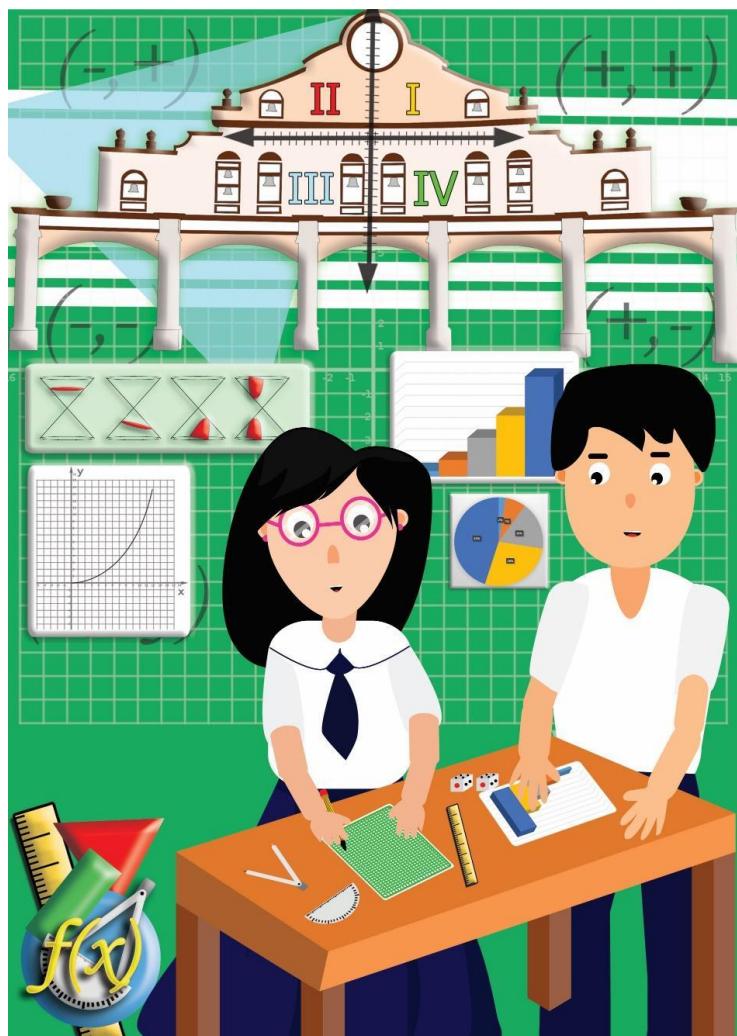


GENERAL MATHEMATICS

Quarter 1: Module 5

Exponential Functions



Writer: Joel C. Hagos, (FHS)

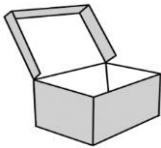
Cover Illustrator: Joel J. Estudillo (SNNHS)



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What I Need to Know

Hello senior high school learners! In this module , you will learn how to:

Represent an exponential function through its (a) table of values, (b) graph, and equation.

Find the domain and range of an exponential function.

Determine the intercepts, zeroes, and asymptotes of an exponential functions.

Solve problems involving exponential functions, equations and inequalities.

You can say that you have understood the lesson in this module if you can already:

1. represent an exponential function through its: (a) table of values, (b) graph, and (c) equation,
2. sketch the graph of exponential function and its transformation,
3. find the domain and range of an exponential function,
4. determine the intercepts, zeroes and asymptotes of an exponential function,
5. solves problems involving exponential functions, equations, and inequalities.



What I Know

Choose the correct letter that corresponds to the exact answer.

1. Which table of values represent this function: $y = 2^x$

A.

x	-3	-2	-1	0	1	2	3
y	1/8	1/4	1/2	-1	-2	-4	-8

B.

x	-3	-2	-2	0	1	2	3
y	8	4	2	1	1/2	1/4	1/8

C.

x	-3	-2	-1	0	1	2	3
y	1/8	1/4	1/2	1	2	4	8

D.

x	-3	-2	-1	0	1	2	3
y	-1/8	-1/4	-1/2	-1	-2	-4	-8

2. Find the value of y when $x = -2$ in the function $y = 2^x$.

A. $-\frac{1}{8}$ B. $-\frac{1}{4}$ C. $\frac{1}{8}$ D. $\frac{1}{4}$

3. Find the domain of the function $f(x) = (\frac{1}{3})^{1-x} - 1$.

A. Domain: $x > -1$ C. Domain: $x = 0$
B. Domain: $x = 1$ D. Domain: Set of all real numbers

4. What is the range of the function $f(x) = (\frac{1}{3})^{1-x} - 1$?

A. Range: $(-1, +\infty)$ C. Range: $(-\infty, -1)$
B. Range: $(-1, \infty)$ D. Range: Set of all real numbers

5. Determine the domain and range of the function $y = 2^{x+3} - 3$.

A. Domain: \mathbf{R} C. Domain: \mathbf{R}
Range: $(-3, -\infty)$ Range: $(-\infty, 3)$
B. Domain: \mathbf{R} D. Domain: \mathbf{R}
Range: $(-3, +\infty)$ Range: $(3, +\infty)$

6. What is the x-intercept of 5^x ?

A. $x = 0$ B. $x = 1$ C. $x = 5$ D. There is no x –intercept.

7. Determine the horizontal asymptote of the function $y = 6^x - 3$.

A. $y = -6$ B. $y = -3$ C. $y = 6$ D. $y = 3$



8. What is the y-intercept of the function $y = 3^{x+3}$?
A. $x = 3$ B. $x = 27$ C. $y = 3$ D. $y = 27$

For item number 9 –10. Refer to the given problem: The initial value of your taxi car is Php 40,000. After 1 year, the value is Php 30,000.

9. What is the percent decrease?
A. 20% B. 25% C. 30% D. 35%
10. Find the value of the car at this same rate after 4 years from the initial value.
A. 12 450.25 B. 12 565.25 C. 12 656.25 D. 12 767.25

**LESSON 1: Exponential Function through its table of values,
graphs and equation.**



What's In

Solve the following:

1. $2^x = 4^{-2+x}$

Answer: _____

2. $3^x > 27$

Answer: _____

3. $4^{x+1} = 8^{x+2}$

Answer: _____

Guide Questions:

1. How did you solve for exponential equation?
2. What are the steps in solving exponential inequality?
3. Do you have any idea how to construct a table of values and graph of the function $f(x) = 2^x$? Discuss your answer.



