9

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DEPARTMENT OF EDUCATION

Mathematics

Quarter 1-Module 9

Solving Problems Involving Quadratic Equations

Week 4

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ADM

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MODULE 9

SOLVING PROBLEMS INVOLVING QUADRATIC EQUATIONS

In the previous lessons, you learned about the different ways in solving quadratic equations, the concepts of the "nature of its roots" and the relationship of its roots and coefficients. Your mastery of the lessons is an important tool to solve many real-world problems related to quadratic equations. In this module, you will learn how to solve word problems that involves quadratic equations.

WHAT I NEED TO KNOW

LEARNING COMPETENCY

The learners will be able to:

- solve problems involving quadratic equations. **M9AL-1e-1**

WHAT I KNOW

Find out how much you already know about quadratic equation. Write the letter that you think is the best answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. The length of the garden is 2 m more than twice its width and its area is $24\,m^2$. Which of the following equations represents the given situation?
   A. $x^2 + x = 12$
   B. $x^2 + 2x = 12$
   C. $x^2 + x = 24$
   D. $x^2 + 2x = 24$

2. A piece of iron rod costs ₱ 60.00. If the rod was 2 meter shorter and each meter costs ₱ 1 more, the cost would remain unchanged. What is the length of the rod ?
   A. 12
   B. 10
   C. -12
   D. -10

3. The hypotenuse of a right-angled triangle is 20 cm. The difference between its other two sides is 4 cm. Find the length of the sides.
   A. 5 cm and 10 cm
   B. 6 cm and 8 cm
   C. 12 cm and 16cm
   D. 8 cm and 12 cm

4. The length of a tarpaulin is 3 feet more than thrice its width and its area is $126\,ft^2$. What is the length of the tarpaulin?
   A. 18 ft
   B. 16 ft
   C. 19 ft
   D. 21 ft

5. The sides of an equilateral triangle are shortened by 12 units, 13 units and 14 units respectively and a right-angle triangle is formed. Find the length of each side of the equilateral triangle.
   A. 13 units
   B. 17 unite
   C. 5 units
   D. 2 units

6. Difference between a number and its positive square root is 12. Find the number.
   A. 8
   B. 9
   C. 22
   D. 16

For Question 7 & 8: Jonna throw an object upward from the top of a 1200 ft. tall building. The height of the object, (measured in feet) t seconds after she threw it is $h = -16t^2 + 160t + 1200$.

7. Where is the object 3 seconds after Jonna threw it?
   A. 1356 ft.
   B. 1536 ft.
   C. 1563 ft.
   D. 1653 ft.

8. How long does it take for the object to hit the ground?
   A. 5 seconds
   B. 12 seconds
   C. 10 seconds
   D. 15 seconds

9. Smith and Johnson together can do a piece of work in 4 days. If they had to work separately, the time taken by Johnson to do the work would be more than that of Smith by 6 days. In how many days can Smith alone do the work?
   A. 6 days
   B. 10 days
   C. 8 days
   D. 7 days

10. An office has two envelope stuffing machines. Working together they can stuff a batch of envelopes in 4 hours. Working Separately, it will take the second machine 6 hour longer than the first machine . How long would it take for the second machine to stuff a batch of envelopes?
    A. 11
    B. 20
    C. 12
    D. 10

WHAT'S IN

Creativity, collaboration and critical thinking

ACTIVITY: MATH TRANSLATOR!

Let us translate word phrases into mathematical equations using the given variable. The first item is done for you.

1. The length of a rectangular tarpaulin is 3ft. more than thrice its width and the area is $126ft^2$. Use **w** to represent the width.

Solution: The product of the length and width of a rectangular tarpaulin represents its area. Hence, length (l) times width (w) = 126, or $lw = 126$ and since the length is 3ft more than its thrice its width, then $l = 3w + 3$.
Therefore, the equation is $(\mathbf{3w + 3})(\mathbf{w}) = \mathbf{126}$.

