

# STATISTICS AND ROBABILITY

## Quarter 3: Module 6

### MEAN AND VARIANCE OF THE SAMPLING DISTRIBUTION OF THE SAMPLE MEAN



Writer:

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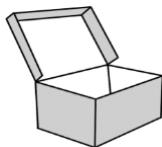
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## What I Need to Know

Hello Grade 11 learners! In this module you will learn how to:

- a. find the mean and variance of the sampling distribution of the sample mean **M11/12/SP-IIId-5** and
- b. define the sampling distribution of the sample mean for normal population when the variance is a. known; and b. unknown **M11/12/SP-IIle-1**.

You can say that you have understood the lesson in this module if you can already:

1. Determine the mean, variance and standard deviation of the sampling distribution of the sample mean and
2. Solve problems involving sampling distribution of the sample mean for normal population when the variance is known; and unknown.



## What I Know

Read each statement carefully. Decide whether the statement in each item is True or False. Write **T** if it is True and **F** if False on your answer sheet.

- 1. If the *population* is normally distributed, the *sampling distribution* of the *sample mean* is normally distributed for any *sample size*.
- 2. The *population mean* is equivalent to the *mean* of the *sampling distribution* of the *sample means* regardless the population is normally distributed or **not**.
- 3. The *variance* is also known as the *standard error of the mean*.
- 4. As the *sample size* increases the *variance* decreases.
- 5. Sample size 30 is considered as a large sample.
- 6. When the *sample size* is equal or greater than 30 ( $n \geq 30$ ) and the population standard deviation ( $\sigma$ ) is NOT known, you may use the sample standard deviation ( $s$ ) for  $\sigma$ .



For items 7-10, choose the letter that best answers the question.

7. Consider the score examples illustrated, in which random samples of size 15 are obtained from  $N(25,7)$ . What is the value of  $\mu_{\bar{x}}$ ?
- A. 3.27      B. 7      C. 15      D. 25
8. The distribution of the scores of the second quarter grades of Grade 11 of Malanday National High School is a population having a mean of 85 and a standard deviation of 12. The sample size of your sampling distribution is 25. Determine the value of  $\sigma_{\bar{x}}$ .
- A. 2.4      B. 5.57      C. 12      D. 85
9. Consider the scores, in which random samples of size 32 are obtained from  $N(35,6)$ . What is the value of  $\sigma^2_{\bar{x}}$ ?
- A. 1.06      B. 1.125      C. 32      D. 35
10. The guidance office personnel of Concepcion Integrated School established that, in every week, the average number of Grade 11 students who commit absenteeism is 9 with a standard deviation of 3.55. They experimented an intervention program for a sample of 30 students who commit the same offense and found out that the sample produced an average of 7.5 absences weekly. Determine the z equivalent of  $\bar{x}$ .
- A. -2.31      B. -1.31      C. 0      D. 1

### LESSON 1: MEAN AND VARIANCE OF THE SAMPLING DISTRIBUTION OF THE SAMPLE MEAN



## What's In

Decide whether the statement in each item is True or False. Write **T** if it is True and **F** if False on your answer sheet.

- \_\_\_\_\_ 1. The **population** is the entire group for study.
- \_\_\_\_\_ 2. **Sample** is a part of the population that you will collect data from.
- \_\_\_\_\_ 3. The number of individuals in your sample depends on the size of the population, and on how precisely you want the results to represent the population as a whole.
- \_\_\_\_\_ 4. Population parameter have the following:  $\bar{x}$ ,  $s^2$  and  $s$ .
- \_\_\_\_\_ 5. Sample statistic contains the following:  $\mu$ ,  $\sigma^2$  and  $\sigma$ .



- How will you distinguish the difference between a population and a sample?
- What are the symbols used for the mean, variance and standard deviation of population parameter?
- What are the symbols used for the mean, variance and standard deviation of sample statistic?

## What's New

Data can help the decision makers in crafting policies and capacitate authorities that will help the community to fight COVID – 19 and restrict its effect.

Classifying cases such as asymptomatic, mild or severe will help medical experts evaluate the factors involved in each case. But it is impossible to observe all the infected all at once. To understand the behavior or nature of this unpredictable virus, they analyze data from different groups of infected individuals.

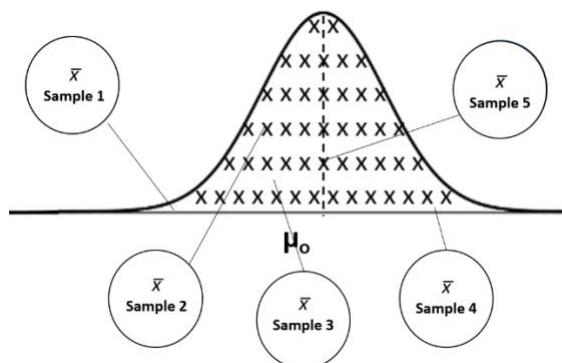
The concept of sampling distribution of the sample mean is helpful in analyzing data; as random samples will have outcomes that differ unpredictably at the beginning but will display patterns in the long run.



## What is It

### MEAN AND VARIANCE OF THE SAMPLING DISTRIBUTION OF THE SAMPLE MEAN

This lesson will focus on the mean and variance of the sampling distribution of the sample mean.



