Senior High School

Department of Education
National Capital Region
SCHOOLS DIVISION OFFICE
MARIKINA CITY

GENERAL MATHEMATICS

Quarter 1: Module 3
Inverse Functions

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What I Need to Know

Hello , Grade 11 learners! In this module, you will learn how to :

Represent real-life situations using one-to one function. **M11GM-Id-1**
Determine the inverse of a one-to-one function. **M11GM-Id-2**
Represent an inverse function through its: (a) table of values, and (b) graph. **M11GM-Id-3**
Find the domain and range of an inverse function. **M11GM-Id-4**
Solve problems involving inverse functions. **M11GM-Ie-2**

You can say that you have understood the lesson in this module if you can already:

1. define one-to-one function.
2. determine whether the function is a one-to-one function or not.
3. represent real – life situations using one to one function.
4. create a table of values of an inverse function.
5. sketch/draw its graph of an inverse function.
6. define inverse of one-to-one functions.
7. find the inverse of one-to-one function applying its properties.
8. find the domain and range of an inverse function.
9. solve problems involving inverse functions.

What I Know

Choose the letter that corresponds to the correct answer.

1. How many times a horizontal line test crosses a one-to-one function?
   A. 0 B. 1 C. 2 D. 3
2. The following are one-to-one functions, EXCEPT:
   A. f(x) = 3 B. f(x) = 2x C. f(x) = x + 1D. f(x) = x – 3
3. The following are one-to-one functions, EXCEPT:
   A person to his/her citizenship
   B. Airport to its airport code.
   C. GSIS member to his/her account number
   D. Student name to his/her LRN

4. Find the inverse function of f(x) = $3x^3 - 8$.

A. $f^{-1}(x) = \sqrt[3]{\frac{x+8}{3}}$ C. $f^{-1}(x) = \sqrt[3]{\frac{x-8}{3}}$

B. $f^{-1}(x) = \sqrt[3]{x+8}$ D. $f^{-1}(x) = \sqrt[3]{x-8}$

5. Determine the f^{-1} in the function $f(x) = 1 + \sqrt{x-2}$.

A. $f^{-1}(x) = (x-1)^2 - 2$ C. $f^{-1}(x) = (x-1)^2 - 2$

B. $f^{-1}(x) = (x-1)^2 + 2$ D. $f^{-1}(x) = (x+1)^2 + 2$

6. Which of the following functions has no inverse function?

A. $f(x) = x$ B. $g(x) = x^2$ C. $h(x) = x^3$ D. $p(x) = \sqrt{x}$

7. Which of the following tables of values shows the inverse function of the given ordered pairs. {(-3,-4), (-2,0), (-1,-2), (0,1)}

A.

x	-4	-2	-2	0
y	-3	0	-1	1

B.

x	-4	0	-2	1
y	-3	-2	-1	0

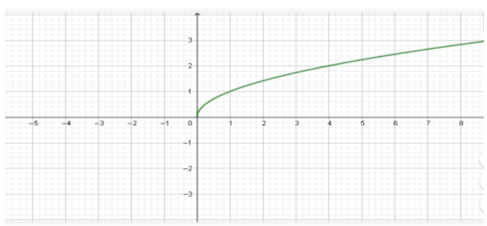
C.

x	-3	-2	-1	0
y	-4	0	-2	1

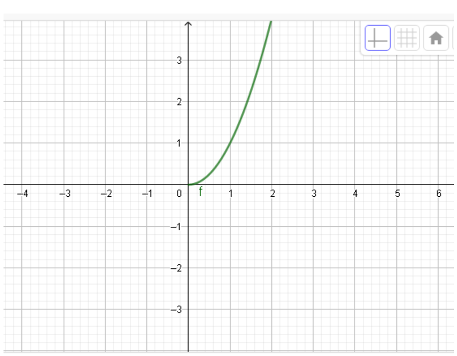
D.

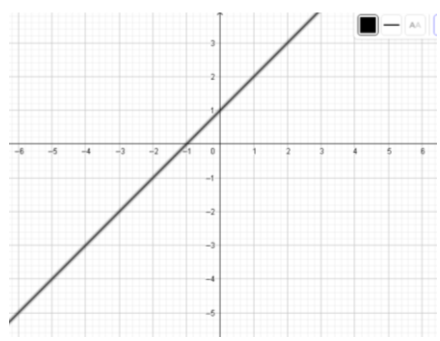
x	-3	-2	-1	1
y	-4	0	-2	0

8. Which graph below shows the inverse of the function $f(x) = \sqrt{x}$?

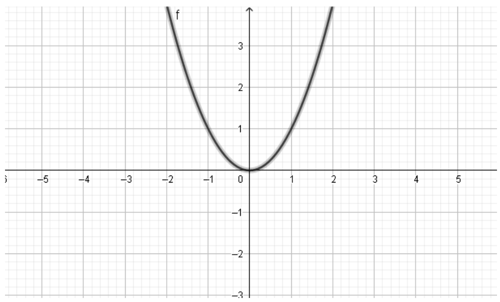


A. B.

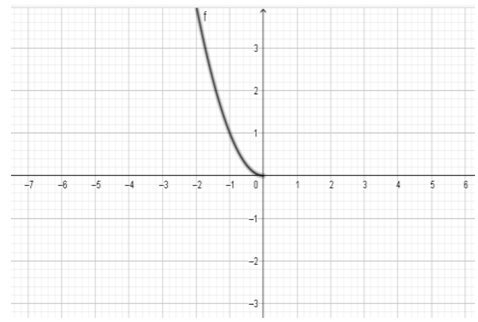




C.



D.



9. Which statement below describes the graph of the function and the graph of its inverse?

A. The graphs are symmetric about the y-axis.
B. The graphs are reflected about the line y = x.
C. The graphs are identical.
D. The graphs are symmetric about the x-axis.

10. Solve the domain of the function g(x) = $\sqrt{x+1}$.

A. [-∞, +∞) B. [1,∞) C. [-1,∞) D. (1,∞)

11. If the domain of f(x) = 2x - 5 is $(-\infty, \infty)$, what is the range of its inverse?

A. [∞,-5) B. [-5,∞) C. (∞,-∞) D. (-∞,∞)

12. If the range of f is (-∞, -3], what is the domain of its inverse?

A. $(-\infty$,-3] B. [-3,-∞) C. $(-\infty$,3) D. (3,∞)

13. The formula to convert degrees Celsius(C) to degrees Fahrenheit (F) is given by the function F =1.8 C + 32. Find the inverse of the function.

A. F^{-1}= 5C-32
C. F^{-1}=C-32$(\frac{9}{5})$
B. F^{-1} = (C-32)$(\frac{5}{9})$
D. F^{-1}=$\frac{5}{9}$(C+32)

14. The formula in getting the Volume (V) of a cube is V = s^3. Find the inverse of the function.

A. $V^{-1} = \sqrt[3]{s}$ B. $V^{-1} = \sqrt[3]{s}$-1 C. $V^{-1} = \sqrt[3]{3s}$ D. $V^{-1} = \sqrt[3]{s} + 3$

15. The formula in getting the circumference C of the circle is C= $2\pi r$, where r is the radius. Find the inverse of the given function.

A. $C^{-1} = \frac{r}{2\pi}$ B. $C^{-1} = \frac{2r}{\pi}$ C. $C^{-1} = \frac{2\pi}{r}$ D. $C^{-1} = \frac{A}{2\pi}$

Lesson 1. Representation of Real Life Situations Using One-to-One Function.

What's In

FACT or **BLUFF**

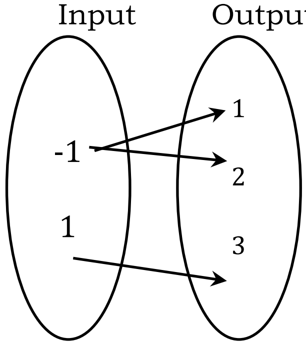
Determine if each statement below is a **Fact** or a **Bluff**. Write your answers on a separate sheet of paper.

