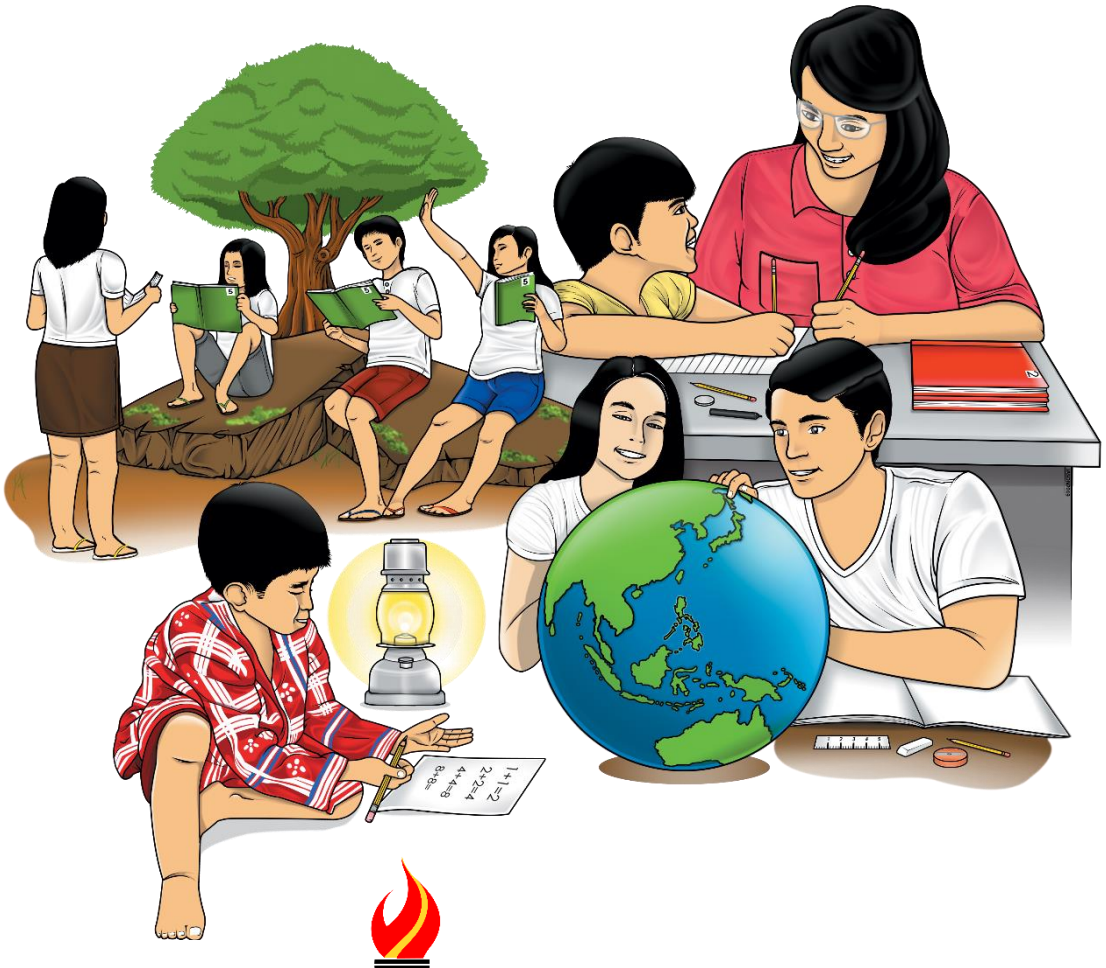
9

DepED
DEPARTMENT OF EDUCATION

Mathematics

Quarter 3 – Module 13: Proving Conditions for Special Right Triangles



ALTERNATIVE DELIVERY MODE
ADM

GOVERNMENT PROPERTY
NOT FOR SALE

Mathematics – Grade 9
Alternative Delivery Mode
Quarter 3 – Module 13: Proving Conditions for Special Right Triangles
First Edition, 2020

Republic Act 8293, section 176 states that: No copyright shall subsist in any work of the Government of the Philippines. However, prior approval of the government agency or office wherein the work is created shall be necessary for exploitation of such work for profit. Such agency or office may, among other things, impose as a condition the payment of royalties.

Borrowed materials (i.e., songs, stories, poems, pictures, photos, brand names, trademarks, etc.) included in this module are owned by their respective copyright holders. Every effort has been exerted to locate and seek permission to use these materials from their respective copyright owners. The publisher and authors do not represent nor claim ownership over them.