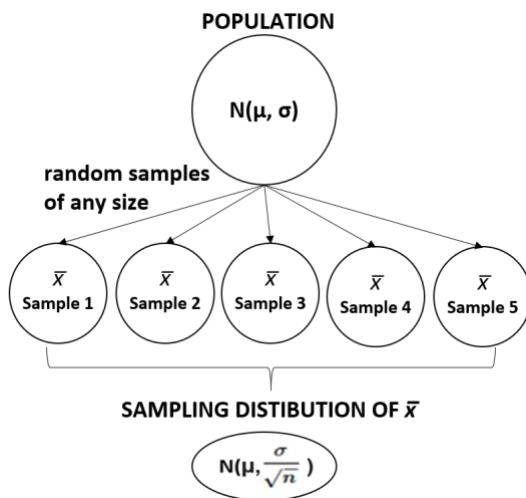
Let say that each sample taken from the population has a size,  $n$ , then each will produce a sample mean,  $\bar{X}$ . As illustrated above, the sample means will be our sampling distribution of the sample means.



The mean of the sampling distribution is denoted by  $\mu_{\bar{x}}$ .

The variance of the sampling distribution is  $\sigma^2_{\bar{x}}$ . Thus, the standard deviation of the sampling distribution is  $\sigma_{\bar{x}}$ .

Let the population be **normally distributed**. If the population is normally distributed, the sampling distribution of the sample mean,  $\bar{x}$ , is normally distributed for any sample size  $n$ .



Thus, the mean of the sampling distribution is equal to the population mean.

$$\mu_{\bar{x}} = \mu$$

The variance of the sampling distribution is equal to the population variance divided by  $n$  (sample size).

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$$

Hence, the standard deviation, also known as the standard error of the mean, of the sampling distribution is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

**Example 1:** Consider the score examples illustrated, in which random samples of size 15 are obtained from  $N(25, 7)$  (Read as, a *normally distributed population whose mean is 25 and standard deviation is 7*). Determine  $\mu_{\bar{x}}$ ,  $\sigma^2_{\bar{x}}$  and  $\sigma_{\bar{x}}$ .

Given:  $n = 15$ ,  $\mu = 25$  and  $\sigma = 7$

Solution:

$\mu_{\bar{x}} = \mu = 25$	$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$ $= 7^2/15$ $= 3.27$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ $= 7/\sqrt{15}$ $= 1.81$
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**Example 2:** The normal distribution of the scores of the first quarter grades of Grade 11 of Calumpang National High School has a  $\mu = 84$  &  $\sigma = 13$ . Find  $\mu_{\bar{x}}$ ,  $\sigma^2_{\bar{x}}$  &  $\sigma_{\bar{x}}$ , if you randomly select (a) 15, (b), 30 & (c) 45 as samples.

Given:  $\mu = 84$  and  $\sigma = 13$

Solution: (a) if  $n = 15$



$\mu_{\bar{x}} = 84$	$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = 13^2/15 = 11.27$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 3.36$
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Solution: (b) if  $n = 32$

$\mu_{\bar{x}} = 84$	$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = 13^2/30 = 5.63$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 2.37$
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Solution: (c) if  $n = 45$

$\mu_{\bar{x}} = 84$	$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = 13^2/45 = 3.76$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 1.94$
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What have you noticed on the variance or standard deviation as the sample size increases?

**Example 3:** There are 35 Grade 11 students got average scores ranging from 90 to 99 in General Mathematics. Ten samples of 3 average scores were taken randomly by their Math teacher to create a sampling distribution of the sample means which is presented below. Determine  $\mu_{\bar{x}}$ ,  $\sigma^2_{\bar{x}}$  and  $\sigma_{\bar{x}}$ .

$\bar{x}$	frequency
91	2
92	2
93	3
95	2
98	1
<b>TOTAL</b>	<b>10</b>

In this kind of problem, you need to add 5 columns to solve for the  $\mu_{\bar{x}}$ ,  $\sigma^2_{\bar{x}}$  and  $\sigma_{\bar{x}}$ . The formulas are as follows:

$$\mu_{\bar{x}} = \sum \bar{x} \cdot P(\bar{x})$$

$$\sigma^2_{\bar{x}} = \sum P(\bar{x}) \cdot (\bar{x} - \mu)^2$$

5 columns were added to solve the problem.

### SOLUTIONS:

$\bar{x}$	frequency	$P(\bar{x})$	$\bar{x} \cdot P(\bar{x})$	$\bar{x} - \mu$	$(\bar{x} - \mu)^2$	$P(\bar{x}) \cdot (\bar{x} - \mu)^2$
91	2	0.20	18.20	-2.30	5.29	1.06
92	2	0.20	18.40	-1.30	1.69	0.34
93	3	0.30	27.90	-0.30	0.09	0.03
95	2	0.20	19.00	1.70	2.89	0.58
98	1	0.10	9.80	4.70	22.09	2.21
<b>TOTAL</b>	<b>10</b>	<b>1.00</b>	<b>93.30</b>			<b>4.21</b>