Try this!

2. The area of a garden is $160m^2$. The length of the garden is 3 m more than twice its width. Use **w** to represent the width.
3. The base of a triangle is 4 dm longer than the altitude and its area is $16dm^2$. Use **h** to represent the altitude
4. The area of a concrete rectangular pathway is $350m^2$ and its perimeter is 90m. Use **w** to represent the width.

Were you able to represent each situation by an equation?

WHAT'S NEW

Communication, creativity and critical thinking

H O P E TO H E A L

In response to the rising number of COVID-19 positive cases in the country, the local government of Quezon City has set-up HOPE 1, an alternative medical facility. It has been identified to complement local government hospitals as they prepare for the possible overloading of their facilities in the coming days.

HOPE 1 will accept referrals from the Quezon City General Hospital, Rosario Maclang Bautista General Hospital and Novaliches District Hospital. The facility will be led by a team of city health professionals. Each room will house several patients and has dividers to isolate one patient from the others.

If the area of a quarantine facility for COVID-19 patients is 120 square meters and the length is 1 less than twice the width. Find the length and the width of the quarantine facility

https://www.designboom.com/architecture/wta-emergency-quarantine-facilities-covid-19-philippines-04-15-2020/

Guide Questions:

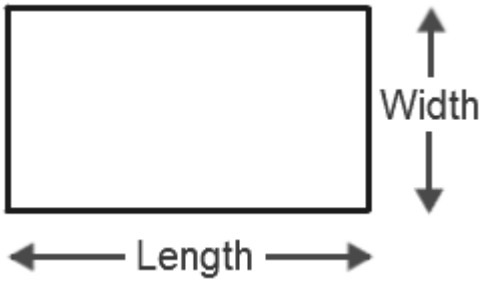
1. What expression represents the width of the room? How about the expression that represents its length?
2. What equation can be formed that would relate the width, length, and the area of the room?
3. How will you describe the equation you formulated?

WHAT IS IT

Communication and critical thinking

WHAT ARE MY DIMENSIONS?

Example 1: The area of a room of HOPE1 for a COVID -19 patient is 120 square meters and the length is 1 less than twice the width. Find the length and the width of the room.



Solution: The product of the length and width represents the area of the room or $(l)(w) = 120$.

Let w = width

$l = 2w - 1$

Equation: $lw = A$

$(l)(w) = 120$	*Equation*
$(2w - 1)(w) = 120$	*Substitution*
$2w^2 - w = 120$	*Using distributive property and simplifying*
$2w^2 - w - 120 = 0$	*Adding the inverse of 120 on both sides*
$(2w + 15)(w - 8) = 0$	*Factoring and principle of zero product property*
$\boldsymbol{w = 8}, w = \frac{-15}{2}$	*Solve for w. (Use the positive value of w since there is no negative value for measurements.)*

Thus, $w = 8$

Solving for l,

$l = (2w - 1)$

$= 2(8) - 1$

$= \mathbf{15}\ \boldsymbol{meters}$

Example *2:* Mang Juan owns a rectangular lot. The perimeter of the lot is 90 m and its area is 450 m^2.

QUESTIONS:

a. What equation represents the perimeter of the lot? How about the equation that represents its area?
b. Using the idea of the sum and product of the roots of a quadratic equation, how would you determine the length and the width of the rectangular lot?

Solutions:

Perimeter

a. $2L + 2W = 90\ m$

$\frac{2(L+W)}{2} = \frac{90\ m}{2}$

$L + W = 45\ m$

Area

$(L)(W) = 450m^2$

b. $L + W = 45$ $\quad EQUATION\ 1$
$LW = 450$ $\quad EQUATION\ 2$

Get $\boldsymbol{L}$ in terms of $\boldsymbol{W}$: $L + W = 450 \rightarrow \boldsymbol{L = 450 - W}$
Substitute to EQUATION 1: $\boldsymbol{LW = 450}$
$(45 - W)(W) = 450 \rightarrow W^2 - 45W + 450 = 0$
Factor: $(W - 30)(W - 15) = 0$
Thus, $\boldsymbol{W = 30; W = 15}$

Example *3:* A piece of pipe costs ₱ 30.00. If the pipe was 1 meter shorter and each meter costs ₱ 1 more, the cost would remain unchanged. What is the length of the pipe?

