Senior High School

Department of Education
SCHOOLS DIVISION OFFICE
MARIKINA CITY

MATHEMATICS

Quarter 2: Module 6
Propositional Logic

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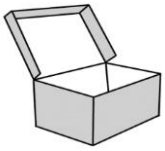


What I Need to Know

Hello Grade 11 learners! In this module, you will learn how to:

- illustrate and symbolize propositions;
- distinguish between simple and compound propositions (M11GM-IIg-3); and
- perform the different types of operations on propositions (M11GM-IIg-4).

You can say that you have understood the lesson in this module if you can already:

1. illustrate and symbolize propositions;
2. distinguish simple and compound propositions; and
3. perform the different types of operations on propositions.

What I Know

Choose the correct letter that corresponds to the exact answer.

1. What is proposition?
   A. Statement that is either true, false, or both
   B. Statement that is both true and false
   C. Statement that is either true or false but not both
   D. Statement that is neither true, false nor both

2. Determine whether the given statement is a proposition or not.
   Everyone is studying propositional logic.
   A. It is a proposition.
   B. It is not a proposition.
   C. Sometimes proposition
   D. Sometimes not proposition

3. Which of these propositions is **TRUE**?
   A. 122 is a perfect square.
   B. If a triangle has an obtuse angle, then it is called an obtuse triangle.
   C. $x < y$ if and only if $y < x$
   D. $\sqrt{12} + \sqrt{27} = 6\sqrt{3}$

4. Which of these propositions is **FALSE**?
   A. A rectangle has four sides.
   B. $3x + 5y = 21$, if $x = 2$ and $y = 3$
   C. The four states of matter are solid, liquid, gas and plasma.
   D. Mars is the closest planet to the sun.

5. Which of the following propositions conveys two or more ideas?
   A. $x + 5 = 12$ if and only if $x = 7$
   B. Atoms are composed of particles called protons, electrons and neutrons.
   C. Angle is a figure formed by two rays.
   D. Mathematics is an interesting subject.

6. What are the four logical connectives?
   A. Conjunctions, conditional, compound, and disjunction
   B. Conjunctions, statements, conditional, and disjunction
   C. Conjunctions, disjunctions, negations, and conditional
   D. Conditional, negation, compound, and disjunction

7. It connects two or more propositions.
   A. Negation
   B. Logical operators
   C. Conjunction
   D. Conditional

8. The statement "I will sing, or I will dance" is an example of which logic connective?
   A. Disjunction
   B. Conjunction
   C. Negation
   D. Conditional

9. The conjunction of the propositions p and q is symbolized by
   A. $p \rightarrow q$
   B. $p \vee q$
   C. $p \wedge q$
   D. $p \leftrightarrow q$

10. Which of the following biconditional statements best described on the given proposition?
    "A right angle is an angle measures 90 degrees."
    A. If an angle measures 90 degrees, then it is a right angle.
    B. If an angle is a right angle, then it measures 90 degrees.
    C. An angle is right angle if and only if it measures 90 degrees.
    D. An angle measures 90 degrees if and only if it is a right angle.

Lesson 1 Illustrating, Symbolizing and Distinguishing Between Simple and Compound Propositions

In this lesson, you will learn more about the concept of propositions. Different types of sentences are illustrated for you to analyze which are propositions and which are not.

What's In

Read each sentence carefully and identify which is Declarative(D), Interrogative (INT), Imperative (IMP) or Exclamatory (E).

1. Wait for me.
2. Do you have your modules already?
3. 8 + 5 = 13
4. You won the prize!
5. 7 is a prime number.
6. Will you please read the statement for me?
7. 8 < 5, if x = 2
8. What a beautiful day!
9. Marikina City is known as the Shoe Capital of the Philippines.
10. I have a great idea!

What's New

Match each tagline listed in column A with its corresponding name of company and agency in column B.