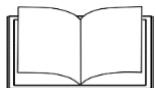


## Explanations:

For $P(\bar{x})$ column.			For $\bar{x}$ , $P(\bar{x})$ column																																										
<table border="1"> <thead> <tr> <th><math>\bar{x}</math></th><th>freq</th><th><math>P(\bar{x})</math></th></tr> </thead> <tbody> <tr> <td>91</td><td>2</td><td><math>2 / 10 = 0.20</math></td></tr> <tr> <td>92</td><td>2</td><td><math>2 / 10 = 0.20</math></td></tr> <tr> <td>93</td><td>3</td><td><math>3 / 10 = 0.30</math></td></tr> <tr> <td>95</td><td>2</td><td><math>2 / 10 = 0.20</math></td></tr> <tr> <td>98</td><td>1</td><td><math>1 / 10 = 0.10</math></td></tr> <tr> <td><b>TOTAL</b></td><td><b>10</b></td><td></td></tr> </tbody> </table> <p>The sum of the frequencies is 10. So each frequency will be divided by 10.</p> <p>What did you notice to the sum of <math>P(\bar{x})</math>?</p>			$\bar{x}$	freq	$P(\bar{x})$	91	2	$2 / 10 = 0.20$	92	2	$2 / 10 = 0.20$	93	3	$3 / 10 = 0.30$	95	2	$2 / 10 = 0.20$	98	1	$1 / 10 = 0.10$	<b>TOTAL</b>	<b>10</b>		<p>For each row, multiply the elements of <math>\bar{x}</math> with <math>P(\bar{x})</math>.</p> <p>Example: <math>91 \times 0.20 = 18.20</math></p> <table border="1"> <thead> <tr> <th><math>\bar{x}</math></th><th><math>P(\bar{x})</math></th><th><math>\bar{x} \cdot P(\bar{x})</math></th></tr> </thead> <tbody> <tr> <td>91</td><td>0.20</td><td>18.20</td></tr> <tr> <td>92</td><td>0.20</td><td>18.40</td></tr> <tr> <td>93</td><td>0.30</td><td>27.90</td></tr> <tr> <td>95</td><td>0.20</td><td>19.00</td></tr> <tr> <td>98</td><td>0.10</td><td>9.80</td></tr> <tr> <td><b>TOTAL</b></td><td><b>1.00</b></td><td><b>93.30</b></td></tr> </tbody> </table> <p><math>\mu_{\bar{x}} = \sum \bar{x} \cdot P(\bar{x})</math></p> <p><math>\mu_{\bar{x}} = 93.30</math></p>	$\bar{x}$	$P(\bar{x})$	$\bar{x} \cdot P(\bar{x})$	91	0.20	18.20	92	0.20	18.40	93	0.30	27.90	95	0.20	19.00	98	0.10	9.80	<b>TOTAL</b>	<b>1.00</b>	<b>93.30</b>
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<table border="1"> <thead> <tr> <th colspan="3">Subtract each element of <math>\bar{x}</math> with <math>\mu_{\bar{x}} = 93.30</math>, to determine each element of <math>\bar{x} - \mu</math>.</th> </tr> <tr> <th><math>\bar{x}</math></th><th><math>\bar{x} - \mu</math></th><th><math>(\bar{x} - \mu)^2</math></th></tr> </thead> <tbody> <tr> <td>91</td><td>-2.30</td><td>5.29 <math>\rightarrow (-2.30)^2</math></td></tr> <tr> <td>92</td><td>-1.30</td><td>1.69 <math>\rightarrow (-1.30)^2</math></td></tr> <tr> <td>93</td><td>-0.30</td><td>0.09 <math>\rightarrow (-0.30)^2</math></td></tr> <tr> <td>95</td><td>1.70</td><td>2.89 <math>\rightarrow (1.70)^2</math></td></tr> <tr> <td>98</td><td>4.70</td><td>22.09 <math>\rightarrow (4.70)^2</math></td></tr> </tbody> </table>			Subtract each element of $\bar{x}$ with $\mu_{\bar{x}} = 93.30$ , to determine each element of $\bar{x} - \mu$ .			$\bar{x}$	$\bar{x} - \mu$	$(\bar{x} - \mu)^2$	91	-2.30	5.29 $\rightarrow (-2.30)^2$	92	-1.30	1.69 $\rightarrow (-1.30)^2$	93	-0.30	0.09 $\rightarrow (-0.30)^2$	95	1.70	2.89 $\rightarrow (1.70)^2$	98	4.70	22.09 $\rightarrow (4.70)^2$	<p>Multiply the corresponding elements of <math>\bar{x}</math> and <math>(\bar{x} - \mu)^2</math> for <math>P(\bar{x}).(\bar{x} - \mu)^2</math>.</p> <table border="1"> <thead> <tr> <th><math>\bar{x}</math></th><th><math>(\bar{x} - \mu)^2</math></th><th><math>P(\bar{x}).(\bar{x} - \mu)^2</math></th></tr> </thead> <tbody> <tr> <td>91</td><td>5.29</td><td>1.06</td></tr> <tr> <td>92</td><td>1.69</td><td>0.34</td></tr> <tr> <td>93</td><td>0.09</td><td>0.03</td></tr> <tr> <td>95</td><td>2.89</td><td>0.58</td></tr> <tr> <td>98</td><td>22.09</td><td>2.21</td></tr> <tr> <td><b>TOTAL</b></td><td></td><td><b>4.21</b></td></tr> </tbody> </table> <p><math>\sigma^2_{\bar{x}} = \sum P(\bar{x}).(\bar{x} - \mu)^2</math></p> <p><math>\sigma^2_{\bar{x}} = 4.21</math></p>	$\bar{x}$	$(\bar{x} - \mu)^2$	$P(\bar{x}).(\bar{x} - \mu)^2$	91	5.29	1.06	92	1.69	0.34	93	0.09	0.03	95	2.89	0.58	98	22.09	2.21	<b>TOTAL</b>		<b>4.21</b>
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If  $\sigma^2_{\bar{x}} = 4.21$ , then what is the value of  $\sigma_{\bar{x}}$ ?