Equation: Let 'x' be the length of the piece of pipe in meters

Since the cost of the piece of pipe is ₱ 30.00, then the price per meter of the pipe is $\frac{30}{x}$.

Based on the problem we will have the following equation,

$$\frac{30}{(x-1)} = 1 + \frac{30}{x}$$

Solution:

$\frac{30}{(x-1)} = 1 + \frac{30}{x}$	*Equation*
$\frac{30}{(x-1)} - \frac{30}{x} = 1$	*Using Addition Property of Equality*
$\frac{30x-30(x-1)}{x(x-1)} = 1$	*Find the LCD*
$\frac{30x-30x+30}{x^2-1} = 1$	*Using distributive property*
$\frac{30}{x^2-1} = 1$	*Simplifying*
$30 = x^2 - 1$	*Using Multiplicative Property of Equality*
$x^2 - 1 - 30 = 0$	
$(x - 6)(x + 5) = 0$	*Factoring and principle of zero product property*
$x = 6$ *or* $x = -5$	

Because length cannot be a negative number, we can ignore "- 5".
So, the length of the given pipe is **6 m**.

Example *4:* Jack and Jill working together can do a work in 6 days. If Jack takes 5 days less than Jill to finish the work, in how many days can Jill alone do it?

Equation: Let x be the time of Jill take to finish the work

x – 5 will be the time Jack take to finish the work.

$$\frac{1}{x} + \frac{1}{x-5} = \frac{1}{6}$$

Solution:

$\frac{x+x-5}{x(x-5)} = \frac{1}{6}$ *Find the LCD*

$\frac{2x-5}{x^2-5x} = \frac{1}{6}$ *Using distributive property and simplify*

$12x - 30 = x^2 - 5x$ *Using Multiplicative Property of Equality*

$x^2 - 17x + 30 = 0$

$x^2 - 15x - 2x + 30 = 0$

$x(x-15) - 2(x-15) = 0$ *Factoring and principle of zero product property*

$(x-2)(x-15) = 0$

$x = 2 \quad or \quad x = 15$

We can ignore "2" because it will make the number of days negative,

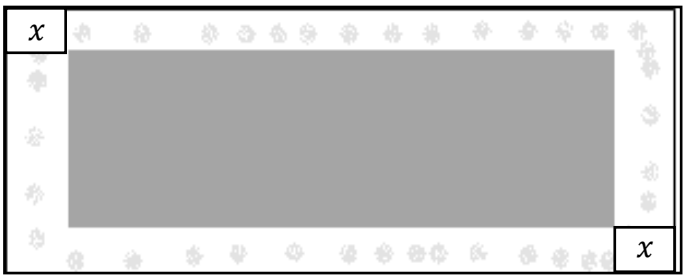
Since x = 15, Jill can work in 15 days, while Jack can work in **10 days**.

WHAT'S MORE

Critical Thinking

Solve the following problems:

1. The square of a number plus the twice number is equal to 1.Find the number.
2. A school has a quadrangle that is 20 ft by 30 ft. The alumni association wants to plant a strip set of sunflowers around the edge of the quadrangle.They have enough seed to cover 336 feet. How wide will the strip be?



WHAT I HAVE LEARNED

In solving problems involving the quadratic equation, you may use the following procedures:

1. Read and analyze the problem carefully.
2. Identify the given and what is asked in the problem
3. Represent the unknown using algebraic expressions.
4. Formulate the mathematical sentence. This will serve as the "working quadratic equation".
5. Find the solution set or roots of the quadratic equation using any method discussed on the previous lessons.
6. To check, substitute the obtained roots to the working quadratic equation.

