example 2

In how many ways can the letters of the word “LOVE” be arranged?

Solution

Since there are four different letters in the word “LOVE”, then

$${}_4P_4 = 4! = 24$$

Therefore, the letters of the word “LOVE” can be arranged in 24 different ways.

What about if you want to know how many ways can you arrange the letters of the word “NONE”? Is the answer the same with that of the word “LOVE” since they have the same number of letters?



What's New

Notice that the letters in the word “LOVE” are distinct or different while the letters in the word “NONE” are not distinct. Why? The letters in the word “NONE” are not distinct because there are two letter “Ns” in it. This will affect the number of ways you can arrange the letters of the word and this type of problem is called **permutation of identical objects**.

Let's analyze this.

N O N E	There are four letters in the word “NONE” and there are two letter “Ns”.
---------	--

N O N E



N O N E

If you interchange the two letter “Ns”, you form the same word. The seemingly two different arrangements are actually one and the same arrangement. Therefore, these are considered one arrangement only.

Let’s solve the problem *How many ways can the letters in the word “NONE” be arranged?*

Step 1 ${}_4 P_4 = 4!$ Since there are four letters, get the permutation of the four letters.

Note: When you get the permutation of the four letters, you will be counting some words twice.

Step 2 $\frac{4!}{2!}$ The permutation of the four letters will be divided by the number of ways to arrange the repeated letter.

Note: The number of ways to arrange the two letter “Ns” is ${}_2 P_2 = 2!$.

Step 3 $\frac{4!}{2!} = \frac{24}{2} = 12$ Simplify.

Therefore, there are 12 ways to arrange the letters of the word “NONE”.

To verify the answer, let us list down all the possible arrangements of the letters of the word “NONE” and remove the repeated words.

NONE

NOEN

NNOE

NNEO

NENO

NEON

ONNE

ONEN

ONEN

ONNE

OENN

OENN

NONE

NOEN

NNOE

NNEO

NENO

NEON

ENNO

ENON

EONN

EONN

ENON

ENNO

Example 3

How many distinguishable arrangements can be formed from the letters of the word “PAGPAPAKATAO”?

Solution

Step 1 ${}_{12}P_{12} = 12!$

Since there are twelve letters, get the permutation of the twelve letters.

Note: When you get the permutation of the twelve letters you will be counting some words twice.

Step 2 $\frac{12!}{3! \times 5!}$

The permutation of the twelve letters will be divided by the product of the number of ways to arrange the repeated letters.

Note: The number of ways to arrange the three letter “Ps” is ${}_3P_3 = 3!$ and the five letter “As” is ${}_5P_5 = 5!$.

Step 3 $\frac{12!}{3! \times 5!} = \frac{479,001,600}{720} = 665,280$ Simplify.

Therefore, there are 665,280 arrangements formed from the letters of the word “PAGPAPAKATAO”.

Activity 1.

Find the number of distinguishable permutations of the letters in each of the given words.

1. BAGUIO
2. REFERENCE
3. MATHEMATICS
4. BOOKKEEPER
5. STATISTICS



What is It

Distinguishable Permutations are permutations that can be distinguished from one another. In the case of a number of things where each is different from the other, such as the letters in the word “BAGUIO”, there is no difference between the number of permutations and the number of distinguishable permutations. But if the original set of things has repetition, then the number of distinguishable permutations of n objects of which n_1 are alike and one of a kind, n_2 are alike and one of a kind, ..., n_k are alike and one of a kind, the number of distinguishable permutations is:

$$\frac{n!}{n_1!n_2!, \dots, n_k!}.$$

Example 4

If there are two cans of orange juice, three cans of lemonade, and five cans of iced tea in a cooler. In how many ways can these drinks be consumed by a costumer?

Solution

Step
1

$${}_{10}P_{10} = 10!$$

Since there are ten (10) drinks, get the permutation of the ten drinks.

Step
2

$$\frac{10!}{2! \times 3! \times 5!}$$

The permutation of the ten drinks will be divided by the product of the number of ways to consume the repeated drinks.

Note: The number of ways to consume the orange juice is ${}_2P_2 = 2!$, the lemonade is ${}_3P_3 = 3!$, and the iced tea is ${}_5P_5 = 5!$.

Step
3

$$\frac{10!}{2! \times 3! \times 5!} = \frac{3,628,800}{1,440} = 2,520$$

Simplify.

Therefore, there are 2,520 possible ways to consume the drinks.