## What's More

**A:** Consider that the population given is normally distributed. Determine  $\mu_{\bar{x}}$ ,  $\sigma^2_{\bar{x}}$  and  $\sigma_{\bar{x}}$ .

1. The population has a mean of 15 and a standard deviation of 4. The sample size of your sampling distribution is 35.
2. Obtained from N (30, 6), the sample size of your sampling distribution is 8.
3. The population has a mean of 115 and a standard deviation of 11. The sample size of your sampling distribution is 30.
4. Consider the scores, in which random samples of size 32 are obtained from N (35,6).
5. The distribution of the scores of the second quarter grades of Grade 11 of Malanday National High School is a population having a mean of 85 and a standard deviation of 12. The sample size of your sampling distribution is 25.



**B:** The sampling distribution of the sample means is given below. Find  $\mu_{\bar{x}}$ ,  $\sigma^2_{\bar{x}}$  and  $\sigma_{\bar{x}}$ .

$\bar{x}$	freq	$P(\bar{x})$	$\bar{x}.P(\bar{x})$	$\bar{x} - \mu$	$(\bar{x} - \mu)^2$	$P(\bar{x}).(\bar{x} - \mu)^2$
75	1					
79	1					
81	2					
83	3					
85	3					
87	4					
92	2					
93	2					
95	1					
98	1					
<b>TOTAL</b>						
$\mu_{\bar{x}} =$		$\sigma^2_{\bar{x}} =$		$\sigma_{\bar{x}} =$		



## What I Have Learned

Fill in the blanks with the appropriate term/s.

If the population is normally distributed, the sampling distribution of the sample mean,  $\bar{x}$ , is normally distributed for any sample size n.

population mean      population standard deviation      population variance

> The mean of the sampling distribution is equal to the \_\_\_\_\_.

> The variance of the sampling distribution is equal to the \_\_\_\_\_ divided by n (sample size).

> The following is equal to

$\mu_{\bar{x}} =$	$\sigma^2_{\bar{x}} =$	$\sigma_{\bar{x}} =$
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## What I Can Do

Given this sample means from the 15 samples taken from the college entrance exam results of a state college, Make your own distribution by assigning corresponding frequency for each sample mean (NOTE: The sum of all the frequency is 15). Then solve for  $\mu_{\bar{x}}$ ,  $\sigma^2_{\bar{x}}$  and  $\sigma_{\bar{x}}$  of the distribution.

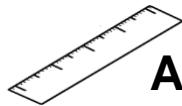
$\bar{x}$	frequency	$P(\bar{x})$	$\bar{x} \cdot P(\bar{x})$	$\bar{x} - \mu$	$(\bar{x} - \mu)^2$	$P(\bar{x}).(\bar{x} - \mu)^2$
55						
60						
67						
75						
89						
<b>TOTAL</b>	<b>15</b>		$\mu_{\bar{x}} =$		$\sigma^2_{\bar{x}} =$	
					$\sigma_{\bar{x}} =$	

### RUBRIC:

POINTS	4	3	2	1
Understands the Problem	Identifies special factors that influences the approach before starting the problem	Understands the problem	Understands enough to solve part of the problem or to get part of the solution	Doesn't understand enough to get started or make progress
Uses Information Appropriately	Explains why certain information is essential to the solution	Uses all appropriate information correctly	Uses some appropriate information correctly	Uses inappropriate information
Applies Appropriate Procedures	Explains why procedures are appropriate for the problem	Applies completely appropriate procedures	Applies some appropriate procedures	Applies inappropriate procedures
Uses Representations	Uses a representation that is unusual in its mathematical precision	Uses a representation that clearly depicts the problem	Uses a representation that gives some important information about the problem	Uses a representation that gives little or no significant information about the problem
Answers the Problem	Correct solution of problem and made a general rule about the solution or extended the solution to a more complicated solution	Correct solution	Copying error, computational error, partial answer for problem with multiple answers, no answer statement, answer labeled incorrectly	No answer or wrong answer based upon an inappropriate plan

source: <https://www.uen.org/rubric/previewRubric.html?id=13>





## Assessment

Read and analyze each item carefully and choose the letter that corresponds to your answer.