NOTE: Solving problems involving quadratic equations does not end in finding and checking the roots. Since, it is a real-life problem you must decide whether both or one or none of the solutions is reasonable.

WHAT I CAN DO

Critical Thinking

Answer the following.

1. An amusement park wants to place a new rectangular billboard to inform visitors of their new attractions. Suppose the length of the billboard to be placed is 4 m longer than its width and the area is $96m^2$. What will be the length and the width of the billboard?
2. A rectangular table has an area of $27ft^2$ and a perimeter of $24ft$. What are the dimensions of the table?

ASSESSMENT

Write the letter of the correct answer on your answer sheet. If your answer is not among the choices, write E together with your final answer.

1. The length of a rectangle is 2 m less than twice the width. If the area of the rectangle is 112 m^2, what is the width?
   A. 5 m B. 8 m C. 9 m D. 14 m

2. A piece of pipe costs ₱ 20.00. If the pipe was 1 meter shorter and each meter costs ₱ 1 more, the cost would remain unchanged. What is the length of the pipe?
   A. -4 C. 5
   B. -5 D. 4

3. The hypotenuse of right-angled triangle is 10 cm. The difference between its other two sides is 2 cm. Find the length of the sides.
   A. 5 cm and 7 cm C. 6 cm and 12 cm
   B. 6 cm and 8 cm D. 8 cm and 10 cm

4. The perimeter and area of a rectangle are 48 cm and 140 cm^2 respectively. Find the length and width of the rectangle.
   A. 12 cm by 10 cm C. 12 cm by 8 cm
   B. 10 cm by 8 cm D. 14 cm by 10 cm

5. The sides of an equilateral triangle are shortened by 2 units, 3 units and 4 units respectively and a right-angle triangle is formed. Find the length of each side of the equilateral triangle.
   A. 3 units C. 5 units
   B. 7 unite D. 12 units

6. Difference between a number and its positive square root is 6. Find the number.
   A. 8 C. 9
   B. 4 D. 16

For Question 7 & 8: Jhondy throw an object upward from the top of a 300 ft. tall building. The height of the object, (measured in feet) *t* seconds after he threw it is $h = -4t^2 + 40t + 300$.

7. Where is the object 5 seconds after Jhondy threw it?
   A. 500 ft. C. 1000 ft.
   B. 160 ft. D. 530 ft.

8. How long does it take for the object to hit the ground?
   A. 5 seconds C. 10 seconds
   B. 12 seconds D. 15 seconds

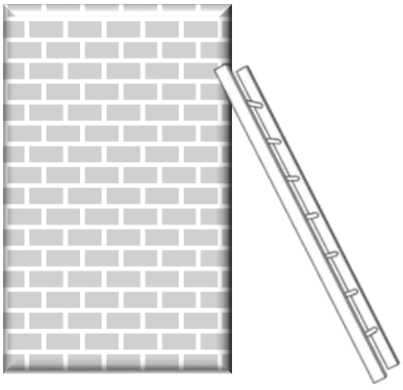
9. John, Luke and Mark can finish painting the fence in 2 hours. If John does the job alone, he can finish it in 5 hours. If Luke does the job alone, he can finish it in 6 hours. How long will it take for Mark to finish the job alone?
   A. 5 hrs. C. 3 hrs.
   B. 7.5 hrs. D. 1 hr.

10. Two pipes working together can fill a tank in 35 minutes. If the large pipe alone can fill the tank in 24 minutes less than the time taken by the smaller pipe then find the time taken by large pipe working alone to fill the tank.
    A. 60 mins. C. 50 mins.
    B. 20 mins. D. 84 mins.