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What's New

Compare the two graphs below:

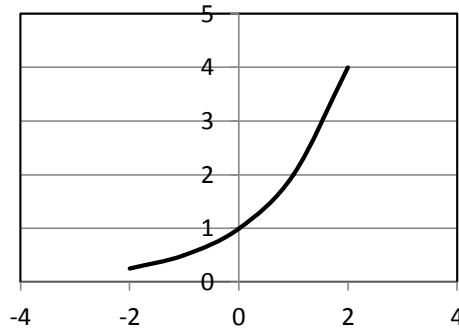


Figure 1: Graph of $f(x) = 2^x$

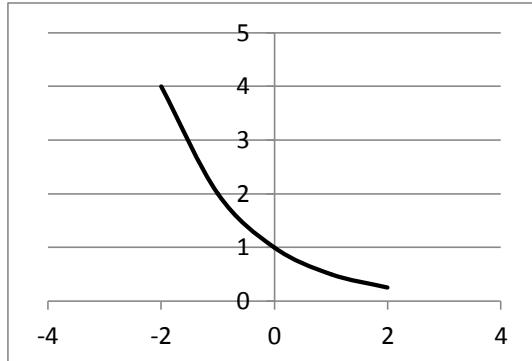


Figure 2: Graph of $f(x) = (\frac{1}{2})^x$

1. What is the domain of each graph?
2. Based on your answer in #1, how would you be able to construct a table of values and sketch the graph of an exponential function?
3. Observing Figure 1, what happens to the function values as the values of x increase (or as you move from left to right)?
4. Observing Figure 2, what happens to the function values as the values of x increase (or as you move from left to right)?
5. Compare the bases of the functions illustrated in Figures 1 and 2. Can you relate these to your observations in #3 and #4?



What is It

Here are some illustrative examples on how to construct a table of values and how to sketch the graph of an exponential function.

Example 1: Construct a table of values and sketch the graph of the $f(x) = 2^x$.

Solution:

Step 1: To construct a table of values, you assign values for x and evaluate the function for the given values of x .

$$f(x) = 2^x$$

x	-4	-3	-2	-1	0	1	2	3	4
f(x)									



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$$\text{If } x = -4, \text{ then } f(-4) = 2^{-4} = \frac{1}{2^4} = \frac{1}{16} \quad \text{if } x = 1, \text{ then } f(1) = 2^1 = 2$$

$$\text{If } x = -3, \text{ then } f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8} \quad \text{if } x = 2, \text{ then } f(2) = 2^2 = 4$$

$$\text{If } x = -2, \text{ then } f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4} \quad \text{if } x = 3, \text{ then } f(3) = 2^3 = 8$$

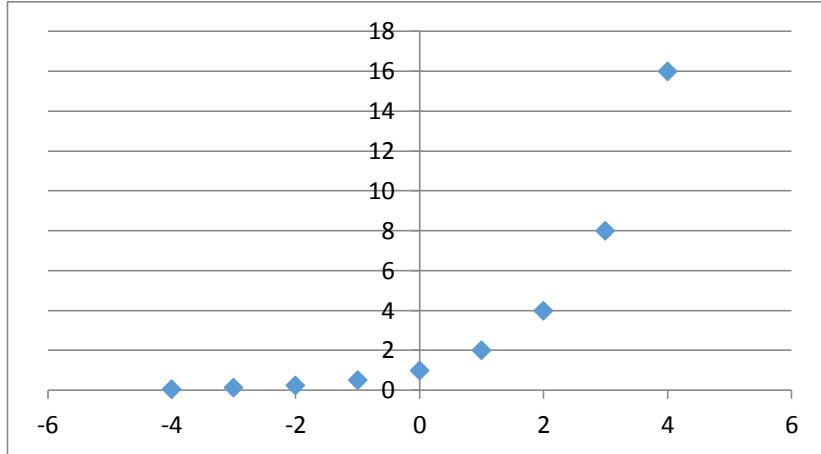
$$\text{If } x = -1, \text{ then } f(-1) = 2^{-1} = \frac{1}{2^1} = \frac{1}{2} \quad \text{if } x = 4, \text{ then } f(4) = 2^4 = 16$$

$$\text{If } x = 0, \text{ then } f(0) = 2^0 = 1$$

The table of values for $f(x) = 2^x$

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16

Step 2: To sketch the graph of the $f(x) = 2^x$, simply plot the points on the Cartesian coordinate system and connect the points.



If you noticed, the given function is defined for all values of x , since $b > 1$, then the graph is increasing. For any real constant c , the line $y = c$ is a **horizontal asymptote** of the function f if any of the two conditions is satisfied:

- (a) As $x \rightarrow \infty, f(x) \rightarrow c$
As $x \rightarrow -\infty, f(x) \rightarrow c$



Example 2: Construct a table of values and sketch the graph of the $g(x) = (\frac{1}{2})^x$

Solution:

Step 1: To construct a table of values, you assign values for x and evaluate the function for the given values of x .

$$g(x) = \left(\frac{1}{2}\right)^x$$

x	-4	-3	-2	-1	0	1	2	3	4
f(x)									

If $x = -4$, then $g(-4) = \left(\frac{1}{2}\right)^{-4} = 2^{-1(-4)} = 16$ if $x = 1$, then $g(1) = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$

If $x = -3$, then $g(-3) = \left(\frac{1}{2}\right)^{-3} = 2^{-1(-3)} = 8$ if $x = 2$, then $g(2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

If $x = -2$, then $g(-2) = \left(\frac{1}{2}\right)^{-2} = 2^{-1(-2)} = 4$ if $x = 3$, then $g(3) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

If $x = -1$, then $g(-1) = \left(\frac{1}{2}\right)^{-1} = 2^{-1(-1)} = 2$ if $x = 4$, then $g(4) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

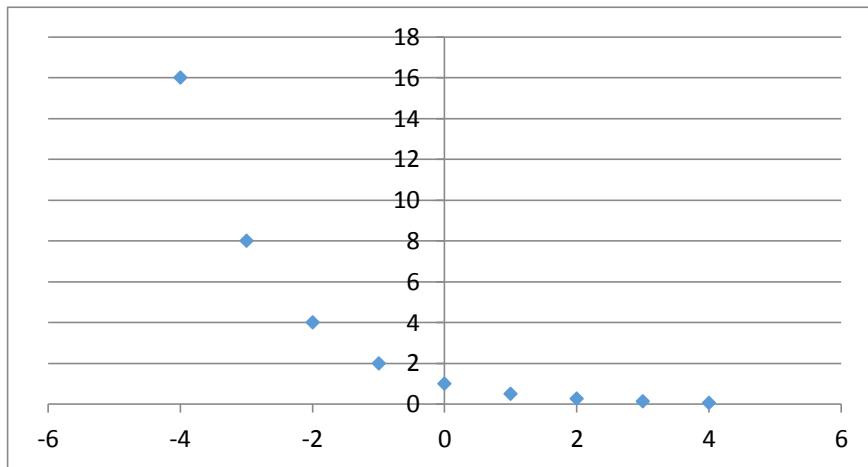
If $x = 0$, then $g(0) = \left(\frac{1}{2}\right)^0 = 2^{-1(0)} = 1$