1. If the population is normally distributed, the sampling distribution of the sample mean is normally distributed for \_\_\_\_\_ sample size.  
A. any                    B. large                    C. medium                    D. small
2. Given that the population is normally distributed, the mean of the sampling distribution is equal to \_\_\_\_\_.  
A.  $\mu$                     B.  $\sigma$                     C.  $\sigma^2$                     D. n
3. If the sampling distribution of the sample mean was derived from a normally distributed population, which of the following statements is true for variance and standard deviation as the sample size becomes larger?  
A. The variance increases and the standard deviation decreases.  
B. The variance decreases and the standard deviation increases.  
C. The variance and the standard deviation decreases.  
D. The variance and the standard deviation increases.
4. Consider the score examples illustrated, in which random samples of size 16 are obtained from  $N(25,9)$ . What is the value of  $\mu_{\bar{x}}$ ?  
A. 2.25                    B. 5.06                    C. 9                            D. 25

For number 5 & 6, use the data in problem number 4.

5. What is the value of  $\sigma^2_{\bar{x}}$ ? Use the choices on no. 4 problem.
6. Find the value of  $\sigma_{\bar{x}}$ . Use the choices on no. 4 problem.
7. The distribution of the scores of the second quarter grades of Grade 11 of Malanday National High School is a population having a mean of 80 and a standard deviation of 14. The sample size of your sampling distribution is 25. Determine the value of  $\sigma_{\bar{x}}$ .  
A. 2.8                    B. 7.84                    C. 14                            D. 80
8. Consider the scores, in which random samples of size 36 are obtained from  $N(29,5)$ . What is the value of  $\sigma^2_{\bar{x}}$ ?  
A. 0.69                    B. 0.83                    C. 0.90                            D. 0.99
9. In number 8 problem, find the value of  $\sigma_{\bar{x}}$ . Use the choices in number 4 problem.
10. What did you notice to  $\sigma_{\bar{x}}$  in relation with  $\sigma^2_{\bar{x}}$ ? If  $\sigma^2_{\bar{x}}$  is less than 1?  
A.  $\sigma_{\bar{x}} < \sigma^2_{\bar{x}}$                     B.  $\sigma_{\bar{x}} > \sigma^2_{\bar{x}}$                     C.  $\sigma_{\bar{x}} = \sigma^2_{\bar{x}}$                     D.  $\sigma_{\bar{x}} = 0$





## Additional Activities

A. Given the data on population, complete the table below.

Obtained from	n	$\mu_{\bar{x}}$	$\sigma^2_{\bar{x}}$	$\sigma_{\bar{x}}$
N(25, 3)	10			
N(32, 6)	15			
N(49, 5)	25			
N(85, 11)	45			
N(125, 18)	60			

B. Given the distribution of the sample means, solve for  $\mu_{\bar{x}}$ ,  $\sigma^2_{\bar{x}}$  and  $\sigma_{\bar{x}}$ .

$\bar{x}$	frequency	$P(\bar{x})$	$\bar{x}.P(\bar{x})$	$\bar{x} - \mu$	$(\bar{x} - \mu)^2$	$P(\bar{x}).(\bar{x} - \mu)^2$
2.5	1					
3	3					
4.5	4					
6	1					
7.2	1					
TOTAL	10		$\mu_{\bar{x}} =$		$\sigma^2_{\bar{x}} =$	
					$\sigma_{\bar{x}} =$	



## LESSON 2: DEFINING THE SAMPLING DISTRIBUTION OF THE SAMPLE MEAN FOR NORMAL POPULATION WHEN THE VARIANCE IS (A) KNOWN; AND (B) UNKNOWN



### What's In

Give what are asked for in the given items.

Find the area under the standard normal distribution curve.

1. between 0 and 0.55  
A. 0.2054      B. 0.2088      C. 0.2123      D. 0.5000
2. between – 0.54 and 0  
A. 0.2054      B. 0.2088      C. 0.2123      D. 0.5000
3. between -1.3 and 0.41  
A. 0.1591      B. 0.4032      C. 0.5000      D. 0.5623
4. to the left of 1.3  
A. 0.0968      B. 0.4032      C. 0.5000      D. 0.9032
5. to the right of 1.3 (Use the choices on number 4.)

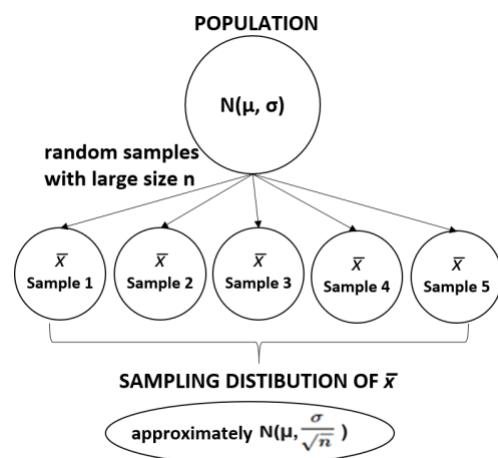


### What's New

#### WHAT IF THE POPULATION IS NOT NORMALLY DISTRIBUTED?

If random samples with **large sample size** will be drawn from a not normally distributed population or any distribution with mean  $\mu$  and standard deviation  $\sigma$ , then the sampling distribution of  $\bar{x}$  will be **approximately** normally distributed with mean  $\mu$  and standard deviation  $\frac{\sigma}{\sqrt{n}}$ .

The general rule is that  $n \geq 30$  is considered as a large sample size.





