9

# Mathematics

## Quarter 1-Module 12

**Solving Quadratic Inequalities**

Week 5

Learning Code - M9AL-Ie-8

ALTERNATIVE DELIVERY MODE

ADM

GOVERNMENT PROPERTY
NOT FOR SALE

Quarter 1 – Module 12 – **New Normal Math for G9**
First Edition 2020


Published by the Department of Education
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Undersecretary: Diosdado M. San Antonio

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# Mathematics

Quarter 1-Module 12

**Solving Quadratic Inequalities**

Week 5

Learning Code - M9AL-Ie-8

DEPARTMENT OF EDUCATION

**MODULE 12**

# SOLVING QUADRATIC INEQUALITIES

Solving quadratic equations was your focus in the previous lessons. But in this module, you will concentrate on finding solutions for quadratic inequalities which will also use the concept of quadratic expression and finding its roots by factoring, completing the square or with the use of the quadratic formula. At the end of this module, it is hoped that you are going to appreciate and utilize this in solving real life problems.

## WHAT I NEED TO KNOW

**LEARNING COMPETENCY**

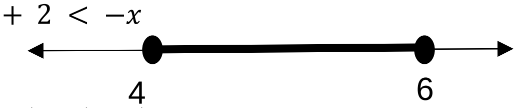
The learners will be able to:

- solve quadratic inequalities .**M9AL-Ie-8**

## WHAT I KNOW

Find out how much you already know about the module. Write the letter that you think is the best answer to each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

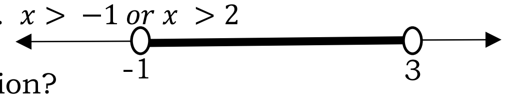
1. A ____________ is an inequality of the form $ax^2 + bx + c < 0$, where a, b and c are real numbers with $a \neq 0$. The inequality symbols >, ≤ and ≥ may also be used.
   A. Quadratic Inequality
   B. Linear Inequality
   C. Inequality
   D. Cubic Inequality
2. Which of the following is NOT example of quadratic inequality?
   A. $x^2 + 4x + 4 \geq 0$
   B. $x^2 - 9 > 0$
   C. $3x + 6 > 0$
   D. $x^2 + 2 < -x$
3. Which of the following is the correct interval notation of the region in the number line at the right?



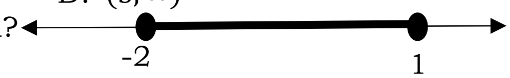

   A. $(4,6)$
   B. $[4,6]$
   C. $(-\infty,4) \cup (6,\infty)$
   D. $(-\infty,4] \cup [6,\infty)$
4. What are the boundary points or critical values of $x^2 - 3x - 4 < 0$?
   A. $-1$ & $4$
   B. $2$ &$1$
   C. $3$ & $4$
   D. $-4,$ & $1$
5. What is the solution set of the quadratic inequality given in number 4?
   A. $[-1,4]$
   B. $(-1,4)$
   C. $(-1,4]$
   D. $(4,2)$

6. $[1, 2]$ is a solution set of quadratic inequality ______________.
   A. $-x^2 + 3x - 2 > 0$
   B. $x^2 + 2x - 3 \geq 0$
   C. $-x^2 + 3x - 2 \geq 0$
   D. $x^2 - 2 \geq 0$

7. Which of the following is the solution set of the inequality $x^2 > x + 2$.
   A. $x < -1$ *or* $x > 2$
   B. $x \geq -1$ *or* $x > 2$
   C. $x < -1$ *or* $x \geq 2$
   D. $x > -1$ *or* $x > 2$




8. Given the number line at the right, which of the following is the interval notation of the region?
   A. $(-1,3)$
   B. $[-1,3]$
   C. $[-\infty, -1)$
   D. $(3, \infty)$




9. What is the interval notation of shaded region?
   A. $(1, -2)$
   B. $[-2, 1]$
   C. $(2, -4)$
   D. $(-2, 1)$

10. What is the solution of the quadratic inequality $x^2 + 4x + 3 \leq 0$?
   A. $[-1, -4]$
   B. $[3, -4]$
   C. $[-1, 6]$
   D. $[-3, -1]$

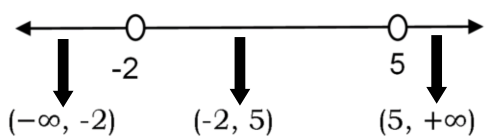
## WHAT'S IN

If a number line is divided into three regions, the two numbers that divide it are the boundaries. If the boundaries are indicated with open circles, then they are not included in the set. If they are indicated with closed circles, then they are part of the set.