The table of values for $g(x) = \left(\frac{1}{2}\right)^x$

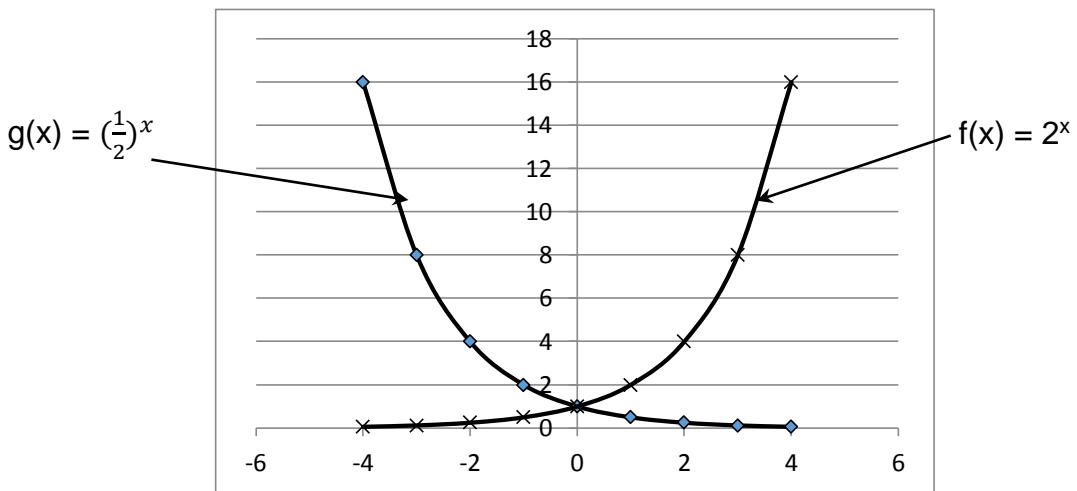
x	-4	-3	-2	-1	0	1	2	3	4
f(x)	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$



Step 2: To sketch the graph of the $g(x) = (\frac{1}{2})^x$, simply plot the points on the Cartesian coordinate system and connect the points.



If you noticed, the given function is defined for all values of x , since $0 < b < 1$, then the graph is decreasing. It can be observed also that when x increases rapidly, the value of function approaches zero. The horizontal asymptote is the line $y = 0$.



Note: If you combine the graph of $f(x)$ and $g(x)$ they are symmetrical with respect to y-axis and it intersect at the point $(0,1)$.

Do all functions mentioned earlier pass the horizontal line test? We see that any horizontal line will pass these graphs at exactly one point, hence we conclude that exponential functions are one-to-one.

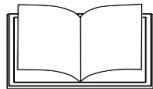


The following summarizes our findings regarding the properties of exponential function and its graph.

Properties of Exponential Functions

Let $F(x) = a \cdot b^{P(x)} + h$ be an exponential function where $P(x)$ is linear.

- A. The line $y = h$ is the horizontal asymptote.
- B. The function is increasing when $b > 1$, but it is decreasing when $0 < b < 1$.
- C. The function is one-to-one.



What's More

Construct the table of values and sketch the graph of each pair of function in one Cartesian plane.

1. $f(x) = 5^x$ and $g(x) = \left(\frac{1}{5}\right)^x$

2. $h(x) = 3^x$ and $k(x) = -3^x$

3. $p(x) = 4^x$ and $q(x) = 4^{x+1}$



What I Have Learned

Fill in the Blank: Complete the sentence below. Write your answer on the space provided.

1. The function $G(x) = 3^x$ has a horizontal asymptote $y = \underline{\hspace{2cm}}$.

2. The graph of the function $f(x) = \left(\frac{1}{3}\right)^x$ is $\underline{\hspace{2cm}}$.

3. If you evaluate $x = -3$ to the function $g(x) = \left(\frac{1}{3}\right)^x$, the value of the function $g(-3) = \underline{\hspace{2cm}}$.



4. The function is _____ when $b>1$, but it is _____ when $0<b<1$.

5. The line $y = h$ is the _____.



What I Can Do

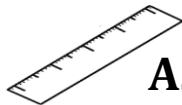
Graph each pair of functions in one Cartesian plane and compare the two functions.

$$F(x) = 7^x + 1 \text{ and } G(x) = 7^x - 2$$

Rubrics

Criteria	4	3	2	1
Mathematical Concept Used	Has shown full understanding of the concepts as evidenced in the work presented. Also used other concepts leading to the same solution	Has shown full understanding of the concepts as evidence in the work presented.	Has shown some degree of understanding of the concepts as evidenced in the work presented.	Has no knowledge of the concepts as evidenced in the work presented.
Accuracy of Computations	The computations were presented logically and done accurately. Alternative solutions/s leading to the same answer has been presented	The computations were presented logically and done accurately.	The computations were presented logically but there were minimal errors in computations.	The computations were incoherent and erroneous.
Presentation	An appropriate mathematical representation is constructed to analyze relationships, extend thinking, and clarify or interpret a phenomenon.	An appropriate and accurate mathematical representation is constructed and refines to solve problems.	An attempt is made construct a mathematical representation to record and communicate problem solving but is not accurate.	No attempt is made to construct a mathematical representation.



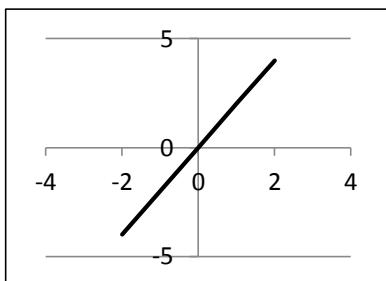


Assessment

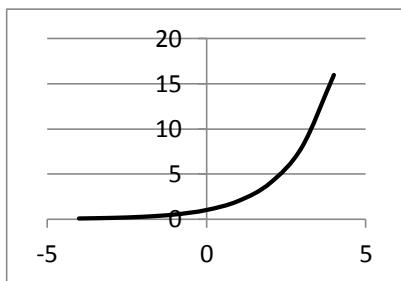
Choose the correct letter that corresponds to the exact answer.

1. Which of the following is a graph of an exponential function?

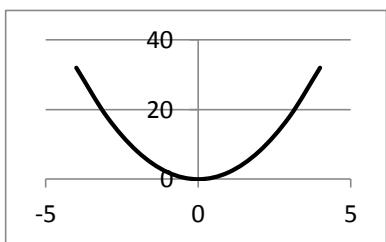
A.



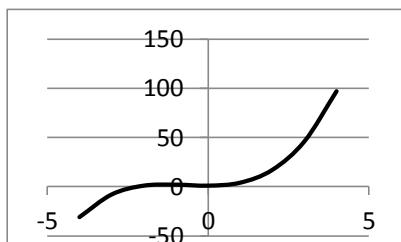
B.



C.



D.



