10

Math

Quarter 2 – Module 1.1

Graphs of Polynomial Functions

SLM

SELF -LEARNING MODULE

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Graphs of Polynomial Functions

Introductory Message

For the facilitator:

Welcome to the Math 10 Self-Learning Module on Graphs of Polynomial Functions.

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator, you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the Math 10 Self-Learning Module on Graphs of Polynomial Functions.

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

CONTENT STANDARD

The learner demonstrates understanding of key concepts of polynomial function.

PERFORMANCE STANDARD

The learner is able to conduct systematically a mathematical investigation involving polynomial functions in different fields.

LEARNING COMPETENCY

Understands, describe and interpret the graph of polynomial functions.
(M10AL-IIa-1)

LEARNING OBJECTIVES:

1. Sketch the graph of polynomial functions using intercepts, some points and the position of the curves from the table of signs.
2. Describe the characteristics of the graph of polynomial functions.
3. Show understanding by interpreting the polynomial graph

INTRODUCTION

Graphing higher degree polynomial functions can be more complicated than graphing linear and quadratic functions. When graphing polynomial functions, we can identify the end behavior, shape and turning points if we are given the degree of the highest term

Polynomial functions of degree 2 or more have graphs that do not have sharp corners; these graphs are called smooth curves. Polynomial functions also display graphs that have no breaks. Curves with no breaks are called continuous. In this module, we need to be familiar with the different concepts to facilitate the graphing of polynomial functions.

REVIEW OF THE PREVIOUS MODULE

In the previous module, you have learned that a polynomial function is a function comprised of more than one power function where the coefficients are assumed to not equal to zero. The term with the highest degree of the variable in polynomial functions is called the leading term. All subsequent terms in a polynomial function have exponent that decrease in value by one.

PRESENTATION OF THE NEW MODULE

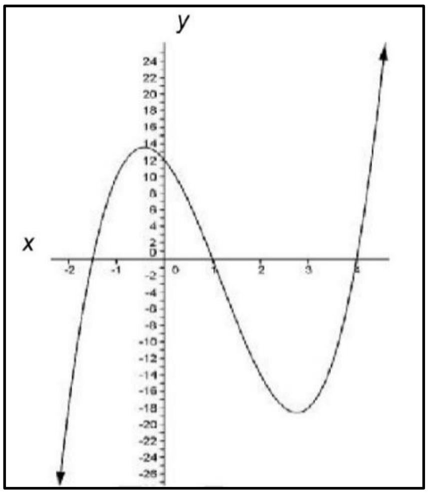
In this section, you need to revisit the lessons on evaluating polynomials, factoring polynomials, solving polynomial equations, and graphing by point-plotting. You can sketch the graph of the polynomial function manually by the use of your knowledge of these topics. You can also use graphing utilities/tools in order to have a clearer view and a more convenient way of describing the features of the graph. You may focus on polynomial functions of degree 3 and higher, since graphing linear and quadratic functions were already taught in previous grade levels. Learning to graph polynomial functions requires your appreciation of its behaviour and other properties.

ACTIVITY

Activity 1: Follow My Path

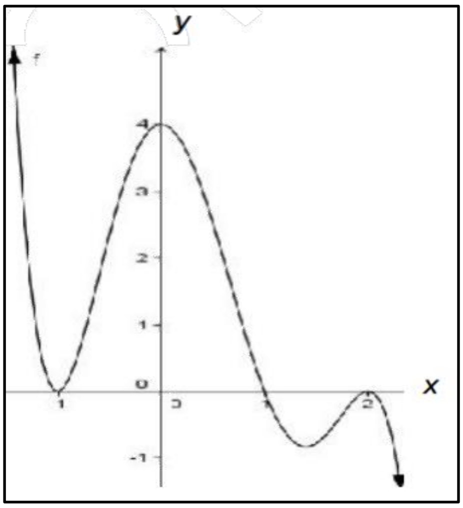
Examine each figure and answer the questions that follow.

Case 1:
The graph that is shown on the right is defined by
$y = 2x^3 - 7x^2 - 7x + 12$ or, in factored form,
$y = (2x + 3)(x - 1)(x - 4)$.



