8

Mathematics
Quarter 1– Module 1A
Factoring Polynomials

ALTERNATIVE DELIVERY MODE

ADM

Mathematics – Grade 8
Alternative Delivery Mode
Quarter 1 – Module 1 Factoring Polynomials
First Edition, 2020

8

Mathematics
Quarter 1 – Module 1A
"Factoring Polynomials"

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

This module has the following parts and corresponding icons:

What I Need to Know

This will give you an idea of the skills or competencies you are expected to learn in the module.

What I Know

This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.

What's In

This is a brief drill or review to help you link the current lesson with the previous one.

What's New

In this portion, the new lesson will be introduced to you in various ways; a story, a song, a poem, a problem opener, an activity or a situation.

What is It

This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.

What's More

This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.

What I Have Learned

This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.

What I Can Do

This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.

Assessment

This is a task which aims to evaluate your level of mastery in achieving the learning competency.

Additional Activities

In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned.

Answer Key

This contains answers to all activities in the module.

At the end of this module you will also find:

References   This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities
   included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module is designed and written to help you factor polynomials completely using different techniques. In all lessons, you are given the opportunity to use your prior knowledge and skills in multiplying and dividing polynomials. Activities are also given to process your knowledge and skills acquired, deepen and transfer your understanding of the different lessons. The scope of this module enables you to use it in many different learning situations. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains the following lessons:

> Lesson 1: Factoring with common monomial factor
>
> Lesson 2: Factoring difference of two squares
>
> Lesson 3: Factoring the Sum and Difference of Two Cubes

After going through this module, you are expected to:

1. determine patterns in factoring polynomials;
2. factor polynomials completely and accurately using the greatest common monomial factor (GCMF);
3. factor the difference of two squares; and
4. factor the sum and difference of two cubes.
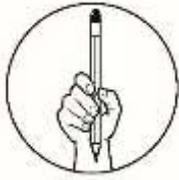
What I Know

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. What is the Greatest Common Factor (GCF) of 24 and 54?
   - A. 1
   - B. 2
   - C. 3
   - D. 6

2. What is the GCF of 20, 24, and 40?
   - A. 1
   - B. 4
   - C. 8
   - D. 20

3. What is the GCF of x^2 and x^9 ?

   - A. x^2
   - B. x^7
   - C. x^{11}
   - D. x^{18}

4. What is the GCF of xy^{10}, x^4y^6, x^9y^9 , and $x^{10}y^{10}$?

   - A. xy
   - B. xy^6
   - C. x^4y^4
   - D. x^4y^6

5. What are the complete factors of the polynomial $7x - 7$?

   - A. $7(x - 1)$
   - B. $7(1 - x)$
   - C. $7x - 1$
   - D. $7(x - 7)$

6. What are the complete factors of $2x^6 - 12x^4$?

   - A. $2(x^4 - 6x^4)$
   - B. $2x^5(x - 6x)$
   - C. $2x^4(x^2 - 6)$
   - D. $x^6(2x^2 - 12)$

7. Which of the following is a perfect square expression?

   - A. $3x^2$
   - B. $4x$
   - C. $9x^3$
   - D. $16x^4$

8. If one factor of the difference of two squares is x + 2, what is the other factor?

   - A. $x - 2$
   - B. $x^2 - 2$
   - C. $x^2 - 2^2$
   - D. $(x - 2)^2$

9. What is the complete factored form of $z^2 - 16$?

   - A. $(z - 4)^2$
   - B. $(z - 8)^2$
   - C. $(z - 4)(z - 4)$
   - D. $(z + 4)(z - 4)$

10. What is the complete factored form of the expression $y^4 - 49$?

A. $(y^2)^2 - (7^2)^2$

B. $(y^2 - 7)^2$

C. $(y^2 + 7)^2$

D. $(y^2 - 7)(y^2 + 7)$

11. Which of the following polynomials has factors $(xy - 1)(xy + 1)$?

A. $x^2 y - 1$

B. $x^2 y^2 - 1$

C. $xy^2 - 1$

D. $x^2 y^2 + 1$

12. Which of the following expressions is a perfect cube?

A. $8x$

B. $27x^2$

C. $64x^6$

D. $125x^4$

13. Which of the following is the complete factored form of the cubic polynomial $x^3 - 8$?

A. $(x - 2)(x^2 + 2x + 4)$

B. $(x + 2)(x^2 - 2x + 4)$

C. $(x - 2)(x^2 + 2x - 4)$

D. $(x - 2)(x^2 - 2x + 4)$

14. Factor completely: $27x^3 + 64y^3$

A. $(3x)^3 + (4y)^3$

B. $(3x + 4y)(9x^2 - 12xy + 16y^2)$

C. $(3x + 4y)(3x^2 - 3xy + 16y^2)$

D. $(3x + 4y)(3x^2 - 12xy + 4y^2)$

15. Your classmate was asked to square $(2x - 3)$, he answered $4x^2 - 9$. Is his answer correct?

A. Yes, because product rule is correctly applied.

B. Yes, because squaring a binomial always produces a binomial product.

C. No, because the answer must be $4x^2 + 9$.

D. No, because squaring a binomial always produces a trinomial product.

Lesson 1
Factoring by Greatest Common Monomial Factor

One of the factoring techniques that you are going to learn in this module is factoring by greatest common monomial factor (GCMF). Concepts such as factors, factoring, and prime factorization have been discussed and have been used in many instances in your previous math classes. Let us try to reactivate what you previously learned by answering the activity below.

What's In

Recall that factor is a number or algebraic expression that divides another number or expressions evenly, that is with no remainder.

Examples:
1. The factors of 4 are 1, 2, and 4 as these can divide 4 evenly.
2. The factors of $2x^2$ are $1, 2, x, x^2, 2x, 2x^2$ as these can divide $2x^2$ evenly.

Activity: Pieces of My Life

Find the possible factors of the given number or expression below. Choose you answers from the box and write it on your answer sheet.

2	x	a	10	z	y
6	b	5	4	3	
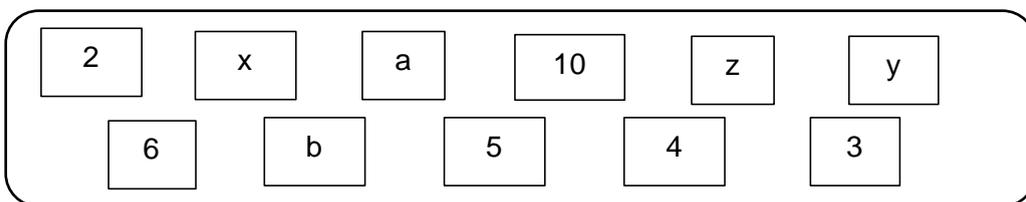
Number/Expression	Factors
1. 8	_____
2. 2x	_____
3. 5ab	_____
4. 12z	_____
5. 20xy	_____

What's New

Consider the rectangle below.

$$L = x + 3$$

$$W = 2$$
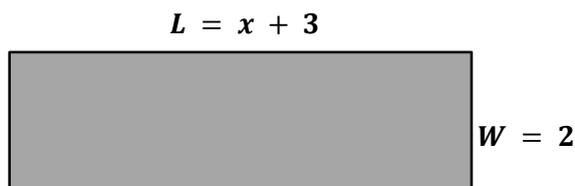
The area of a rectangle is the product of the length and the width, or $A_{rectangle} = L \cdot W$.