1. A function is a rule of correspondence between two non-empty sets; that to each element of the first set corresponds one and only one element of the second set.

2. Each diagram below illustrates a function.

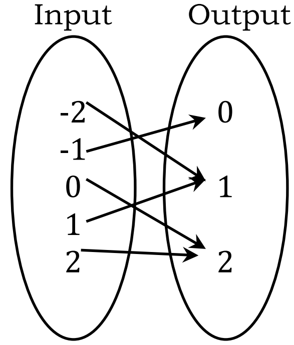
A. One-to-many function

Input Output



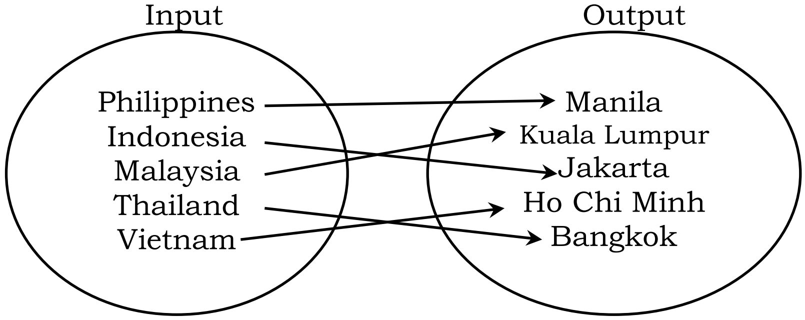
B. Many-to-one function

Input Output



C. One-to-one function

Input Output



3. In the function f(x) = 2x – 3, the value of f(2) = -1.

4. The table of values below is not a function.

Input	-1	1	2	3
Output	4	5	5	4

What's New

Consider the two diagrams below answer the questions that follow.

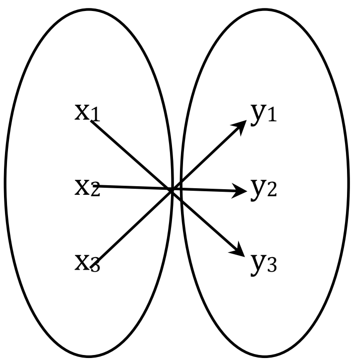


Figure 1

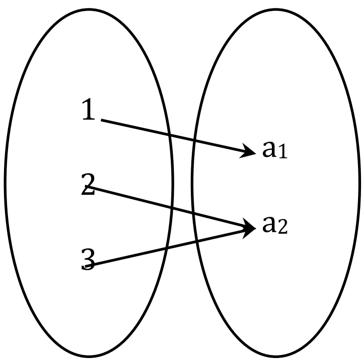


Figure 2

Fill in the table based on the given diagram above.

DIAGRAM	DOMAIN	RANGE	KINDS OF RELATION
1.			
2.			

1. What are the domains and ranges of the two diagrams?
2. Are there similarities and differences between the two diagrams?
3. Do the two diagrams represent a one-to-one function?
4. Can you give a relationship between two variables where each input has a specific output?

What is It

A function is a one-to-one if no two different elements in the domain D have the same element in the range R. The definition of a one-to-one function can be written algebraically as follows:

Let x_1 and x_2 be elements of the domain D.
A function **f** is one-to-one if $x_1 \neq x_2$ implies that $f(x_1) \neq f(x_2)$.

Examples: Determine if each item below represents a one-to-one function.

1.{(0,1), (1,2), (2,3), (3,4)}

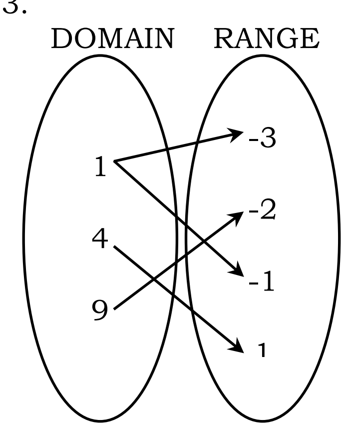
One-to-one function since no two different elements in domain have the same element in the range
.

2.

x	3	2	1	0	-1	-2
y	4	3	2	1	0	-1

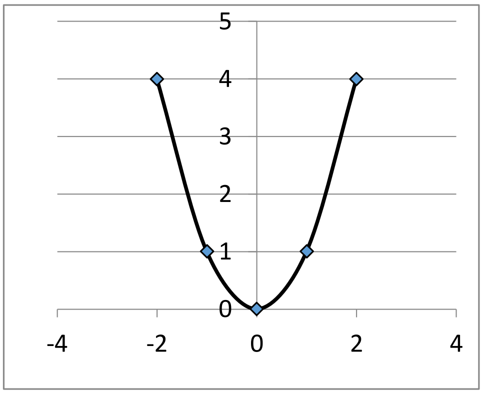
One-to-one function since no two different elements in domain have the same element in the range.

3.



Notice that the domain1 is mapped into -3 and -1 of the range; therefore, it is not a one – to – one function.

4.



Using the horizontal line test, the line intersects the graph more than once, then the function is not a one-to-one; or since two values of x are used to map with one value in y, then the function represented by the graph is not one-to-one.

Note:
Horizontal Line Test:
- If the horizontal line test intersects the graph more than once, then the function is not one-to-one.
- If the horizontal line test intersects the graph of the function at exactly one point, then the function is one-to-one.

Here are some illustrative examples of one-to-one functions in real-life situations.

1: The relation between car and vehicle identification number.
Solution: No two or more cars have the same Vehicle Identification Number.

2: Mapping of names of students from their LRN.
Solution: No two students have the same LRN.

3: The relation pairing a GSIS member to his/her GSIS number.
Solution: Each GSIS member is assigned to a unique GSIS number.

What's More

Determine if the following is a one-to-one function. Write a brief explanation.

x	-3	-2	-1	0	1	2	3
y	-27	-8	-1	0	1	8	27

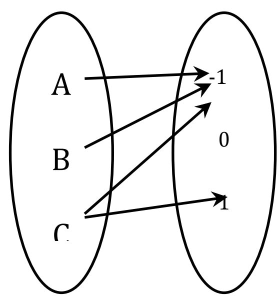
1. The function $f(x) = x^3$

2. The relation between Books in General Mathematics to their authors.

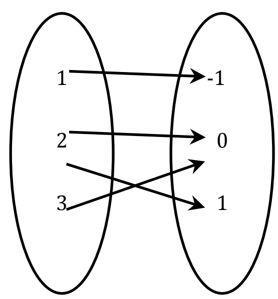
3. A cellphone number belongs to one person.

4. An ID number belongs to one person.

5.



6.



What I Have Learned

Fill in each blank with an appropriate word to complete the paragraph.

A function is a ______1______ if no two different elements in Domain have the same element in Range. A function f(x) is one-to-one if __2__ is not equal to __3__ then $f(x_1)$ is not equal to __4__. If the _______5______ intersects the graph in more than once, then the function is __________6___________.

What I Can Do

Give your own 3 examples of one-to-one functions and explain each why you think it is a one-to-one function. Write your answers in the box.

Rubrics for the Performance Task:

Score	Description
10 points	Three different correct examples and explanations
8 points	Three different correct examples without explanations
5 points	Two different correct examples with explanations
3 points	One exact example with explanation
No point	No output

Assessment

Read each item carefully and choose the letter that corresponds to the exact answer. Write your answer in a sheet of paper.

1. In how many points does a horizontal line test cross the graph of a one-to-one function?

   A. 0 B. 1 C. 2 D. 3

2. Which of the following sets of ordered pairs demonstrates a one-to-one function?

   A. {(1,2),(1,3),(1,4)} C. {(1,5),(2,6),(3,7)}
   B. {(-1,0),(0,-1,(0,1)} D. {(2,4),(3,4),(4,4)}

3. The following are one-to-one functions, EXCEPT:

   A. $f(x) = 3$ B. $f(x) = 2x$ C. $f(x) = x + 1$ D. $f(x) = x - 3$

4. In a basketball team, a player corresponds to his jersey number. This situation illustrates _____________.

A. Exponential Function
B. One-to-one function
C. Not one-to-one function
D. Power function

5. The following statements represent a one-to-one function, EXCEPT

A. A car to its key
B. Pag-Ibig ID number to its member
C. Country to its capital
D. Students to their current ages

Additional Activities

Determine whether the following is a one-to-one function or not. Write your brief explanation.