Example 5

How many different eight-digit numbers can be written using the digits 1, 2, 3, 4, 4, 5, 5, and 5?

Solution

Step 1

$${}_8P_8 = 8! = 40,320$$

Since there are eight (8) digits, get the permutation of the eight digits.

Step 2

$$\frac{40,320}{2! \times 3!}$$

The permutation of the eight digits will be divided by the product of the number of ways to arrange the repeated digits.

Note: The number of ways to arrange the digit 4 is ${}_2P_2 = 2!$ and the digit 5 is ${}_3P_3 = 3!$.

Step 3

$$\frac{{}_8P_8}{2! \times 3!} = \frac{40,320}{12} = 3,360$$

Simplify.

Therefore, there are 3,360 different eight-digit numbers.

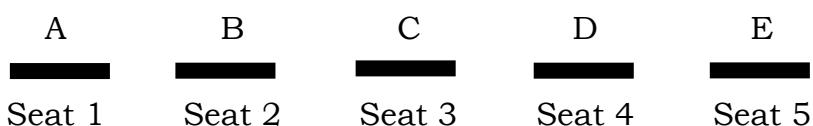
Activity 2.

Find the number of permutations in each situation.

1. Lisa has three vases of the same kind and two candle stands of the same kind. In how many ways can she arrange these items in a line?
2. Find the number of distinguishable permutations of the digits of the number 348,838.
3. What is the number of possible arrangements of nine books on a shelf where four Algebra books are of the same kind, three Geometry books are of the same kind, and two Statistics books are of the same kind?
4. A clothing store has shirts in four sizes: small, medium, large, and extra-large. If it has two small, three medium, six large, and two extra-large shirts in stock, in how many ways can these shirts be sold if each is sold one after the other?
5. How many different nine-digit numbers can be written using the following digits: 2,2,2,7,7,8,8,8, and 9?

The arrangements discussed previously are linear in nature. There are some arrangements which are circular in nature such as sitting in a roundtable, making a necklace with different colored beads, and the like. The number of ways of counting associated with the circular arrangement gives rise to **circular permutation (P)**.

Suppose there are five chairs around a table to be occupied by five persons A, B, C, D, and E, in how many ways can they arrange themselves? These five persons A, B, C, D, and E can arrange themselves in $5!$ ways if they are to be arranged in a row. There is a start and there is an end.



In a circular permutation, there is nothing like a start or an end. The picture illustrates the five chairs around a table. The five persons can be arranged in different ways.



<https://www.houzz.com/products/6-piece-outdoor-teak-dining-set-60-round-table-5-celo-stacking-arm-chairs-prvw-vr~126789186>

How many person/s will occupy a seat?

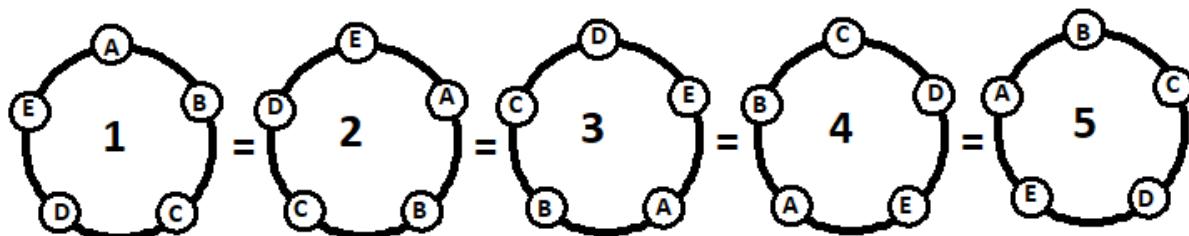
Why?

1 st seat	1 person	A particular chair is to be designated as the first seat which serves as the starting point, and to be occupied by the designated first person, say D.
2 nd seat	4 persons	Since the designated first seat is already occupied by the designated first person (D), then the next chair, either clockwise or counterclockwise, can now be occupied by any of the four remaining persons A, B, C, or E. Let us say B occupies the second seat.

3 rd seat	3 persons	The third seat next to B can be occupied by any of the remaining three persons A, C, or E, say A.
4 th seat	2 persons	The fourth seat next to A can be occupied by any of the two remaining persons C or E, say E.
5 th seat	1 person	The fifth seat is to be occupied by the remaining person C.