**Recall:**

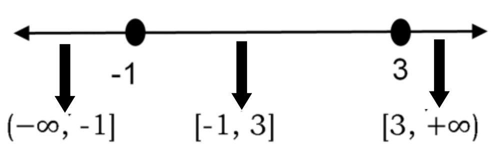
Write the interval notation for each region in the number lines.

1.)



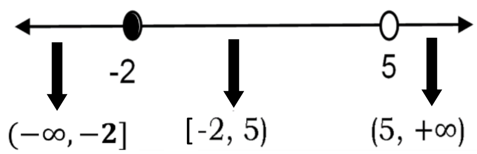

These are the interval notations for each region. We use parenthesis if the boundaries are not included.

2.)




We use brackets to indicate that the boundaries are included. We always use parenthesis for the negative and positive infinities.

3.)




**TRY IT!**

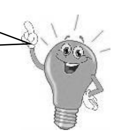
Write the interval notation for each region in the number lines.



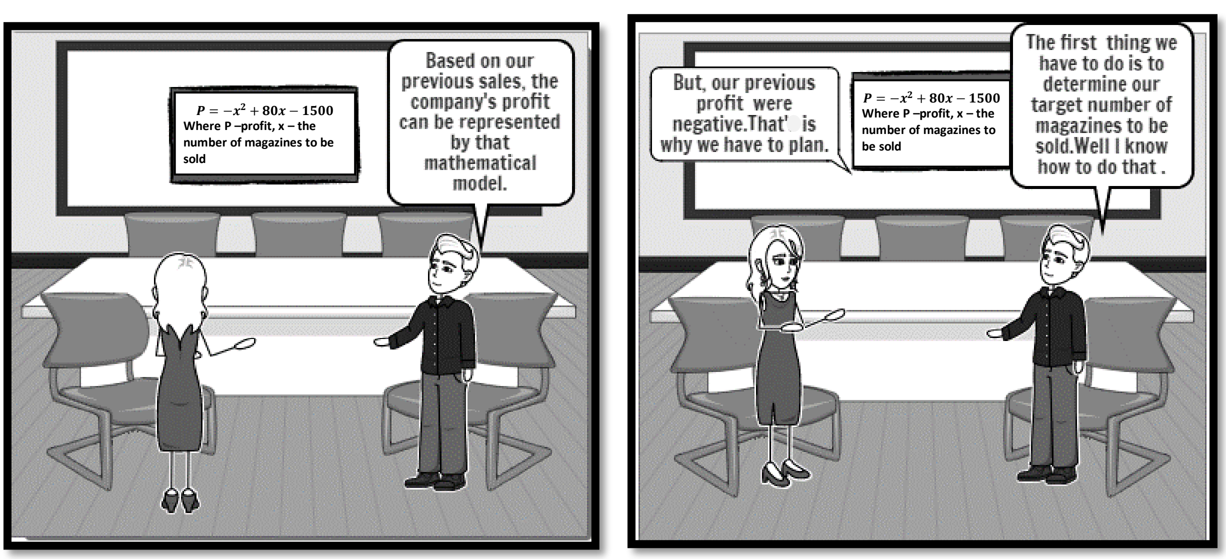


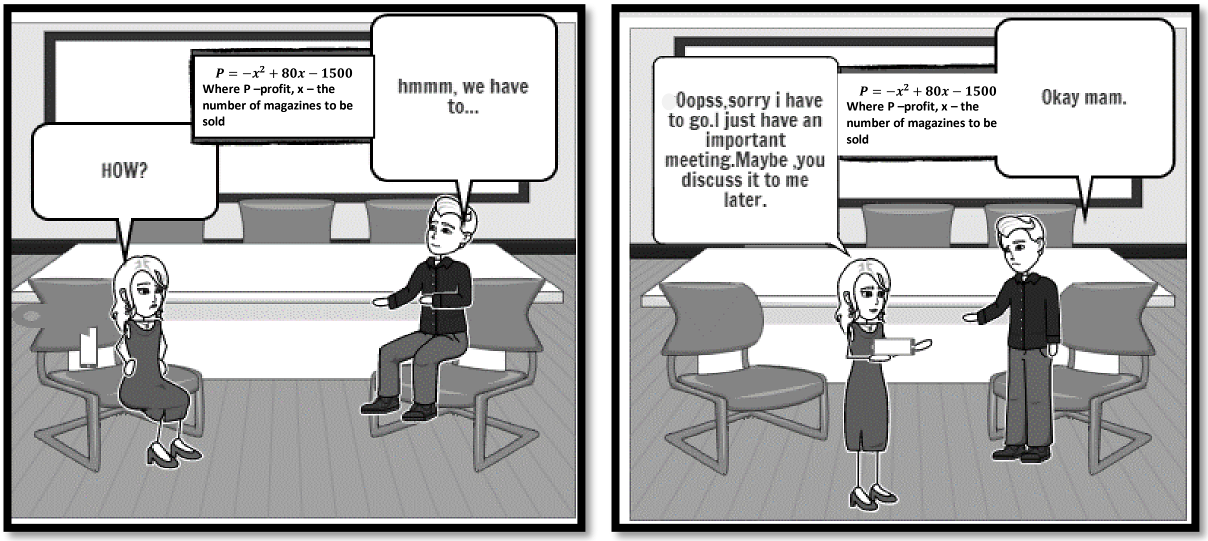
## WHAT'S NEW

Communication, Creativity, Character Building and Collaboration



Read and analyze the conversation of a boss and an employee






1. What do you think would be the suggestion of the employee?
2. Was the action of the boss correct?
3. As a group, make a comic strip having the possible suggestion of the employee on how to get the target number of magazines to be sold in order to have a positive profit.
   **You may use "comic strip maker" online.**

## WHAT IS IT

Communication and Collaboration

**QUADRATIC INEQUALITIES**

A quadratic inequality is an inequality of the form $ax^2 + bx + c < 0$, where a, b and c are real numbers and $a \neq 0$. The inequality symbols >, ≤ and ≥ may also be used.