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writers:	Zenaida S. Evia, John Alvin A. Ferrer
Editor:	Elma A. Panuncio, Maita G. Camilon
Validators:	Remylinda T. Soriano, Angelita Z. Modesto George B. Borromeo
Illustrator:	Zenaida S. Evia, John Alvin A. Ferrer
Layout Artist:	Zenaida S. Evia, John Alvin A. Ferrer, Alfaith G. Poblete
Management Team:	Malcolm S. Garma, Genia V. Santos Dennis M. Mendoza Maria Magdalena M. Lim Aida H. Rondilla Lucky S. Carpio

Printed in the Philippines by _____

Department of Education - National Capital Region

Office Address: Misamis St., Brgy. Bago Bantay, Quezon City
Telefax: (632) 8926-2213 /8929-4330 /8920-1490 and 8929-4348
E-mail Address: ncr@deped.gov.ph

9

Mathematics

Quarter 3 – Module 13: Proving Conditions for Special Right Triangles

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

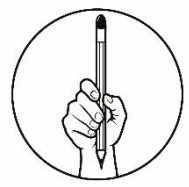
This module was designed and written with you in mind. It is here to help you master the **Proving Conditions for Special Right Triangles**. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

The module is comprised only of one lesson:

- Proving conditions for special right triangles

After going through this module, you are expected to:

- prove the conditions for special right triangles. **M9GE-IIIg-h-1**



What I Know

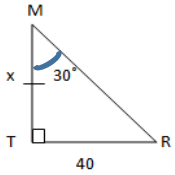
Find out how much you already know about the module. Write the letter that you think is the best answer to each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answers as you go through this module.

1. In a 30°-60°-90° triangle, which side is the shortest?
   a. the hypotenuse
   b. the side opposite the 30° angle
   c. the side opposite the 60° angle
   d. the included side of the acute angle

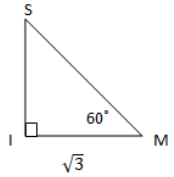
2. In a 30°-60°-90° triangle, which side is the longer side?
   a. the hypotenuse
   b. the side opposite the 30° angle
   c. the side opposite the 60° angle
   d. the included side of the acute angle

3. In a 30°-60°-90° triangle, the length of the hypotenuse (h) is equal to the length of the _____.
   a. shorter leg times 2
   b. longer leg times $\sqrt{3}$
   c. shorter leg times 3
   d. longer leg times the length of the shorter leg

4. The value of x in ΔMRT is _____.
   a. 40 c. 20
   b. $40\sqrt{3}$ d. $\sqrt{3}$



5. The length of the hypotenuse in ΔSIM is _____.
   a. $2\sqrt{3}$ c. $4\sqrt{3}$
   b. $3\sqrt{3}$ d. $5\sqrt{3}$

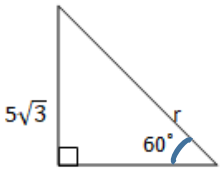


6. What relationship exists between the length of the shorter leg and the length of the hypotenuse in a 30°-60°-90° triangle?
   a. the length of the hypotenuse is 2 times the length of the shorter leg.
   b. the length of the hypotenuse is $\sqrt{2}$ times the length of the shorter leg.
   c. the length of the hypotenuse is $\sqrt{3}$ times the length of the shorter leg.
   d. none of the above

7. Which side is opposite the 90° angle in a 30°-60°-90° triangle?
   a. shorter leg
   b. longer leg
   c. hypotenuse
   d. shorter and longer leg

8. Which side is opposite the 60° angle in a 30°-60°-90° triangle?
   a. hypotenuse
   b. shorter leg
   c. both a and b
   d. longer leg

9. In the given triangle at the right, the value of r is ____
   a. 5
   b. $5\sqrt{3}$
   c. 10
   d $10\sqrt{3}$



10. If a 30°-60°-90° triangle has a hypotenuse of 20 cm., what is the length of the shorter leg?
   a. 8 cm. b. 10 cm c. $8\sqrt{3}$ cm. d. $10\sqrt{3}$ cm

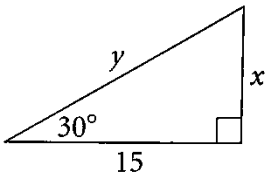
11. The legs of a right triangle have lengths 5 cm and $5\sqrt{3}$ cm. What is the length of the hypotenuse?
   a. 15 cm b. 12 cm c. 10 cm d. 25 cm

12. Which of the following triangles are always similar?
   a. any two right triangles
   b. any two acute triangles
   c. any two isosceles right triangles
   d. any two obtuse triangles

13. What is the altitude of an equilateral triangle whose side is of length 2p?
   a. p b. 2p c. $\sqrt{3}$p d. $2\sqrt{3}$p

14. How far from the base of a building must a 10-foot ladder be placed to lean against the building and makes an angle of 60° with the ground?
   a. 5 ft. b. $5\sqrt{3}$ ft. c. $12\sqrt{3}$ft. d. 24ft.