1. $f(x) = 2x + 1$

Answer: ____.

2.

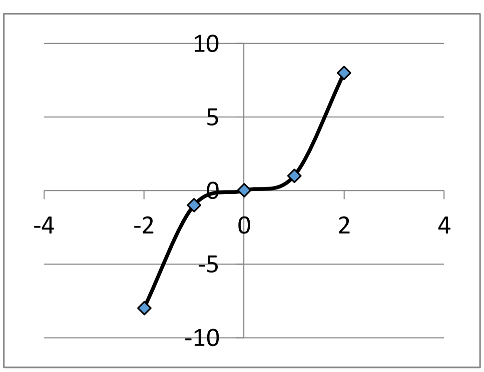
x	1	2	3	4	5
f(x)	2	2	4	4	5

Answer: ____.

3. $y = \{(0,0),(1,-1),(2,-2),(3,-3),(4,-4)\}$

Answer: ____.

4.



Answer: ____.

Lesson 2. Determining Inverse of a One-to-One Function.

What's In

A. State whether each item is a one-to-one function or not.

1. f = {(-1 , 2),(1 ,- 4),(5 , 6),(8 , 6),(9 , -3)}

2. p = {(-1 , 2),(0 , 4),(2 , -4),(3 , 6),(4 , 0)}

3. $f(x) = 2x^3 + x^2 + 2$

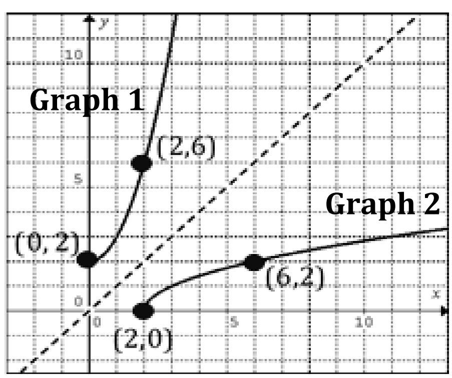
4. $p(x) = 2x + 1$

5. f = {(14 , 2),(15 , 4),(18 , -4),(24 , 6),(78 , 0)}

B. Based on your answers above, when is a function considered one-to-one?

What's New

Consider the given graph below and answer the questions that follow.



(a) What do you notice about the coordinates of the points of the first graph and coordinates of the points of the second graph?
(b) The line drawn in dashes represents the equation $y = x$. What do you notice about the appearance of the two graphs shown on the grid?
(c) Do the two graphs represent one-to-one functions?
(d) How would you be able to "transform" the first graph into the second graph?

What is It

Inverse of One-to-One Functions

Let f be a one-to-one function with domain A and range B. Then the inverse of *f*, denoted by f^{-1}, is a function with domain B and range of A defined by $f^{-1}(y) = x$ if and only if $f(x) = y$ for any y in B.

A function has an inverse function if and only if it is a one-to-one function.

Property of an inverse of a one-to-one function

Given a one-to-one function f(x) and its inverse $f^{-1}(x)$, then the following are true if and only if:

a. the inverse of $f^{-1}(x)$ is f(x).
b. $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}, and
c. $f^{-1}(f(x)) = x$ for all x in the domain of f.

These are the *steps* in finding the inverse of a one-to-one function:

1. Write the function in the form y = f(x);
2. Interchange the x and y variables;
3. Solve for y in terms of x; and
4. Check and verify if $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$.

Example 1. Find the inverse of f(x) = 2x + 1.
Solution:

$f(x) = 2x + 1$	Given
$y = 2x + 1$	Rewrite the function in the form y = f(x)
$x = 2y + 1$	Interchange x and y variables
$x - 1 = 2y$	Solve for y in terms of x applying APE
$\frac{x-1}{2} = y$ or $y = \frac{x-1}{2}$	Using MPE

Thus, the inverse of f(x) = 2x + 1 is $f^{-1}(x) = \frac{x-1}{2}$.

Checking: f(x) = 2x + 1 and $f^{-1}(x) = \frac{x-1}{2}$

$(f \circ f^{-1})(x) = f(f^{-1}(x))$	$(f^{1} \circ f)(x) = f^{-1}(f(x))$
$= f\left(\frac{x-1}{2}\right)$	$= f^{-1}(2x + 1)$
$= \cancel{2}\left(\frac{x-\cancel{1}}{\cancel{2}}\right) + \cancel{1}$	$= \frac{\cancel{2}x+\cancel{1}-\cancel{1}}{\cancel{2}}$
$= x$	$= x$

Thus, the composition of a one-to-one function f and f^{-1} always result to the identity function $(f \circ f^{-1})(x) = x$ and $(f^{1} \circ f)(x) = x$

Example 2: Find the inverse of $p(x) = x^3 - 5$.

Solution:

$p(x) = x^3 - 5$	Given
$y = x^3 - 5$	Rewrite the function in the form y = f(x)
$x = y^3 - 5$	Interchange x and y variables
$x + 5 = y^3$	Solve for y in terms of x applying APE
$y = \sqrt[3]{(x+5)}$	

Thus, the inverse of $p(x) = x^3 - 5$ is $p^{-1}(x) = \sqrt[3]{(x+5)}$.

Checking: $f(x) = x^3 - 5$ and $f^{-1}(x) = \sqrt[3]{(x+5)}$

$(f \circ f^{-1})(x) = f(f^{-1}(x))$	$(f^{1} \circ f)(x) = f^{-1}(f(x))$
$= f\left(\sqrt[3]{(x+5)}\right)$	$= f^{-1}(x^3 - 5)$
$= \left(\sqrt[\cancel{3}]{(x+5)}\right)^{\cancel{3}} - 5$	$= \sqrt[3]{(x^3 - \cancel{5} + \cancel{5})}$
$= x + \cancel{5} - \cancel{5}$	$= \sqrt[\cancel{3}]{x^{\cancel{3}}}$
$= x$	$= x$

Thus, the composition of a one-to-one function f and f^{-1} always result to the identity function $(f \circ f^{-1})(x) = x$ and $(f^{1} \circ f)(x) = x$

Example 3: Find the inverse function of $f(x) = \frac{2}{x-3}$.

Solution:

$f(x) = \frac{2}{x-3}$	Given
$y = \frac{2}{x-3}$	Rewrite the function in the form y = f(x)
$x = \frac{2}{y-3}$	Interchange x and y variables
$(y-3)(x) = \frac{2}{y-3}(y-3)$	Multiply both side by y – 3
$xy - 3x = 2$	Solve for y in terms of x applying APE
$xy = 3x + 2$	Multiply both side by $\frac{1}{x}$
$y = \frac{3x+2}{x}$	

Thus, the inverse function of $f(x) = \frac{2}{x-3}$ is $f^{-1}(x) = \frac{3x+2}{x}$.