2. Construct a table of values for the function $y = 3^x$

A.

x	-3	-2	-1	0	1	2	3
f(x)	1/27	1/9	1/3	-1	-3	-9	-27

B.

x	3	2	1	0	-1	-2	-3
f(x)	1/27	1/9	1/3	1	3	9	27

C.

x	-3	-2	-1	0	1	2	3
f(x)	27	9	3	1	-3	-9	-27

D.

x	-3	-2	-1	0	1	2	3
f(x)	1/27	1/9	1/3	1	3	9	27

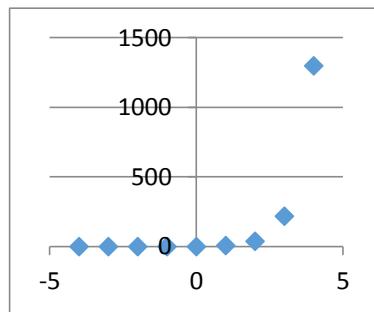
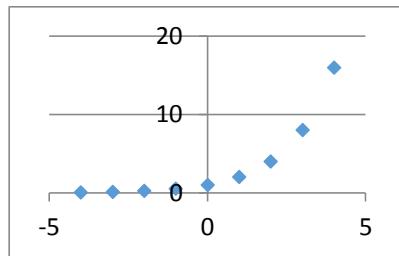


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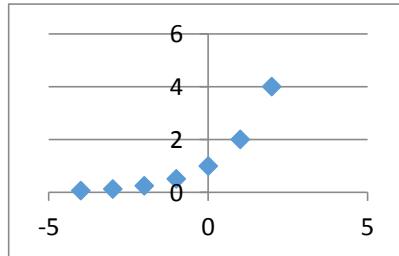
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3. Identify which among is the graph of $y = 3^x$

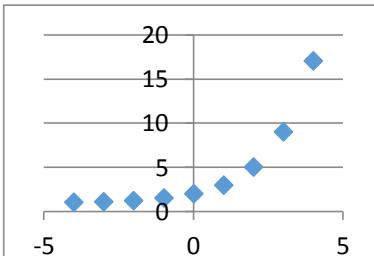
A. B.



C.

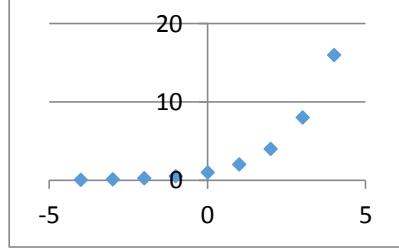


D.

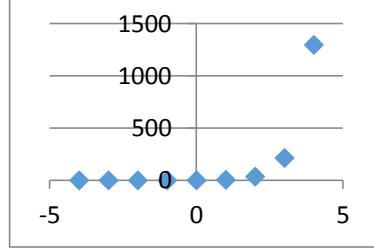


4. Which of the following is the graph of the function $f(x) = 6^x$

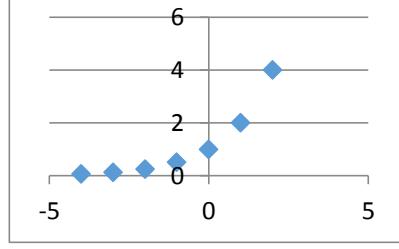
A.



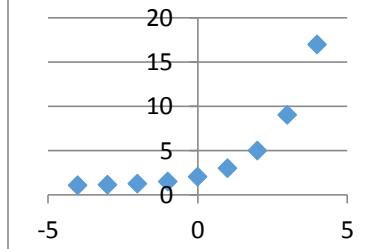
B.



C.



D.



5. What is the value of y when $x = 0$ in the function $y = 6^x$?

- A. $-\frac{1}{6}$ B. 0 C. 1 D. 6

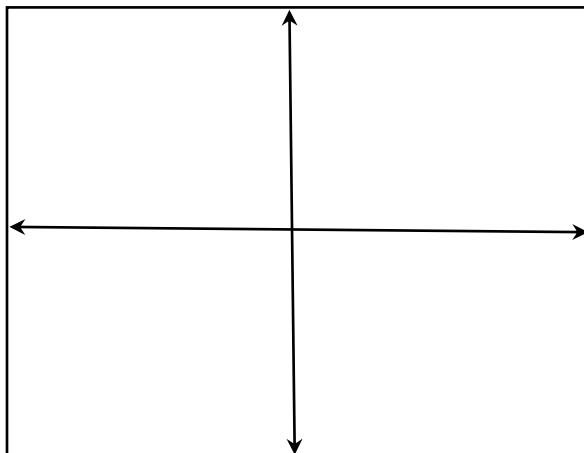




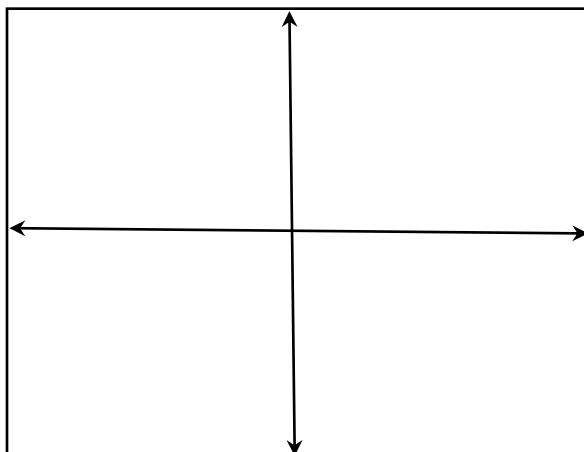
Additional Activities

Construct a table of values and sketch the graph of each pair of function in one Cartesian plane.

1. $P(x) = 4^x + 2$ and $Q(x) = 4^x - 2$



2. $H(x) = 3^{x+1}$ and $K(x) = (\frac{1}{3})^{x+1}$



LESSON 2: The Domain and Range of an Exponential Function.



What's In

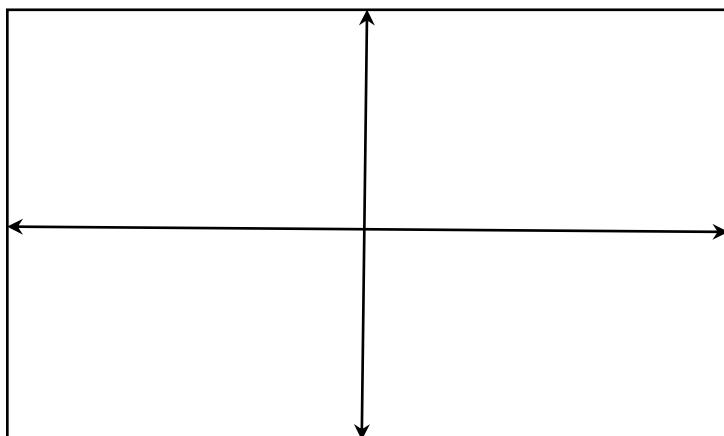
Complete the table of values and sketch the graph of each pair of function. $f(x) = 3^{x+1}$ and $g(x) = 3^{x-1}$

$$f(x) = 3^{x+1}$$

x	-4	-3	-2	-1	0	1	2	3
f(x)								

$$g(x) = 3^{x-1}$$

x	-4	-3	-2	-1	0	1	2	3
g(x)								



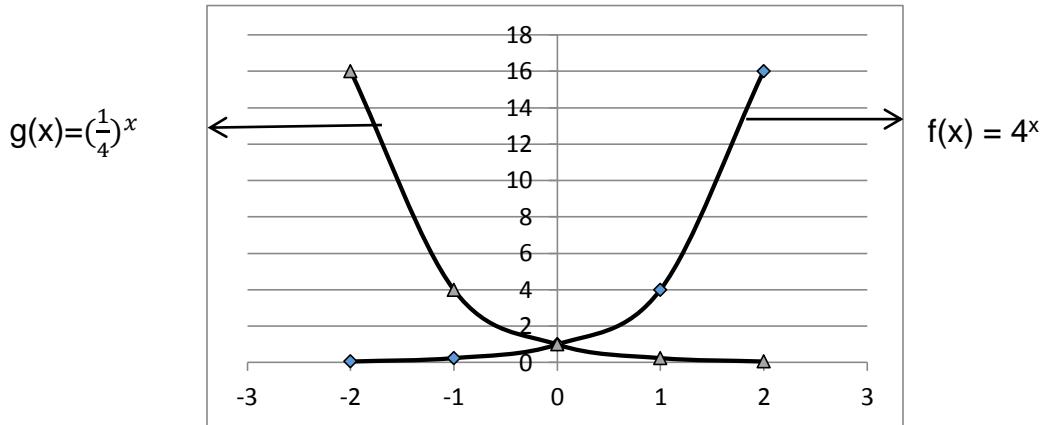
Guide Questions:

1. What is the difference of the two functions?
2. What Base on your graph, what inferences can you make?
3. Do you think that the two functions have same domain and range? Why?