15. In the triangle at the right, find the value of y.
   a. 5
   b. 3
   c. $5\sqrt{3}$
   d. $10\sqrt{3}$



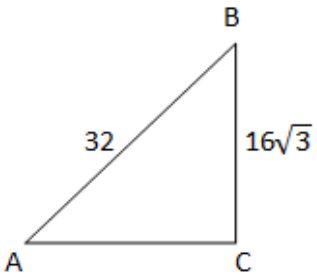
Lesson 1 Proving Conditions for Special Right Triangles

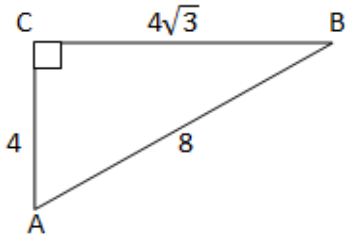
You have learned similarity in right triangles formed inside other right triangles. You also learned that there are theorems that can be used to show similarity between these right triangles. In this module, you will learn that there are right triangles whose sides and angles form a certain relationship that affects their measurements.

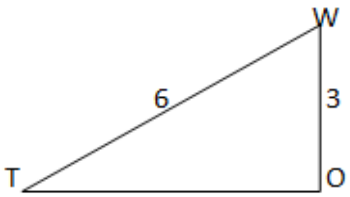
What's In

Collaboration and Critical Thinking

Use a protractor to find the measures of the angles of each triangle.







What's New

Communication, Critical Thinking, and Collaboration

Read the selection below.

Parenting for Tomorrow

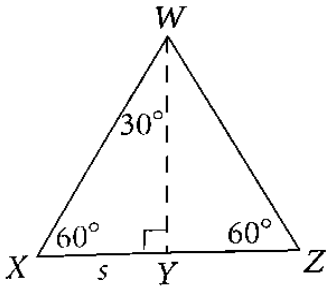
The U.N. affirms this when it states that the first right of every learner is the right to be wanted. A learner should never be the result of chance but should always be the product of a responsible gift of love of his/her parents to each other, and to all of humanity and the Creator Himself. Likewise, all parents should prepare their own little world—their home. In this process, the learners should learn a sense of true love, to learn more math in homes. To work on this, the Department of Education advises us to have" Online Learning Continuity Plan".

To start with, lets' try to prove the 30°-60°-90° right triangle.

From the story, it states that the first right of every child is the right to be wanted and should never be the result of chance but should always be the product of a responsible gift of love of his/her parents. Even if the child is in their home, they must continue to learn mathematics. So, now let's study math by proving another type of special right triangle, the 30°-60°-90° triangle.

In the 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

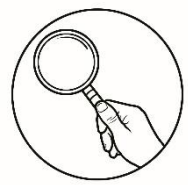
length of the hypotenuse = 2(length of the shorter leg)
length of the $longer\ leg\ =\ \sqrt{3}$ (length of the $shorter\ leg$)



Proof:

For $30° - 60° - 90°$, ΔWXY in equilateral ΔWXZ ,
$\overline{WY}$ is the perpendicular bisector of $\overline{XZ}$.
Thus, |XY| = ½ |XZ| = ½ |XW|, or |XW| = 2|XY| = 2s.
Also,

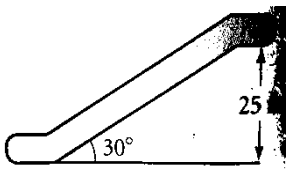
$\|XY\|^2 + \|YW\|^2 = \|XW\|^2$	Use Pythagorean Theorem
$s^2 + \|YW\|^2 = (2s)^2$	Express |XW| in terms of s, thus |XW| = 2s
$\|YW\|^2 = 4s^2 - s^2$	Subtract s^2 from each side
$\|YW\|^2 = 3s^2$	Simplify
$\|YW\| = s\sqrt{3}$	Find the square root of each side



What is It

The 30°-60°-90° Right Triangle Theorem, like the 45°-45°-90° Triangle Theorem, lets you find the lengths of the two sides of the right triangle when you know the length of the third side.