Checking: $f(x) = \frac{2}{x-3}$ and $f^{-1}(x) = \frac{3x+2}{x}$

$(f \circ f^{-1})(x) = f(f^{-1}(x))$	$(f^{1} \circ f)(x) = f^{-1}(f(x))$
$= f\left(\frac{3x+2}{x}\right)$	$= f^{-1}\left(\frac{2}{x-3}\right)$
$= \frac{2}{\frac{3x+2}{x}-3}$	$= \frac{3\left(\frac{2}{x-3}\right)+2}{\frac{2}{x-3}}$
$= 2 \cdot \frac{x}{\cancel{3x}+2-\cancel{3x}}$	$= \frac{\cancel{6}+2x-\cancel{6}}{\cancel{x-3}} \cdot \frac{\cancel{x-3}}{2}$
$= \frac{\cancel{2}x}{\cancel{2}}$	$= \frac{\cancel{2}x}{\cancel{2}}$
$= x$	$= x$

Thus, the composition of a one-to-one function f and f^{-1} always result to the identity function $(f \circ f^{-1})(x) = x$ and $(f^{1} \circ f)(x) = x$

What's More

A. State if the functions are inverses.

1. $f(x) = 2x$
   $g(x) = \frac{1}{2}x$
   ____

2. $g(x) = 3 + \frac{3}{2}x$
   $h(x) = \frac{2x-6}{3}$
   ____

3. $u(x) = \frac{x+2}{x+3}$
   $v(x) = \frac{-3x+2}{x-1}$
   ____

B. Find the inverse of each function.

1. $f(x) = \sqrt{x} - 5$ ________________
2. $g(x) = \frac{1}{x} - 3$ ________________
3. $h(x) = 2x^3 - 1$ ______________

What I Have Learned

Fill in each blank with the appropriate term or expression .

From the definition of one-to-one function, we can write that a given function f(x) is one-to-one if ____________1______________. The contrapositive from the definition is if f(A) = _____2_______ then ___3______. The inverse function of f is denoted by __4____, is a function with ______5_______ and range A defined by $f^{-1}(y)$ = __6__ if and only if _______7_______ for any y in B.

List the steps in finding the inverse of a function.

1. Write the function ___________8________________.
2. ___________9___________variables.
3. Solve for ________________10________________________.
4. The property of an inverse of a one-to-one function

Given a one-to-one function f(x) and its inverse _________11_________, then the following are true:

A. The inverse of $f^{-1}(x)$ ___12__________.

B. _________13_______ = x for all in the ________14______________.

C._________15_____________ = x for all x in the domain of f.

What I Can Do

Solve the following:

1. What is the inverse of f(x) = $\sqrt[4]{2x}$?

2. Find f(x) if $f^{-1}(x) = \frac{1}{x-2}$.

Rubrics for problem solving

Score	Description
15 points	Complete solutions and correct answer
10 points	Incomplete solutions and correct answer
5 points	Incomplete solutions and incorrect answer
No point earned	No output at all

Assessment

Choose the letter that corresponds to the correct answer.

1. What is the inverse function of p(x) = $x^3 - 2$.

A. $p^{-1}(x) = \sqrt[3]{x+2}$ C. $p^{-1}(x) = \sqrt[3]{x-2}$

B. $p^{-1}(x) = \frac{2}{x-2}$ D. $p^{-1}(x) = \frac{x^3}{2}$

2. Find the inverse function of f(x) = $\sqrt{x+2}$.

A. $f^{-1}(x) = x - 2$ C. $f^{-1}(x) = x^2 - 2$

B. $f^{-1}(x) = x^2 + 2$ D. $f^{-1}(x) = 2x - 1$

3. Which of the following is true on the properties of f and its inverse function?

A. The domain of f^{-1} is the range of g. C. The range of f^{-1} is f

B. The domain of f^{-1} is the range of f. D. $(f \circ f^{-1})(x) \neq (f^{-1} \circ f)(x)$

4. Find the inverse of the function f(x) = x – 5.

A. $f^{-1}(x) = -x - 5$ C. $f^{-1}(x) = x - 5$

B. $f^{-1}(x) = x + 5$ D. $f^{-1}(x) = \frac{x}{5}$

5. Evaluate $g^{-1}(2)$ from $g^{-1}(x) = x + 2$.

A. 1 B. 2 C. 3 D. 4

Additional Activities

Find the inverse function of the following.

1. f(x) = x – 6

2. g(x) = $\sqrt{2x}$

3. h(x) = $\frac{x+2}{x-2}$

Lesson 3. Represent Inverse Function Through Its Table of Values and Graph

What's In

A. State if the two functions are inverses.

1. $g(x) = \frac{3}{-x-5}$

   $h(x) = -\frac{3}{x} - 5$

2. $f(x) = 2 + (x - 2)^3$

   $g(x) = \sqrt[3]{x-2}$

B. Find the inverse of each function.

1. $f(x) = 5x - 2$
2. $g(x) = -2x + 1$

What's New

Given the table of values below, answer the questions that follow.

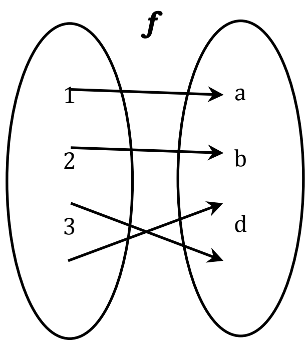
x	-1	0	1	2	3
y	0	1	2	3	4

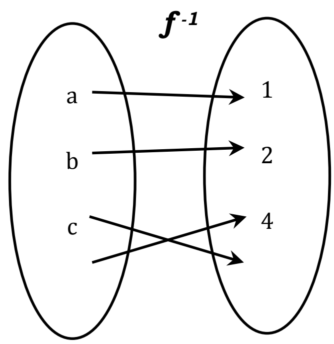
x	0	1	2	3	4
y	-1	0	1	2	4

1. What have you noticed on the tables of values shown above?
2. If you are to plot the points or the coordinates from the table of values in one Cartesian plane, what inferences can you make?
3. If we sketch the graphs of the two tables of values, how would they look like?

What is It

Inverse functions are functions that "reverse" each other.
In the given example below, function f takes 1 to a, 2 to b, 3 to c, and 4 to d.





The inverse function f which is denoted by f^{-1} will reverse this mapping from the diagram. Function f^{-1} takes a to 1, b to 2, c to 3, and d to 4.

From this, we could say that the formal definition of inverse functions is

$$\mathbf{f(a) = b \Leftrightarrow f^{-1}(b) = a}$$

Let's go deeper by working and analyzing the illustrative examples below.

Example 1: Given the function f(x) = x + 1, construct the table of values for f and f^{-1}, then sketch the graph of the function and the inverse function. The assigned values of x = -1, 0, 1, 3, and 5.

Solution:
To construct the table of values, evaluate the assigned values of x in the function f(x) = x+1

x = -1	x = 0	x = 1	x = 3	x = 5
f(-1) = x+ 1	f(0) = x+ 1	f(1) = x+ 1	f(3) = x+ 1	f(5) = x+ 1
f(-1) = -1+1	f(0) = 0+1	f(1) = 1+1	f(3) = 3+ 1	f(5) = 5+ 1
f(-1) = 0	f(0) = 1	f(1) = 2	f(3) = 4	f(5) = 6
(-1,0)	(0,1)	(1,2)	(3,4)	(5, 6)

x	-1	0	1	3	5
f(x)	0	1	2	4	6

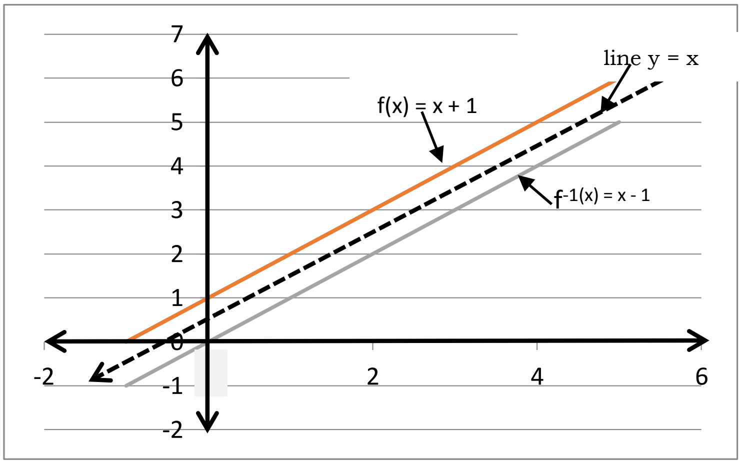
f =

Now, construct the table of values of the inverse of the function. Reverse the input and the output.

x	0	1	2	4	6
$f^{-1}(x)$	-1	0	1	3	5

f^{-1} =

To sketch the graph of the function and the inverse, simply plot the coordinates to the Cartesian plane.