What's New



What is the relation between the graphs $f(x)$ and $g(x)$?
Can you determine the domain and range of the two functions?



What is It

In here, we will be discussing on how to determine the domain and range of an exponential function.

Theorem: Domain and Range of an exponential function

Let $f(x) = a \cdot b^{P(x)} + h$ be an exponential function where $P(x)$ is linear. Then

$$\text{Domain}(f) = \mathbb{R}$$

$$\text{Range}(f) =$$



Example 1: Find the domain and range of the function $y = 3^x$

Solution:

Domain (f) = \mathbb{R} Since 3^x is defined for any real number x.

Range(f) = $(0, +\infty)$ Using the theorem, the value of $a = 1$, $b = 3$ and $h = 0$. Since $a > 0$, then the range must be $(h, +\infty)$

Example 2: Let $g(x) = 5^{x+1} - 3$. Find the domain and range of g.

Solution:

Domain (f) = \mathbb{R} The domain of g is \mathbb{R} .

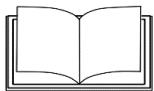
Range (f) = $(-3, +\infty)$ Using the theorem, the value of $a = 1$, $b = 5$ and $h = -3$. Since $a > 0$, then the range must be $(h, +\infty)$

Example 3: Given the function $H(x) = -3(\frac{1}{3})^{x-2} + 2$, Find the domain and range of h.

Solution:

Domain (f) = \mathbb{R} The domain of h is \mathbb{R} .

Range (f) = $(-\infty, 2)$ Using the theorem, the value of $a = -3$, $b = \frac{1}{3}$ and $h = 1$. Since $a < 0$, then the range must be $(-\infty, h)$



What's More

Find the domain and range of the given exponential functions.

1. $f(x) = (\frac{1}{4})^x$

2. $g(x) = -2^{3-x} + 5$

3. $h(x) = 2 + 2 \bullet 5^{2x-1}$

4. $k(x) = 3^{x+1} - 4$

5. $p(x) = 5 - 3 \bullet 2^{2-2x}$





What I Have Learned

Complete the statement/s. Write your answer on the space provided.

1. The domain of the function $y = 5^x + 2$ is _____.
2. The range of the function $y = 5^x + 2$ is _____.
3. From the function $g(x) = 3 + 2 \cdot 4^{x+1}$ the value of $a =$ _____.
4. What is the value of h from the $g(x)$ in number 3? $h =$ _____.
5. Let $f(x) = a \cdot b^{P(x)} + h$ be an exponential function where $P(x)$ is linear. Then

- Domain(f) = _____
- Range(f) = _____



What I Can Do

A. Consider the following functions, find the domain and range.

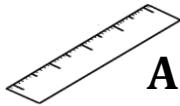
1. $f(x) = \left(\frac{1}{2}\right)^{x+2}$
2. $g(x) = 1 - 5 \cdot 6^x$

B. Give your own examples of exponential functions and determine the domain and range.

Rubrics

Score	Description
15 points	90-100% Correct answers with concrete explanation and there's corresponding neatness on the output.
10 points	60-89% Correct answers with an explanation and there's corresponding neatness on the output.
5 points	Incomplete answer with 50% incorrect answers and there's no corresponding neatness on the output.
No point earned	No output at all





Assessment

Choose the letter that corresponds to the exact answer.

1. What is the range of the function $f(x) = \left(\frac{1}{3}\right)^{1-x} - 1$?
A. Range: $(-1, +\infty)$ C. Range: $(+\infty, -1)$
B. Range: $(-\infty, 1)$ D. Range: Set of all real numbers
2. Determine the domain and range of the function $y = 2^{x+3} - 3$.
A. Domain: \mathbf{R} C. Domain: \mathbf{R}
Range: $(-3, -\infty)$ Range: $(-\infty, 3)$
B. Domain: \mathbf{R} D. Domain: \mathbf{R}
Range: $(-3, +\infty)$ Range: $(3, +\infty)$
3. Solve for the domain and range of the function $y = 2 + 2 \cdot 3^{x+1}$.
A. Domain: \mathbf{R} C. Domain: \mathbf{R}
Range: $(2, -\infty)$ Range: $(-\infty, 2)$
B. Domain: \mathbf{R} D. Domain: \mathbf{R}
Range: $(-2, +\infty)$ Range: $(2, +\infty)$
4. What is the domain of the function $y = 3^x$?
A. Domain: $x = 3$ C. Domain: $(0, +\infty)$
B. Domain: $y = 3$ D. Domain: Set of all real numbers
5. What is the range of the function $p(x) = 4^{x+1} - 2$?
A. $(4, +\infty)$ B. $(-2, +\infty)$ C. $(-\infty, -2)$ D. $(+\infty, -4)$



Additional Activities

Find the domain and range of the following functions.

1. $f(x) = 6^x$
2. $g(x) = 6^x + 5$
3. $h(x) = 3 - \left(\frac{1}{6}\right)^{x+1}$
4. $k(x) = 2 \bullet 5^x - 3$
5. $p(x) = -1 \bullet \left(\frac{1}{2}\right)^{x+1} - 4$

LESSON 3: Intercepts, Zeroes and Asymptote of Exponential Functions.



What's In

Use the theorem in finding the Domain and Range of the following functions:

1. $p(x) = 5^x$ and $g(x) = -2 \bullet 5^x - 1$

Domain (f) = _____
Range (f) = _____

Domain(g) = _____
Range(g) = _____

2. $f(x) = 2^x + 2$ and $h(x) = 2^x - 3$

Domain(f) = _____
Range(f) = _____

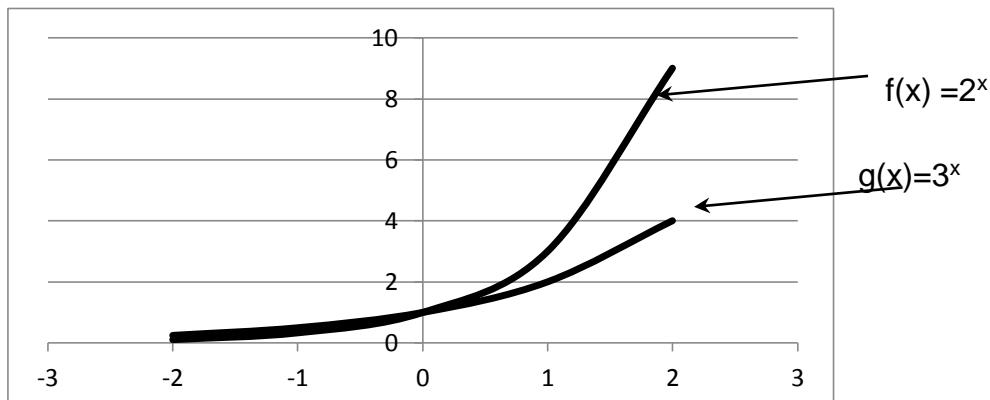
Domain(h) = _____
Range(h) = _____

Guide Questions:

1. How did you use the theorem in finding the domain of the functions?
2. How did you use the theorem in finding the range of the functions?
3. Do you think what is the intercepts and horizontal asymptotes of the Given functions above?



What's New



The Two functions



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Domain: Set of all real numbers

Range: Set of all positive real numbers

How do you determine the y-intercept of the function?

Can you apply it to the two functions shown above?



What is It

You have already covered the horizontal asymptote of an exponential function of the form $f(x) = ab^{g(x)} + h$, where $g(x)$ is a linear function.

Properties of exponential functions:

For any exponential function $f(x) = ab^{x-c} + h$:

- (a) Domain is the set of all real numbers.
- (b) Range: $(h, +\infty)$ if $a > 0$, $(-\infty, h)$ if $a < 0$
- (c) $f(x)$ is one-to-one
- (d) y-intercept: $f(0) = ab^{-c} + h$
- (e) Horizontal Asymptote: $y = h$
- (f) Behavior
 - $a > 0$ and $b > 1$: increasing
 - $a > 0$ and $0 < b < 1$: decreasing
 - $a < 0$ and $0 < b < 1$: increasing
 - $a < 0$ and $b > 1$: decreasing

Example 1: Given the function $f(x) = 2^x$ and $g(x) = 5^x$. Find the domain, range, intercepts, and horizontal asymptote.

Solution: For $f(x) = 2^x$ and $g(x) = 5^x$

Domain: Set of all real numbers

Range: Set of all positive real numbers.

Intercepts: Both $f(x)$ and $g(x)$ the $y = 1$, and there is no x-intercept.

Horizontal Asymptote: $y = 0$

Example 2: Determine the domain, range, intercepts, and horizontal asymptote of the function $p(x) = 3^{-x}$.

Solution: Notice that $p(x) = 3^{-x}$ can be written in the form $p(x) = \left(\frac{1}{3}\right)^x$, thus the base $b < 1$ and the function $p(x)$ is decreasing.

Domain: Set of real numbers

Range: Set of all positive real numbers

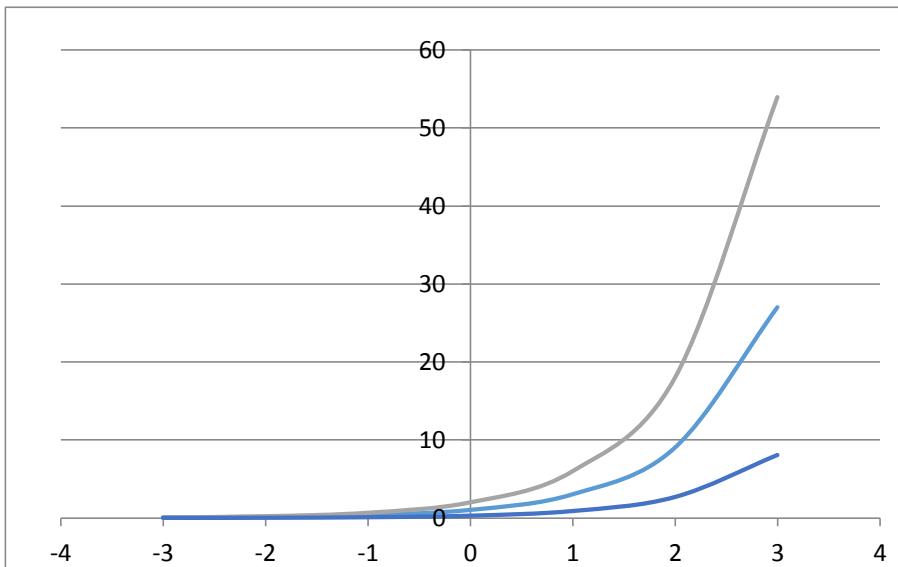
Intercepts: y-intercept is 1, there is no x-intercept

Horizontal Asymptote: $y = 0$

Example 3: Given the following functions, table of values and their graphs, find their domain, range, intercepts and horizontal asymptote:

$$F(x) = 3^x, G(x) = 2(3^x) \text{ and } H(x) = 0.3(3^x)$$

X	-3	-2	-1	0	1	2	3
F(x)	0.037	0.111	0.333	1	3	9	27
G(x)	0.074	0.222	0.667	2	6	18	54
H(x)	0.011	0.033	0.1	0.3	0.9	2.7	8.1



Domain: The domain of the three functions is a set of real numbers.

Range: The range of the three functions is $y > 0$ or set of all positive real numbers.

Intercepts: y-intercept of $F(x) = 3^x$ is 1, y-intercept of $G(x) = 2(3^x)$ is 2 and while the y-intercept of $H(x) = 0.3(3^x)$ is 0.3. There are no x-intercepts for these three functions since there is no value of x that can make each function value equal to zero is no x-intercept



Horizontal Asymptote: If you notice from the graph the horizontal asymptote of the three functions is $y = 0$.

Example 4: Consider the function $f(x) = \left(\frac{1}{3}\right)^{1-x} - 2$, find the domain, range, intercept and horizontal asymptote.

Solution: In this function, $a = 1$, $b = \frac{1}{3}$, $h = -2$ and $P(x) = 1-x$.

Domain: Set of all real numbers

Range: $(-2, +\infty)$

Intercept: y -intercept is -1.67

Horizontal Asymptote: $y = -2$. Notice that in this form of exponential function $f(x) = a \cdot b^{P(x)} + h$, the value of h is the horizontal asymptote.

Example 5: Let $f(x) = 5 - 2 \cdot 4^{x+1}$. Determine the domain, range, intercepts and horizontal asymptote.

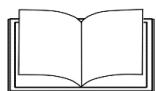
Solution: Notice that $f(x) = 5 - 2 \cdot 4^{x+1}$ can be written as $f(x) = -2 \cdot 4^{x+1} + 5$. Thus the value of $a = -2$, $b = 4$, $h = 5$ and $P(x) = x + 1$.

Domain: Set of all real numbers

Range: $(-\infty, 5)$

Intercept: y -intercept is $y = -3$

Horizontal Asymptote: $y = 5$



What's More

Determine the domain and range, y -intercept, and horizontal asymptotes of the following functions.

1. $f(x) = 5^x - 2$

Domain:

Range:

Intercept:

Horizontal Asymptote:

4. $P(x) = 5^{2-x} - 1$

Domain:

Range:

Intercept:

Horizontal Asymptote:



$$2. g(x) = \left(\frac{1}{5}\right)^{x+1}$$

Domain:

Range:

Intercept:

Horizontal Asymptote:

$$5. Q(x) = -2\left(\frac{1}{2}\right)^{x+3} - 6$$

Domain:

Range:

Intercept:

Horizontal Asymptote:

$$3. h(x) = -3^{2x-1} + 3$$

Domain:

Range:

Intercept: y-intercept is

Horizontal Asymptote:

What I Have Learned

Fill in the blank with the correct answer.