Example 3. An escalator lifts people to the second floor, 25 ft. above the first floor. The escalator rises at a 30° angle. How far does a person travel from the foot to the top of the escalator?

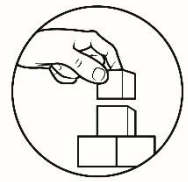


Solution:

Let h be the length of the escalator.
25 ft. is the length of the shorter leg opposite the 30° angle.

h = 2 (length of the shorter leg)
h = 2(25 ft)
h = 50 ft.

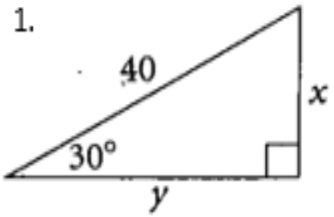
A person travels 50 ft from the foot to the top of the escalator.



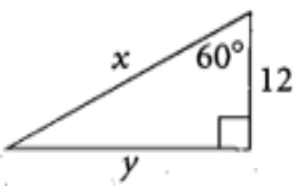
What's More

Activity 1: Find the value of each variable in each given triangle below.

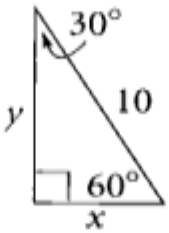
1.



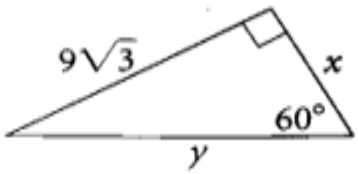
2.



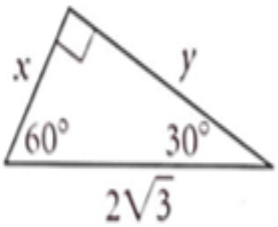
3.



4.

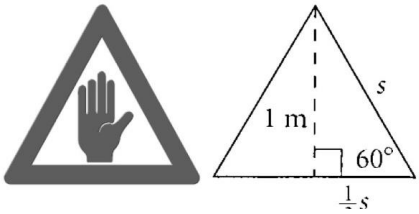


5.



Activity 2:

Road Signs. The warning sign at the right is an equilateral triangle. The height of the sign is 1m. Find the length s of each side of the sign to the nearest tenth of a meter.

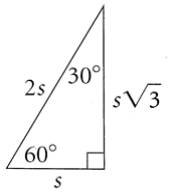


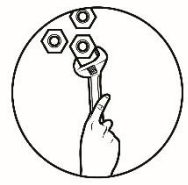
What I Have Learned

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is √3 times the length of the shorter leg.

length of the hypotenuse = 2(length of the shorter leg)

length of thelonger leg = √3(length of the shorter leg)

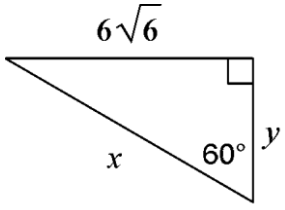




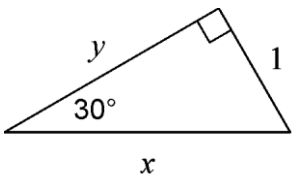
What I Can Do

I. Complete the measures of the dimensions each triangle. Use your answer sheet.

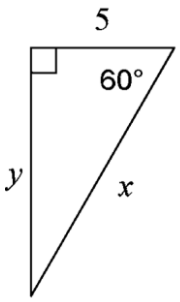
1)



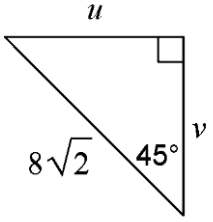
2)



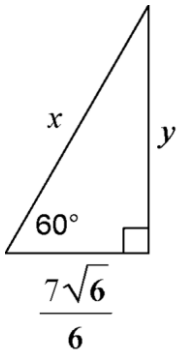
3)



4)

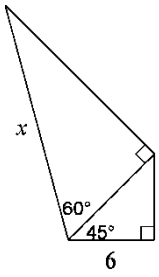


5)



II. Analyze each figure, choose the letter of the correct value of the indicated variable.

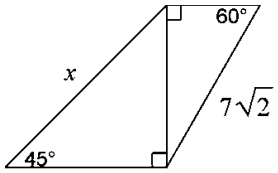
1)