Note: The graphs of f and f^{-1} are symmetric with respect to the line y = x.

Example 2: Given the function $g(x) = x^2 - 4$, complete the table of values below and sketch the graph of function g and g^{-1}.

x	0	1	2	3	4
g(x)					

g =

Solution:
To complete the table of values, evaluate the assigned values of x in the function $g(x) = x^2 - 4$.

x = 0	x = 1	x = 2	x = 3	x = 4
$f(0) = x^2 - 4$	$f(1) = x^2 - 4$	$f(2) = x^2 - 4$	$f(3) = x^2 - 4$	$f(4) = x^2 - 4$
$f(0) = (0)^2 - 4$	$f(1) = (1)^2 - 4$	$f(2) = (2)^2 - 4$	$f(3) = (3)^2 - 4$	$f(4) = 4^2 - 4$
$f(0) = -4$	$f(1) = -3$	$f(2) = 0$	$f(3) = 5$	$f(4) = 12$
(0,-4)	(1, -3)	(2, 0)	(3,5)	(4, 12)

x	0	1	2	3	4
g(x)	-4	-3	0	5	12

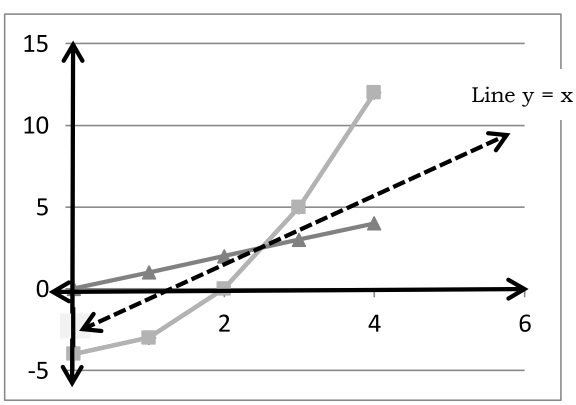
g =

Now, construct the table of values of the inverse of the function. Reverse the input and the output

x	-4	-3	0	5	12
g(x)	0	1	2	3	4

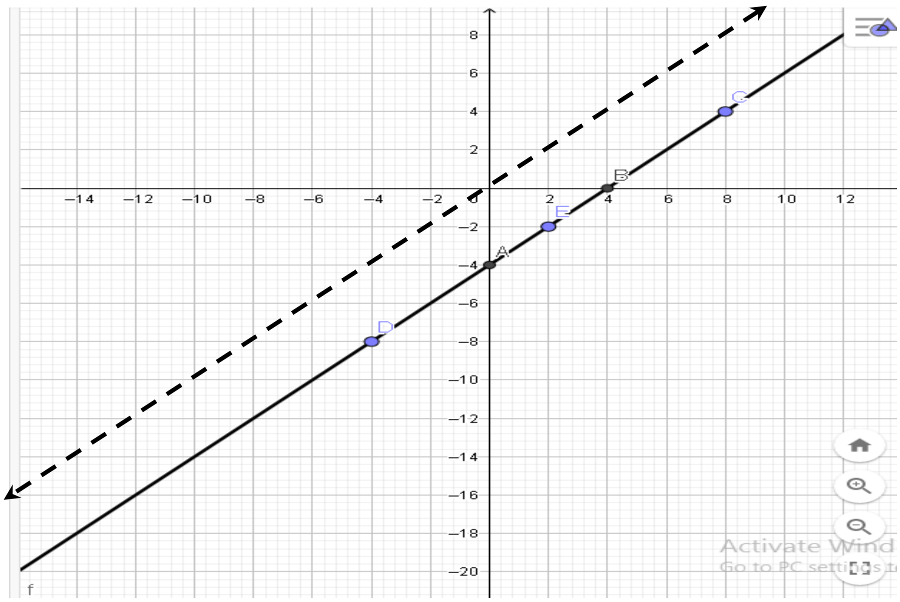
g^{-1} =

To sketch the graph of the function and the inverse, simply plot the coordinates to the Cartesian plane.



Note: the graphs of f and f^{-1} are symmetric with respect to the line y = x.

Example 3: Consider the function f(x) = x + 4 in the graph shown below. Graph the inverse of f using the line y = x provided below.



What's More

Find the inverse of each function and complete the table of values.

x					
y					

1. f(x) = 3 – 5x $\qquad$ $f^{-1}(x)$ = ________________

x	-2	-1	0	1	2
y					

2. g(x) = 2x + 4 $\qquad$ $g^{-1}(x)$ = ________________

x	1	3	5	7	9
y					
x					
y					

3. Graph the functions f(x) = 3x – 1 and $f^{-1}(x) = \frac{x-1}{3}$.

What I Have Learned

Do the following.

1. Given the set of ordered pairs g = {(1,2),(2,3),(3,4),(4,5)} , construct a table of values of g^{-1}.

2. From the definition of inverse function, determine the inverse of **f(a) = b**⇔ ____________.

3. Give the inverse function of the table of values given below.

x	-1	0	1	2
f(x)	3	5	7	9

What I Can Do

A function D(t) given below is showing the distance in kilometers a Grab taxi has traveled in t minutes.

t(time)	20	40	60	80
D(t) (kilometer)	10	30	50	70

1. Find $d^{-1}(70)$.
2. In one coordinate plane, sketch the graphs of function D(t) and D^{-1} then, indicate the line of reflection.

Assessment

1. Which of the following table of values represent f(x) = 2x +1 if the values of x are -2,-1,0,1,2?

A.

x	-2	-1	0	1	2
f(x)	0	1	2	3	4

B.

x	0	1	2	3	4
f(x)	-2	-1	0	1	2

C.

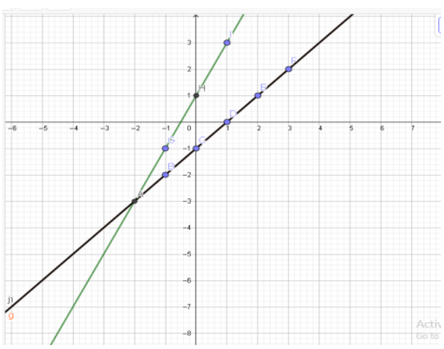
x	-3	-1	1	3	5
f(x)	-2	-1	0	1	2

D.

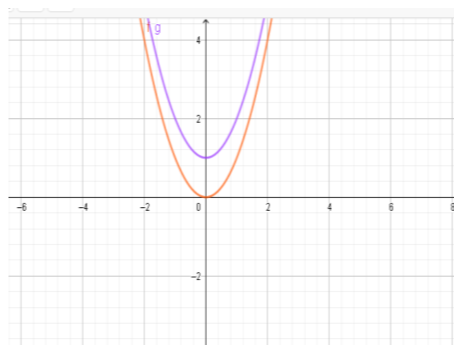
x	-2	-1	0	1	2
f(x)	-3	-1	1	3	5

2. Which of the following graphs represents the function f(x) = 2x+1 and its inverse?

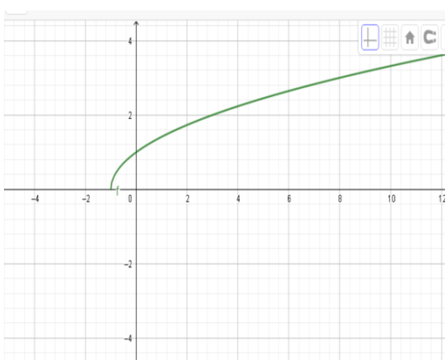
A.



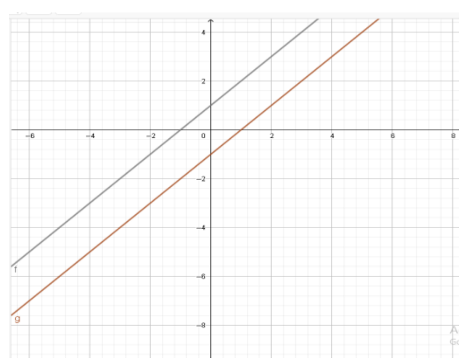
B.



C.



D.



For items 3-5, refer to the table of values below.

x	-2	-1	0	1	2
g(x)	0	1	3	5	7

3. Determine the domain of $g^{-1}(x)$?
   A. {-2,-1,0,1,2}   B. {0,1,3,5,7}   C.{-1,0,3,5}   D.{1,2,5,7}

4. What is the range of $g^{-1}(x)$?
   A. {-2,-1,0,1,2}   B. {0,1,3,5,7}   C.{-1,0,3,5}   D.{1,2,5,7}

5. If g(x) = 7 when x = 2, what is $g^{-1}(7)$?
   A. -1   B. 0   C. 1   D. 2

Additional Activities

A. Find the inverse of the following functions. Construct the table of values and sketch the graph of the given function and its inverse. You may assign your own values of x.

1. f(x) = 3x $f^{-1}(x)$ = ______________________

x					
f(x)					
x					
f(x)					

2. f(x) = $\frac{3x}{5}$ $f^{-1}(x)$ = ______________________

x					
f(x)					
x					
f(x)					

Lesson 4. The Domain and Range of An Inverse Function.

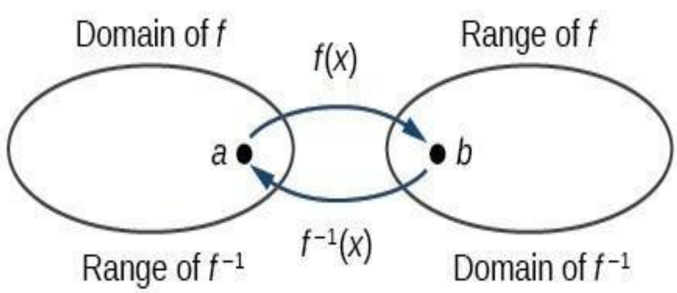
What's In

Given the functions $F(x) = 4x + 1$ and $G(x) = 4x - 1$, perform and answer the following:

A. Construct a table of values for function F and G.
B. Sketch the graph of the function F and G.
C. Are the two functions inverses of each other? Why?

What's New

Consider the illustration below and answer the questions that follow.



1. In f(x), what is its domain? Range?
2. In $f^{-1}(x)$, what is its domain? Range?
3. If the range of f(x) is the domain of $f^{-1}(x)$, what is the domain of f(x)
4. If the range of $f^{-1}(x)$ is the domain of f(x). What is the domain of $f^{-1}(x)$?
5. What did you observe about the domain and range of a function and its inverse?

What is It

Definition: The <u>range</u> of the function *f* is the <u>domain of *f*$^{-1}$</u>. Likewise, the <u>domain</u> of f is the <u>range of f^{-1}.</u>

Properties of a function and its inverse:
1. The domain of f^{-1} is the range of f.
2. The range of f^{-1} is the domain of f.
3. The graphs of f and f^{-1} are symmetric with respect to the line y = x.
4. $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$ for all $x \in$ Dom (f), $x \in$ Dom (f^{-1})

For better understanding, more examples are presented below.

Example 1: Find the domain and range of the two functions.
$f(x) = x + 3$ and $f^{-1}(x) = x - 3$

Solution: Since $f(x) = x + 3$ and $f^{-1}(x) = x - 3$ are both linear functions, then the domain and range are the set of all real numbers. In symbol,

$f(x) = x + 3$	and	$f^{-1}(x) = x - 3$
Domain:$\{x/x \in R)\}$		Domain:$\{x/x \in R\}$
Range:$\{y/y \in R\}$		Range: :$\{y/y \in R\}$

Example 2: Find the domain and range of the inverse of the function $g(x) = \sqrt{x-2}$.

Solution:
To find the inverse of $g(x) = \sqrt{x-2}$

$g(x) = \sqrt{x-2}$	Given
$x = \sqrt{y-2}$	Interchange x to y
$x^2 + 2 = y$	Solve y in terms of x by squaring both side.
$y = x^2 + 2$ or $g^{-1}(x) = x^2 + 2$	

Thus, the $g^{-1}(x) = x^2 + 2$.

To Solve the domain of g(x) = $\sqrt{x-2}$, solve for x – 2 $\geq$ 0.

g(x) = $\sqrt{x-2}$,	Given
x – 2 $\geq$ 0	Solve x in the radicand by equating to greater than or equal to zero
x $\geq$ 2	

The domain of g(x) is [2,∞) while the range of g(x) is the domain of $g^{-1}(x)$ which is $[0,\infty)$. We have just considered half of the parabola above the x-axis since the function g(x) = $\sqrt{x-2}$ refers to the principal root.

Therefore, the Range of g(x) is $[0,\infty)$.

The domain of $g^{-1}(x)$ is $[0,\infty)$ while the range of $g^{-1}(x)$ which is [2,∞). In symbol,

g(x) = $\sqrt{x-2}$	$g^{-1}(x) = x^2 + 2$
Domain:{x/x$\geq$ 2}	Domain: {x/x $\geq$ 0}
Range:{y/y$\geq$ 0}	Range: {y/y$\geq$ 2}

Example 3: Given f(x) = $\frac{1}{x+2}$, whose domain and range are $(-\infty,-2) \cup (-2,+\infty)$ and $(-\infty,0) \cup (0,+\infty)$ respectively,
a. Determine its inverse function.
b. Determine the domain and range of f^{-1}.

To find the inverse of the function f,
Solution:
a.

f(x) = $\frac{1}{x+2}$	Given
y = $\frac{1}{x+2}$	Rewrite the function in the form y = f(x)
x = $\frac{1}{y+2}$	Interchange x and y variables
x(y+2) = 1	Solve for y in terms of x applying MPE
y = $\frac{1}{x} - 2$ or $f^{-1}(x) = \frac{1}{x} - 2$	

b.
Since the domain and range of f is given, to determine the domain and range of f^{-1}, we will apply the properties of a function and its inverse stated below.
1. The domain of f^{-1} is the range of f.
2. The range of f^{-1} is the domain of f.

$f(x) = \frac{1}{x+2}$	$f^{-1}(x) = \frac{1}{x} - 2$
Domain: $(-\infty,-2) \cup (-2,\infty)$	Domain: $(-\infty,0) \cup (0,\infty)$.
Range: $(-\infty,0) \cup (0,\infty)$.	Range: $(-\infty,-2) \cup (-2,\infty)$

Therefore, the domain f^{-1} is $(-\infty,0) \cup (0,\infty)$ and the range of f^{-1} is $(-\infty,-2) \cup (-2,\infty)$

What's More

Find the domain and range of the following functions. Then find the inverse function and find its domain and range.

1. $f(x) = x + 2$
   Domain of f :______________
   Range of f : ______________
   $f^{-1}(x)$ = _________________
   Domain of f^{-1}:_____________
   Range of f^{-1}: _____________

2. $g(x) = \sqrt{x-5}$
   Domain of g:_________________
   Range of g: __________________
   $g^{-1}(x)$ = ______________________
   Domain of g^{-1}:_______________
   Range of g^{-1}:________________

What I Have Learned

Fill in the blank with the appropriate word to complete the sentence.

1. The domain of g(x) is ________ of its inverse g^{-1}.

2. The _________ of f is the domain of f^{-1}.

3. We ___________ the domain and range of the given function to find the domain and range of its inverse.

What I Can Do

Find the domain and range of the following function. Then find the inverse function and write its domain and range.

$f(x) = \sqrt{2x - 6}$
$f^{-1}(x) =$ ________________

Domain of f: ___________________ Range of f:_____________________

Domain of f^{-1}: __________________ Range of f^{-1}: ___________________

Rubrics for problem solving

Score	Description
15 points	Complete solutions with correct answers
10 points	Correct answers with incomplete solutions.
5 points	50% correct answer with incomplete solution
No point earned	No output at all

Assessment

Write the letter that corresponds to the correct answer on a clean sheet of paper.