A	B
1. We Find Ways	A. Manila Water
2. Care in Every Drop	B. Metrobank
3. We've Got It All for You!	C. Department of Tourism
4. You're in Good Hands	D. Banco de Oro (BDO)
5. It's more fun in the Philippines	E. SM Malls

Write **S** if the statement conveys a single idea and **C** if the statement conveys two or more ideas.

1. Manila is the capital of the Philippines.
2. Either I will cook, or I will buy food for my lunch.
3. If you are 60 years old and above, then you're entitled to have a Senior Citizen Card.
4. One-peso coins were tossed.
5. She wrote a poem, and they recited it in front of the class.

What is It

A **proposition** is a declarative sentence that is either true or false, but not both. If a proposition is true, then its truth value is true, which is denoted by T. On the other hand, if its truth value is false, it is denoted by F.

We can use propositional variables to represent propositions and usually denoted by small letters such as p, q, r,…

Example:

We are discussing propositions.

p: We are discussing propositions.

may be read as

p is the proposition "We are discussing propositions."

If sequence of propositions is given, we denote the propositions by p_1, p_2, …

Examples:

Tell whether each of the following statements is a proposition or not. If it is a proposition, give its truth value.

p: 7 is an odd number.
q: You look awesome!
r: Please follow the given instructions.
s: $\sqrt{625} + \sqrt{4} = 17$
p_1: Marikina is one of the cities in the National Capital Region.
p_2: f(x) = $\frac{\sqrt{x+2}}{x+6}$ is rational
p_3: What is the range of the function?

Recall that proposition must be a declarative sentence and its truth value is either true or false but not both at the same time.

p: This is a declarative sentence and 7 is an odd number; hence p is a true proposition.
q: This statement is exclamatory; hence it is not a proposition.
r: This sentence is imperative; hence it is not a proposition.
s: The given mathematical equation when translated into words reads as "the sum of square root of four and square root of six hundred twenty-five is seventeen". It is a declarative sentence thus s is false proposition.
p_1: This is a declarative sentence. Marikina is one out of the 14 cities in the National Capital Region, hence p_1 is a true proposition.

p_2: This is a declarative statement. The numerator of the given function is not a polynomial function and the function p_2 is not rational; therefore, the statement is false. This is a false proposition.

p_3: This is an interrogative sentence; hence it is not a proposition.

A **simple proposition** is a proposition that expresses only one thought.

Examples:

1. General mathematics is one of the core subjects in Senior High School.
2. He participated actively in class discussion.
3. (-15) + 6 = - 9
4. He has a green thumb.
5. The weather forecaster announced the expected weather conditions tomorrow.

A **compound proposition** is a proposition made up of simple propositions and joined by using logical connectives.
Some of the logical connectives are:

Word	Connective	Symbol
"and"	conjunction	^
"or"	disjunction	∨
"not"	negation	¬
"if…then"	conditional	→
"if and only if"	bi-conditional	↔

Examples:

1. If two angles are supplementary, then they will form a linear pair.
2. My relatives went to the park and they had an amazing moment.
3. A triangle is equilateral if and only if all angles measure 60^0.
4. Either logic is fun and interesting, or it is boring.
5. The number 6 is even and less than 10.

Explanation:

1. p_1 : Two angles are supplementary.
   p_2 : Two angles will form a linear pair.

It is compound because the two simple propositions are joined by the connective "if then".

2. p_1 : My relatives went to the park.
   p_2 : They had an amazing moment.

It is compound because the two simple propositions are joined by the connective "and".

3. p_1 : A triangle is equilateral.
   p_2 : All angles measure 60^0.

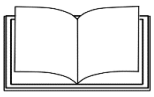
It is compound because the two simple propositions are joined by the connective "if and only if".

4. p_1 : Logic is fun and interesting.
   p_2 : Logic is boring.

It is compound because the two simple propositions are joined by the connective "or".

5. p_1 : 6 is even number.
   p_2 : 6 is less than 10.

It is compound because the two simple propositions are joined by the connective "and".



What's More

Determine whether each of the following statements is a proposition or not. Write **P** if the statement is a proposition and **NP** if not a proposition. If your answer is proposition, identify if it is simple or compound proposition.