1. The _____ is the set of all real numbers.
2. The range is the set of _____.
3. It is a one-to-one function. It satisfies the _____.
4. The y-intercepts is_____. There is no x-intercept.
5. The horizontal asymptote is the line _____ (or the x-axis). There is no vertical asymptote.
6. The function is increasing if _____, and is _____ if $0 < b < 1$.



What I Can Do

Graph the following functions using one Cartesian plane and find the domain and range, y-intercept and horizontal asymptote.

a. $f(x) = \left(\frac{1}{6}\right)^x$

b. $g(x) = \left(\frac{1}{7}\right)^x$

c. $h(x) = \left(\frac{1}{8}\right)^x$



Scoring Rubrics					
Criteria	Beginning 1	Developing 2	Accomplished 3	Exemplary 4	Score
Neatness	The output was poorly created and difficult to read	The output was sloppy and difficult to read	The output is generally neat and readable	The output was done very neat and easy to read	
Accuracy	Majority of the data are incorrect and somehow irrelevant	The data are inaccurately represented contain major errors, or are missing	Data representation contains minor errors	All data are accurately represented	
Content (Application in real life)	Student has a complete lack of understanding of concept	Students has limited understanding of a concept	Students understanding of the concept is evident	Students understanding of concept is clearly evident	
Time Management	The output was submitted late 3-5 days after the deadline	The output was submitted late 1-2 days after the deadline	The output was submitted on time	The output was submitted ahead of time.	





Assessment

Choose the letter that corresponds to the exact answer.

1. What is the horizontal asymptote of the function $g(x) = 7^x - 2$?
A. $y = -7$ B. $y = -2$ C. $y = 2$ D. $y = 7$
2. The following properties of exponential functions are true EXCEPT:
 - A. The domain is the set of all real numbers.
 - B. The function is increasing if $b > 1$.
 - C. It is a one-to-one function if it satisfies the Horizontal Line Test.
 - D. The Horizontal asymptote of b^x is the line $x = 0$.
3. Which of the following is not true on the two functions $f(x) = 2^x + 2$ and $g(x) = 2^x - 3$?
 - A. The domain of the two functions is the set of all real numbers.
 - B. The y-intercepts are 3 and -2
 - C. The range of $f(x) = 2^x + 2$ is all $y > 2$ and the range of $g(x) = 2^x - 3$ is all $y > -3$.
 - D. The Horizontal asymptotes are $y = -2$ and $y = 3$
4. Which of the following is not true on the function $p(x) = 5^x$ and $g(x) = 6^x$?
 - A. Domain: Set of all real numbers
 - B. Range: Set of all positive real numbers
 - C. y-intercept is 1. There is no x-intercept
 - D. Horizontal Asymptote are $y = 5$ and $y = 6$
5. What is the x-intercept of 5^x ?
A. $x = 0$ B. $x = 1$ C. $x = 5$ D. There is no x –intercept.



Additional Activities

Complete the table of values below. Sketch the graph in one Cartesian plane. And identify its domain, range, y-intercept, horizontal asymptote.

$$F(x) = 3^x$$

$$G(x) = 3^{x+1} + 2$$

$$H(x) = 3^{x+1} - 2$$



x	-3	-2	-1	0	1	2	3
F(x)							
G(x)							
H(x)							

LESSON 4: Problems Involving Exponential Function, Equation and Inequality.



What's In

Solve the following:

1. $9^x = 81$

Solution

2. $4^{2x} - 2 = 62$

Solution

3. $(\frac{1}{2})^{5x} < (\frac{1}{2})^{3x-4}$

Solution

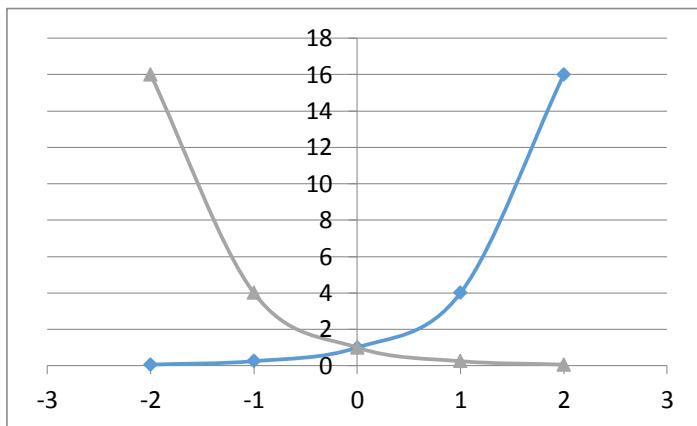
Guide Questions:

- How did you solve exponential equation for numbers 1 and 2?
- Can you give one properties/rules in solving exponential inequality?
- How do we solve problems involving exponential function, equation, and inequality?





What's New



Follow up Questions:

1. What color of the graph that show increasing and decreasing?
2. Which of the two graphs shows an exponential growth or exponential decay?
3. What is an exponential growth or an exponential decay?



What is It

Let us define first what exponential growth/decay is.

Definition:

Exponential growth / decay is a specific way that a quantity may increase / decrease over time.

To solve problems on exponential growth and decay, we have to be mindful of exponential growth and decay functions. Here are some illustrative examples for your guide as you learn on how to solve problems on exponential growth/decay.

Example 1: You invest 2,500 at a rate of 8% interest compounded annually. How much is the investment worth after 4 years?

Note: This problem is an example of exponential growth since we look for the final value. The growth is being compounded once the term is completed.



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Rate Compounded Annually -Exponential Growth.

$A = P(1 + r)^t$ where A = Ending Amount, P = Principal or starting amount, r = growth rate(in decimal), t = Number of years.

Solution: Given: P = 2,500, r = 8% or 0.08, and t = 4 years

$$\begin{aligned}A &= P(1 + r)^t \\A &= 2,500(1 + 0.08)^4 \\A &= 3,401.22\end{aligned}$$

Thus, the worth of investment after 4 years is Php 3,401.22

Example 2: Jonathan decided to buy a used Toyota car that costs Php 60,000. Jonathan heard that the Toyota car may depreciate at a rate of 12% per year. What is the total value of car in 5 years?

Note: If you notice the Purchase Toyota car is depreciated, thus this problem has something to do with exponential decay.

Rate Compounded Annually -Exponential Decay

$A = P(1 - r)^t$ where A = Ending Amount, P = Principal or starting amount, r = growth rate(in decimal), t = Number of years.

Solution: P = 60,000, r = 12% or 0.12, and t = 5 years

$$\begin{aligned}A &= P(1 - r)^t \\A &= 60,000(1 - 0.12)^5 \\A &= 31,663.92\end{aligned}$$

Thus the amount of Toyota car will be worth Php 31,663.92 in 5 years.

Example 3: Miguel has 30,000 to invest for 3 years and he is interested with an interest rate of 6%. How much money will he have at the end of 5 years if the interest rate compounds monthly?