## What is It

### SAMPLING DISTRIBUTION OF THE SAMPLE MEAN FOR NORMAL POPULATION WHEN THE VARIANCE IS KNOWN

To solve problems involving sampling distribution of the sample mean, we will use a test statistic for the mean of a population ( $\mu$ ) called z – test. It is used:

- when the sample size is equal or greater than 30 ( $n \geq 30$ );
- when the population variance ( $\sigma^2$ ) or the population standard deviation ( $\sigma$ ) is known; and
- when  $n < 30$ , but it is *indicated* that the population is normal and  $\sigma^2$  is known.

The z test formula is

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Where:  $\bar{x}$  is the sample mean       $\sigma$  is the population deviation  
 $\mu$  is the population mean       $n$  is the sample size

**NOTE:** Consider the table below to be not confused.

$Z = \frac{x - \mu}{\sigma}$	This formula is used if we are converting an individual score or observation to its z value.		Each score (x) in the population, has a corresponding z-value.
$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$	This is used to convert a sample mean ( $\bar{x}$ ) to a z test.		The sample yielded from the population will produce a sample mean ( $\bar{x}$ ), which will be converted to a z test value.

**Example 1:** The guidance office personnel of Concepcion Integrated School established that in every week, the average number of Grade 11 students who commit absenteeism is 9 with a standard deviation of 3.55. A sample of 30 students who commit the same offense were tested and resulted an average of 7.5 absences weekly. What is the probability that mean of the sample will be less than 7.5?



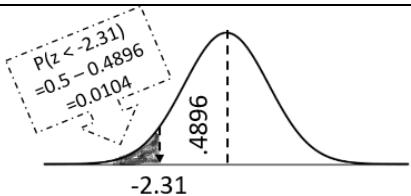
From the given:  
 $n \geq 30$  ✓  
 $\sigma^2$  or  $\sigma$  ✓  
 Conditions were satisfied.

GIVEN:  $\mu = 9$ ;  $\sigma = 3.55$ ;  $n = 30$ ;  $\bar{x} = 7.5$

FIND:  $z$

SOLUTION: 
$$Z = \frac{\bar{x} - \mu}{\sigma} = \frac{7.5 - 9}{\frac{3.55}{\sqrt{n}}} = \frac{-1.5}{\frac{3.55}{\sqrt{30}}} = -2.31$$

Label the curve and determine the probability.



$$\begin{aligned} P(\bar{x} < 7.5) &= P(z < -2.31) \\ &= 0.5 - 0.4896 \\ &= 0.0104 \text{ or } 1.04\% \end{aligned}$$

The chance that the sample of 30 students will commit an average of less than 7.5 absences is 1.04%.

**Example 2:** The distributor of a popular milk tea outlet near a school projected that the students' average consumption of milk tea daily is 417.50 ml with a variance of 576. The owner of the outlet tests the distributor's claim by testing a sample of orders from 31 students, whose mean is 425.72 ml. Determine the probability that students' average consumption of milk tea is greater than 425.72 ml.

From the given:  
 $n \geq 30$  ✓  
 $\sigma^2$  or  $\sigma$  ✓  
 Conditions were satisfied.

GIVEN:  $\mu = 417.50$ ;  $\sigma^2 = 576$ ;  $n = 31$ ;  $\bar{x} = 425.72$

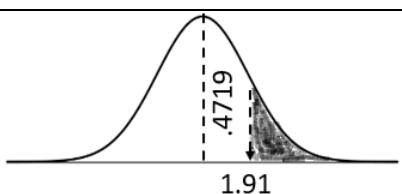
FIND:  $z$

SOLUTION: 
$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{425.72 - 417.50}{\frac{24}{\sqrt{31}}} = 1.91$$

$$\begin{aligned} \sigma^2 &= 576 \\ \sqrt{\sigma^2} &= \sqrt{576} \\ \sigma &= 24 \end{aligned}$$



Label the curve and determine the probability.



$$\begin{aligned}P(\bar{x} > 425.72) &= P(z > 1.91) \\&= 0.5 - 0.4719 \\&= 0.0281 \text{ or } 2.81\%\end{aligned}$$

The probability that the 31 students will have an average consumption more than 425.72 ml is 2.81%.

**NOTE:** When the sample size is equal or greater than 30 ( $n \geq 30$ ) and the population standard deviation ( $\sigma$ ) is NOT known, you may use the sample standard deviation ( $s$ ) for  $\sigma$ . As  $s$  will be a population approximated for  $\sigma$  when  $n \geq 30$ .

**Example 3:** A soda factory claims that each bottle of their beverage drink has a volume of 355 ml all the time. A consumer representative inspector wanted to check this claim. He took a sample of 35 bottles, and found that the mean is 342.25 ml and a standard deviation of 42 ml. What is the probability that the mean of the sample will be less than 342.25 ml?