We use the computation $P = 1 \times 4 \times 3 \times 2 \times 1 = 4!$ to know the number of ways five people can be seated in a roundtable. After simplifying the solution, we conclude that there are 24 ways to arrange five people in a roundtable.

Let's analyze the situation using the illustration.



Notice that the five arrangements are all the same but in different angles or views. The arrangements are just rotating clockwise or counterclockwise. There are five times as many as it should be.

$$\text{Then, } P = \frac{5!}{5} = \frac{5 \times 4!}{5} = 1 \times 4! = 4! = 24$$

Therefore, five people can be arranged around in 24 different ways.

If n objects are arranged in a circle, then there are $\frac{n!}{n}$ or $(n-1)!$ permutations of the n objects around the circle.

Example 6

Ten boy scouts are to be seated around a campfire. How many ways can they be arranged?

Solution

$$P = \frac{10!}{10} = \frac{10 \times 9!}{10} = 9! = 362,880$$

Therefore, there are 362,880 ways to arrange the ten boy scouts around a campfire.

Example 7

Eight people are to be seated at a roundtable. One of them is to be seated close to the window. How many arrangements are possible?

Solution

$$P = n! = 8! = 40,320$$

Note: If n objects on a circle are arranged relative to a fixed point, then there are $n!$ permutations even though the objects are on a circle, the permutations are linear since a reference point has been established.

Therefore, there are 40,320 arrangements possible.

Example 8

How many different ways can four keys, no two of which are the same, be arranged on a key-ring that has a clasp?

Solution

$$P = \frac{n!}{2} = \frac{4!}{2} = \frac{24}{2} = 12$$

Note: This is no longer a case of an ordinary circular permutation since objects are arranged with respect to a fixed point, the clasp, and is reflective. In this case, there are $\frac{n!}{2}$ permutations.

Therefore, there are 12 ways to arrange the four keys, no two of which are the same, on a key-ring.

Activity 3.

Find the number of permutations of the given objects in each of the following situations.

1. How many seating arrangements are possible for five people at a roundtable?
2. In how many different ways can four keys, no two of which are the same, be arranged on a key-ring that has no clasp?
3. Twelve beads, no two of which are the same, are to be strung in a necklace with a clasp. In how many ways can it be done?
4. How many ways can five boys and five girls be seated alternately at a circular table?
5. How many ways can seven people be seated at a roundtable relative to the door of a room?



What's More

Now, it's your turn to answer the following activities.

Activity 4.

Matching Type: Match each problem at the left to the permutation notation at the right to answer the said problems. Write the letter of the correct answer on the blank before the item number.

PROBLEM

- ___ 1. How many ways can six beads, no two of which are the same, be arranged on a bracelet without clasp?
- ___ 2. Using the digits 5, 6, 6, 6, 6, 6, 7, 7, 8, and 9, how many 10-digit numbers can be formed?
- ___ 3. In how many ways can six people be seated in a round table?
- ___ 4. In how many ways can five identical red cards, two identical blue cards, and three identical black cards be arranged in a row?
- ___ 5. How many ways can six keys, no two of which are the same, be arranged on a key ring with a clasp?

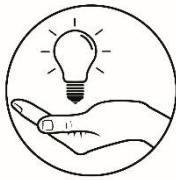
PERMUTATION NOTATION

- A. $\frac{10!}{5! \times 3! \times 2!}$
- B. $(6-1)!$
- C. $\frac{10!}{5! \times 3!}$
- D. $\frac{6!}{2}$
- E. $\frac{(6-1)!}{2}$
- F. $\frac{10!}{5! \times 2!}$

Activity 5.

Answer the following questions.

1. A couple wants to plant some shrubs around a circular walkway. They have 10 shrubs, no two of which are the same. In how many ways can the shrubs be planted?
2. In how many ways can six colored beads, no two beads are of the same color, be threaded on a string with a clasp?
3. Five people are to be seated around a circular table. If two people insist to seat next to each other, how many arrangements are possible?
4. How many ways can the letters of each given word be arranged in a row?
 - a. MAKADIYOS
 - b. MAKATAO
 - c. MAKAKALIKASAN
 - d. MAKABANSA



What I Have Learned

Summing up, let us list down what you have learned in our discussions.

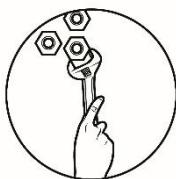
The number of distinguishable permutations of n objects of which n_1 are alike and one of a kind, n_2 are alike and one of a kind, ..., n_k are alike and one of a kind, the number of distinguishable permutations is:

$$\frac{n!}{n_1!n_2!, \dots, n_k!}.$$

If n objects are arranged in a circle, then there are $\frac{n!}{n}$ or $(n-1)!$ permutations of the n objects around the circle.