Note: Since the interest rate compounds at different intervals during the year, we need to use another formula: Rate Compounded Annually (n =1) – Exponential Growth

$A = P(1 + \frac{r}{n})^{nt}$ where A = Ending Amount, P = Principal or starting amount, r = growth rate(in decimal), t = Number of years.n= number of times the interest is compounded per year

Solution: Given: P = 30,000, r = 0.06, t = 5, n = 12(compounded monthly).

$$\begin{aligned}A &= P(1 + \frac{r}{n})^{nt} \\A &= 30,000(1 + \frac{0.06}{12})^{(5)(12)} \\A &= 40,465.50\end{aligned}$$

Thus, Miguel will have Php 40,465.50 at the end of 5 years, if the rate compounds monthly.

Example 4: A bacteria triples every 5 hours.(a) If there are 80 bacteria now, how many will there be after 12 hours?(b) Find the bacteria population 3 hours earlier.

Note: This problem is related to geometric progression. Here, the initial amount will grow/decay at the constant ratio "b".



Formula for Geometric Progression: $A = ab^x$

Where: A = Ending amount

a = Beginning amount

b = Growth/Decay ratio

x = No. of years / terms or $\frac{\text{time we are interested in}}{\text{time bacteria}}$

A. If there are 80 bacteria now, how many will there be after 12 hours?

Solution:

Given: a = 80, b = 3 and x = $\frac{12}{5}$

$$A = ab^x$$

$$A = 80(3)^{\frac{12}{5}}$$

$$A = 80(3)^{2.4}$$

$$A = 1,117.33$$

Thus the number of bacteria at the end of 12 hours is 1,117.

B. Find the bacteria population 3 hours earlier.

Note: Same formula, -3 for the time, since we want the number of bacteria 3 hours earlier.

Solution:

$$A = ab^x$$

$$A = 80(3)^{-\frac{3}{5}}$$

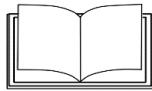
$$A = 41.38$$

Thus the number of bacteria 3 hours earlier is 41.



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What's More

Solve the following problems:

1. At present the number of bacteria in a certain culture doubles every hour. If there were 60 bacteria present in the culture initially, how many bacteria will be present at the end of 6 hour?
2. A culture of bacteria triples every 10 hours.
 - A. If there are 120 bacteria now, how many will there be after 20 hours?
 - B. Find the bacteria population 4 hours earlier.
3. Joseph invests Php3,500 at a rate of 8% interest compounded annually. How much is the investment worth after 3 years?
4. The value of a new branded cellphone is Php38,000. If the value of the branded cellphone decreases 10% per year, what will be the value of the branded cellphone after 5 years?

What I Have Learned

Fill in the Blank:

1. Exponential _____ is a specific way that a quantity may increase / decrease over time.
2. Compounding Formulas: Rate Compounded Annually: Exponential Growth: _____, Exponential Decay: _____.
3. Compounding Formulas: Rate Compounded Multiple Times Per Year: Exponential Growth: _____, Exponential Decay: _____.
4. Formula for Geometric Progression: _____.
5. In geometric progression, we say it is growth function if _____.

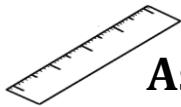




What I Can Do

- Albert is a chemist and he has 50 grams of a substance that has a **half-life** of 8 hours, how much will there be after 16 hours?
- Find a bank account balance if the account starts with Php.100, has an annual rate of 4%, and the money left in the account for 12 years.

Scoring Rubrics					
Criteria	Beginning 1	Developing 2	Accomplished 3	Exemplary 4	Score
Neatness	The output was poorly created and difficult to read	The output was sloppy and difficult to read	The output is generally neat and readable	The output was done very neat and easy to read	
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Assessment

Choose the letter that corresponds to the exact answer.

1. You decide to buy an old jeepney that costs Php 25,000. You've heard that the jeepney may depreciate at a rate of 8% per year. What will the jeepney be value in 5 years?
A. 13, 477.04 B. 14, 477.04 C. 15, 477.04 D. 16, 477.04
2. Suppose that a Senior High School graduating class had 400 students graduating the first year, but after that, the number of students graduating declines by a certain percentage. If the number of students graduating will be 300 in 2 years, what is the decay rate?
A. 10.6% B. 11.05% C. 12.5% D. 13.4%1.
3. Solve: $4^{x+1} = 4^7$
A. $x = 4$ B. $x = 5$ C. $x = 6$ D. $x = 7$
4. Solve: $(\frac{1}{4})^x = 16$
A. $x = -2$ B. $x = -1$ C. $x = 1$ D. $x = 2$
5. Solve: $3^{3x+4} > 3^{2x}$
A. $x > -5$ B. $x > -4$ C. $x > -3$ D. $x < -4$



Additional Activities

Mr. Santos deposit Php.16000 in a bank account. Find the balance after 2 years for each of the following situations:

- a. The account pays 3.5% annual interest compounded monthly.
- b. The account pays 1.5% annual interest compounded quarterly.
- c. The account pays 6% annual interest compounded yearly.

SUMMATIVE TEST

Choose the letter of the correct answer.

1. Find the value of y when $x = -2$ in the function $y = 2^x$.

A. $-\frac{1}{8}$ B. $-\frac{1}{4}$ C. $\frac{1}{8}$ D. $\frac{1}{4}$

2. Which of the following represents a table of values for $y = 3^x$?

A.

x	0	1
f(x)	1	2

B.

x	0	1
f(x)	1	3

C.

x	0	1
f(x)	1	5

D.

x	0	1
f(x)	1	4

3. 1. Which of the following is the domain of the function $g(x) = 3^x$?

A. Domain: $x > 3$ C. Domain: $x = 0$
B. Domain: $x = 3$ D. Domain: Set of all real numbers

4. Find the range of the function $f(x) = 4^x$.

A. Range: $x > 4$ C. Range: $x = 0$
B. Range: $x = 4$ D. Range: Set of all positive real numbers

5. Determine the domain and range of the function $y = 2^{x+3} - 3$.

A. Domain: \mathbb{R} C. Domain: \mathbb{R}
Range: $(-3, -\infty)$ Range: $(-\infty, 3)$
B. Domain: \mathbb{R} D. Domain: \mathbb{R}
Range: $(-3, +\infty)$ Range: $(3, +\infty)$

6. Find the horizontal asymptote of the function $y = 4^x$.

A. $y = -4$ B. $y = -1$ C. $y = 0$ D. $y = 4$

7. What is the y – intercept of the function $y = 4^x$?

A. 1 B. 2 C. 3 D. 4

8. Which of the following is true on the function $g(x) = 3^{x+1}$?

A. The y -intercept is 1. C. The y -intercept is 3.
B. The y -intercept is 2. D. The y -intercept is 4.



For item number **9 and 10**: Refer to the given problem: A bacteria triples every 6 hours.

9. If there are 50 bacteria now, how many will there be after 24 hours?

- A. 3,050 B. 4,050 C. 5,050 D. 6,050

10. Find the bacteria population 3 hours earlier.

- A. 28.87 B. 30 C. 36.78 D. 43.09





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