Questions:
1. What is the area of the rectangle?
2. Is the area of the rectangle a polynomial?
3. What is the relationship between the area of the rectangle and its sides?
4. What can you say about the width of the rectangle comparing it to the area?
5. What do you call the process of rewriting the polynomial as a product of polynomial factors?

What is It

Suppose we will make use of the area of the rectangle in the previous section which is $2x + 6$. Now, working backward, we have to find the length and the width of the rectangle.

Notice that $2x + 6$ can be written as:

$$2 \cdot x + 2 \cdot 3$$

Notice also that 2 is common to both terms. So, by rewriting it we have,

$$2x + 6 = 2 \cdot x + 2 \cdot 3 = 2(x + 3)$$

Recall that by distributive property, $2(x + 3)$ will go back to its original form $2x + 6$. Hence,

$$2(x + 3) = 2x + 6$$

Note!
When you factor, see to it the product of these factors is always the original expression or polynomial.

This means that, $2(x + 3)$ is the completely factored form of $2x + 6$.

Based on the example above, you have noticed that the method of factoring used is finding a number or expression that is common to all the terms in the original expression, that is, 2 is a common factor to both $2x$ and 6. Since there is no other factor, other than 1, which is common to all terms in the given expression, 2 is called the *greatest common monomial factor* (GCMF).

To further illustrate the concept of GCMF, try to explore the following examples:

Example 1. Find the GCF of each pair of monomials.
        a. $4x^3 \; and \; 8x^2$ b. $15y^6 \; and \; 9z$

Solution:
    a. $4x^3 \; and \; 8x^2$

        Step 1. Factor each monomial.
$$4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$$
$$8x^2 = 2 \cdot 2 \cdot 2 \cdot x \cdot x$$

        Step 2. Identify the common factors.
$$4x^3 = \boxed{2} \cdot \boxed{2} \cdot x \cdot \boxed{x} \cdot \boxed{x}$$
$$8x^2 = \boxed{2} \cdot \boxed{2} \cdot 2 \cdot \boxed{x} \cdot \boxed{x}$$

        Step 3. Find the product of the common factors.
$$2 \cdot 2 \cdot x \cdot x = 4x^2$$

        Hence, $4x^2$ is the GCMF of $4x^3 \; and \; 8x^2$.

    b. $15y^6 \; and \; 9z$

        Step 1. Factor each monomial.
$$15y^6 = 3 \cdot 5 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$$
$$9z = 3 \cdot 3 \cdot z$$

        Step 2. Identify the common factors.
$$15y^6 = \boxed{3} \cdot 5 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$$
$$9z = \boxed{3} \cdot 3 \cdot z$$

        Step 3. Find the product of the common factors.

        Note that 3 is the only common factor.

        Hence, 3 is the GCMF of $15y^6 \; and \; 9z$

Notice that in the examples above, prime factorization is used to find the GCMF of the given pair of monomials. The next examples illustrate how the GCMF is used to factor polynomials.

Example 2. Write $6x + 3x^2$ in factored form.

Step 1. Determine the number of terms.

In the given expression, we have 2 terms: $6x$ and $3x^2$.

Step 2. Determine the GCF of the numerical coefficients.

coefficient	factors	common factors	GCF
3	$1, 3$	$1, 3$	3
6	$1, 2, 3$		

Step 2. Determine the GCF of the variables. The GCF of the variables is the one with the least exponent.

$$GCF(x, x^2) = x$$

Step 3. Find the product of GCF of the numerical coefficient and the variables.

$$(3)(x) = 3x$$

Hence, $3x$ is the GCMF of $6x$ and $3x^2$.

Step 4. Find the other factor, by dividing each term of the polynomial $6x + 3x^2$ by the GCMF $3x$.

$\rightarrow \quad \dfrac{6x}{3x} + \dfrac{3x^2}{3x}$ Divide each term by the GCMF

$\rightarrow \quad \dfrac{3x \cdot 2}{3x} + \dfrac{3x \cdot x}{3x}$ Rewrite each term as a product

$\rightarrow \quad \mathbf{2 + x}$

Step 5. Write the complete factored form
$$6x + 3x^2 = \mathbf{3x\,(2 + x)}$$

Example 3: Write $12x^3y^5 - 20x^5y^2z$ in complete factored form.

Step 1. Determine the number of terms.

There are two terms in the given expression $12x^3y^5 - 20x^5y^2z$, $12x^3y^5 \ and \ 20x^5y^2z$.

Step 2. Determine the GCF of the numerical coefficient.

coefficient	factors	common factors	GCF
12	$1, 2, 3, 4, 6, 12$	$1, 2, 3, 4$	4
20	$1, 2, 4, 5, 10, 20$		

Step 2. Determine the GCF of the variables. The GCF of the variables is the one with

the least exponent and is common to every term.

$GCF(x^3y^5, x^5y^2z) = x^3\,y^2$

Step 3. Find the product of GCF of the numerical coefficient and the variables.

$$4 \cdot x^3 y^2 = 4 x^3 y^2$$

This means that, $4 x^3 y^2$ is the GCMF of the two terms $12x^3y^5$ and $20x^5y^2z$.

Step 4. Find the other factor, by dividing each term of the polynomial $12x^3y^5 - 20x^5y^2z$ by the GCMF $4 x^3 y^2$.

$$\rightarrow \quad \frac{12x^3y^5}{4 x^3 y^2} - \frac{20x^5y^2z}{4 x^3 y^2}$$

$$\rightarrow \quad \frac{4x^3y^2 \cdot 3y^3}{4 x^3 y^2} - \frac{4x^3y^2 \cdot 5x^2z}{4 x^3 y^2}$$

$$\rightarrow \quad 3y^3 - 5x^2z$$

Step 5. Write the complete factored form

$$12x^3y^5 - 20x^5y^2z = \mathbf{4 x^3 y^2 (3y^3 - 5x^2z)}$$

Example 4: Write $12x^3 - 18xy + 24x$ in complete factored form.

Step 1. Determine the number of terms.
There are three terms in the expression
$12x^3 - 18xy + 24x$: $12x^3, 18xy, 24x$

Step 2. Determine the GCF of the numerical coefficient.

coefficient	factors	common factors	GCF
12	$1, 2, 3, 4, 6, 12$		
18	$1, 2, 3, 6, 9, 18$	$1, 2, 3, 6$	6
24	$1, 2, 3, 4, 6, 8, 12, 24$		

Step 2. Determine the GCF of the variables. The GCF of the variables is the one with the least exponent and is common to every term.
$$GCF(x^3, xy, x) = x$$

Step 3: Find the product of GCF of the numerical coefficient and the variables.

$$(6)(x) = 6x$$
Hence, $6x$ is the GCMF of $12x^3, 18xy, 24x$.

Step 4. Find the other factor, by dividing each term of the polynomial
$12x^3 - 18xy + 24x$ by the GCMF $6x$.

$$\rightarrow \quad \frac{12x^3}{6x} - \frac{18xy}{6x} + \frac{24x}{6x}$$

$$\rightarrow \quad \frac{6x \cdot 2x^2}{6x} - \frac{6x \cdot 3y}{6x} + \frac{6x \cdot 4}{6x}$$

$$\rightarrow \quad 2x^2 - 3y + 4$$

Step 5: Write the complete factored form.

$$12x^3 - 18xy + 24x = \mathbf{6x\,(2x^2 - 3y + 4)}$$

Example 5. Write $28x^3z^2 - 14x^2y^3 + 36yz^4$ in complete factored form.

Step 1. Determine the number of terms.

There are three terms in the expression $28x^3z^2 - 14x^2y^3 + 36yz^4$: $28x^3z^2, 14x^2y^3, and\ 36yz^4$.

Step 2. Determine the GCF of the numerical coefficient.

coefficient	factors	common factors	GCF
28	$1, 2, 4, 7, 14, 28$		
14	$1, 2, 7, 14$	$1, 2$	2
36	$1, 2, 3, 4, 6, 9, 12, 18, 36$		

Step 2. Determine the GCF of the variables. The GCF of the variables is the one with the least exponent and is common to every term.

$$GCF(\,x^3z^2, x^2y^3, yz^4\,) = 1$$

Note that there are no factors common to all the three terms, this means that $x^3z^2, x^2y^3,\ and\ yz^4$ are relatively prime. Hence, the GCF is 1.

Step 3: Find the product of GCF of the numerical coefficient and the variables.

$$(2)(1) = 2$$

Hence, 2 is the GCMF of $12x^3, 18xy, 24x$.

Step 4. Find the other factor, by dividing each term of the polynomial $28x^3z^2 - 14x^2y^3 + 36yz^4$ by the GCMF 2.

$$\rightarrow \quad \frac{28x^3z^2}{2} - \frac{14x^2y^3}{2} + \frac{36yz^4}{2}$$

$$\rightarrow \quad \frac{2 \cdot 14x^3z^2}{2} - \frac{2 \cdot 7x^2y^3}{2} + \frac{2 \cdot 18yz^4}{2}$$

$$\rightarrow \quad 14x^3z^2 - 7x^2y^3 + 18yz^4$$

Step 5: Write the complete factored form.

$$28x^3z^2 - 14x^2y^3 + 36yz^4 = \mathbf{2\,(14x^3z^2 - 7x^2y^3 + 18yz^4)}$$

Below is the summary of the steps of factoring the Greatest Common Monomial Factor.

1. Determine the number of terms.
2. Find the greatest common factor of the numerical coefficients.
3. Find the variable with the least exponent that appears in each term of the polynomial. It serves as the GCF of the variables.
4. Get the product of the greatest common factor of the numerical coefficient and the variables. It serves as the **greatest common monomial factor** of the given polynomial.
5. Find the other factor by dividing the given polynomial by its **greatest common monomial factor** and write the final factored form of the polynomial.
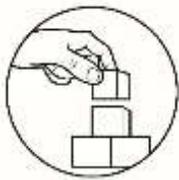
What's More

Activity 1: Break the Great!

Directions: Determine the Greatest Common Monomial Factor (GCMF) of each polynomial and write its factored form. Write your answer on a separate sheet of paper.

Polynomial	GCMF	Factored Form
1. $x^2 + 2x$		
2. $5x^2 - 10x^3$		
3. $25x^2y^3 + 55xy^3$		
4. $10c^3 - 80c^5 - 5c^6 + 5c^7$		
5. $12m^5n^2 - 6m^2n^3 + 3mn$		

Questions:
1. How did you find the GCMF of the numerical coefficients of each term?
2. How did you find the GCMF of the variables in each term?
3. What did you do to the obtained GCF of the numerical coefficients and the GCF of the variables?
4. How did you find the remaining factors?
5. Did you have any difficulty in finding the GCF of the terms?
6. Did you have any difficulty in finding the remaining factor/s of polynomials after GCF is obtained? If so, why? If none, what helped you factor those expressions correctly?

Activity 2: You Complete Me

Directions: Write a polynomial factor in the blank to complete each statement. Write the answers on your answer sheet.

1. $7p^2 - 7p$ = $7p \,(\underline{\hspace{2cm}})$

2. $18xy + 3y$ = $(\underline{\hspace{2cm}}) \,(6x + 1)$

3. $15t^3 - 15t^2 + 20t$ = $5t \,(\underline{\hspace{2.5cm}})$

4. $17x^5 - 51x^4 - 34\,x$ = $(\underline{\hspace{2cm}}) \,(x^4 - 3x^3 - 2)$

5. $35x^5y^2 + 21x^4y + 14x^3y^2$ = $7x^3y \,(\underline{\hspace{2.5cm}})$

Questions:

1. Which was easier: finding the remaining factor given the GCF, or finding the GCF given the other factor? Why?
2. What did you do to find the GCF given the remaining factors?

What I Have Learned

Reflect on the activities you have done in this lesson by completing the following statements. Write your answers on your journal notebook.

The 3 things that I have learned are

_____
_____
_____

The one thing that I want to know more about is

_____

Factoring Difference of Two Squares

This lesson emphasizes another factoring technique which is factoring the difference of two squares. For you to be well guided on this lesson, recall first the topics about perfect square and special product of sum and difference of two terms in Grade 7.

What's In

Powerful 2!

Recall: Perfect squares are numbers or expressions that can be expressed to the power of 2.

Examples:
1. $4 = 2 \cdot 2 = 2^2$
2. $9x^2 = 3x \cdot 3x = (3x)^2$

Thus, 4 and $9x^2$ are perfect square.

Determine which of the following is a perfect square. Write **P** if it is a perfect square and **N** if it is not. Write your answer on your answer sheet.

1. 16
2. 12
3. 25
4. $8x^2$
5. $36y^4$

Questions:
1. Which items are perfect squares?
2. Which items are not perfect squares?
3. What did you do to determine whether the numbers are perfect squares?

Squares of Plus and Minus

Determine the product of each sum and difference of two terms. Write your answers on your answer sheet.

1. $(x + 1)(x - 1)$
2. $(x + 4)(x - 4)$
3. $(x + 3)(x - 3)$
4. $(x + 7)(x - 7)$
5. $(x + 9)(x - 9)$

Questions:
1. How did you get the product of sum and difference of two terms?
2. Have you observed any pattern?
3. How did you find the activity?
4. Were you able to get the correct answers? If not, what difficulty did you encounter?

What is It

Recall the topic about special product particularly the product of the sum and difference of two terms. It states that the product of $(a + b)$ and $(a - b)$ is equal to the difference of two squares which is $a^2 - b^2$.

$$(a + b)(a - b) = a^2 - b^2$$

Notice that when the sum and product of two terms are multiplied (meaning, the first and second term of the factors are the same but they differ with the operation, one is + while the other is −), the result will always be the difference of the squares of the two terms (that is, you square the first term, square the second term and the operation between them is −). Thus, to factor the **_difference of two squares,_** you just have to reverse the pattern.

$$a^2 - b^2 = (a + b)(a - b)$$

In order to use this factoring technique, recall the concept of **_perfect squares_**. For example, 16, x^2, $25a$, $9y^4$, and $\frac{1}{4}$. These are all perfect squares. Why? Look at and study the illustration below.

$$16 = 4 \cdot 4 = (4)^2$$
$$x^2 = x \cdot x = (x)^2$$
$$25a^2 = 5a \cdot 5a = (5a)^2$$
$$9y^4 = 3y^2 \cdot 3y^2 = (3y^2)^2$$
$$\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^2$$

You see from the examples that a perfect square is a number or expression which can be written as an exact square of a rational quantity.

Let us now see how to factor difference of two squares by examining at the given examples below.

Example 1: Write $x^2 - 9$ in completely factored form.

Step 1. Express the first and the second terms in exponential form with a power of 2.

$$x^2 = x \cdot x = (x)^2$$
$$9 = 3 \cdot 3 = (3)^2$$

Step 2: Subtract the two terms in exponential form following the pattern $a^2 - b^2$.
$$(x)^2 - (3)^2$$

Step 3: Factor completely following the pattern $a^2 - b^2 = (a + b)(a - b)$

$$(x + 3)(x - 3)$$

Hence, the complete factored form is, $x^2 - 9 = (x)^2 - (3)^2 = (x + 3)(x - 3)$.

Example 2: Write $16a^6 - 25b^2$ in completely factored form.

Step 1. Express the first and the second terms in exponential form with a power of 2.

$$16a^6 = 4a^3 \cdot 4a^3 = (4a^3)^2$$
$$25b^2 = 5b \cdot 5b = (5b)^2$$

Step 2. Subtract the two terms in exponential form following the pattern $a^2 - b^2$.

$$(4a^3)^2 - (5b)^2$$

Step 3: Factor completely following the pattern $a^2 - b^2 = (a + b)(a - b)$

$$(4a^3 + 5b)(4a^3 - 5b)$$

Hence, the complete factored form of $16a^6 - 25b^2$ is,

$$16a^6 - 25b^2 = (4a^3)^2 - (5b)^2 = (4a^3 + 5b)(4a^3 - 5b)$$

Example 3: Write $a^2 b^4 - 81$ in completely factored form.

Step 1: Express the first and the second terms in exponential form with a power of 2.

$$a^2 b^4 \quad = \quad ab^2 \cdot ab^2 \quad = \quad (ab^2)^2$$
$$81 \quad = \quad 9 \cdot 9 \quad = \quad (9)^2$$

Step 2: Subtract the two terms in exponential form following the pattern $a^2 - b^2$.

$$(ab^2)^2 - (9)^2$$

Step 3: Factor completely following the pattern $a^2 - b^2 = (a + b)(a - b)$

$$(ab^2 + 9)(ab^2 - 9)$$

Thus, the complete factored form of $a^2 b^4 - 81$ is,

$$a^2 b^4 - 81 = (ab^2)^2 - (9)^2 = (ab^2 + 9)(ab^2 - 9)$$

Based on the examples above, these are the steps in factoring the difference of two squares:
Step 1: Express the first and the second terms in exponential form with a power of 2.
Step 2: Subtract the two terms in exponential form following the pattern $a^2 - b^2$.
Step 3: Factor completely following the pattern $a^2 - b^2 = (a + b)(a - b)$.

Note that there are cases where after expressing each term as a power of 2, the resulting numbers or expressions can still be factored further. This is when the results are still perfect squares. Hence, there is a need to inspect thoroughly and ensure that all terms are completely factored.

Consider the following examples:

Example 4: Write $x^4 - 81$ in completely factored form.

Solution:
$$x^4 - 81 = (x^2)^2 - (9)^2 = (x^2 + 9)(x^2 - 9)$$

The two factors are $(x^2 + 9)(x^2 - 9)$. Notice the second factor $x^2 - 9$ is a *difference of two squares,* thus, it can still be factored out. That is,

$$x^2 - 9 = (x)^2 - (3)^2 = (x + 3)(x - 3).$$

This means that, the complete factored form of $x^4 - 81$ is

$$x^4 - 81 = (x^2)^2 - (9)^2 = (x^2 + 9)(x^2 - 9)$$
$$= (x^2 + 9)(x + 3)(x - 3).$$

In the example above, $x^2 + 9$ is called the *sum of two squares*. It cannot be factored!

Example 5: Write $3w^2 - 48$ in completely factored form.

Solution:

At first glance, it seems like the given binomial is not factorable using sum and difference of two terms since the terms are not perfect squares. Note however, that the first and second terms of the binomial have a common factor of 3. Hence, the binomial can be factored using a combination of GCMF and the sum and difference of two terms.

Find the GCF of the terms and write it in factored form.

$$3w^2 - 48 = 3\,(w^2 - 16)$$

Observe that $w^2 - 16$ is a difference of two squares. Hence, it can be factored as

$$w^2 - 16 = (w)^2 - (4)^2 = (w + 4)(w - 4)$$

Thus, the complete factored form of $3w^2 - 48$ is

$$3w^2 - 48 = 3\,(w + 4)(w - 4)$$

Based on the examples presented above, can you now completely factor the difference of two squares independently?
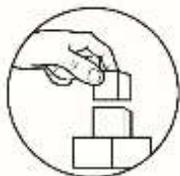
What's More

Activity 1: Tell Me!

Tell whether or not the given binomial is a difference of two squares. If it is, write D. If it is not, write N. Write your answer on your answer sheet.

1. $a^2 - 81$
2. $c^2 - 18$
3. $d^2 - 25$
4. $25e^2 - 16$
5. $r^2 + 9s^4$

Activity 2: Missing You

Find the missing terms of the factors. Write your answer on your answer sheet.

1.	$a^2 - 81$	=	$(a + \underline{\quad})(a - \underline{\quad})$
2.	$p^2 - q^2$	=	$(\underline{\quad} + \underline{\quad})(p - q)$
3.	$c^2 - d^2$	=	$(c + d)(\underline{\quad} - \underline{\quad})$
4.	$49e^2 - 81f^2$	=	$(7e + \underline{\quad})(\underline{\quad} - 9f)$
5.	$100g^2 - 121h^2$	=	$(\underline{\quad} + 11h)(10g - \underline{\quad})$

Activity 3: Whole to Parts

Directions: The factors of the following polynomials are given below. Choose the right factors that correspond to each given polynomial and write your answers on your answer sheet.

$(x - 9)$	$(5x - 1)$	$(4x - 9y)$	$(9x + 20y^2)$	$(2x - 7)$	$4x^2 - 9$
$(4x + 9y)$	$(2x + 7)$	$(x + 9)$	$(5x + 1)$	$(9x - 20y^2)$	

1. $x^2 - 81$ = _____
2. $4x^2 - 49$ = _____
3. $16x^2 - 81y^2$ = _____
4. $25x^2 - 1$ = _____
5. $81x^2 - 400y^4$ = _____

Activity 4: When I Broke It, I Found It!

Directions: Supply the missing terms to factor the given polynomials completely. Write your answer on your answer sheet.

1. $3x^2 - 12y^2 = 3(x^2 -$ ____$)$

$(x +$ ____$)($ ____$- 2y)$

Final Factors: _____

2. $1 - 16x^8 = (1 +$ ____$)($ ____$- 4x^4)$

$(1 +$ ____$)($ ____$- 2x^2)$

Final Factors: _____

3. $a^4 - 625b^8 = ($ ____$+ 25b^4)(a^2 -$ ____$)$

$(a +$ ____$)($ ____$- 5b^2)$

Final Factors: _____

17 CO_Q1_Mathematics8_Module1A

What I Have Learned

Directions: Read and answer as directed. Write your answers on your answer sheet.

1. How will you factor difference of two squares? Write the step–by – step process.

2. Give an example of binomial where the resulting factors after using the pattern for sum and difference of two squares are still factorable using the same method. Outline your step-by-step process in getting the complete factored form.

Lesson 3	Factoring Sum and Difference of Two Cubes

In this lesson, you will learn how to factor the sum and difference of two cubes. However, for you to do that, you must recall the concept of perfect cube and how to express mathematical expression to the power of 3 when you were in Grade 7. To refresh your learning on this matter, try to answer the following activity.

What's In

Perfect cubes are numbers or expressions that can be expressed to the power of 3. Say, $8x^6$. There are two things that we need to manipulate, the constant 8 and the variable x^6. The constant 8 can be expressed as $8 = 2 \cdot 2 \cdot 2 \ or \ 2^3$ and the variable x^6 can be rewritten as $x^6 = (x^2)^3$ using the law of exponent $((a^m)^n = a^{m \cdot n}$ or the Power Rule.

Thus, it follows that $8x^6$ can be expressed as $8x^6 = (2^3)(x^2)^3 \ or \ (2x^2)^3$.

Activity: Power of 3!

Directions: Express the following in exponential form with a power of 3. Write your answers on you answer sheet.

1. 27
2. $\frac{1}{8}$
3. $64y^3$
4. $125x^3$
5. $27x^6y^{12}$

Questions:

1. How did you find the activity?
2. Were you able to correctly express each expression as a power of 3?
3. Did you encounter any difficulty in performing the activity? If so, what did you do to overcome this difficulty?

What's New

Activity: See the Pattern

Given below are expressions in factored form in which one of the factors is a binomial and the other one is a trinomial. Follow the process in multiplying them and compare the product to the factors and give your observations. Write your answer on your answer sheet.

Factored Form	Steps	Process
1. $(x - 3)(x^2 + 3x + 9)$	$x^2(x - 3) + 3x(x - 3) + 9(x - 3)$	Distributing $(x - 3)$ to each of the terms in the given trinomial (Distributive property)
	$x^3 - 3x^2 + 3x^2 - 9x + 9x - 27$	Distributing x^2 to $(x - 3)$, $3x$ to $(x - 3)$, and 9 to $(x - 3)$ (Distributive property)
Product	$x^3 - 27$	By simplification

Compare the product $x^3 - 27$ to its factors $(x - 3)(x^2 + 3x + 9)$. What are your observations?

Factored Form	Steps	Process
2. $(x + 3)(x^2 − 3x + 9)$	$x^2(x + 3) − 3x(x + 3) + 9(x + 3)$	Distributing $(x + 3)$ to each of the terms in the given trinomial (Distributive property)
	$x^3 + 3x^2 − 3x^2 - 9x + 9x + 27$	Distributing x^2 to $(x + 3)$, $−3x$ to $(x + 3)$, and 9 to $(x + 3)$ (Distributive property)
Product	$x^3 + 27$	By simplification

Notice that the operation of the binomial factor was changed to plus (+) and also the first operation in the trinomial factor was changed to minus (−). Compare the product $x^3 + 27$ to its factors $(x + 3)(x^2 − 3x + 9)$. What are your observations?

What is It

The activity above deals with the product of a binomial and a trinomial which could be a sum or difference of two cubes. To illustrate, let us have the example below:

$$(x − 2)(x^2 + 2x + 4) \quad = \quad x^2(x - 2) + 2x(x - 2) + 4(x - 2)$$

$$= \quad x^3 − 2x^2 + 2x^2 - 4x + 4x − 8$$

$$= \quad x^3 − 8 \rightarrow \text{Difference of two cubes}$$

To get the factored form of the difference of cubes, reverse the process as shown below.

$$x^3 − 8 = x^3 − 2^3 = (x - 2)(x^2 + 2x + 4)$$

To get the binomial factor, subtract the base of the first term by the base of the second term.

First term: x^3 its base is x
Second term: 2^3 its base is 2
Binomial Factor: $(x - 2)$

To get the trinomial factor:

First term: Square the first term of the binomial factor $x - 2$
$$\rightarrow x^2$$

Second term: Multiply the terms of the binomial factor $x - 2$.

$$\rightarrow 2x$$

CO_Q1_Mathematics8_Module1A

Third term: Square the second term of the of the binomial factor $x - 2$.

$$\rightarrow 2^2 = 4$$

Hence, the trinomial factor is $x^2 + 2x + 4$. (Note that since the binomial factor is connected by a $-$, then, the middle term should be its additive inverse or $+$ and in factoring sum or difference of two cubes, the operation of the third term of the trinomial factor is always $+$).

This suggests the following rule for factoring a difference of cubes.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Let us now use the pattern in the examples below.

Example 1: Factor $y^3 - 27$.

Look for the two terms a and b by expressing every term to the power of 3.

$y^3 = (y)^3$ and $27 = (3)^3$

Using the pattern, $a = y$ and $b = 3$. By substituting to $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$:

$$
\begin{aligned}
y^3 - 27 \quad = \quad y^3 - 3^3 \quad &= \quad (y - 3)(y^2 + 3y + 3^2) \\
&= \quad (y - 3)(y^2 + 3y + 9)
\end{aligned}
$$

Example 2: Factor $8x^3 - 64$.

First, examine if the terms have a greatest common monomial factor (GCMF). Note that $8x^3$ and 64 have a GCMF of 8. Hence, we can rewrite the expression as: $8(x^3 - 8)$, where $x^3 - 8$ is a difference of two cubes.

Look for the two terms a and b by expressing every term to the power of 3.

$x^3 = (x)^3$ and $8 = 2^3$

Following the pattern, $a = x$ and $b = 2$, by substituting this to $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, we have:

$$
\begin{aligned}
8x^3 - 64 \quad = \quad 8[(x)^3 - (2)^3] \quad &= \quad 8(x - 2)(x)^2 + 2(x) + 2^2) \\
&= \quad \mathbf{8(x - 2)(x^2 + 2x + 4)}
\end{aligned}
$$

Example 3: Factor $27m^4 - 8mn^6$

Examine first whether the terms $27m^4$ and $8mn^6$ contain a GCMF. Notice that the given binomial cannot be factored directly using difference of two cubes since there is a variable m in both terms which is not a perfect cube. Hence, factoring by taking out the GCMF must be applied first.

$$27m^4 - 8mn^3 = m(27m^3 - 8n^6)$$

The factor $27m^3 - 8n^3$ is a difference of two cubes. Hence, the pattern can be applied.

$$27m^3 = (3m)^3 \ and \ 8n^6 = (2n^2)^3$$

Using the pattern, $a = 3m$ and $b = 2n^2$. By substituting to $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$, we have:

$$27m^3 - 8n^6 \quad = \quad (3m)^3 - (2n^2)^3 \quad = \quad (3m - 2n^2)((3m)^2 + 3m\,(2n^2) + (2n^2)^2)$$

$$= \quad \mathbf{(3m - 2n^2)(\ 9m^2 + 6mn^2 + 4n^4)}$$

Putting all the factors together, the complete factored form $27m^4 - 8mn^6$ is:

$$27m^4 - 8mn^6 = m\,(27m^3 - 8n^6) = \mathbf{m\,(3m - 2n^2)(\ 9m^2 + 6mn^2 + 4n^4)}$$

In the same manner, the sum of two cubes can be factored using a pattern similar to the difference of cubes. It is the result of a multiplication like the following:

$$(x + 2)(x^2 - 2x + 4) \quad = \quad x^2(x + 2) - 2x\,(x + 2) + 4\,(x + 2)$$

$$= \quad x^3 + 2x^2 - 2x^2 - 4x + 4x + 8$$

$$= \quad x^3 + 8 \rightarrow \text{Sum of two cubes}$$

This means that in order to get the complete factored form of the sum of cubes, we will just do the reverse process and we have,

$$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$$

To get the binomial factor, add the base of the first term to the base of the second term.

First term: x^3 its base is x
Second term: 2^3 its base is 2
Binomial Factor: $(x + 2)$

To get the trinomial factor:

First term: Square the first term of the binomial factor $x + 2$.
$$\rightarrow x^2$$

Second term: Multiply the terms of the binomial factor $x + 2$.

$$\rightarrow 2x$$

Third term: Square the second term of the binomial factor $x + 2$.

$$\rightarrow 2^2 = 4$$

Hence, the trinomial factor is : $x^2 - 2x + 4$. (Note that since the binomial factor is connected by a $+$, then, the middle term should be its additive inverse or $-$ and in factoring sum or difference of two cubes, the operation of the third term of the trinomial factor is always $+$).

This suggests the following rule for factoring a difference of cubes.

$$\mathbf{a^3 + b^3 = (\ a + b)(\ a^2 - ab + b^2\ )}$$

Example 4: Factor $1 + 8k^3$

Look for the two terms a and b by expressing every term to the power of 3.

$$1 = (1)^3 \text{ and } 8k^3 = (2k)^3$$

So, $a = 1$ and $b = 2k$, substituting it to $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$,
we have:

$$1 + 8k^3 = (1 + 2k)[(1)^2 - 1(2k) + (2k)^2]$$
$$= (1 + 2k)(1 - 2k + 4k^2)$$

Example 5: Factor $5h + 40hk^3$

In this case you need to consider factoring the greatest common monomial factor to determine the sum of cubes.
$$5h + 40hk^3 = 5h(1 + 8k^3)$$

Note that $1 + 8k^3$ is already factored in Example 4 as $(1 + 2k)(1 - 2k + 4k^2)$. Hence, the final factored form of $5h + 40hk^3 = 5h(1 + 2k)(1 - 2k + 4k^2)$.
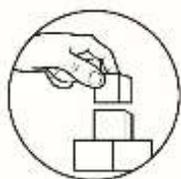
What's More

Activity 1: Cube to the Left, Cube to the Right

You learned in the previous activity about the sum and difference of two cubes. Determine whether the following polynomials are sum of two cubes (STC), difference of two cubes (DTC), or neither sum nor difference of two cubes (NSND). Write your answers on your answer sheet.

1. $16 + a^3$

2. $27b^3 - 64$

3. $125 + v^6$

4. $1000 - y^3$

5. $1 + a^3b^3$

Activity 2: The Missing Parts

Complete the following products. Write your answers on your answer sheet.

1. $(x + 3)(x^2 - 3x + 9)$
$= (x + 3)x^2 - (\quad)3x + (\quad)9$
$= x^3 + \_\_ - 3x^2 - 9x + \_\_ + 27$
$= x^3 + \_\_\_$

2. $(x - 3)(x^2 + 3x + 9)$
$= (\quad)x^2 + (x - 3)3x + (x - 3)\_\_$
$= \_\_ - 3x^2 + 3x^2 - \_\_ + 9x - 27$
$= \_\_ - 27$

3. $(x + y)(x^2 - xy + y^2)$
$= (x + y)\_\_ - (\quad)xy + (x + y)\_\_$
$= x^3 + \_\_ - x^2y - \_\_ + xy^2 + \_\_$
$= \_\_ + y^3$

4. $(x - y)(x^2 + xy + y^2)$
$= (\quad)x^2 + (\quad)xy + (\quad)y^2$
$= \_\_ - x^2y + x^2y - xy^2 + \_\_ - y^3$
$= x^3 - \_\_$

Activity 3: Break the Cubes

Factor each completely. Write your answers on your answer sheet.