The boundary or boundaries are determined by solving for the roots of the quadratic equation either by factoring or with the use of the quadratic formula. After finding the roots (or boundaries), it is best to place it on a line graph to easily find the test points.

**Example 1:** Solve the inequality $x^2 + 3x – 4 > 0$.

***Solution:***

***Step 1.*** Express the quadratic inequality as quadratic equation in standard form. Then solve for the roots of this equation. In this case, by factoring.

$$x^2 + 3x - 4 = 0$$
$$(x + 4)\ (x - 1) = 0$$
$$x = -4 \quad or \quad x = 1$$

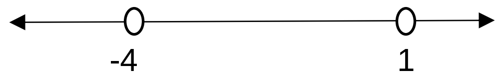
***Step 2***. Place the roots as boundaries by drawing circles on a line graph. The boundaries depend on the inequality symbol used.

Note that when substituting $x = -4$ or $x = 1$ in the inequality, the mathematical statements are false.

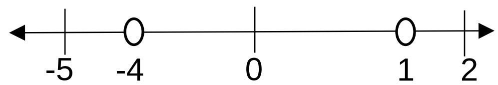
$$(-4)^2 + 3(-4) - 4 > 0 \qquad\qquad (1)^2 + 3(1) - 4 > 0$$
$$16 - 12 - 4 > 0 \qquad\qquad 1 + 3 - 4 > 0$$
$$0 > 0 \quad \text{is FALSE} \qquad\qquad 0 > 0 \quad \text{is FALSE}$$

It means that the roots are not solutions to the inequality.

We use open circles for -4 and 1, because the roots are not part of the solution set.



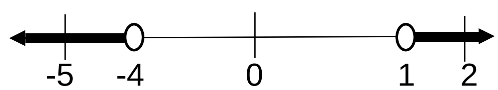

***Step 3***. Take test points in each region of the number line separated by the boundaries. The test points used are preferably the closest integers to the boundaries, and 0.




In this example, we can use $-5$, 0, and 2. Substitute these values to the inequality and determine if the points are included in the solution set.

| | | |
|---|---|---|
| $(-5)^2 + 3(-5) - 4 > 0$<br>$25 - 15 - 4 > 0$<br>$6 > 0$<br>Since $6 > 0$, $x = -5$ is a solution. | $(0)^2 + 3(0) - 4 > 0$<br>$0 + 0 - 4 > 0$<br>$-4 \ngtr 0$<br>Since $-4 \ngtr 0$, $x = 0$ is NOT a solution. | $(2)^2 + 3(2) - 4 > 0$<br>$4 + 6 - 4 > 0$<br>$6 > 0$<br>Since $6 > 0$, $x = 2$ is a solution. |

***Step 4***. If the test point is a solution, shade the region it is located. It means that all numbers in that interval are part of the solution set.




***Step 5***. Write the solution to the inequality using the interval notation.

The solution to the inequality, $x^2 + 3x - 4 > 0$ are all real numbers $x$, where $x < -4$ and $x > 1$. These numbers are in the intervals $(-\infty, -4)$ and $(1, \infty)$. We write the as the union of those sets. Hence, the solution set is $(-\infty, \mathbf{-4}) \cup (\mathbf{1}, +\infty)$.

**Example 2:** Solve the inequality $2x^2 - x \geq 15$

***Solution:***

***Step 1.*** Express as a quadratic equation, then find the roots.

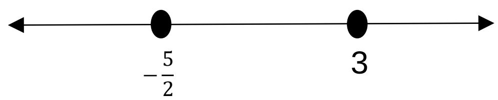
$$2x^2 - x - 15 = 0$$
$$(2x + 5)(x - 3) = 0$$
$$x = -\frac{5}{2} \quad or \quad x = 3$$

***Step 2.*** Draw a line graph with the respective boundaries. We check if the boundaries are solutions to the inequality.

$$2\left(-\frac{5}{2}\right)^2 - \left(-\frac{5}{2}\right) \geq 15$$
$$2\left(\frac{25}{4}\right) + \frac{5}{2} \geq 15$$
$$\frac{25}{2} + \frac{5}{2} \geq 15$$
$$15 \geq 15 \quad \text{is TRUE}$$

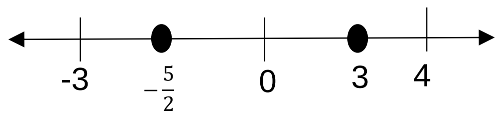
$$2(3)^2 - (3) \geq 15$$
$$2(9) - 3 \geq 15$$
$$18 - 3 \geq 15$$
$$15 \geq 15 \quad \text{is TRUE}$$

Observe that the mathematical statements are true. Thus, the boundaries are included in the solution and we will use closed circles.



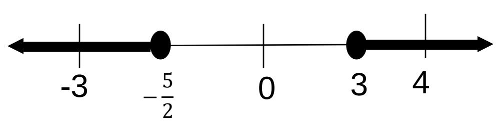

***Step 3.*** Take test points in each region of the number line. Then check of they are solutions to the inequality or not.

For this example, we take $-3$, 0, and 4 as the test points.




| | | |
|---|---|---|
| $2(-3)^2 - (-3) \geq 15$<br>$2(9) + 3 \geq 15$<br>$18 + 3 \geq 15$<br>$21 \geq 15$<br>Since $21 \geq 15$, $x = -3$ is a solution. | $2(0)^2 - (0) \geq 15$<br>$2(0) \geq 15$<br>$0 \ngeq 15$<br>Since $0 \ngeq 15$, $x = 0$ is NOT a solution. | $2(4)^2 - (4) \geq 15$<br>$2(16) + 4 \geq 15$<br>$32 + 4 \geq 15$<br>$36 \geq 15$<br>Since $36 \geq 15$, $x = 4$ is a solution. |

***Step 4.*** Shade the regions on the number line that includes the solution to the inequality.




***Step 5.*** Write the solution in interval notation.

The solution to the inequality $2x^2 - x \geq 15$ is $\left(-\infty, -\frac{\mathbf{5}}{\mathbf{2}}\right] \cup [\mathbf{3}, +\infty)$.

Based on this and the previous example, we can conclude that:

- If the inequality symbol used is $>$ or $<$, we draw open circles.
- If the inequality symbol used is $\geq$ or $\leq$, we draw closed circles.

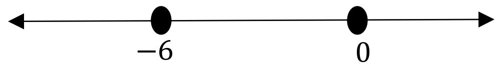
**Example 3:** Solve the inequality $x^2 + 6x \leq 0$.

***Solution:***

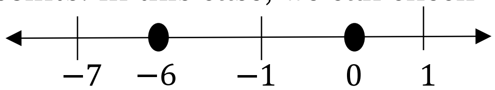
***Step 1.*** Express as a quadratic equation, then find the roots.

$$x^2 + 6x = 0$$
$$x(x + 6) = 0$$
$$x = 0 \quad or \quad x = -6$$

***Step 2.*** Draw a line graph with the respective boundaries. Since the symbol used is $\leq$, we draw closed circles.

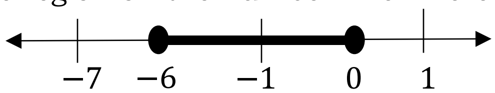



***Step 3.*** Take test points. In this case, we can check $-7$, $-1$, and $1$.




| | | |
|---|---|---|
| $(-7)^2 + 6(-7) \leq 0$<br>$49 - 42 \leq 0$<br>$7 \nleq 0$<br>Since $7 \nleq 0$, $x = -7$ is NOT a solution. | $(-1)^2 + 6(-1) \leq 0$<br>$1 - 6 \leq 0$<br>$-5 \leq 0$<br>Since $-5 \leq 0$, $x = -1$ is a solution. | $(1)^2 + 6(1) \leq 0$<br>$1 + 6 \leq 0$<br>$-5 \nleq 0$<br>Since $7 \nleq 0$, $x = 1$ is NOT a solution. |

***Step 4.*** Identify the region on the number line where the solution is located.




***Step 5.*** Write the final answer in interval notation.

The solution to the inequality $x^2 + 6x \leq 0$ is $[\mathbf{-6, 0}]$.

**Example 4:** Solve the inequality $9 - x^2 > 0$.

Note that this inequality can be expressed as such,

$$-x^2 + 9 > 0$$
$$-(x^2 - 9) > 0$$
$$x^2 - 9 < 0$$

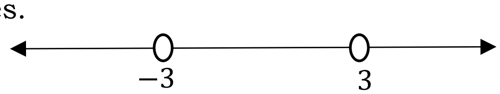
Remember that when multiplying or dividing the whole inequality by $-1$, the symbol used is reversed.

***Solution:***

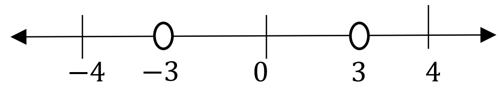
***Step 1.*** Express as a quadratic equation. In this example we can solve the equation by extracting the roots.

$$x^2 - 9 = 0$$
$$x^2 = 9$$
$$\sqrt{x^2} = \sqrt{9}$$
$$x = -3 \quad or \quad x = 3$$

***Step 2.*** Draw a line graph with the respective boundaries. Since the symbol used is $>$ in the original given and $<$ when we rewrote the inequality, we draw open circles.

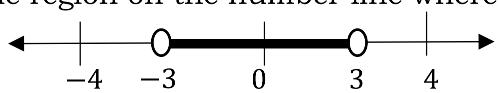


***Step 3.*** Take test points, in this case, $-4$, $0$, and $4$. Determine if they are solutions.



| | | |
|---|---|---|
| $9-(-4)^2>0$<br>$9-16>0$<br>$-7 \ngtr 0$<br>Since $-7 \ngtr 0$, $x=-4$ is NOT a solution. | $9-(0)^2>0$<br>$9>0$<br>Since $9>0$, $x=0$ is a solution. | $9-(4)^2>0$<br>$9-16>0$<br>$-7 \ngtr 0$<br>Since $7 \nleq 0$, $x=4$ is NOT a solution. |

***Step 4.*** Identify the region on the number line where the solution is located.



***Step 5.*** Write the final answer in interval notation.

The solution to the inequality $9-x^2>0$ is $(\mathbf{-3,3})$.

## WHAT'S MORE

In a separate sheet, solve each quadratic inequality. State the solution set using interval notation and sketch its graph if possible.

**Note: Show the step by step procedures.**

1. $x^2+6x+9>0$
2. $x^2+10x+25\le 0$
3. $x^2-2x-8\le 0$
4. $x^2-16<0$
5. $3x-x^2>0$

## WHAT I HAVE LEARNED

A quadratic inequality is any inequality that can be expressed in any of the forms:

$$ax^2+bx+c<0 \qquad ax^2+bx+c>0$$
$$ax^2+bx+c\le 0 \qquad ax^2+bx+c\ge 0$$

where a, b and c are real numbers and $a\neq 0$.

**Steps in Solving a Quadratic Inequality**

1.) Express the quadratic inequality as a quadratic equation in the form of $ax^2+bx+c=0$ and then solve for x

2.) Locate the numbers found in step one on a number line. They serve as the boundary points. The number line will be divided into regions.

3.) Choose one number from each region as a test point. Substitute the test point to the original inequality.

4.) If the inequality holds true for the test point, then that region belongs to the solution set, otherwise, it is not part of the solution set of the inequality.

5.) Write the solution set as interval notation.

## WHAT I CAN DO

Solve the quadratic inequalities and express the solution sets using interval notation.

1.) $-x^2 + 6x + 7 \geq 0$

2.) $2x^2 - 7x + 3 > 0$

3.) $x^2 - 2x - 11 \leq 0$

4.) $x^2 - 2x + 3 > 0$

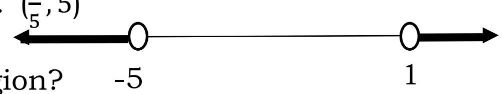
5.) $2x^2 + x - 6 \leq 0$

## ASSESSMENT

Write the letter of the correct answer on your answer sheet. If your answer is not found among the choices, write the correct answer.

1. A _______ inequality is any inequality that can be expressed in any of the forms $ax^2 + bx + c < 0$, $ax^2 + bx + c > 0$, $ax^2 + bx + c \leq 0$, $ax^2 + bx + c \geq 0$, where a, b and c are real number and $a \neq 0$.
   A. Quadratic
   C. Quartic
   B. Linear
   D. Cubic
2. Which of the following is an example of quadratic inequality?
   A. $x + 5x + 9 \geq 0$
   C. $x + 2 > 0$
   B. $x^2 - 16 > 0$
   D. $x^3 + 1 < -x$
3. Which of the following is the correct interval? notation for the shaded region in the number line?


   A. $(-3,4)$
   C. $(-\infty,-3) \cup (4,\infty)$
   B. $[-3,4]$
   D. $(-\infty,-3] \cup [4,\infty)$
4. Which of the following are the boundary points or critical values of $4x^2 < 9$
   A. $-\frac{3}{2}$ & $\frac{3}{2}$
   C. $\frac{3}{2}$ & $\frac{3}{2}$
   B. $\frac{2}{3}$ & $\frac{3}{2}$
   D. $-\frac{3}{2}$ & $2$
5. Referring to the inequality at item #4, which of the following is the solution set?
   A. $(-\frac{3}{2}, \frac{3}{2})$
   C. $(-\frac{3}{2}, 2)$
   B. $[-\frac{3}{2}, \frac{3}{2}]$
   D. $(\frac{1}{2}, \frac{3}{2})$
6. $[-1, \frac{5}{2}]$ is a solution set of quadratic inequality _____________.
   A. $3x^2 + 2x - 5 > 0$
   C. $5 - 2x^2 \geq -3x$
   B. $3x^2 + 2x - 5 \leq 0$
   D. $5 - 2x2 < -3x$
7. Which of the following is the solution set of the inequality $5x^2 + 10 \geq 27x$?
   A. $(-\infty, \frac{2}{5}] \cup [5,\infty)$
   C. $[\frac{2}{5}, 5]$
   B. $(-\infty, \frac{2}{5}] \cap [5,\infty)$
   D. $(\frac{2}{5}, 5)$
8. Given the number line at the right, which of the following is the interval notation for the given region?




   A. $(-5,1)$
   C. $(-\infty,-5) \cup (1,\infty)$
   B. $[-5,1]$
   D. $(-\infty,-5] \cup [1,\infty)$

9. Given the number line at the right, which of the following is the interval notation of shade region?


   A. (2, -8)
   B. (2, 8)
   C. (8, 2]
   D. [2,8]

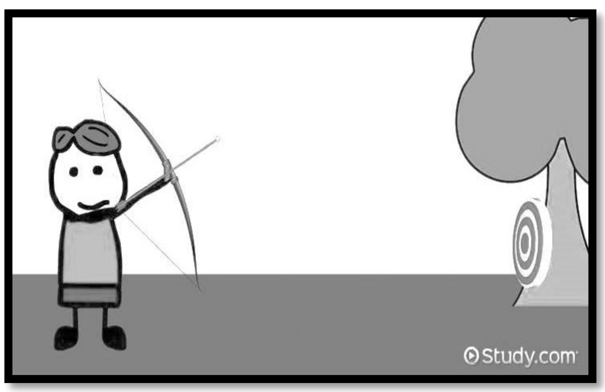
10. What is the solution of the quadratic inequality $4x^2 + 4x + 1 > 0$?
   A. $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$
   B. $(-\infty, -\frac{1}{2}) \cap (-\frac{1}{2}, \infty)$
   C. $(-\frac{1}{2}, 2)$
   D. $(-\infty, -\frac{1}{2}] \cup [-\frac{1}{2}, \infty)$

## ADDITIONAL ACTIVITIES

Critical Thinking and Collaboration

**A. Think it up!**

Kevin shot an arrow straight upward with a velocity of 80 meters per second from an altitude of 20 meters. For how many seconds will this arrow be more than 100 meters high?




(Use the position equation $S = -16t^2 + v_0 t + s_0$)

Where:
S is the altitude after t seconds
$s_0$ is the initial height
$v_0$ is the initial velocity straight upward.

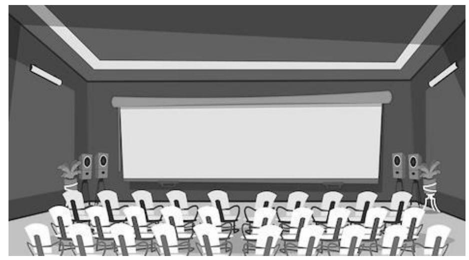
**B. Reflect!**

If you shop on a regular basis in supermarket you will notice that prices vary from one week to another. Why is it so?

## PROBLEM – BASED WORKSHEET

### A. The Conference Hall

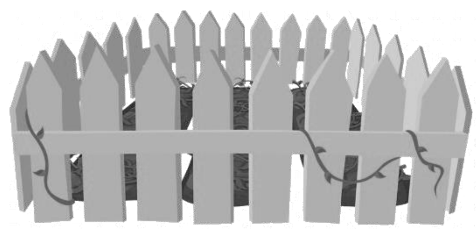
The floor of a conference hall can be covered completely with tiles. Its length is 36 feet longer that its width. The area of the floor is less than 765 sq. feet.



1. If $x$ represents the length of the floor, what mathematical sentence would represent the area of the floor?
2. What is the maximum length of the floor?
3. What is the maximum width of the floor?

### B. Gardening

Angel has 40 meters of metal fencing material to fence three sides of a garden. She wants her garden to have an area of at most 200 square meters. A tall wooden fence serves as her fourth side



1. If y represents the length of one of the two equal sides of the garden, what mathematical sentence would represent the area of the garden?

2. What could be the range of the length of the side opposite the tall wooden fence?

## E-Search

You may also check the following link for your reference and further learnings on solving quadratic equations.

- https://www.youtube.com/watch?v=_gWjLKsFOPE
- https://www.youtube.com/watch?v=1aYMo1NIfFA
- https://www.youtube.com/watch?v=OVEK6JI8Mcc
- https://www.mathsisfun.com/algebra/inequality-quadratic-solving.html
-

## REFERENCES

Dugopolski, Mark.2006.*Elementary and Intermediate Algebra 2nd edition.*MCGraw-Hill.New York City

https://www.mathsisfun.com/algebra/inequality-quadratic-solving.html
E-Math Intermediate Algebra by O. Oronce et.al
Math time 9
https://study.com/academy/lesson/conservation-of-energy-in-projectile-motion-examples-analysis.html
https://www.freepik.com/free-vector/woman-with-long-hair-teaching-online_7707557.htm
https://www.freepik.com/free-vector/kids-having-online-lessons_7560046.htm
https://www.freepik.com/free-vector/illustration-with-kids-taking-lessons-online-design_7574030.htm

## ANSWER KEY

### WHAT'S IN
1. $(-\infty,-10]$ $[-10,1]$ $[1,+\infty)$
2. $(-\infty,4]$ $[4,9]$ $[9,+\infty)$
3. $(-\infty,-3)$ $(-3,6)$ $(6,+\infty)$
4. $(-\infty,-11)$ $(-11,-5)$ $(-5,+\infty)$

### WHAT I CAN DO
1. $[-1,7]$
2. $\left(-\infty,\frac{1}{2}\right) \cup (3,+\infty)$
3. $[1-2\sqrt{3}, 1+2\sqrt{3}]$
4. $(-\infty,+\infty)$
5. $[-2,\frac{3}{2}]$

### WHAT'S MORE
1. All real numbers except $-3$
2. $x=-5$
3. $[-2,4]$
4. $[-4,4]$
5. $(0,3)$

### ADDITIONAL ACTIVITIES

$$\frac{1}{2}(5-\sqrt{5}) < t < \frac{1}{2}(5+\sqrt{5})$$

### WHAT I KNOW

1. A
2. C
3. B
4. A
5. B
6. C
7. A
8. A
9. B
10. D

### ASSESSMENT

1. A
2. B
3. B
4. A
5. A
6. C
7. A
8. C
9. B
10. A

### PROBLEM – BASED WORKSHEET

A. 1. If x is the length of the floor, then, its width is $x - 36$.
Since, most floor of the conference hall are rectangular in form, its area can be determine using the formula; A = (length) (width)

2. The mathematical sentence that will help us solve the problem is,

$$(\boldsymbol{x})(\boldsymbol{x}-\mathbf{36}) < \mathbf{765} \quad \text{or} \quad \boldsymbol{x}^2 - \mathbf{36}\boldsymbol{x} - \mathbf{765} < \mathbf{0}$$

To solve the resulting mathematical sentence, apply the rule on solving quadratic inequality, that is, replacing the inequality symbol by an equal sign to solve for the value of $w$.

$$x^2 - 36x - 765 = 0 \rightarrow (x+15)(x-51) = 0 \rightarrow x = -15 \text{ or } \boldsymbol{x} = \mathbf{51}$$

3. 15 **feet**

B.

1. If y represents one of the two equal sides, the other side is $40 - 2y$ , since 40 meters stands for the perimeter to be used for fencing materials. Thus, the area can be found using the formula; A = (length) (width), and the mathematical sentence that will solve the problem is,

$$(\boldsymbol{y})(\mathbf{40} - \mathbf{2}\boldsymbol{y}) \leq \mathbf{200} \quad \text{or} \quad \boldsymbol{y}^2 - \mathbf{20}\boldsymbol{y} \geq \mathbf{100}$$

2. Solving the inequality for the value of y.

$$y^2 - 20y + 100 = 0 \rightarrow (y-10)(y-10) = 0 \rightarrow \boldsymbol{y} = \mathbf{10}$$

Therefore, the range of the two equal sides is **0 < y < 20** meters, while the range of the **side opposite the tall wooden fence** must be (**0, 40**) meters.