If n objects on a circle are arranged relative to a fixed point, then there are $n!$ permutations. Even though the objects are on a circle, the permutations are linear since a reference point has been established.

If n objects on a circle are reflective, then there are $\frac{(n-1)!}{2}$ permutation.



What I Can Do

In this part of the module, you will apply the concepts of solving permutation with different restrictions. Consider the situation:

There are three identical red books, one white book, and one black book on the shelf. How many ways can the books be arranged if the white and black books are separated?

STEP 1 Find the total number of arrangements.

$$P_{total} = \frac{5!}{3!}$$

$$P_{total} = 20 \text{ ways}$$

STEP 2 Find the total number of arrangements if the white and black books are NOT separated.

Note: If the white and black books are not separated or together, then they are considered as one and can be arranged in
 ${}_2 P_2 = 2! = 2 \text{ ways.}$

$$P_{not \ separated} = \frac{4!}{3!} \times 2$$

$$P_{not \ separated} = 8 \text{ ways}$$

STEP 3 Find the difference between the total number of arrangements and the number of arrangements if the white and black books are not separated or together.

$$P_{separated} = P_{total} - P_{not \ separated}$$

$$P_{separated} = 20 - 8$$

$$P_{separated} = 12 \text{ ways}$$

Therefore, there are 12 ways to arrange the books if the white and black books are separated.

Congratulations, I know that you are ready to apply what you have learned in this module.

Activity 6.

1. In how many ways can five people be arranged in a circle such that two of them sit together?

STEP 1	Find the total number of arrangements. <i>Note: Two people who sit together are considered as one.</i>	
STEP 2	Find the number of arrangements of the two people who sit together.	
STEP 3	Find the product of the total number of arrangements and the number of arrangements if the two people sit together.	

2. Eight members of the faculty board, A, B, C, D, E, F, G, and H are seated at a roundtable. How many ways can they be seated if:
- the members can sit anywhere?
 - A, B, C sit beside each other?
 - E and G refuses to sit together?
 - H has a reserved seat?

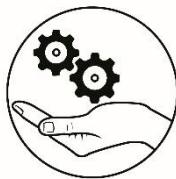


Assessment

DIRECTION: Let us determine how much you have learned from this module. Read, analyze and answer each item carefully. Write the letter of the correct answer on the blank before the item number.

- ___ 1. What concept is involved when the letters of the word “FAIR” taken 4 at a time equals 24?
A. Combinatio B. FPC C. Permutation D. Probability
n
- ___ 2. In how many ways the letters of the word “RUNNING” be arranged?
A. 5,040 B. 2,520 C. 1,260 D. 840
- ___ 3. There are four identical green flags, two identical white flags, and two identical red flags. Using all eight flags, how many signals can be made?
A. 252 B. 420 C. 840 D. 1,680
- ___ 4. How many six-digit numbers can be made using the digits from 9, 4, 4, 3, 4, and 2?
A. 720 B. 360 C. 120 D. 60
- ___ 5. A department store sells three same jackets, three same shirts, two same ties, and two same pairs of pants. How many different suits consisting of jacket, shirt, tie, and pants are possible?
A. 25,200 B. 19,120 C. 18,900 D. 4,200

- ___ 6. How many different ten-digit numerals can be written using the digits 1, 2, 3, 4, 4, 5, 5, 6, 6, and 9?
A. 453,600 B. 45,360 C. 4,536 D. 536
- ___ 7. Find the number of distinguishable permutations of the letters of the word “ATI-ATIHAN”.
A. 3,628,800 B. 518,400 C. 302,400 D. 15,120
- ___ 8. In how many ways can three identical issues of a Mathematics magazine, two identical Statistics magazines, and five identical Science magazines be put in a stack?
A. 1,260 B. 2,520 C. 5,040 D. 30,240
- ___ 9. In how many ways can eight people be seated around a circular table?
A. 362,880 B. 40,320 C. 5,040 D. 720
- ___ 10. How many ways can 11 people be seated around a circular table if two insist to seat next to each other?
A. 181,440 B. 362,880 C. 725,760 D. 1,451,520
- ___ 11. Six keys, no two of which are the same, are to be placed on a circular key chain. How many different arrangements are there?
A. 60 B. 30 C. 15 D. 10
- ___ 12. Seven beads, no two of which are the same, will be put together to form a bracelet without a lock. How many ways can the beads be arranged?
A. 5,040 B. 2,520 C. 720 D. 360
- ___ 13. How many ways can eight beads, no two of which are the same, be arranged on a chain with a clasp?
A. 40,320 B. 20,160 C. 5,040 D. 2,520
- ___ 14. In how many ways can five people be seated in a round table?
A. 24 B. 120 C. 720 D. 5,040
- ___ 15. Nine people are to be seated at a roundtable. One of them is to be seated close to the door. How many arrangements are possible?
A. 479,001,600 B. 39,916,800 C. 3,628,800 D. 362,880

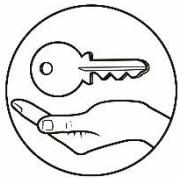


Additional Activity

Activity 7.

Answer the following questions.

1. How many ways can three identical red bulbs, four identical yellow bulbs, and two identical blue bulbs be arranged in a string of Christmas tree lights with nine sockets?
2. Eight beads, no two of which are the same, are strung in a necklace with a clasp. In how many ways can the beads be arranged?
3. How many ways can four one-hundred-peso bills and five fifty-peso bills be distributed among nine winners in a parlor game?



Answer Key

What I Know		Activity 1	Activity 2	Activity 3	Activity 4	Assessment		Activity 3	Activity 4	Activity 5	Activity 6	Assessment		Activity 7	Activity 8	Assessment		Activity 9	
1. C		1. 720	1. 10	1. 24	Activity 3			1. 24	Activity 2	Activity 1	What I Know			ACTIVITY 5	ACTIVITY 7	ACTIVITY 9	ACTIVITY 12	ACTIVITY 15	
2. B		2. 7560	2. 60	2. 3		4.	151,200	3. 1,260	3. 239,500,800	3. 4,989,600	2. 7560	1. 10	1. 362,880	2. 360	1. 362,880	2. 360	1. 362,880		
3. A		3. 4,989,600	3. 1,260	3. 239,500,800		5.	50,400	4. 360,360	4. 2,880	4. 151,200	3. 4,989,600	2. 60	1. 720	1. 10	1. 362,880	2. 360	1. 362,880		
4. D		4. 151,200	4. 2,880	4. 50,400		5.	5,040	5. 5,040	5. 5,040	5. 5,040	4. 151,200	3. 4,989,600	2. 60	1. 720	1. 10	1. 362,880	2. 360	1. 362,880	
5. C		5. 5,040	5. 5,040	5. 5,040		6.	A	6. A	6. A	6. A	5. 5,040	4. 151,200	3. 4,989,600	2. 60	1. 720	1. 10	1. 362,880	2. 360	1. 362,880
6. D						7.	B	7.	B	7.									
7. D						8.	C	8.	C	8.									
8. D						9.	B	9.	B	9.									
9. B						10.	C	10.	C	10.									
10. A						11.	D	11.	D	11.									
11. C						12.	B	12.	B	12.									
12. B						13.	D	13.	D	13.									
13. D						14.	B	14.	B	14.									
14. B						15.	C	15.	C	15.									
15. C																			

References:

- Circular Permutation.* (2020, July 13). Retrieved from <https://study.com>.
- Circular Permutation.* (2020, July 4). Retrieved from <https://superprok.co.uk.com>.
- Combination and Permutation.* (2020, July 9). Retrieved from <https://mathsisfun.com>.
- Gladys C. Nivera, M. R. (2015). *Grade 10 Mathematics Patterns and Practicalities*. Makati City: Don Bosco Press, Inc.
- Mirla S. Esparrago, N. V. (2015). *Next Century Mathematics*. Quezon City: Phoenix Publishing House, Inc.
- Obaña, G. G., & Mangaldan, E. R. (2009). *Making Connections in Mathematics*. Manila: Vicarish Publication & Trading, Inc.
- Permutation and Combination.* (2020, July 13). Retrieved from <https://eis.ouw.edu.au.com>.
- Permutation with Repetition.* (2020, July 20). Retrieved from <https://emathematics.net.com>.
- Permutation with Repetition and Circular Permutation.* (2020, July 4). Retrieved from <https://brilliant.org.com>.
- Permutation with Restrictions.* (2020, July 4). Retrieved from <https://toppr.com>.

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