1. $x^3 + 27$

2. $8y^3 - 27$

3. $1 + x^3 y^3$

4. $64 - p^6$

5. $-2m^5 + 250m^2$

What I Have Learned

Directions: Read and answer the following questions. Write your answers on your answer sheet.

1. How did you completely factor the sum and difference of two cubes? Write the process of each and their rule or pattern.
2. There are cases in which the given expression cannot immediately be factored using patterns in the sum and difference of cubes. How will you utilize the patterns in the sum and difference of two cubes in this case?
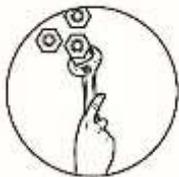
What I Can Do

Packing Breakables

An online seller of glass water tumblers is about to ship the order of his client via a local courier service provider. To ensure that the items will not be damaged during the shipping, the tumblers were secured in a small box and is to be placed in a larger box filled with styrofoam chips.
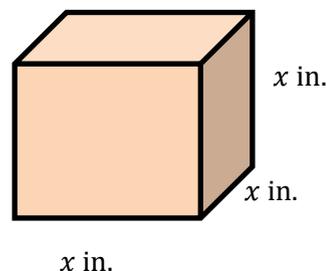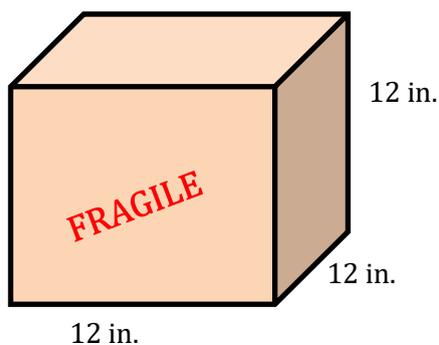
Question:

1. Write a polynomial that describes the amount of space in a larger box that must be filled with styrofoam chips.
2. Factor the polynomial.

Assessment

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. What is the process of finding the factors of an expression which is the reverse process of multiplication?
   A. factoring
   B. special product
   C. rationalization
   D. continuous division

2. What is the Greatest Common Factor of 12 and 24?
   A. 2   B. 4   C. 12   D. 24

3. Which of the following pair of numbers has a GCF of 6?
   A. 2 and 3   B. 8 and 12   C. 6 and 10   D. 12 and 18

4. What is the GCF of $2a^3$ and a^6?
   A. a^3 B. $2a^3$ C. a^6 D. $2a^6$

5. What is the GCF of a^5b^8, a^4b^6, a^3b^9, and $a^{12}b^{10}$?
   A. a^5b^8 B. a^3b^6 C. $a^{12}b^8$ D. $a^{12} b^6$

6. All of the following are factors of $12x^2$ Except one. What is it?
   A. 12   B. 12x   C. $12x^2$ D. $12x^3$

7. What is the GCF of the expression $4x^4 + 6x$?
   A. 2   B. x   C. 2x   D. $2x^4$

8. If one factor of $4x^2 + 6$ is 2, what is the other factor?
   A. 2x + 3   B. 4x + 6   C. 2x + 6   D. $2x^2 + 3$

9. If one factor of $6ab^2 - 12 a^2b^3$ is $1 - 2ab$, what is the other factor?
   A. 6ab   B. $6a^2b$ C. $6ab^2$ D. $6a^2b^2$

10. Which of the following is a perfect square?
    A. $12x^4$ B. $16x^2y^2$ C. $25x^5y^6$ D. $36x^6y^7$

11. Which of the following expressions has factors $(2x - y)$ and $(2x + y)$?
    A. $2x^2 + y^2$ B. $2x^2 - y^2$ C. $4x^2 + y^2$ D. $4x^2 - y^2$

12. Using the pattern for factoring the sum of cubes, we know that factoring $8 + b^3$
gives

    A. $(2 - b)(4 - 2b + b^2)$ C. $(2 + b)(4 - 2b + b^2)$

    B. $(2 - b)(4 + 2b + b^2)$ D. $(2 + b)(4 + 2b + b^2)$

13. What is the complete factored form of $27x^3 - 64y^3$?

    A. (3x − 4y)(9x² + 12xy + 16y²)   C. (3x − 4y)(9x² - 12xy + 16y²)

    B. (3x + 4y)(9x² + 12xy + 16y²)   D. (3x + 4y)(9x² - 12xy + 16y²)

14. What is the complete factored form of 10 + 270y³?

    A. 10(1 + 3y)(1 + 3y +9y²)   C. 10(1 - 3y)(1 + 3y +9y²)

    B. 10(1 + 3y)(1 - 3y +9y²)   D. 10(1 - 3y)(1 - 3y +9y²)

15. The area of a rectangular garden is $9t^2 − 64$ square units. If one side is $3t − 8$,
what is the other side?

    A. 3t − 8   B. 3t + 8   C. t- 8   D. t + 8

Additional Activities

Activity: Do More!

    Factor completely each of the given expressions and look for the answers in the rectangle. Write your answer on your answer sheet.

$2xy^2$	$3a^5b^2(13b - 9a^2 + 18a^3b^3)$	$3a^2b(4ab - 1)$		$3x^2(4x^2 - 1)$	$(a + 1)(a^2 + a + 3)$	$3x(2x - 3)$ $(2x + 3)$	$(a + 1)(a^2 - a + 3)$	$(f^m - g^{2n})(f^{2m} + f^m g^{2n} + g^{4n})$	$(2h^2 + 3j^3)$ $(4h^4 - 6h^2j^3 + 9j^6)$
		$x^2(6x - 1)$				$(4ab - 1)$			$(a - 1)(a^2 - a)$
$(4x^2 - 2)$		$(4xyz - 2)$	$x^2 + y^3$		$(x - y^2)$	$8(4 - 5m^2)$ $(4 + 5m^2)$			$(xy^2 - z)(xy^2 + z)$ $(x^2y^4 + z^2)$
						$(a + 1)(a^2+3)$			$3x(4x - 1)$
		$9a^2b(3ab - 1)$				$(x^2 - 11y^4)$ $(x^2 + 11y^4)$			$(b^2 - 3c^2)$ $(b^4 + 3b^2c^2 + 9c^4)$