From the given:

$$n \geq 30 \quad \checkmark$$

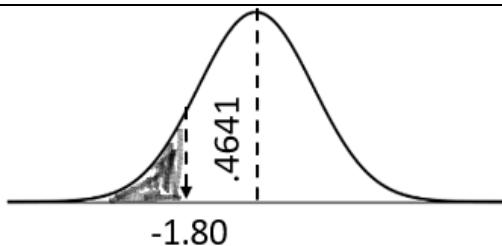
But since  $\sigma$  is NOT given, the value of  $s$  was used for  $\sigma$ .

$$\text{GIVEN: } \mu = 355; \quad n = 35; \quad \bar{x} = 342.25; \quad s = 42$$

$$\text{FIND: } z$$

$$\text{SOLUTION: } z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{342.25 - 355}{\frac{42}{\sqrt{35}}} = -1.80$$

Label the curve and determine the probability.

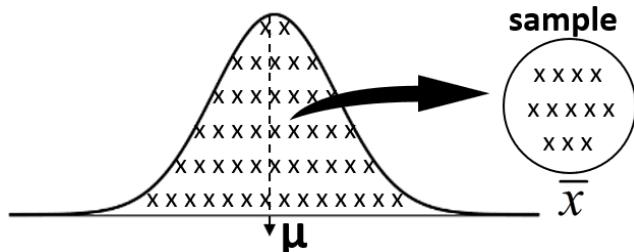


$$\begin{aligned}P(\bar{x} < 342.25) &= P(z < -1.80) \\&= 0.5 - 0.0359 \\&= 0.359 \text{ or } 3.59\%\end{aligned}$$



## SAMPLING DISTRIBUTION OF THE SAMPLE MEAN FOR NORMAL POPULATION WHEN THE VARIANCE IS UNKNOWN

If the variance,  $\sigma^2$ , or standard deviation,  $\sigma$ , is unknown the sample mean,  $\bar{x}$ , cannot be converted to standard normal.



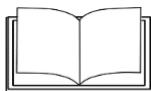
Given the mean,  $\bar{x}$ , and the variance,  $s^2$ , to test if the sample derived from a normal population with known mean  $\mu$  and unknown variance  $\sigma^2$ , the test statistic to be used is  $t$  – test.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Where:  $\bar{x}$  is the sample mean       $s$  is the sample standard deviation  
 $\mu$  is the population mean       $n$  is the sample size

Converting a sample mean  $\bar{x}$  to its  $t$ -test equivalent is as simple as the process of converting it to a  $z$ -test value. The result will be evaluated differently because  $t$ -table will be used instead of the  $z$ -table. Examples on analyzing sampling distribution of the sample mean for normal population when the variance is unknown will be discussed thoroughly on the succeeding topics where the  $t$ -distribution will be discussed.





## What's More

Evaluate the following problems.

1. The average score of all Grade 11, from Concepcion Integrated School, in their assessment in Statistics and Probability is 73.50 with a standard deviation of 24. A sample of 15 students were selected. Assuming that the distribution is normal, what is the probability that the mean of the sample will be greater than 85?
2. A soda factory claims that each bottle of their beverage drink has a volume of 1 L all the time. A consumer representative inspector wanted to check this claim. He tested a sample of 45 bottles, and found that the mean is 0.95 L and a standard deviation of 0.126 L. What is the probability that the mean of the sample will be less than 0.95 L?
3. The average COVID – 19 cases per barangay in a particular city is 25.31 with a standard deviation of 3.87. Assuming a normal distribution, and a sample of 3 barangays were selected, what is the probability that the mean of the sample will be less than 20 cases?
4. The weights of a group of Grade 11 students of Jesus Dela Pena High School are normally distributed with a mean of 58 kg and a standard deviation of 8 kg.
  - (a) If samples of 8 are drawn from this group, how many percent would have means between 50 kg and 65 kg?
  - (b) If a sample of 3 students were gathered, what is the probability that their mean weight is above 70 kg?



## What I Have Learned

- To solve problems involving sampling distribution of the sample mean for normal population when the variance is **known**, we will use a test statistic called \_\_\_\_\_.
- To solve problems involving sampling distribution of the sample mean for normal population when the variance is **unknown**, we will use a test statistic called \_\_\_\_\_.





## What I Can Do

From the period of August 8 to August 13, 2020, the **average** daily percentage of recoveries for each barangay in Marikina was **39.15** and a **standard deviation** of **3.13**. (SOURCE: Marikina PIO)

	Aug 8, 2020	Aug 10, 2020	Aug 11, 2020	Aug 12, 2020	Aug 13, 2020
PATIENTS RECOVERED	466	441	473	476	428
CONFIRMED TOTAL CASES	943	1016	1023	1030	1039
DAILY AVERAGE RATE OF RECOVERY	49.42%	43.41%	46.24%	46.21%	41.19%

Perform the given tasks. Each task satisfied will have a score of 5 and if it is not satisfied the score will be 3.

TASKS	Satisfied	Not Satisfied
1. Formulate a problem where the sample mean and the sample size will yield a z - value between 0 and +3.		
2. Provide a computation in finding the z -value.		
3. Lable the computed z value on the curve.		
4. Determine the probability if it is less than the sample mean.		
5. Determine the probability if it is greater than the sample mean.		

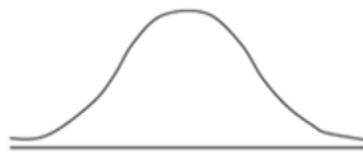


## PRESENTATION:

1. If a sample of \_\_\_\_ days will be selected, what will be the probability that the mean rate will be greater than /less than \_\_\_\_?