1. Read a lot to improve your reading skill.
2. $8^2 = 64$
3. A square has four sides, and all its sides are equal.
4. If you are absent, you have to submit an excuse letter to your teachers.
5. Please submit your outputs on time.
6. Do you have your module activities ready?
7. The news will be aired soon in different television channels.
8. Well done!
9. She will go to the mall.
10. $20 < x < 30$, $x = 25$ or $20 < x < 30$, $x = 10$

What I Have Learned

Answer the following questions.

1. What is proposition? Cite 2 examples of proposition and give its truth value.
2. How do you represent proposition? Give an example.
3. What is the difference between simple proposition and compound proposition? Cite an example for each proposition.

What I Can Do

Read the dialogue below and list down all the propositions you can find.

A conversation between two friends online.

Ryan: How are you, Paul?

Paul: I'm good, but I am worried because until now I find it difficult to navigate the Google Classroom.

Ryan: Oh! That's really a problem Paul! It will be difficult to participate on the online discussion as well as accessing the online quiz if you don't know what to click or where to find the attached files in the Google Classroom. Send a message to your adviser and ask her to teach you.

Paul: My adviser already taught us how to use the Google Classroom a week ago. It was easy when she explained the process, but when I tried doing it by myself, I got lost.

Ryan: Then practice with your siblings, so you'll remember how to navigate it.

Paul: That's a good idea! I'll ask my brother to help me navigate the Google Classroom. How about you? Did you have difficulty in navigating it?

Ryan: Uh, at first I found it complicated. So, I watched video tutorials in YouTube and tried navigating it for 2 days. Now, I can say, I'm ready to attend the class in Google Classroom.

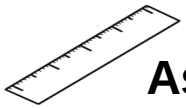
Paul: Oh, you have no problem learning it because you are a tech wizard.

Ryan: Not really, I just find it cool learning something new. Aside from that, I need to learn it because Google Classroom is one of the platforms that we have to use this school year.

Paul: You're right. I just have to practice more until I get the hang of using the Google Classroom.

Ryan: That's the spirit Paul! Just keep on practicing and you'll be better at it.

Paul: Thank you for encouraging me Ryan.



Assessment

Choose the correct letter that corresponds to the exact answer.

1. Determine whether the given statement is a proposition or not.
   Students learned the basic concepts of propositional logic.
   A. It is not a proposition.
   B. It is a proposition.
   C. Sometimes not proposition
   D. Sometimes a proposition

2. What is compound proposition?
   A. It conveys single idea.
   B. It conveys two or more ideas.
   C. It is a command sentence.
   D. It is a question.
3. How do you define proposition?
   A. Statement that is both true and false.
   B. Statement that is neither true, false, nor both.
   C. Statement that is either true, false, or both.
   D. Statement that is either true or false, but not both.
4. Why is the given statement **NOT** a proposition?

   Please read it carefully.

   A. It is an expression.
   B. It is a meaningless interjection.
   C. It is a question.
   D. It is a command sentence.
5. What is a simple composition?
   A. It conveys two or more ideas.
   B. It conveys single idea.
   C. It is a command sentence.
   D. It is a question.

Determine whether each of the following statements is a proposition or not. Write **P** if the statement is a proposition and **NP** if not a proposition. If your answer is proposition, identify if it is simple or compound proposition.

1. Hooray!
2. I will narrate the story and you will act it.
3. If x = 1 and y = 2 then x + y = 3.
4. The teacher reminds his class regarding the school activities.
5. She sold the bag at 50% less than its original price.

Additional Activities

Choose the letter that corresponds to the exact answer.

1. Which of these statements is simple proposition?
   A. The only odd prime number is 2.
   B. Either I will clean my room, or I will take a nap.
   C. I will eat breakfast and I will drink coffee.
   D. If today is Friday, then tomorrow is Saturday.
2. Which of these propositions is **TRUE**?
   A. Man is immortal.
   B. A triangle has four angles.
   C. All fish can swim.
   D. A triangle does not have three sides.