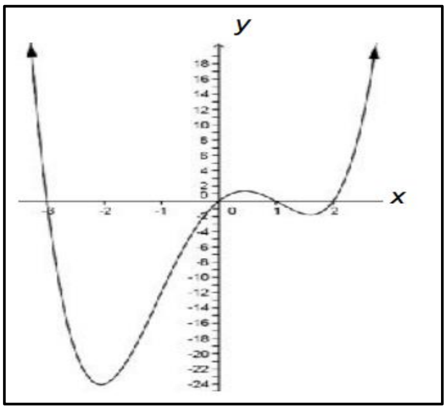
Questions:
a. Is the leading coefficient a positive or a negative number?
b. What is the degree of the polynomial? Is it even or odd?
c. How do you describe the end behaviors of the graph?

Case 2:
Drawn on the right is the graph of
$y = -x^5 + 3x^4 + x^3 - 7x^2 + 4$ or, in factored form,
$y = -(x + 1)^2(x - 1)(x - 2)^2$.

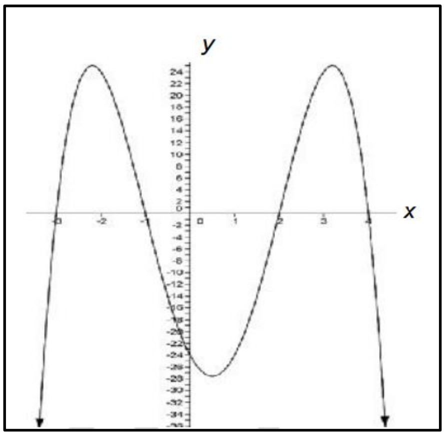


Questions:
a. Is the leading coefficient a positive or a negative number?
b. Is the degree of the polynomial even or odd?
c. How do you describe the end behaviors of the graph?

Case 3
The graph on the right is defined by
$y = x^4 - 7x^2 + 6x$ or, in factored form,
y = x(x + 3)(x – 1)(x – 2)^2.
Questions:
a. Is the leading coefficient a positive number or a negative number?
b. Is the polynomial of even or odd degree?
c. What are the end behaviors of the graph?



Case 4
Drawn on the right is the graph of
y = - x^4+2x^3+13X^2 -14x-24 or in factored form,
y = -(x + 3)(x + 1)(x – 2)(x – 4).
Questions:
a. Is the leading coefficient a positive or a negative number?
b. Is the degree of the polynomial even or odd?
c. What are the end behaviors of the graph?



ANALYSIS

Polynomial Functions	Leading Coefficient: n> 0 or $n < 0$	Degree: Even or Odd	Behavior of the graph: Rising or Falling		Possible Sketch
			left	right	
1. y = 2x^3 – 7x^2 – 7x + 12	n>0	odd	falling	rising	
2. $y = -x^5 + 3x^4 + x^3 - 7x^2 + 4$					
3. $y = x^4 - 7x^2 + 6x$					
4. y= - x^4+2x^3+13X^2 -14x-24					

ABSTRACTION

Activity 2: Watch Out My Behavior!

Given the polynomial function y=(x+4)(x+2)(x-1)(x-3), complete the table and answer the questions that follow.

Value of x	Value of y	Relation of y value to 0: y>0, y=0,or y<0?	Location of the point (x,y): above the x-axis,on the

			x- axis,or below the x-axis?
-5			
-4			
-3	-24	Y<0	Below the x-axis
-2			
0			
1	0	Y=0	On the x-axis
2			
3			

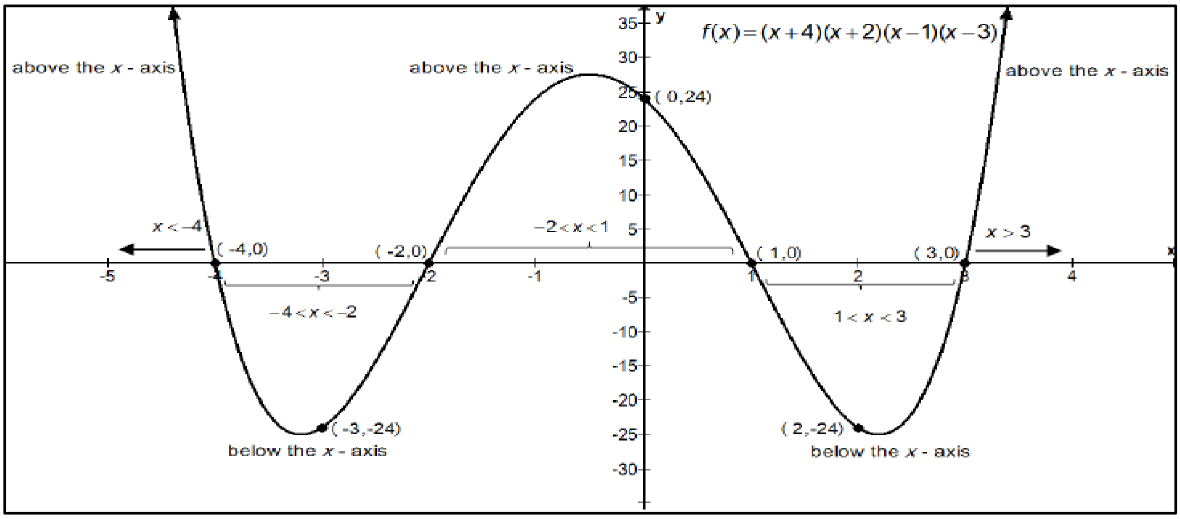
Questions:

1. Does the graph pass through the x-axis? At what point?
2. If x < -4, what can you say about the graph?
3. If -4 < x < -2, what can you say about the graph?
4. If -2 < x<1<1, what can you say about the graph?
5. If 1< x<3, what can you say about the graph?
6. If x > 3, what can you say about the graph?

To instantly locate the curve and sketch the graph, the **TABLE OF SIGNS** will be of great help.

	Intervals				
	X<-4	-4<X<-2	-2<X<1	1<X<3	X>3
Test Value	-5	-3	0	2	4
X+4	-	+	+	+	+
X+2	-	-	+	+	+
x-1	-	-	-	+	+
x-3	-	-	-	-	+
y=(x+4)(x+2)(x-1)(x-3)	+	-	+	-	+
Position of the curve Relative to the x-axis	above	below	above	below	above

The graph of y=(x+4)(x+2)(x-1)(x-3)



- **Key Points**

The previous task is important for you since it has something to do with the **x – intercepts of a graph.** These are the values of x when y = 0, thus, the point(s) where the graph intersects the x-axis can be determined.

To recall the relationship between factors and x-intercepts .consider these examples:

a. Find the intercepts of y = $x^3 - 4x^2 + x + 6$.

Solution:

To find the x-intercept/s, set y=0. Use the factored form. That is, $y = x^3 - 4x^2 + x + 6$

$y = (x + 1)(x - 2)(x - 3)$Factor Completely.

$0 = (x + 1)(x - 2)(x - 3)$ Equate y to 0.

$x + 1 = 0 \quad or \quad x - 2 = 0 \quad or \quad x - 3 = 0$ Equate each factor to 0.

$x = -1 \qquad x = 2 \qquad x - 3$ to determine x.

> The x-intercepts are -1, 2, and 3. This, means that the graph will pass through (-1,0), (2,0), and (3,0).

Finding the y-intercept, simply set x=0 in the given polynomial. That is,

$y = x^3 - 4x^2 + x + 6$

$y = 0^3 - 4(0)^2 + 0 + 6$

y = 6

> The y- intercept is 6. This means the graph will also pass through (0,6).

b. Find the intercepts of $y = x^4 + 6x^3 - x^2 - 6x$

Solution:

For the x-intercept(s), find x when y=0. Use the factored form.

That is,

$$y = x^4 + 6x^3 - x^2 - 6x$$

$y = x(x+6)(x+1)(x-1)$ Factor completely

$0 = x(x+6)(x+1)(x-1)$ Equate y to 0.

$x = 0 \quad or \quad x+6=0 \quad or \quad x+1=0 \quad or \quad x-1=0$ Equate each factor to 0.

$x=-6 \qquad x=-1 \qquad x=1$ determine x.

The x-intercepts are -6,-1,0, and 1. This, means the graph will pass through (-6,0), (-1,0), (0,0) , and (1,0).

Finding the y- intercept requires us to set x= 0 in the given polynomial.

That is,

$$y = x^4 + 6x^3 - x^2 - 6x$$

$$y = (0^4) + 6(0^3) - (0^2) - 6(0)$$

$$y = 0$$

The y- intercept is 0. This means the graph will also pass through (0,0).

You have been provided with some illustrative examples in solving for the x- and y- intercepts. It is an important step in graphing a polynomial function. These intercepts are used to determine the points where the graph intersects or touches the x-axis and the y-axis but these points are not enough to draw the graph of polynomial functions.

The Leading Coefficient Test can help you determine the end behaviors of the graph of a polynomial function as x increases or decreases without bound. The intercepts can be solved using the Rational Root Theorem.

Other useful strategy is to determine whether the graph crosses or is tangent to the x-axis at each x- intercept. This strategy involves the concept of multiplicity of a zero of a polynomial function. Multiplicity expresses how many times a particular number is a zero or root for the given polynomial.

APPLICATION

Activity 3: Sign On and Sketch Me

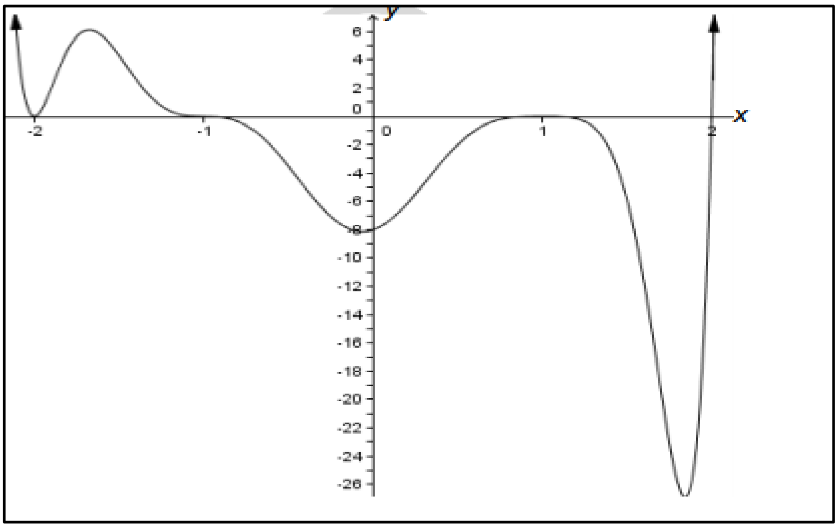
For each of the following functions, give
a. the x-intercept(s)
b. the intervals when the x-intercepts are used to partition the number line
c. the table of signs
d. a sketch of the graph

1. $y=(2x+3)(x-1)(x-4)$
2. $y= -x^3 + 2x^2 +11x -2$

Answer the following questions:
a. What is the formation of the graph as x decreases without bound?
b. In which interval(s) is the graph (i) above and (ii) below the x-axis?
c. What happens to the graph as x increases without bound?
d. What is the leading term of the polynomial function?
e. What is the leading coefficient?
f. What is the degree of the function?

ENRICHMENT

Given the function $y=(x+2)^2(x+1)^3(x-1)^4(x-2)$ and its graph, complete the table below and answer the questions that follow.



Root or Zero	***Multiplicity***	***Characteristic of multiplicity:(odd of even)***	***Behavior of the graph relative to the x-axis at this root:(crosses or is tabgent to)***
-2			
-1			
1			
2			

Questions:

a. What do you notice about the graph when it passes through a root of even multiplicity?
b. What do you notice about the graph when it passes through a root of odd multiplicity?

ANSWER KEY

Activity 1: Follow My Path!

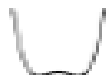
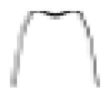
Case 1:
a. positive b. odd degree c. falling to the left rising to the right

Case 2:
a. negative b. odd degree c. rising to the left falling to the right

Case 3:
a. positive b. even degree c. rising to the left rising to the right

Case 4:
a. negative b. even degree c. falling to the left falling to the right

Summary table:

Sample Polynomial Function	Leading Coefficient: $n > 0$ or $n < 0$	Degree: *Even* or *Odd*	Behavior of the Graph: *Rising* or *Falling*		Possible Sketch
			Left-hand	Right-hand	
1. $y = 2x^3 - 7x^2 - 7x + 12$	$n > 0$	odd	falling	rising	
2. $y = -x^5 + 3x^4 + x^3 - 7x^2 + 4$	$n < 0$	odd	rising	falling	
3. $y = x^4 - 7x^2 + 6x$	$n > 0$	even	rising	rising	
4. $y = -x^4 + 2x^3 + 13x^2 - 14x - 24$	$n < 0$	even	falling	falling	

Activity 2: Watch Out my behavior?

Value of x	Value of y	Relation of y-value to 0: $y > 0, y = 0,$ or $y < 0$?	Location of the Point (x,y): *above the x-axis, on the x-axis, or below the x-axis?*
-5	144	$y > 0$	above the x-axis
-4	0	$y = 0$	on the x - axis
-3	-24	$y < 0$	below the x-axis
-2	0	$y = 0$	on the x - axis
0	24	$y > 0$	above the x-axis
1	0	$y = 0$	on the x - axis
2	-24	$y < 0$	below the x-axis
3	0	$y = 0$	on the x - axis
4	144	$y > 0$	above the x-axis

Answers to the Questions:

1. (-4,0), (-2,0), (1,0), and (3,0)
2. The graph is above the x-axis.
3. The graph is below the x-axis.
4. The graph is above the x-axis.
5. The graph is below the x-axis.
6. The graph is above the x-axis.

Activity 3: Sign on and Sketch me

1. $y=(2x+3)(x-1)(x-4)$

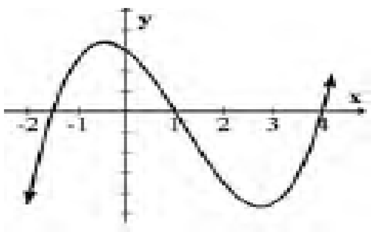
(a) $-\frac{3}{2}$, 1, 4

(b) $x<-\frac{3}{2}$, $-\frac{3}{2}<x<1$, $1<x<4$, $x>4$

(c)

	Intervals			
	$x<-\frac{3}{2}$	$-\frac{3}{2}<x<1$	$1<x<4$	$x>4$
Test Value	-2	0	2	5
$2x+3$	-	+	+	+
$x-1$	-	-	+	+
$x-4$	-	-	-	+
$y=(2x+3)(x-1)(x-4)$	−	+	−	+
position of the curve relative to the x-axis	below	above	below	above

(d)



2. $y=-x^3+2x^2+11x-12$ or $y=-(x+3)(x-1)(x-4)$

(a) -3, 1, 4

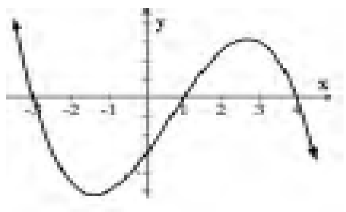
(b) $x<-3$, $-3<x<1$, $1<x<4$, $x>4$

(c)

	Intervals			
	$x<-3$	$-3<x<1$	$1<x<4$	$x>4$
Test Value	-4	0	2	5
$x+3$	-	+	+	+
$x-1$	-	-	+	+
$x-4$	-	-	-	+
$y=-(x+3)(x-1)(x-4)$	+	-	+	-
position of the curve relative to the x-axis	above	below	above	below

Note: Observe that there is one more factor, -1, that affects the final sign of y. For example, under $x<-3$, *the sign of y is positive because* $-(-)(-)(-)=+$.

(d)



Answers to the Questions:

1. For $y=(2x+3)(x-1)(x-4)$
   a. Since there is no other x-intercept to the left of $-\frac{3}{2}$, then the graph falls to the left continuously without end.
   b. (i) $-\frac{3}{2}<x<1$ and $x>4$ (ii) $x<-\frac{3}{2}$ and $1<x<4$
   c. Since there is no other x-intercept to the right of 4, then the graph rises to the right continuously without end.
   d. leading term: $2x^3$
   e. leading coefficient: 2, degree: 3

2. For $y=-x^3+2x^2+11x-12$ or $y=-(x+3)(x-1)(x-4)$
   a. Since there is no other x-intercept to the left of -3, then the graph rises to the left continuously without end.
   b. (i) $x<-3$ and $1<x<4$ (ii) $-3<x<1$ and $x>4$
   c. Since there is no other x-intercept to the right of 4, then the graph falls to the right continuously without end.
   d. leading term: $-x^3$
   e. leading coefficient: -1, degree: 3

Enrichment:

Root or Zero	*Multiplicity*	*Characteristic of multiplicity:(odd of even)*	*Behavior of the graph relative to the x-axis at this root:(crosses or is tabgent to)*
-2	2	even	Tangent to x-axis
-1	3	odd	Crosses the x-axis
1	4	even	Tangent to x-axis
2	1	odd	Crosses the x-axis

Answers:

a. The graph is tangent to the x-axis
b. The graph crosses the x-axis

REFERENCES

Learner's Module (LM) in Math 10, pp. 99-108
Teacher's Guide (TG) in Mathematics 10, pp. 87-90

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