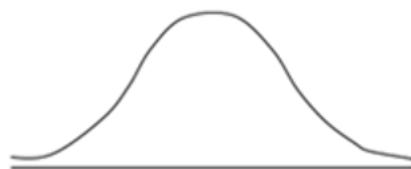
2. Solve for z:

4. Determine the probability if it is less than

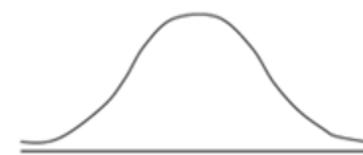


$$P(z < \underline{\hspace{2cm}}) = \underline{\hspace{2cm}}$$

3. Label the z-value on the curve.



5. Determine the probability if it is greater than



$$P(z > \underline{\hspace{2cm}}) = \underline{\hspace{2cm}}$$





## Assessment

Read and analyze each given statement. Figure out the letter of your choice and write it on your answer sheet.

1. What test statistic must be used to evaluate problems involving sampling distribution of the sample mean for normal population when the variance is known?

- A.  $\sigma$       B. t      C.  $\bar{x}$       D. z

2. What test statistic must be used to evaluate problems involving sampling distribution of the sample mean for normal population when the variance is unknown? Use the choices in problem number 1.

3. The guidance office personnel of Concepcion Integrated School established that, in every week, the average number of Grade 11 students who commit absenteeism is 9 with a standard deviation of 3.55. They experimented an intervention program for a sample of 30 students who commit the same offense, what is probability that the mean will be less than 7.5?

- A. 1.04%      B. 48.96%      C. 58.96%      D. 98.96%

4. The average COVID – 19 cases per barangay in a particular city is 25 with a standard deviation of 4. Assuming a normal distribution, and a sample of 3 barangays were selected, what is the probability that the mean of the sample will be greater than 20 cases?

- A. 1.54%      B. 48.46%      C. 58.46%      D. 98.46%

5. The weights of a group of Grade 11 students of Marikina Science High School are normally distributed with a mean of 58 kg and a standard deviation of 8 kg. If a sample of 4 students were gathered, what is the probability that their mean weight is above 65 kg?

- A. 4.01%      B. 20%      C. 45.99%      D. 95.99%





## Additional Activities

Solve.

A wellness clinic claims that their diet and wellness program provides an average loss of 24 pounds in 25 days, having a variance of 20.25. The clinic introduced a new program which strictly monitors the salt, sugar and saturated fat intake. If random samples of 10 patients were tested, what is the probability that the sample mean will be:

- (a) greater than 27 lbs?
- (b) less than 20 lbs?
- (c) between 22 and 26 lbs

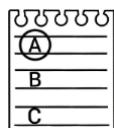


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- <https://www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/more-significance-testing-videos/v/z-statistics-vs-t-statistics>





## Answer Key

What's In	What's More	What I Have Learned	Assessment	Additional Activities
1. B	1. 322%	z - test	1. D	(a) 174%
2. A	2. 0.39%	t - test	2. B	(b) 0.25%
3. D	3. 8.7%		3. A	(c) 84.14%
4. D	4 (a) 99.11%		4. D	
5. A	(b) 4.7%		5. A	

Exercise B				
$\bar{x}$	frequency $P(x)$	$x \cdot P(x)$	$\bar{x} - \mu$	$(\bar{x} - \mu)^2$
TOTAL	10	1.00	$\bar{x} = 4.27$	$\sigma^2_x = 1.98$
7.2	1	0.10	0.72	2.93
6	4	0.40	1.80	1.73
4.5	3	0.30	0.90	2.27
3	2.5	0.25	-1.77	3.13
2.5	1	0.10	-1.77	0.31
				$\mu = 1.41$

Additional Activities - Exercise A		
5. 125	5.4	2.32
4. 85	2.69	1.64
3. 49	1	1
2. 32	2.4	1.55
1. 25	0.9	0.95
2.5	1	0.25
		$\sigma^2_x = 1.41$

Assessment		
6. A	7. A	8. A
1. A	2. A	3. C
		$\sigma^2_x = 5.67$
		$\mu = 5.67$

What I Have Learned				
$\bar{x}$	freq	$P(x)$	$x \cdot P(x)$	$\bar{x} - \mu$
TOTAL	20	1.00	86.55	$\sigma^2_x = 32.15$
98	1	0.05	4.90	11.45
95	2	0.10	9.30	8.45
93	2	0.20	17.40	6.45
92	4	0.15	12.75	5.45
87	3	0.15	12.45	1.55
83	2	0.10	8.10	-3.55
81	1	0.05	3.95	-7.55
79	1	0.05	3.40	-11.55
75	1	0.05	3.75	-11.55
			$\mu = 86.55$	$\sigma^2_x = 32.15$

Lesson 1 What's More - Exercise B						
1. T	6. T	What's In	What's More - Exercise A	What I Know	What's In	What's More
2. F	7. D	1. T	1. 15	0.46	0.68	4. T
3. T	8. A	3. T	2. 30	4.5	2.12	9. B
4. T	9. B	4. F	4. 35	1.13	2.01	10. A
5. F	10. A	5. F	5. 85	5.76	2.4	
6. T						
7. D						
8. A						
9. B						
10. A						



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