3. Which of these propositions is **FALSE**?
   A. Some birds cannot fly.
   B. Moon is made up of water.
   C. Sun is made up of hydrogen and helium.
   D. Square has four right angles.
4. Which of the following statements is a compound proposition?
   A. The bell rang and the music began.
   B. They speak English at school.
   C. It will rain today.
   D. He plays basketball.
5. Which of the following statements is **TRUE**?
   A. Composite numbers are whole numbers that have more than two factors.
   B. 3 + 2 = 8 and 5 – 2 = 7
   C. 1 > 3 and 3 is a positive integer.
   D. -2 > 3 or 3 is a negative integer.

Lesson 2 Performing the Different Types of Operations on Propositions

What's In

Complete the statement below using "and", "or", "if then" and "if and only if".

1. They arrived early _____ they got really good seats.
2. They will start the program at exactly 7:00 a.m._____ they will wait for the judges.
3. ____ Ana is in Grade 11, _____ she is a senior high school student.
4. It will rain _____the clouds are heavy.
5. _____ a person is yawning, _____ he is sleepy.
6. I studied Mathematics _____ World Literature.
7. The boiling point of water is 100°C _____ it is at the normal atmospheric pressure.
8. An equiangular triangle has three equal sides, _____ it is the same as an equilateral triangle.
9. She will cook dinner, _____ she will go out to a restaurant.
10. Blackberries are very nutritious _____ they are great in pies.

What's New

Read and analyze each sentence carefully. Give your understanding by answering the given questions.

1. If you are late, then you will miss the bus.

2. Study hard or you will fail.
3. A polygon is a triangle if and only if it has exactly three sides.
4. She gave the instructions and they executed it properly.
5. If today is Wednesday, then tomorrow is Thursday.

Questions:

- Are there any logical connectives in each sentence? What are they?
- Separating the sentences using logical connectives, does it make the sentence meaningful?
- How many ideas are there in each sentence?
- What are the single ideas of each sentence?

What is It

Many mathematical statements are constructed by combining one or more propositions. These new propositions are connected by using **logical operators or logical connectives**. They are words or symbols that serve as functions that may affect the validity of the resulting compound sentence.

NEGATION

The negation operator makes a proposition contradict its original meaning. The logical operator negation is symbolized by "~" which means "**not**" and is defined through its truth table.

Table 1. The Truth Matrix of the Negation of a Proposition

p	**~p**
T	F
F	T

Examples:

State the negation of the following propositions.

1. p: 8 is a positive number. **~p : 8 is not a positive number.**
2. q: I do not like to dance. **~q : I like to dance.**
3. m: x + y = 9 **~m : x + y ≠ 9**

CONJUNCTION

Two simple propositions that are connected using the word "**and**". The operator conjunction means "**and**". It is symbolized by ∧ and is defined through its truth table.

Table 2. The Truth Matrix for the Conjunction of Two Propositions

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Examples:

Let p and q be the propositions.

1. p: 6 is an even integer. q: 7 is an odd integer.

   p ∧ q: 6 is an even integer and 7 is an odd integer

2. p: Amy is on the volleyball team. q: John is on the basketball team.

   p ∧ q: Amy is on the volleyball team and John is on the basketball team.

3. p: The number r is odd. q: The number s is prime.

   p ∧ q: The number r is odd and the number s is prime.

DISJUNCTION

Two simple propositions that are connected using the word "**or**'. The operator conjunction means the inclusive "**or**". It is symbolized by **v** and is defined through its truth table.

Table 3. The Truth Matrix for the Disjunction of Two Propositions

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

Examples:

Let p and q be the propositions.

1. p: $\sqrt{2}$ is not a rational number. q: $\sqrt{4}$ is a rational number.

   p v q: $\sqrt{2}$ is not a rational number or $\sqrt{4}$ is a rational number.

2. p: I will pass this subject. q: I will fail this subject.

   p v q: I will pass this subject, or I will fail this subject.

3. p: You work hard. q: You can finish the job.

p v q: You work hard, or you can finish the job.