1. Given the table of values of g below. Which of the following is the domain of g^{-1}?

x	2	4	6	8
g(x)	3	5	7	9

A. {3,5,7,9}　　C. {2,3,4,5}
B. { 2,4,6,8}　　D. {6,7,8,9}

2. Solve the domain of function g(x) = $\sqrt{3x+2}$

A. $[-\frac{2}{3}, +\infty)$　　B. $[\frac{2}{3}, +\infty)$　　C. $[-\frac{3}{2}, +\infty)$　　D. $[\frac{3}{2}, -\infty)$

3. If the domain of f(x) = $x^3 - 5$ is $(-\infty, \infty)$, what is the range of its inverse?

A. [∞,-5)　　B. [-5,∞)　　C. (-∞,∞)　　D. (5,∞)

4. Find the domain of the INVERSE of the function given the set of ordered pairs {(-5, 3), (2, -4), (3, 1), (0, 2)}.

A. {-5,1,2,3}　　B. {3,-4,1,2}　　C. {-4,1,2,3}　　D. {-5.-4,2,3}

5. What is the domain of the inverse of the function f(x) = $\frac{x^2-x-2}{2x^2+x-1}$?

A. {x/x ∈R)　　B. {x/x=1,$\frac{1}{2}$}　　C. {x/x≠ $\frac{1}{2}$}　　D. {x/x≠ $-1, \frac{1}{2}$}

Additional Activities

Find the inverse of the following functions , then determine the domain and range.

1. B(x) = 3x – 4　　　　B^{-1}(x) =________________

Domain:___________　　　　Domain: _______________

Range: ___________　　　　Range: ________________

2. $G(x) = x^3 + 1$ $G^{-1}(x) =$ _______________

Domain: __________ Domain: _______________

Range: ___________ Range: ________________

Lesson 5. Solving Problems Involving Inverse Functions.

What's In

Find the inverse of the following:

1. $f(x) = 6x - 5$
2. $g(x) = x^3$

What's New

Consider the Formula of Fahrenheit below and answer the questions that follow.

$$F = \frac{9}{5}C + 32$$

1. If the boiling point of water is 100 degrees Celsius what is its temperature in Fahrenheit?

2. Is the water temperature of 100 degrees Celsius the same as 212 degrees Fahrenheit?

What is It

Let us consider some examples of word problems involving inverse function.

Example 1: The formula $F = \frac{9}{5}C + 32$ gives a Fahrenheit temperature F as a function based on Celsius temperature C, while the formula $C = \frac{5}{9}(F - 32)$ gives a Celsius temperature C as a function based on Fahrenheit temperature.

Observe that in the first formula, F = 212 when C = 100, and in the second formula, C = 100 when F = 212 . They are examples of inverses.

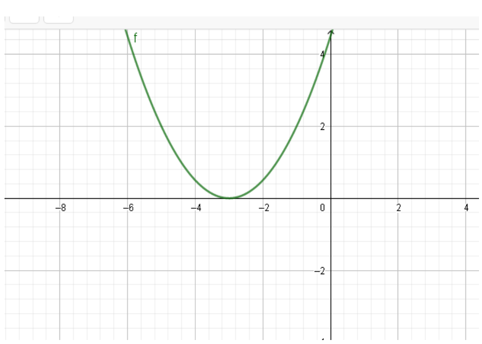
Since the inverse of the function $F = \frac{9}{5}C + 32$ is $f^{-1}(F) = \frac{5}{9}(F - 32)$, $f^{-1}(F)$ represents Celsius temperature based on degrees Fahrenheit and f(C) represents Fahrenheit temperature based on degrees Celsius. Because 212 degrees Fahrenheit is the same as 100 degrees Celsius, we can represent them as $f(100) = 212$ and $f^{-1}(212) = 100$.

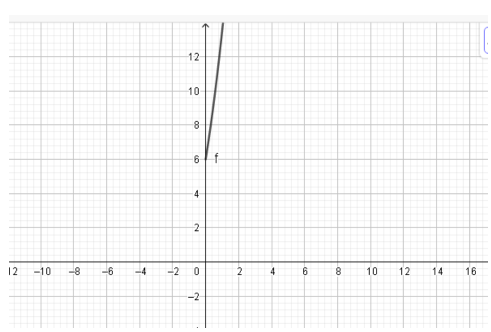
Example 2: Steve asked you to think of a nonnegative number, then add three to the number, square the number, multiply the result by 2 ,and divide the result by 4. If the result is 18, what is the original number? Construct an inverse function that will provide the original number if the result is given.

Solution: We first construct the function that will compute the final number based on the original number. Following the given instructions, we came up with this function:

$$f(x) = (x + 3)^2 \cdot 2 \div 4 = \frac{2(x+3)^2}{4}$$

The first graph below can not be considered a one-to-one function because it does not satisfy the horizontal line test, however, the instruction asked for a nonnegative original number . Therefore, the domain of the function must be restricted to x≥0, as shown in the second graph below.





The function with domain x≥0 is then a one-to-one function, and we can find its inverse.

$f(x) = \frac{2(x+3)^2}{4}$	Given
$y = \frac{2(x+3)^2}{4}$	Rewrite the function in the form y = f(x)
$x = \frac{2(y+3)^2}{4}$	Interchange x and y variables
$\frac{4x}{2} = (y+3)^2$	MPE
$\sqrt{2x} = y+3$	Solve for y in terms of x by squaring both side
$y = \sqrt{2x} - 3$ or $f^{-1}(x) = \sqrt{2x} - 3$	

To find the original number, evaluate the inverse function at x = 18

$f^{-1}(x) = \sqrt{2x} - 3$
$f^{-1}(18) = \sqrt{2(18)} - 3$
$f^{-1}(18) = \sqrt{36} - 3$
$f^{-1}(18) = 6 - 3$
$f^{-1}(18) = 3$

Thus, the original number is 3.

Example 3. Engineers have determined that the same maximum force t in tons a particular bridge can carry is related to the distance d in meters between its supports by using the function below:

$$t(d) = (\frac{12.5}{d})^3$$

A. How far should the supports be if the bridge is to support 6.5 tons?
B. Find the inverse function to determine the result.

Solution:
A. The equation of the function is $t = (\frac{12.5}{d})^3$.
To lessen confusion in this case, let us not interchange d and t as they denote specific values.

Solve in terms of t instead of d :

$t = (\frac{12.5}{d})^3$
$\sqrt[3]{t} = \frac{12.5}{d}$
$d = \frac{12.5}{\sqrt[3]{t}}$

The inverse function is $d(t) = \frac{12.5}{\sqrt[3]{t}}$.
Substitute the function at t = 6.5

$d = \frac{12.5}{\sqrt[3]{t}}$
$d = \frac{12.5}{\sqrt[3]{6.5}}$
d = 6.70

Thus, the support should be placed at most 6.70 meters apart.

What's More

Solve. The function f(d) = 5d + 10 gives the temperature in degrees Celsius inside the earth as a function d, the depth in kilometers.

A. Find the temperature in 3km, 6km, 200km.

B. What is the inverse function of temperature?

What I Have Learned

Consider the problem below and answer the questions that follow.

A 300-liter tank of oil is being drained at the constant rate of 20 liters per minute.

a. Write a function V for the number of liters in the tank after t minutes.
b. Write the inverse function of V.

What I Can Do

Solve.

Mr. Seichi is calculating the speed of a tsunami. The function is

$$f(s) = \sqrt{9.8\,d}$$

where d is ocean depth in meters. In this case, the ocean was 1500 meters deep. How fast was the wave (in m/s^2)? What is the inverse of f(s)?

Rubrics

Score	Description
15 points	Complete solutions with correct answers
10 points	Correct answers with incomplete solutions.
5 points	50% correct answer with incomplete solution
No point earned	No output at all

Assessment

Write the letter that corresponds to the correct answer.