ADDITIONAL ACTIVITIES

Critical Thinking and Creativity

A. A ladder 5 m long is leaning against a wall. The distance from the bottom of the ladder to the wall is 1 m less than the distance from the top of the ladder to the ground. How far is the bottom of the ladder from the wall?



B. Formulate problems that involve quadratic equations. Provide an illustrations and guide questions, then solve in as many ways as possible.

PROBLEM – BASED WORKSHEET

A.

Painting the House

After the ECQ, Chino and Philo are planning to paint a house together. Chino thinks if he works alone, it would take him 5 hours more than the time Philo takes to paint the entire house. Working together, they can complete the job in 6 hours.

1. If Philo can finish the job in x hours, how long will it take Chino to finish the job?
2. What should be the equation to solve the given situation?
3. How would you represent the amount of work that Chino can finish in 1 hour?
4. What amount of work can Philo finish in 5 hours?
5. How long does it take Chino to paint the house working alone?

B.

Car Wash

Mario and Kenneth work in a car wash station. The time that Mario takes in washing a car alone is 16 minutes less than the time that Kenneth takes in washing the same car. If both of them work together in washing the car, it will take them 15 minutes."

1. If Mario can finish the job in y minutes, what expression will represent Kenneth's rate?
2. What should be the equation to solve the problem?
3. How long will it take Mario to finish washing the car working alone?
4. How long will it take Kenneth to finish washing the car working alone?
5. Working together, what amount of the job can they finish in 10 minutes?

E-Search

You may also check the following link for your reference and further learnings on solving problems involving quadratic equations:

- http://sites.millersville.edu/bikenaga/basic-algebra/quadratic-word-problems/quadratic-word-problems.html
- https://www.vivaxsolutions.com/maths/gcseqdrtcwrdproblems.aspx
- https://www.youtube.com/watch?v=IGGnn9oa4QY
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https://www.polk.k12.ga.us/userfiles/313/Two%20step%20word%20problem%20rubric.pdf

PROBLEM – BASED WORKSHEET

A. If x represents Philo's completion of the job, then Chino can finish the job in $(\boldsymbol{x+5})$ ***hours.***

Chino's rate of work is $\frac{1}{x+5}$, thus, the amount of work that he can finish in 1 hour is represented by the expression = (rate) (time) = $\left(\frac{1}{x+5}\right)(1\ hour) = \frac{1}{x+5}$ while Philo's rate is $\frac{1}{x}$, the working equation is,

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

Multiplying both sides by the least common denominator. [LCD = $6x(x+5)$]

$$\begin{aligned} 6(x+5) + 6x &= x(x+5) \\ 12x + 30 &= x^2 + 5x \\ x^2 - 7x - 30 &= 0 \\ (x-10)(x+3) &= 0 \end{aligned}$$

$\boldsymbol{x = 10}$ or $x = -3$

The equation has two solutions. However, we only consider the positive value of x since the situation involves measures of time. Thus, it takes **15 hours for Chino** to paint the house working alone, while it takes **Philo 10 hours**. Also, for 5 hours, ***half*** of the house has already painted by Philo.

B. If Mario can finish the job in y minutes, thus his rate is $\frac{1}{y}$, it follows that Kenneth's rate

is $\frac{1}{y+16}$. The working equation is, $\frac{1}{y} + \frac{1}{y+16} = \frac{1}{15}$

Multiplying both sides by the least common denominator. [$15y(y+16)$]

$$\begin{aligned} 15(y+16) + 15y &= y(y+16) \\ 30y + 240 &= y^2 + 16y \\ y^2 - 14y - 240 &= 0 \\ (y-24)(y+10) &= 0 \end{aligned}$$

$\boldsymbol{y = 24}$ or $y = -10$

We only consider the positive value of x since the situation involves measures of time. Thus, it takes **24 minutes for Mario** to finish the job working alone, while it takes **Kenneth 40 minutes** working alone. Working together, the amount of work they can finish in

10 minutes is $\frac{1}{24}(10) + \frac{1}{40}(10) = \frac{5}{12} + \frac{1}{4} = \frac{5+3}{12} = \frac{8}{12}$ or $\frac{2}{3}$.

ASSESSMENT

1. B	3. B	5. B	7. A	9. B
2. C	4. D	6. C	8. D	10. A

ADDITIONAL ACTIVITIES

A. 3 meters

B. Use the given Rubric to check your work.
(https://www.polk.k12.ga.us/userfiles/313/Two%20step%20word%20problem%20rubric.pdf)

Rubric: Writing Math Word Problems

	4 pts	3 pts	2 pts	1 pts
Requirements	**4** Student has all required word problems for two steps problem.	**3** Student has created 1 step problems.	**2** Student created the problem but did not solve it.	**1** Student could not create a word problem.
Content	**4** Appropriate content is used for each word problem. Student clearly understands the mathematical concepts.	**3** Appropriate content is used for each word problem. Student shows some understanding of the mathematical concepts.	**2** Appropriate content may be used. Student shows little understanding of the mathematical concepts.	**1** Appropriate content is not observed. Student does not demonstrate an understanding of the mathematical concepts.
Solution	**4** Solution is correct and all work is shown.	**3** Solution is correct, however there is no work shown for the problem.	**2** Solution is incorrect; however the student showed their work and made an error in calculations.	**1** Solution is incorrect and student either didn't show work or worked the problem incorrectly.
Organization	**4** The word problem is written in clear and coherent language. It is easy to follow and read.	**3** The word problem is written in clear and coherent language. It is easy to read most of the time.	**2** The word problem is not written in clear and coherent language. It is easy to follow and read some of the time.	**1** The word problem is not written in clear and coherent language, or may not be observed. It is difficult to read and follow.
Mechanics	**4** Proper language, capitalization and punctuation are present with no mistakes.	**3** Proper language, capitalization and punctuation are present with no more than two mistakes.	**2** Proper language, capitalization and punctuation may be used, but more than two mistakes.	**1** Proper language, capitalization and punctuation are not observed.

ANSWER KEY

WHAT I KNOW

1.	A	3.	C	5.	B	7.	B	9.	A
2.	A	4.	D	6.	D	8.	D	10.	C

WHAT'S IN

2. $(2w+3)(w) = 160$
3. $16 = \frac{(h+4)(h)}{2}$
4. $Perimeter: l + w = \frac{90}{2} = 45$
   $Area: lw = 350\ ; \quad l = 45 - w$

$$350 = (45 - w)(w)$$

WHAT I CAN DO

1. $length = 12m, width = 8m$
2. The dimensions of the table are $3ft$ *and* $9ft$, respectively.

WHAT'S MORE

1. Let x be the unknown number

$$x^2 + 2x = 1 \qquad | \qquad x^2 + 2x - 1 = 0;\ a=1,\ b=2,\ c=-1$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-1)}}{2(1)}$$

$$\frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

2. Let x be the width of the strip set of sunflowers

$20 + 2x =$ width of the quadrangle with strip set of sunflowers

$30 + 2x =$ length of the quadrangle with strip set of sunflowers

Area of the Large rectangle $= l \times w$

$= (30 + 2x)(20 + 2x)$

Area of Quadrangle $= l \times w$

$= 30 \times 20$

$= 600$

Area of the Large rectangle − Area of Quad-rangle = Area of strip

$$(30 + 2x)(20 + 2x) - 600 = 336$$

SOLUTION:

$$(30 + 2x)(20 + 2x) - 600 = 336$$

$$4x^2 + 100x + 600 - 600 = 336$$

$$4x^2 + 100x - 336 = 0$$

$$x^2 + 25x - 84 = 0$$

$$(x + 28)(x - 3) = 0$$

$$x = -28 \text{ or } x = 3$$

since there is no negative value for measurements, the strip set of sunflowers around the edge of the quadrangle is ***3 feet*** *wide.*