Conditional (If – then Statement)

Two simple propositions that are connected using the words "**if... then...**" and is symbolized by →.

p→q (if p then q)

If-then statement in which p is a hypothesis while q is a conclusion and is defined through its truth table.

Table 4. The Truth Matrix for the Conditional of Two Propositions

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Examples:

Let p and q be the propositions.

1. p: 3 is an integer. q: 2 is a rational number.
   p→q: If 3 is an integer then 2 is a rational number.

2. p: I will pass the interview. q: I will get the job.
   p→q: If I will pass the interview then I will get the job.

3. p: 9 > 7. q: 3 + 6 = 9.
   p→q: If 9 > 7, then 3 + 6 = 9.

Bi-conditional (If and only if Statement)

The operator biconditional means "**if and only if**," and is symbolized by ↔. It is denoted by "**iff**".

p↔q (p if and only if q)

It is defined to be true whenever both parts have the same truth value and is false otherwise. It is defined through its truth table.

Table 5. The Truth Matrix for the Bi-Conditional of Two Propositions

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

Examples:

Let p and q be the propositions.

1. p: A polygon is a triangle. q: A polygon has exactly three sides.
   p↔q: A polygon is a triangle if and only if it has three sides.

2. p: 5,175 is divisible by 5. q: 5,175 is divisible by 3 and 5
   p↔q: 5,175 is divisible by 5 if and only if 5,175 is divisible by 3 and 5.

3. p: You passed the final test. q: You got a rating of 75% or higher.
   p↔q: You passed the final test if and only if you got a rating of 75% or higher.

What's More

I. Match each proposition in Column A to its respective symbol in Column B.

A	B
1. Negation	a. ∧
2. Conditional	b. v
3. Bi-conditional	c. ~
4. Conjunction	d. →
5. Disjunction	e. ↔

II. Let p and q be the propositions.

p: 12 is an even integer.
q: 17 is an odd integer.

Write each of the following propositions in terms of p, q, and logical connectives.

1. 12 is an even integer and 17 is an odd integer.
2. 12 is not an even integer or 17 is an odd integer.
3. 17 is not an odd integer.
4. If 17 is not an odd integer then 12 is an even integer.
5. 12 is an even integer if and only if 17 is an odd integer.

What I Have Learned

Write all the ten (10) words related in logical operators found in the given cross word puzzle. Clues are given below:

C	S	M	S	C	C	M	P	A	C	C	F
O	I	A	I	O	O	O	O	S	O	O	I
N	M	N	M	M	N	C	U	D	N	N	L
J	P	I	P	P	N	N	N	F	N	N	E
U	L	L	L	O	R	O	D	G	E	E	V
N	E	A	Y	U	E	I	Q	H	C	C	O
C	S	I	M	N	C	E	W	J	T	T	I
T	S	D	C	D	T	T	N	K	O	I	L
I	A	F	O	B	O	I	O	L	R	N	R
O	Z	G	M	E	R	A	I	I	S	M	U
N	I	F	T	H	E	N	T	M	D	E	S
P	X	H	P	A	S	D	C	P	S	I	N
L	I	F	O	U	P	S	N	L	D	N	O
K	C	F	U	T	R	O	U	I	N	Y	I
J	V	S	A	I	O	P	J	C	O	O	T
H	B	C	N	N	A	S	S	D	F	G	I
G	N	O	D	F	D	N	I	O	C	H	S
F	M	N	A	U	E	O	D	A	I	J	O
D	Q	J	N	L	C	T	N	O	R	S	P
A	Q	U	W	E	R	T	Y	L	U	I	O
N	E	G	A	T	I	O	N	T	Y	E	R
E	C	N	O	N	D	I	T	I	O	I	P
L	A	N	O	I	T	I	D	N	O	C	F

1. It makes a proposition contradictory to its original meaning.
2. Proposition that conveys single idea.
3. Propositions connected using the word “and”.
4. Words such as if and only if, and, or, and if then…
5. Two or more propositions joined by using logical connectives.
6. Propositions that are connected by using if … then.
7. Word that is used to connect bi conditional statements.
8. Word that is used to connect conditional statements.
9. Declarative sentence that can be classified as true or false but not both.
10. Propositions that are connected by using “or”.

What I Can Do

In your notebook or bond paper, write a paragraph consisting of five (5) sentences with different logical operators and express it in symbols with a theme:

"Keep Your Education Move: Learn Together and Be Engaged with Distance Learning"

Assessment

Match each proposition in Column A to its respective symbol in Column B.

Let p, q and r be the propositions.

p: Paola has a headache.
q: Paola misses the contest.
r: Paola wins the grand prize.

A	**B**
1. If Paola has a headache then she misses the contest.	A. $q \leftrightarrow p$
2. Paola has a headache, and she misses the contest.	B. $\sim q \wedge r$
3. Paola misses the contest if and only if she has a headache.	C. $\sim q$
4. Paola wins the grand prize, and she has a headache.	D. $\sim p \wedge q$
5. Paola did not have a headache, and she misses the contest.	E. $p \wedge q$
6. Paola did not miss the contest and she wins the grand prize.	F. $\sim r$
7. If Paola did not have a headache, then she wins.	G. $\sim r \leftrightarrow q$
8. Paola did not miss the contest.	H. $\sim p \rightarrow r$
9. Paola did not win the grand prize.	I. $p \rightarrow q$
10. Paola did not win the contest if and only if she misses the contest.	J. $r \wedge p$

Additional Activities

Let p, q and r be the propositions.

p: Anne bought a box of pizza last night.
q: Carol finished her project.
r: Sally watched the musical play this morning.

Write each of the following propositions in terms of p, q, r, and logical connectives.

1. Anne bought a box of pizza last night and Carol finished her project.
2. Carol did not finish her project if and only if Sally watched the musical play this morning.
3. If Anne did not buy a pizza last night then Carol did not finish her project.
4. Either Carol finished her project or Sally watched the musical play this morning, but not both.
5. Sally did not watch the musical play this morning.

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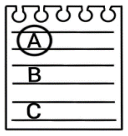

Answer Key

Answer Key (Module 6 – Lesson 1)

Pre-assessment

1. C	2. A	3. B	4. D	5. A
6. C	7. B	8. A	9. C	10. C

What's In

1. IMP	2. INT	3. D	4. E	5. D
6. INT	7. D	8. E	9. D	10. E

What's New

1. D	2. A	3. E	4. B	5. d
1. S	2. C	3. C	4. S	5. C

What's More

1.	NP	
2.	P	SIMPLE
3.	P	COMPOUND
4.	P	COMPOUND
5.	NP	
6.	NP	
7.	P	SIMPLE
8.	NP	
9.	P	SIMPLE
10.	P	COMPOUND

Assessment

1. B		2. B	3. D	4. D	5. B

1.	NP	
2.	P	COMPOUND
3.	P	COMPOUND
4.	P	SIMPLE
5.	P	SIMPLE
6.	P	SIMPLE
7.	P	SIMPLE
8.	P	COMPOUND
9.	P	SIMPLE
10.	P	SIMPLE

Additional Activities

1. A	2. C	3. B	4. A	5. A

Answer Key (Module 6 – Lesson 2)

What's In

1. and
2. or
3. If, then
4. if and only if
5. If, then
6. and
7. if and only if
8. and
9. or
10. and

What's More

I. 1. C 2. D 3. E 4. A 5. B

II. 1. p ∧ q 2. ~p ∨ q 3. ~q 4. ~q → p 5. p ↔ q

What I Have Learned

1. negation
2. simple
3. conjunction
4. connectors
5. compound
6. conditional
7. if and only if
8. if then
9. proposition
10. disjunction

Assessment

1. I
2. E
3. A
4. J
5. D
6. B
7. H
8. C
9. F
10. G

Additional Activities

1. p ∧ q
2. ~q ↔ r
3. ~p → ~q
4. q ∨ r
5. ~r

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