A) $\frac{3\sqrt{3}}{2}$ B) 3

C) $12\sqrt{2}$ D) $\sqrt{3}$

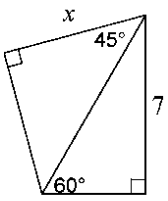
2)



A) 28 B) $\frac{7\sqrt{6}}{6}$

C) $7\sqrt{3}$ D) $7\sqrt{6}$

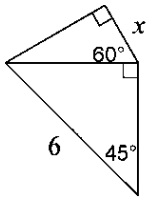
3)



A) $14\sqrt{3}$ B) $\frac{21}{2}$

C) $\frac{7\sqrt{6}}{3}$ D) $\frac{14\sqrt{3}}{3}$

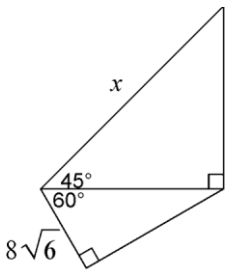
4)



A) $\frac{3}{2}$ B) $4\sqrt{6}$

C) 4 D) $\frac{3\sqrt{2}}{2}$

5)



A) 12 B) $12\sqrt{6}$

C) $8\sqrt{2}$ D) $32\sqrt{3}$

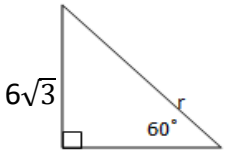
Assessment

Read and answer each question accurately. Write the letter of the correct answer on your answer sheet.

1. Which angle in a 30°-60°-90° triangle is opposite of the hypotenuse?
   a. 30° c. 90°
   b. 60° d. none of the above

2. Which angle is opposite of the longer leg in a 30°-60°-90° triangle?
   a. 30° c. both a and b
   b. 90° d. 60°

3. In the given triangle, the value of r is.
   a. 6 c. 12
   b. $6\sqrt{3}$ d $12\sqrt{3}$



4. If a 30°-60°-90° triangle has a hypotenuse of length 30 cm., what is the length of the shorter leg?
   a. 10 cm. b. 15 cm c. $15\sqrt{3}$ cm. d. $10\sqrt{3}$ cm

5. The legs of a 30°-60°-90° triangle have lengths 7 cm and $7\sqrt{3}$ cm. What is the length of the hypotenuse?
   a. 12 cm b. 13 cm c. 14 cm d. 15 cm

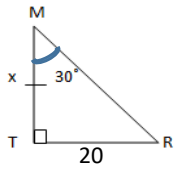
6. Which of the following triangles are always similar?
   a. any two right triangles c. any two equilateral triangles
   b. any two acute angles d. any two obtuse angles

7. In a 30°-60°-90° triangle, which side is the shortest?
   a. the hypotenuse
   b. the side opposite the 30° angle
   c. the side opposite the 60° angle
   d. the included side of the acute angle

8. The side opposite the 60-degree angle in a 30°-60°-90° triangle is _____.
   a. the hypotenuse c. the *lo*nger leg
   b. the s*horter leg* d. any of the two legs

9. In a 30°-60°-90° triangle, the length of the hypotenuse (h) is equal to _____.
   a. the length of longer leg times the length of shorter leg
   b. twice the length of the shorter leg
   c. the length of longer leg times $\sqrt{3}$
   d. the length of shorter times 3

10. The value of x in ΔMRT at the right is _____.



a. 20 c. 10
b. $20\sqrt{3}$ d. $10\sqrt{3}$

11. The length of the hypotenuse in the figure at the right is _____.

a. $4\sqrt{3}$ c. $6\sqrt{3}$
b. $5\sqrt{3}$ d. $8\sqrt{3}$

12. What relationship exists between the length of the shorter leg and the length of the hypotenuse in a 30°-60°-90° triangle?

a. the length of the hypotenuse is 2 times the length of the shorter leg
b. the length of the hypotenuse is $\sqrt{2}$ times the length of the shorter leg
c. the length of the hypotenuse is $\sqrt{3}$ times the length of the shorter leg
d. none of the above

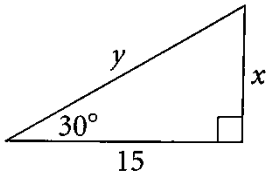
13. What is the altitude of an equilateral triangle whose side is of length 4p?

a. 2p b. 4p c. $2\sqrt{3}$p d. $4\sqrt{3}$p

14. How far from the base of a building must a 20-foot ladder be placed to lean against the building and makes an angle of 60° with the ground?

a. 10 ft. b. $10\sqrt{3}$ft. c. $20\sqrt{3}$ft. d. 24ft.