    1. $27a^3b^2 - 9a^2b$

    2. $39a^5b^3 - 27a^7b^2 + 54a^8b^5$

    3. $a^2(a + 1) + a(a + 1) + 3(a + 1)$

    4. $12x^3 - 27x$

    5. $128 - 200m$

Answer Key

Lesson 1: Factoring with common monomial factor

What I know
1. D
2. B
3. A
4. B
5. A
6. C
7. D
8. A
9. D
10. D
11. B
12. C
13. A
14. B
15. D

What's In
Activity:
1. 2 and 4
2. 2 and x
3. 5, a, and b
4. 2, 3, 4, 6, and z
5. 2, 5, 10, x, and y

What's New
1. Yes
2. The product of the sides is the area of the rectangle
3. The width of the rectangle is a factor of its area.
4. Factoring

What's More
Act. 1: Break the Great
1. GCF: x ; Factored Form: $x(x+2)$
2. GCF: $5x^2$; Factored Form: $5x^2(1 - 2x)$
3. GCF: $5xy^3$; Factored Form: $5xy^3(5x + 11)$
4. GCF: $5c^3$; Factored form $5c^3(2 - 16c^2 - c^4)$
5. GCF: $3mn$; Factored Form $3mn(4m^3n - 2mn^2 + 1)$

Act. 2: You Complete Me
1. $p - 1$
2. $3y$
3. $3t^2 - 3t + 4$
4. $17x$
5. $5x^2y + 3x + 2y$

What I have Learned
Answers may vary

Lesson 2: Factoring difference of two squares

What's In
Act. 1: Power of 2!
1. P
2. N
3. P
4. N
5. P

Act. 2: Missing You
1. $(a + 9)(a - 9)$
2. $(b + d)(b - d)$
3. $(c + d)(c - d)$
4. $(7e + 9f)(7e - 9f)$
5. $(10g + 11h)(10g - 11h)$

Act. 3: Whole to Parts
1. $(x - 9)(x + 9)$
2. $(2x - 7)(2x + 7)$
3. $(4x - 9y)(4x + 9y)$
4. $(5x - 1)(5x + 1)$
5. $(9x + 20y^2)(9x - 20y^2)$

What' New
1. $x^2 - 1$
2. $x^2 - 16$
3. $x^2 - 9$
4. $x^2 - 49$
5. $x^2 - 81$

Act. 4: When I broke it, I Found it.
1. $3x^2 - 12y^2 = 3(x^2 - 4y^2)$
   $= 3(x + 2y)(x - 2y)$
2. $1 - 16x^8 = (1 + 4x^4)(1 - 4x^4)$
   $= (1 + 4x^4)(1 + 2x^2)(1 - 2x^2)$
3. $a^4 - 625b^8 = (a^2 + 25b^4)(a^2 - 25b^4)$
   $= (a^2 + 25b^4)(a - 5b^2)(a + 5b^2)$

What I have Learned
Answers may vary

What's More
Act. 1. Tell Me
1. D
2. N
3. D
4. D
5. N

Lesson 3: Factoring sum and difference of two cubes

What's In
1. 3^3
2. $(\frac{1}{2})^3$
3. $(4y)^3$
4. $(5x)^3$
5. $(3x^2y^4)^3$

What's More
Act. 1: Cube to the Left, Cube to the Right
1. NSND
2. DTC
3. STC
4. DTC
5. STC

Act. 2: Missing Parts
1. $(x + 3)x^2 - (x + 3)3x + (x + 3)9$
   $x^3 + 3x^2 - 3x^2 - 9x + 9x + 27$
   $x^3 + 27$
2. $(x - 3)x^2 + (x - 3)3x + (x - 3)9$
   $x^3 - 3x^2 + 3x^2 - 9x + 9x - 27$
   $x^3 - 27$
3. $(x + y)x^2 - (x + y)xy + (x + y)y^2$
   $x^3 + x^2y - x^2y - xy^2 + xy^2 + y^3$
   $x^3 + y^3$
4. $(x - y)x^2 + (x - y)xy + (x - y)y^2$
   $x^3 - x^2y + x^2y - xy^2 + xy^2 - y^3$
   $x^3 - y^3$

Lesson 3: Factoring sum and difference of two cubes

What's More

Act. 3: Break the Cube
1. $(x + 3)(x^2 - 3x + 9)$
2. $(2y - 3)(4y^2 + 6y + 9)$
3. $(1 + xy)(1 - xy + x^2y^2)$
4. $(2 + p)(2 - p)(16 + 4p^2 + p^4)$
5. $-2 m^2 (m - 5)(m^2 + 5m + 25)$

What I Can Do

1. $1,728 - x^3$
2. $(12-x)(144 + 12x + x^2)$

Additional Activity

1. $9a^2b(3ab - 1)$
2. $3a^5b^2(13b - 9a^2 + 18a^3b^3)$
3. $(a + 1)(a^2 + a + 3)$
4. $3x(2x - 3)(2x + 3)$
5. $8(4 - 5m^2)(4 + 5m^2)$
6. $(xy^2 - z)(x^2y^4 + z)(x^2y^4 + z^2)$
7. $(x^2 - 11y^4)(x^2 + 11y^4)$
8. $(b^2 - 3c^2)(b^4 + 3b^2c^2 + 9c^4)$
9. $2h^2 + 3j^3)(4h^4 - 6h^2j^3 + 9j^6)$
10. $(f^m - g^{2n})(f^{2m} + f^mg^{2n} + g^{4n})$

Assessment
1. A
2. C
3. D
4. A
5. B
6. D
7. C
8. D
9. C
10. B
11. D
12. C
13. A
14. B
15. B

References

Alferes, M.S., Duro, M.A. (2004). *MSA Intermediate Algebra.* Quezon City. Quezon City MSA Academic Advancement Institute

Glorial, C.J., et.al. (2010). *21st Century Mathematics.* Quezon City. Phoenix Publishing House Inc.

Oronce O.A and Mendoza M.O. (2014). *e – Math Algebra.* Metro Manila. Rex Books Store

Rees K.R., Sparks. F.W., and Rees C.S. (1990). *College Algebra Tenth Edition.* Pennsylvania Plaza.McGraw-Hill Publishing Company

Links

Interactive Mathematics (2018). *Special Products.* Retrieved from http://www.intmath.com/factoring-fractions/1-special-products.php

Math Planet (nd). *Special products of polynomials.* http://www.mathplanet.com/education/algebra-1/factoring-and-polynomials/special-products-of-polynomial

Paul's Online Notes (2003). *Factoring Polynomials.* Retrieved from http://tutorial math.lamar.edu/Classes/Alg/Factoring.aspx

Purple Math (nd). *Factoring Polynomials.* Retrieved from http://www.purplemath.com/modules/rtnladd2.htm