1. Marwin is an engineer and he wants to determine that the same maximum force t in tons that a particular bridge can carry is related to the distance d in meters between its supports by the following function:

$$t(d) = \left(\frac{16.5}{d}\right)^3$$

How far should the supports be if the bridge is to support 8.5 tons?

A. 4.12 B. 5. 12 C. 6. 12 D. 7.12

2. Suppose I am travelling at 50 miles per hour and I want to know how far I have gone in x hours. This can be represented by the function
$f(x) = 50x$. Find the inverse of the given function.

A. $f^{-1}(x) = 50-x$ B $f^{-1}(x) = 50+x$ C. $f^{-1}(x) = \frac{x}{50}$ D. $f^{-1}(x) = \frac{50}{x}$

3. If 144 degrees Fahrenheit is the same as 48 degrees Celsius, we can represent them as:

A. $f(48) = 144$ and $f^{-1}(144) = 48$
B. $f^{-1}(48) = 144$ and $f(144) = 48$
C. $f(48) = 144$ and $f^{-1}(-144) = -48$
D. $f^{-1}(144) = 144$ and $f(48) = 48$

For item 4 – 5, please refer to the problem below.

Patrick asked you to think of a nonnegative number, add four to the number, square the number, multiply the result by 3 and divide the result by 4, then the result is 64.

4. Construct an inverse function that will provide the original number if the result is given.
A. $f^{-1}(x) = \sqrt{x}-4$ B. $f^{-1}(x) = \sqrt{4}-x$ C. $f^{-1}(x) = \sqrt{2x}-4$ D. $f^{-1}(x) = \sqrt{3x}-4$

5. What is the original number?
A. 2 B. 3 C. 4 D. 5

Additional Activities

Analyze and solve the given problem below.

The formula for converting Celsius to Fahrenheit is given as $F = \frac{9}{5}C + 32$ where C is the temperature in Celsius and F is the temperature in Fahrenheit. If the temperature in thermometer reads 101.3°F, what is that in ℃?

SUMMATIVE TEST:

Write the letter that corresponds to the exact answer.
1. Which of the following is the simplest way to determine a one-to-one function?
A. Horizontal Line Test B. Graph C. Straight Line D. Vertical Line
2. The following are one-to-one functions, EXCEPT:
A. $f(x) = 5$ B. $f(x) = 5x$ C. $f(x) = x - 1$ D. $f(x) = x – 4$
3. The following relations show one-to-one function, EXCEPT:
A. A car to its key
B. Books to their authors
C. Sim cards to cellphone numbers
D. SSS ID number to SSS member

4. Find the inverse of function f(x) = $2x^3 - 4$.

A. $f^{-1}(x) = \sqrt[3]{\frac{x+4}{2}}$ C. $f^{-1}(x) = \sqrt[3]{\frac{x-4}{2}}$

B. $f^{-1}(x) = \sqrt[3]{x+4}$ D. $f^{-1}(x) = \sqrt[3]{x-2}$

5. Determine the f^{-1} in the function f(x) = $2 + \sqrt{x-3}$.

A. $f^{-1}(x) = (x-2)^2 - 3$ C. $f^{-1}(x) = (x-2)^2 - 3$

B. $f^{-1}(x) = (x-2)^2 + 3$ D. $f^{-1}(x) = (x+2)^2 + 3$

6. Which of the following functions has no inverse function?

A. f(x) = x+3 B. g(x) = $2x^2$ C. h(x) = x^3 D. p(x) = $\sqrt{x}$

7. Which of the following tables of values shows the inverse function of the given ordered pairs. {(-5,-4), (-3,0), (-1,-2), (1,1)}

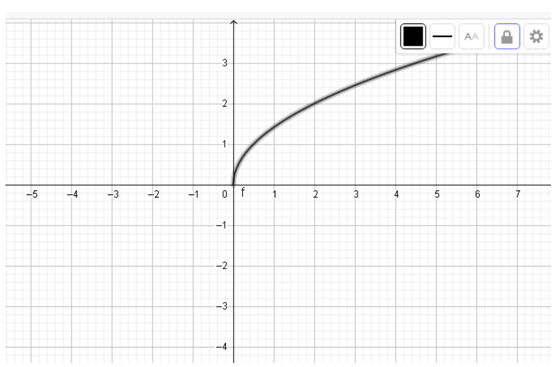
x	-5	-3	-1	0
y	-4	0	-2	1
x	-5	-3	-1	1
y	-4	0	-2	1

A. C.

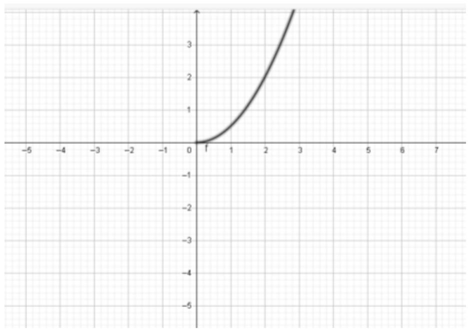
x	-5	-2	-2	0
y	-3	0	-1	1
x	-4	0	-2	1
y	-3	-2	-1	0

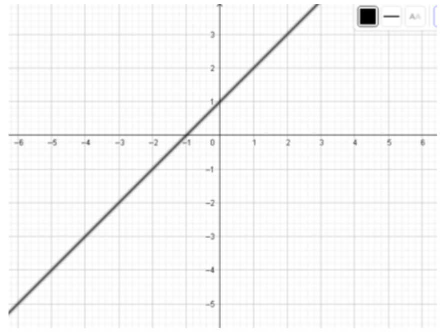
B. D.

8. Which graph shows the inverse of the function f(x) = $\sqrt{2x}$ as shown in the graph below?

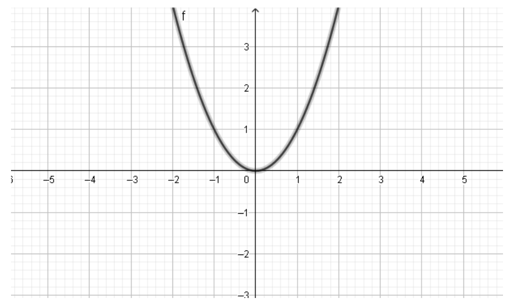


A. B.





C. D.



9. How do you describe the graph of the function and the graph of its inverse?
   A.The graphs are symmetric about the y-axis.
   B.The graphs are reflected about the line y = x.
   C.The graphs are identical.
   D.The graphs are symmetric about the x-axis.

10. Solve the domain of the function g(x) = $\sqrt{x-1}$.
   A. [-∞, +∞) B. [1,∞) C. [1,∞) D. (-1,∞)

11. If the domain of f(x) = x - 3 is $(-\infty, \infty)$, what is the range of its inverse?

   A.[∞,-3) B. [-3,∞) C. (∞,-∞) D. (-∞,∞)

12.If the range of f is $(-\infty, -2]$, what is the domain of its inverse?
   A. (−∞,-2] B. [-2,-∞) C. (−∞,2) D. (2,∞)

13. The formula to convert degrees Fahrenheit (F) to degrees is Celsius(C) given by the function C =$\frac{5}{9}$(F- 32). Find the inverse of the function to convert Celsius to Fahrenheit
   A. F= $5C$-32
   C. F= $\frac{9}{5}$(F-32)
   B. F = $(C$-32)($\frac{5}{9}$)
   D. F = $\frac{9}{5}$C+32

14. Using the inverse function in number 13 ,convert 65 ℃ to °F
   A. 42°F B. 45°F C. 120°F D. 237.6°F

15. Jonathan is calculating the speed of a tsunami. The function is $f(s) = \sqrt{8.5\,d}$ where d is ocean depth in meters. In this case, the ocean is 1200 meters deep. What is the inverse of f(s)?

A. $f^{-1}(s) = \frac{d^2}{8.5}$ B. $f^{-1}(s) = \frac{8.5}{d^2}$ C. $f^{-1}(s) = d^2 + 8.5$ D. $f^{-1}(s) = d^2 - 8.5$

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