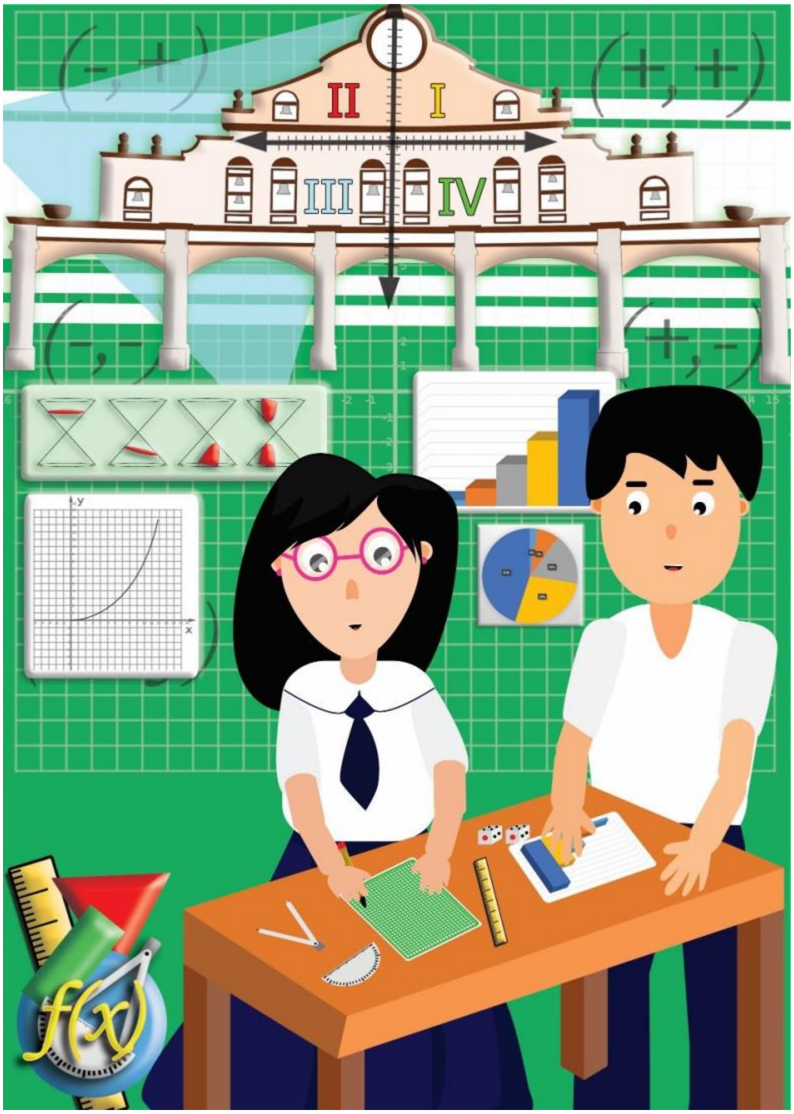
Senior High School

Department of Education
National Capital Region
SCHOOLS DIVISION OFFICE
MARIKINA CITY

STATISTICS AND PROBABILITY

Quarter 3: Module 7
Central Limit Theorem



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What I Need to Know

Good day Grade 11 learners! In this module, you are going to learn how to:

Illustrate the Central Limit Theorem **M11/12SP-IIIe-2**,
Define the sampling distribution of the sample mean using the Central Limit Theorem **M11/12SP-IIIe-3** and
Solve problems involving sampling distribution of the sample mean **M11/12SP-IIIf-1**.

You can say that you have understood the lesson in this module if you can already:

1. illustrate the Central Limit Theorem,
2. use Central Limit Theorem in defining the sample distribution of the sample mean,
3. solve problems involving sampling distribution of the sample means and
4. identify the steps in solving sampling distribution of the sample mean.

What I Know

Let us see how far you will go and how much you know about this pre-test. Let's get started!

Read and analyze each item carefully. On a separate sheet of paper, write the letter that corresponds to the correct answer.

1. If the mean of the sampling distribution is 6.5, which of the following statement best describe the population mean?
   A. The population decreases by 6.5.
   B. The population is greater than 6.5.
   C. The population mean is also equal to 6.5.
   D. The population mean and mean of the sampling distribution of the means cannot be compared.
2. As the sample size n increases, the standard deviation of the sampling distribution __________.
   A. Increases C. stays the same
   B. decreases D. not enough information is given
3. Which of the following description does **NOT** illustrate the Central Limit Theorem?
   A. The Central Limit Theorem is used to approximate the distribution of the sample mean over the distribution of the population mean.

B. If the sample size n, where n is sufficiently large is drawn from any population with mean μ and a standard deviation σ,then the sampling distribution of sample means approximates a normal distribution.
C. Whenever the population is not normally distributed, or if we do not know the distribution, the Central Limit Theorem allows us to conclude that the distribution of sample means will be normal if the sample size is sufficiently large.
D. Given a random variable X with mean μ and variance σ^2,then regardless of whether the population distribution of X is normally distributed or not, the shape of the distribution of the sample means taken from the population approaches a normal distribution.

For items **4 – 5,** refer to the problem below.

A population of 25 year- old female has a mean salary of P20,800 with a standard deviation of P1200.

4. If a random sample of 10 is considered from the population, what is the probability that their salary is greater than P20900?
   A. 2.59 B.0.0319 C. 0.4952 D. 0.0048
5. If a random sample of 9 is taken, what is the probability that their salary is greater than P20750 but less than 20,875?
   A. 0.1270 B.0.0753 C. 0.0517 D. 0.0236

LESSON 1: Illustrating the Central Limit Theorem

What's In

In the previous module, you have studied ***finding the mean and the variance of the sampling distribution of the sample means,*** let us recall by answering the following problem.

Activity 1:

Consider all the possible samples of size 2 that can be drawn from the population 1, 2, 3, 4, 5, and 6. Create a sampling distribution of the sample means. Compute the following:

1. Mean of the sampling distribution of the sample means
2. Variance of the sampling distribution of the sample means
3. Standard deviation of the sampling distribution of the sample means

What's New

Supposed you are the regional manager of a grocery chain in-charge of many stores in the region. Your project is to optimize the weekly re-stocking of a certain product. You want to know how many of that certain product to be ordered weekly for each store so that you can minimize the amount of inventory that will be idle in store shelves. How will you do it?

Visiting every single store in your region and getting the number of their sales weekly for that particular product is time consuming and costly. The Central Limit Theorem will help you balance the time and cost of collecting data to draw conclusions.

What is It

Central Limit Theorem

The central limit theorem is one of the most powerful and useful ideas in all of statistics. There are two alternative forms of the theorem, and both alternatives are concerned with drawing finite samples size n from a population with a known mean, μ, and a known standard deviation, σ. The first alternative says that if we collect samples of size n with a "large enough n," calculate each sample's mean, and create a histogram of those means, then the resulting histogram will tend to have an approximate normal bell shape. The second alternative says that if we again collect samples of size n that are "large enough," calculate the sum of each sample and create a histogram, then the resulting histogram will again tend to have a normal bell-shape.

Therefore, If random samples of size **n** are drawn from a population, then as **n** becomes larger, the sampling distribution of the mean approaches the normal distribution, regardless of the shape of the population distribution.

Importance and Practical Rules Commonly Used

1. If the original population is not itself normally distributed, here is a common guideline: For samples of size n greater than 30, the distribution of the sample means can be approximated reasonably well by normal distribution. (There are exceptions, such as population with very non-normal distributions requiring sample sizes larger than 30, but such exceptions are relatively rare). The approximation gets better as the sample size n becomes larger.

2. If the original population is itself normally distributed, then the sample means will be normally distributed for any sample size n (not just the values of n larger than 30).

Hence, as the sample size increases, the sampling distribution of the mean, can be approximated by a normal distribution with mean and standard deviation. It means that if the sample size n is randomly selected from a population with mean and variance σ^2, the sampling distribution of the sample means will approach a normal distribution even when the original population is not normally distributed, as long as the sample size n is sufficiently large.

Central Limit Theorem Principles

When selecting a simple random sample from a population with μ and standard deviation σ, it is essential to know these principles:

1. If $n \geq 30$, then the sample means have a distribution that can be approximated by a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.
2. If $n \leq 30$ and the original population has a normal distribution, then the sample means have normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.
3. If $n \leq 30$ but the original population does not have normal distributions, then this method does not apply.

Defining the Sampling Distribution of the Sample Means using the Central Limit Theorem

Let us define or describe the sampling distribution of the sample means from an infinite population through an example.

Example: The scores of individual students on a national achievement test have a normal distribution with a mean $\mu = 20.8$ and a standard deviation $\sigma = 1.9$. At Feliciano Delos Santos High School, 81 students took the test. If the scores of the students at this school have the same distribution as the national scores,

a. describe the sampling distribution of the scores of these students by computing its mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$ (also known as sample error of the population mean) and

b. how is the standard deviation of the sample means changed when the number of students who took the test is increased from 81 to 250?

Solution:

a. Given: $\mu = 20.8$, $\sigma = 1.9$ and $n = 81$

The mean of the sampling distribution $\mu_{\bar{x}} = 20.8$ since $\mu_{\bar{x}} = \mu$.

If the number of students who took the test is 81, the standard deviation of the sampling distribution is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.9}{\sqrt{81}} = \frac{1.9}{9} = 0.21$.

b. If the number of students who took the test is increased from 81 to 250, the standard deviation is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.9}{\sqrt{250}} = 0.12$.

From the computation, the standard deviation of the sample means decreased when the number of students who took the test is increased from 81 to 250.

What's More

Activity: Solve for the following items and describe.

1. A random sample of 75 measurements is obtained from a population with a mean $\mu = 38$ and standard deviation $\sigma = 9$. Describe the sampling distribution of the sample means by computing $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$

2. A molding machine prepares a truck spare part with a target diameter $\mu = 3.425$ centimeters. The machine has some variability so the standard deviation of the diameter is $\sigma = 0.002$ centimeter. A sample of 16 spare parts is inspected every after half hour for process control purposes and records are kept of the sample mean diameter. Describe the sampling distribution of the sample means by computing its mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$.

What I Have Learned

Let us see what you have learned in this module by completing the following statements.

1. If we collect samples of size *n* with a "large enough *n*," calculate each sample's mean, and create a histogram of those means, then the resulting histogram will tend to have an approximate normal _______________shape.
2. If random samples of size **n** are drawn from a population, then as **n** becomes larger, the sampling distribution of the mean approaches the _______________ distribution, regardless of the shape of the population distribution.
3. For samples of size n greater than _______________, the distribution of the sample means can be approximated reasonably well by normal distribution.
4. The ____________ of the population mean is always equal to the mean of the true population from which the population sample were drawn.
5. The _______________ or sample error of the population mean is always equal to the standard deviation of the parent population divided by the square root of the sample size n.

What I Can Do

Read and analyze the following problems carefully and answer. Show your solution if necessary.

1. If the population of ABM students of Don Orlando National High School has a mean of 15.63, what is the mean of the sampling distribution of the means?
2. If the mean of the sampling distribution is 21.60, what is the mean of the population?
3. The number of driving miles before a certain type of tire begins to show wear is on the average 18,200 miles with a standard deviation σ = 3,500 miles. If Jack Car Rental Agency buys 25 of these tires for replacement purposes and parts each one on a different car, compute the mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$ of the sampling distribution of the sample means.

Assessment

Solve.

1. A certain type of thread is manufactured with a mean tensile strength of 78.3 kilograms and a standard deviation of 5.6 kilograms. Assuming that the population is infinite, how is the standard error of the mean changed when the sample size is increased from 64 to 196?
2. A soft drink machine dispenses drinks with an average content of 240 ml with a standard deviation of 15 ml. A random sample of 40 soft drink dispensed is checked. Compute the mean and standard deviation of the sampling distribution of the sample means.

Additional Activities

A randomly selected from the population means and the standard deviations are given with sample size n, solve for the mean and standard deviation of the sampling distributions of the means.

	n	μ	σ	$\mu_{\bar{x}}$	$\sigma_{\bar{x}}$
1.	6	8.5	2		
2.	4	6	1.6		
3.	15	6	0.9		
4.	24	21	2.5		
5.	46	31	4		

LESSON 2: SOLVING PROBLEMS INVOLVING SAMPLING DISTIBUTION OF THE SAMPLE MEANS

What's In

Begin this lesson by recalling what you have learned in Lesson 1.
1. In your own words, state the **Central Limit Theorem.**
2. How do you define or describe a sampling distribution of the sample means?
3. How do you compute the mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$ of the sampling distribution of the sample means?
4. Mr. Cruz, a teacher of Statistics and Probability subject has 200 students. The population mean is μ = 71.18 and the standard deviation of the population is σ = 10.73. A random of 15 samples is drawn. Describe the sampling distribution of the sample means by computing its mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$.

What's New

In the previous module, you have learned how to use the z – table and find the area that corresponds to z – values. Using the z – table (appended in module 4), find the area that corresponds to

1. z = 2

2. z = 1.5
3. z = -2.4
4. z = 1.77
5. z = -1.05

In this lesson, you will use the sampling distribution of the mean to obtain information about the sample mean.

What is It

The Central Limit Theorem is of fundamentals importance in statistics because it justifies the use of normal curve methods for a wide range of problems. This theorem applies automatically to sampling from infinite population. It also assures that no matter what the shape of the population distribution of the mean is, the sampling distribution of the sample means is closely normally distributed whenever n is large.

Working Procedures

When working with an individual value from a normally distributed population, use

$$Z = \frac{X-\mu}{\sigma}.$$

When working with a mean for some sample (or group), be sure to use the value of $\sigma/\sqrt{n}$ for the standard deviation of the sample means. Use

$$Z = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

Where: X bar = sample means
μ = population mean
σ = population standard deviation
n = sample size

Example: The average sugar content of a certain one liter ice cream brand is 375 milligrams and the standard deviations is 12 milligrams. Assume the variable is normally distributed.

a. If a one liter ice cream is selected, what is the probability that the sugar content will be greater than 380 milligrams?

b. If a sample of 25 one liter ice cream is selected, what is the probability that the sugar content of the mean of the sample will be greater than 380 milligrams?

c. If a sample of 4 one liter ice cream is selected, what is the probability the sugar content of the sample means is greater than 370 mg. but less than 385 mg?

Solution:

a. Given: $\mu = 375,\ \sigma = 12, X = 380$
Required: P(X > 380)
Since we deal with individual data, use the formula

$$Z = \frac{X-\mu}{\sigma}$$

$$= \frac{380-375}{12}$$

$$= \frac{5}{12}$$

= 0.42 (nearest hundredths)

Find P(X > 380) by getting the areas under the normal curve.

P(X > 380) = P(z > 0.42)

= 0.5000 – 0.1628

= 0.3372

So, the probability that the sugar content of the mean of the sample is greater than 380 milligrams is 0.3372 or 33.72 %.

b. Given: $\mu = 375,\ \sigma = 12,\ \underline{X} = 380, n = 25$

Required: P(X > 380)

Since we deal with data of sample means, use the formula

$$Z = \frac{\underline{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{380-375}{\frac{12}{\sqrt{25}}}$$

$$= \frac{5}{\frac{12}{5}}$$

= 2.08 (nearest hundredths)

Find P(X > 380) by getting the areas under the normal curve.

P(X > 380) = P(z > 2.08)

= 0.5000 – 0.4980

= 0.002

So, the probability that the sugar content of the selected 25 one liter ice cream is greater than 380 milligrams is 0.002 or 2 %.

c. Given: $\mu = 375,\ \sigma = 12,\ \underline{X} = 370\ and\ \underline{X} = 385, n = 4$

Required: P(385 < X < 370)

Since we deal with data of sample means, use the formula

$$Z = \frac{X-\mu}{\frac{\sigma}{\sqrt{n}}}$$

For $\underline{X} = 370$,

$$Z = \frac{X-\mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{370-375}{\frac{12}{\sqrt{4}}}$$

$$= \frac{-5}{\frac{12}{2}}$$

$$= \frac{-5}{6}$$

= .83 (nearest hundredths)

For $\underline{X} = 385$,

$$Z = \frac{X-\mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{385-375}{\frac{12}{\sqrt{4}}}$$
$$= \frac{10}{\frac{12}{2}}$$
$$= \frac{10}{6}$$
= 1.67 (nearest hundredths)

Find P(385 < X < 370) by getting the areas under the normal curve.

$$P(385 < X < 370) = P(1.67 < z < .83)$$
$$= 0.4525 - 0.2969$$
$$= 0.1556$$

So, the probability that the sugar content of the selected 4 one liter ice cream is greater than 370 mg. but less than 385 mg is 0.1556 or 15.56 %.

What's More

Solve for the following items.

The average number of milligrams (mg) of cholesterol in a cup of a certain buko salad desert made by Company ABC is 660 mg. with the standard deviation of 35 mg. Assume that the variable is normally distributed.

a. What is the probability that the cholesterol content will be more than 670 mg. if a cup of buko salad is selected?

b. What is the probability that the mean of the sample will be larger than 670 mg. if a sample of 10 cups of buko salad is selected?

What I Have Learned

In your own words, summarize what you have learned in this lesson.

What I Can Do

Apply the knowledge you have gained in this module by answering the problem below. Use a separate sheet of paper for your answer.

The average time it takes to a group of college students to complete certain examination is 46.2 minutes. The standard deviation is 8 minutes. Assume that the variable is normally distributed.

a. What is a probability that a randomly selected college student will complete the examination in less than 43 minutes?

b. If 50 randomly selected college students take the examination, what is the probability that the mean time it takes the group to complete the test will be less than 43 minutes?

Assessment

Solve for the following items.

A certain group of welfare recipients receives an average benefits of P4000 per month with a standard deviation of P200.

1. If a random sample of recipient is taken, what is the probability that the mean benefit of this recipient is greater than P4100?
2. If a random sample of 36 recipient is taken, what is the probability that their mean benefit is greater than P4050?

Additional Activities

Solve for the following items using the application of Central Limit Theorem.

Assume that women's height are normally distributed with the mean given by μ = 63.6 in. and standard deviation given by σ = 2.5 in. (Data is based from National Health Survey.)

1. If 1 woman is randomly selected, find the probability that her height is less than 64 in.
2. If 100 women are randomly selected, find the probability that they have a mean height greater than 63 in.
3. If 1 woman is randomly selected, find the probability that her height is between 63.5 and 64.5 in.

SUMMATIVE TEST

Read and analyze each item carefully. Then on a separate sheet of paper, write the letter of the correct answer for each item.

1. If the mean of the sampling distribution of the mean is 6.5, which of the following statement best describe the population mean?
   A. The population decreases by 6.5.
   B. The population is greater than 6.5.
   C. The population mean is also equal to 6.5.
   D. The population mean and mean of the sampling distribution of the means cannot be compared.
2. As the sample size n increases, the standard deviation of the sampling distribution __________.
   A. Increases
   B. decreases
   C. stays the same
   D. not enough information is given
3. Which of the following description does NOT illustrate the Central Limit Theorem?
A. The Central Limit Theorem is used to approximate the distribution of the sample mean over the distribution of the population mean.
B. If the sample size n, where n is sufficiently large is drawn from any population with mean μ and a standard deviation σ,then the sampling distribution of sample means approximates a normal distribution.
C. Whenever the population is not normally distributed, or if we do not know the distribution, the Central Limit Theorem allows us to conclude that the distribution of sample means will be normal if the sample size is sufficiently large.
D. Given a random variable X with mean μ and variance σ^2,then regardless of whether the population distribution of X is normally distributed or not, the shape of the distribution of the sample means taken from the population approaches a normal distribution.

For items **4 – 5**, refer to the problem below.

A population of 25 year old female has a mean salary of P20,800 with a standard deviation of P1200.

4. If a random sample of 10 is considered from the population, what is the probability that their salary is greater than P20900?

A. 2.59 B.0.0319 C. 0.4952 D. 0.0048

5. If a random sample of 9 is taken, what is the probability that their salary is greater than P20750 but less than 20,875?

A. 0.1270 B.0.0753 C. 0.0517 D. 0.0236

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Answer Key

What I Know

1. C 2. B 3. A 4. D 5. A

LESSON 1

What's In

1. 3.5 2. 2.92 3. 1.7

WHATS' MORE

2. $\mu_{\bar{x}} = 38, \sigma_{x} = 1.04$ 2. $\mu_{\bar{x}} = 3.425, \sigma_{\bar{x}} = 0.0005$

What I have Learned

1. bell 4. sample mean
2. normal 5. standard deviation of the sample mean
3. 30

What I Can Do

1. $\mu_{\bar{x}} = 15.63$ 2. $\mu_{\bar{x}} = 21.60$ 3. $\mu_{\bar{x}} = 18200, \sigma_{x} = 700$

Assessment

1. The standard error of the mean changed from 0.7 to 0.4. As the sample size increases, the standard error of the mean decreases.
2. $\mu_{\bar{x}} = 240, \sigma_{x} = 2.37$

Additional Activity

1. $\mu_{\bar{x}} = 8.5, \sigma_{x} = 0.82$ 4. $\mu_{\bar{x}} = 21, \sigma_{\bar{x}} = 0.51$
3. $\mu_{\bar{x}} = 6, \sigma_{\bar{x}} = 0.8$ 5. $\mu_{\bar{x}} = 31, \sigma_{x} = 0.59$
3. $\mu_{\bar{x}} = 6, \sigma_{x} = 0.23$

LESSON 2:

WHATS' IN

1. Varied answer
2. By computing the mean and standard deviation of the sample means
3. To compute the mean of the sample means, use $\mu_{\bar{x}} = \mu$. To compute the standard deviation of the sample means, use $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.
4. $\mu_{\bar{x}} = 71.18, \sigma_{x} = 2.77$

WHATS' NEW

1. 0.4772 2. 0.4332 3. 0.4918 4. 0.4616 5. 0.3531

WHATS' MORE

a. 0.3859 b. 0.1841

What I have Learned

Varied answer

What I Can Do

a. 0.3446 b. 0.0023

Assessment

1. 0.3085 2. 0.0668

Additional Activity

1. 0.4364 2. 0.0080 3. 0.1566

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