15. In the triangle at the right, find the value of x.



a. 5 c. $5\sqrt{3}$
b. 3 d. $10\sqrt{3}$

Additional Activities

Critical Thinking and Creativity

Directions: Write a reflective learning on the Pythagorean Theorem by answering the questions that follow. You may express your answers in more critical and creative presentation of your great learning. Have fun and enjoy! Use your answer sheet.

A sailboat is a boat propelled partly or entirely by sails and is smaller than a sailing ship. Distinctions in what constitutes a sailing boat and ship vary by region and maritime culture. A real-world application for special right triangles could be sail boats. You can find sail boats with 30-60-90-degree triangles. If you want to find the length of the hypotenuse of the triangle, just use the right formula and you can find it.

1. Give the word or words that best describe the story.

2. What is the essence of the story?

3. Write at least 3 sample problems based on to the story.

PROBLEM – BASED LEARNING WORKSHEET

A sailboat is a boat propelled partly or entirely by sails and is smaller than a sailing ship. Distinctions in what constitutes a sailing boat and ship vary by region and maritime culture. A real-world application for special right triangles could be sail boats. You can find sail boats with 30-60-90-degree triangles. If you want to find the length of the hypotenuse, just use the right formula and you can find it.

Let's Analyze

1. Are the sizes of the headsail and mainsail different?

2. What kind of triangle represents the headsail and mainsail of the sailing boat?

3. What theorem can be used to find the length of the longest side of the headsail of the sailing boat?

References

Ponsones R., Remoto-Ocampo S., *Math Ideas and Life Applications Second Edition*

Coronel A., *Growing Up With Math,* Copyright 14

Mathematics Learner's Material First Edition

Wilcutt R., *Essentials for Algebra Concepts and Skills*, Houghton Mifflin Company.

E-Search

You may also check the following link for your reference and further learnings on solving quadratic equation using completing square.

- https://mathbitsnotebook.com/Geometry/Similarity/SMProofs.html
- https://tutors.com/math-tutors/geometry-help/similar-triangles
- https://www.ck12.org/geometry/Similarity-by-AA-SSS-SAS/lesson/AA-Similarity-BSC-GEOM/
- https://www.khanacademy.org/math/geometry/hs-geo-similarity/hs-geo-triangle-similarity-intro/v/similarity-postulates
- https://www.youtube.com/watch?v=VXlFEilh-cw
- www.unicef.org
- www.parentingfortomorrow.com
- https://www.pcma.org/social-distancing-future-events

Answer Key

WHAT I KNOW

1. B	6. A	11.C
2. C	7. C	12.C
3. A	8. D	13.C
4. B	9. C	14.A
5. A	10.B	15.D

WHAT'S IN

1. $m\angle A = 60, m\angle B = 30, m\angle C = 90$
2. $m\angle A = 60, m\angle B = 30, m\angle C = 90$
3. $m\angle T = 30, m\angle W = 60, m\angle O = 90$

WHAT'S MORE

Activity 1

1. $x = 20, y = 20\sqrt{3}$
2. $x = 24, y = 12\sqrt{3}$
3. $x = 5, y = 5\sqrt{3}$
4. $x = 9, y = 18$
5. $x = \sqrt{3}, y = 3$

Activity 2

$S = \frac{2\sqrt{3}}{3}$ m

WHAT I CAN DO

I.

1) $x = 12\sqrt{2}, y = 6\sqrt{2}$
2) $x = 2, y = \sqrt{3}$
3) $x = 10, y = 5\sqrt{3}$
4) $u = 8, v = 8$
5) $x = \frac{7\sqrt{6}}{3}, y = \frac{7\sqrt{2}}{2}$

II.

1) C 4) D
2) C 5) D
3) C

ASSESSMENT

1. C	6. C	11.A
2. D	7. B	12.A
3. C	8. C	13.C
4. B	9. B	14.A
5. C	10.B	15.C

ADDITIONAL ACTIVITIES

1. Sailboat, Right Triangle
2. Even boats use the idea of Pythagorean Theorem
3. Let the Students give their own e.g.

PISA

1. Yes
2. Right Triangle
3. Pythagorean Theorem

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph