

# Mathematics

## Quarter 4 – Module 3: Trigonometric Function of Special Angles



**Mathematics – Grade 9  
Alternative Delivery Mode  
Quarter 4 – Module 3: Trigonometric Functions of Special Angles  
First Edition, 2020**

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**9**

# **Mathematics**

## **Quarter 4 – Module 3:**

### **Trigonometric Functions for Special Angles**

## **Introductory Message**

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

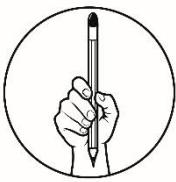
Thank you.



## What I Need to Know

The learners will be able to:

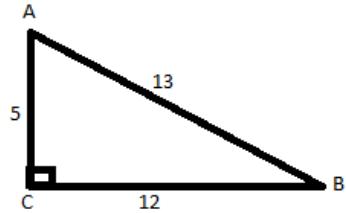
- Solve problems involving trigonometric functions of special angles.  
**(M9GE-IVC-44.2)**



## What I Know

Answer each of the following items. Write the letter that corresponds to the correct answer on your answer sheet. After taking and checking your answers on this short test, take note of the items that you were not able to answer correctly and look for the right answers as you go through this module.

For items 1 – 5, refer to  $\triangle ACB$  at the right.



1. The value of  $\sin A$  is \_\_\_\_?

- a.  $\frac{12}{13}$       b.  $\frac{5}{13}$       c.  $\frac{13}{12}$       d.  $\frac{13}{5}$

2. The value of  $\cos B$  is \_\_\_\_?

- a.  $\frac{12}{5}$       b.  $\frac{12}{13}$       c.  $\frac{5}{13}$       d.  $\frac{13}{12}$

3. The value of  $\tan A$  is \_\_\_\_?

- a.  $\frac{12}{5}$       b.  $\frac{12}{13}$       c.  $\frac{5}{12}$       d.  $\frac{13}{12}$

4. The value of  $\cot B$  is \_\_\_\_?

- a.  $\frac{12}{13}$       b.  $\frac{13}{5}$       c.  $\frac{12}{5}$       d.  $\frac{13}{12}$

5. The value of  $\sec B$  is \_\_\_\_?

- a.  $\frac{13}{5}$       b.  $\frac{5}{13}$       c.  $\frac{12}{13}$       d.  $\frac{13}{12}$

6. What is the value of  $\sin 45^\circ$ ?

a.  $\frac{\sqrt{2}}{2}$

b.  $\frac{\sqrt{3}}{2}$

c.  $\sqrt{2}$

d.  $\sqrt{3}$

7. What is the value of  $\cot 30^\circ$ ?

a.  $\frac{\sqrt{2}}{2}$

b.  $\frac{\sqrt{3}}{2}$

c.  $\sqrt{2}$

d.  $\sqrt{3}$

8. What is the value of  $\sec 60^\circ$ ?

a.  $\frac{2\sqrt{3}}{3}$

b.  $\frac{\sqrt{3}}{3}$

c. 1

d. 2

9. What is the value of  $\cot 45^\circ + \tan 60^\circ$ ?

a.  $\frac{4\sqrt{3}}{3}$

b.  $1 + \sqrt{3}$

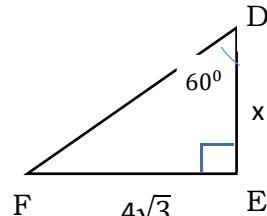
c. 2

d.  $\frac{2\sqrt{3}+\sqrt{2}}{2}$

For items 10 – 11, refer to  $\triangle DEF$  at the right,  $m\angle D = 60^\circ$  and  $|EF| = 4\sqrt{3}$  cm.

10. How long is DE?

a.  $2\sqrt{3}$  cm   b. 4 cm   c. 8 cm   d.  $4\sqrt{6}$  cm



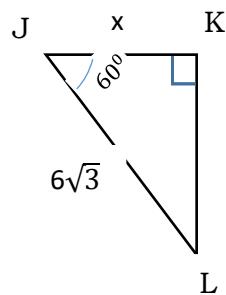
11. What is the measure of  $\angle F$ ?

a.  $30^\circ$    b.  $45^\circ$    c.  $60^\circ$    d.  $90^\circ$

For items 12 -13, refer to  $\triangle JKL$  at the right,  $m\angle J = 60^\circ$  and  $|JL| = 6\sqrt{3}$  cm.

12. How long is JK?

a. 3 cm   b.  $2\sqrt{3}$  cm   c.  $3\sqrt{3}$  cm   d. 6 cm



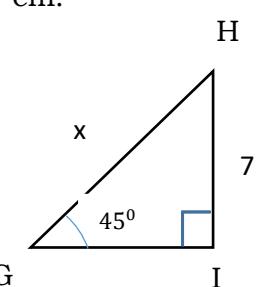
13. What is the measure  $\angle K$ ?

a.  $30^\circ$    b.  $45^\circ$    c.  $60^\circ$    d.  $90^\circ$

For items 14 -15, refer to  $\triangle HIG$  at the right,  $m\angle G = 45^\circ$  and  $|HI| = 7$  cm.

14. How long is GH?

a. 7 cm   b.  $7\sqrt{2}$  cm   c. 14 cm   d.  $14\sqrt{2}$  cm



15. What is the measure of  $\angle H$ ?

a.  $90^\circ$    b.  $60^\circ$    c.  $45^\circ$    d.  $30^\circ$

# Lesson 1

## Find and Solve Problems Involving Trigonometric Functions of Special Angles

In this module, you will learn how to evaluate the trigonometric expressions involving special angles. These special angles are the measures of the acute angles in the special right triangles, the isosceles right triangle ( $45^\circ$ - $45^\circ$ - $90^\circ$ ) and the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle. The following activities will guide you to learn and master this topic.



### What's In

The concept of “special right triangles” is the common basis in evaluating trigonometric expressions. In this case, let us have a recall to illustrate this.

**A.** Find the values of the following using calculator. Round-off to 4 decimal places.

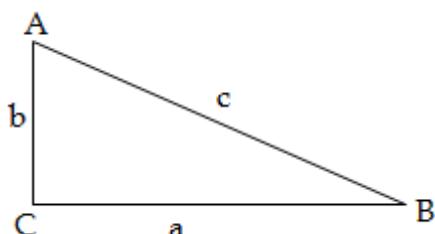
1. $\cos 40^\circ$	2. $\sin 85^\circ$	3. $\tan 35^\circ$	4. $\cos 45^\circ$	5. $\tan 68^\circ$
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**B.** Given are the trigonometric functions values of angle A, find the measure of angle A to the nearest whole number in degree.

1. $\sin A = 0.529$	2. $\cos A = 0.493$	3. $\tan A = 1.8$	4. $\sin A = 0.256$	5. $\tan A = 0.725$
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**C.** Let a and b be the legs and c be the hypotenuse of the right triangle ACB. Find the missing length of the side of the right triangle.

- |  |
|--|
| 1. $a = 3 \text{ cm}$ , $b = 4 \text{ cm}$ , $c = ?$<br>2. $a = 6 \text{ cm}$ , $b = 8 \text{ cm}$ , $c = ?$<br>3. $a = 5 \text{ cm}$ , $b = 12 \text{ cm}$ , $c = ?$<br>4. $a = 6 \text{ cm}$ , $c = 10 \text{ cm}$ , $b = ?$<br>5. $b = 24 \text{ cm}$ , $c = 26 \text{ cm}$ , $a = ?$ |
|--|

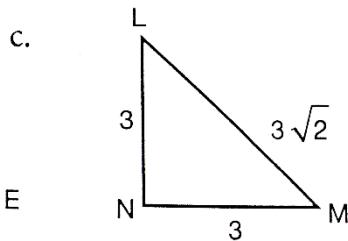
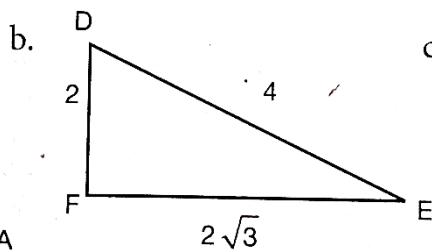
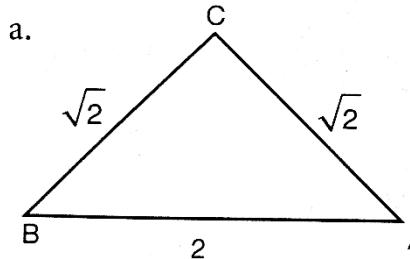




## What's New

Do the following activities and answer the items that follow.

Use a protractor to find the measures of the angles of each triangle.



1.  $\angle A =$  \_\_\_\_\_  
2.  $\angle B =$  \_\_\_\_\_  
3.  $\angle C =$  \_\_\_\_\_

1.  $\angle D =$  \_\_\_\_\_  
2.  $\angle E =$  \_\_\_\_\_  
3.  $\angle F =$  \_\_\_\_\_

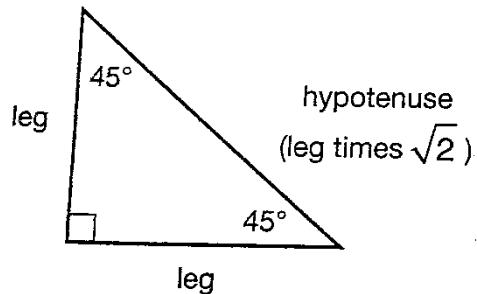
1.  $\angle L =$  \_\_\_\_\_  
2.  $\angle M =$  \_\_\_\_\_  
3.  $\angle N =$  \_\_\_\_\_

The triangles shown above are the special right triangles namely the  $45^\circ-45^\circ-90^\circ$  triangle and the  $30^\circ-60^\circ-90^\circ$  triangle. The 30-degree, 45-degree, and the 60-degree angles are the special angles. To determine the values of the trigonometric functions of these special angles, we can use geometric methods.

### KEY CONCEPTS

In Geometry, the following sides of special right triangles are related as follows:

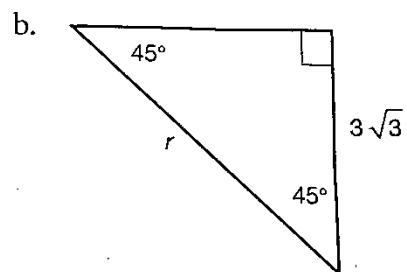
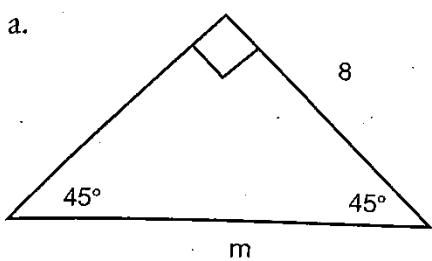
#### 45°-45°-90° Right Triangle Theorem



In a  $45^\circ-45^\circ-90^\circ$  triangle, the legs are congruent; the length of the hypotenuse is  $\sqrt{2}$  times the length of a leg.

$$\text{Hypotenuse} = \sqrt{2} \text{ leg}$$

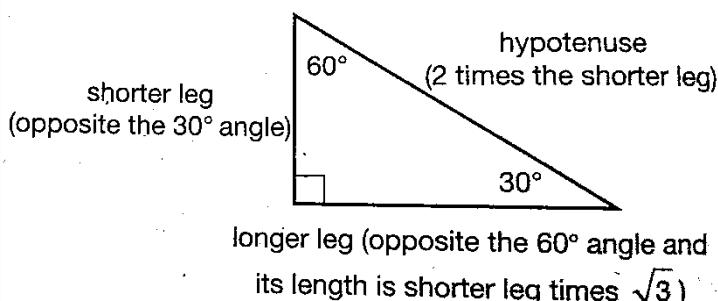
Example: Find the length of the hypotenuse of the given triangles.



Solution:

<p>a. Hypotenuse = m Leg = 8 Hypotenuse = <math>\sqrt{2}</math> leg <math>m = \sqrt{2} * 8 = 8\sqrt{2}</math></p>	<p>b. Hypotenuse = r Leg = <math>3\sqrt{3}</math> Hypotenuse = <math>\sqrt{2}</math> leg <math>r = \sqrt{2} * 3\sqrt{3} = 3\sqrt{6}</math></p>
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### 30°-60°-90° Right Triangle Theorem

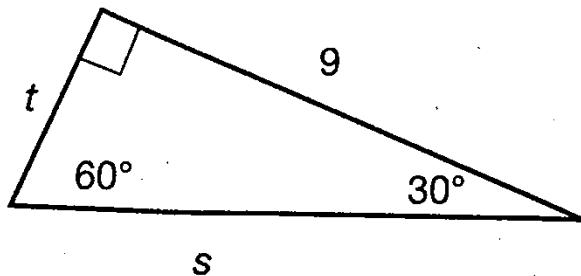


In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg; the length of the longer leg is  $\sqrt{3}$  times the length of the shorter leg.

$$\text{Hypotenuse} = 2 \text{ shorter leg}$$

$$\text{Longer leg} = \sqrt{3} \text{ shorter leg}$$

Example: Find the lengths of the sides represented by the variables s and t.



Solution:

$$\text{Hypotenuse} = s$$

$$\text{Shorter leg} = t$$

$$\text{Longer leg} = 9$$

Longer leg = $\sqrt{3}$ shorter leg $9 = t\sqrt{3}$ $t = \frac{9}{\sqrt{3}} \left(\frac{\sqrt{3}}{3}\right)$ $t = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$	hypotenuse = 2 shorter leg $s = 2t$ $s = 2 * 3\sqrt{3}$ $s = 6\sqrt{3}$
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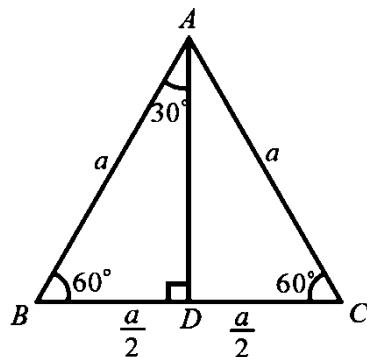
### **The Trigonometric Functions of Special Angles**

In Trigonometry,  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$  are called special angles. These special angles are frequently seen in applications and we can use Geometry to determine the values of the trigonometric functions of these angles.

- **Values of the Trigonometric Functions of  $30^\circ$  and  $60^\circ$**

Let  $\Delta ABC$  be an equilateral triangle whose side length is  $a$  (see the figure below). Draw  $\overline{AD}$  perpendicular to  $\overline{BC}$  at D. Point D is the midpoint of  $\overline{BC}$ . So,  $\Delta ABC$  is bisected into two congruent right triangles ADB and ADC.

Then,  $|BD| = |DC| = \frac{a}{2}$  and  $m\angle BAD = m\angle CAD = 30^\circ$



Now, in right triangle ADB,  $m\angle BAD = 30^\circ$  and  $|BD| = \frac{a}{2}$ .

In right triangle ADB, the length of  $\overline{AD}$  can be obtained using the Pythagorean formula.

$ AB ^2 =  AD ^2 +  BD ^2$	$\frac{3a^2}{4} =  AD ^2$
$a^2 =  AD ^2 + \left(\frac{a}{2}\right)^2$	$\sqrt{\frac{3a^2}{4}} = \sqrt{ AD ^2}$
$a^2 - \frac{a^2}{4} =  AD ^2$	$\sqrt{\frac{3a^2}{4}} =  AD $
$\frac{4a^2 - a^2}{4} =  AD ^2$	$\frac{\sqrt{3}a}{2} =  AD $

Hence, we can find the values of the trigonometric functions of  $30^\circ$  from the right triangle ADB.

$\sin 30^\circ = \frac{ BD }{ AB } = \frac{\frac{a}{2}}{a} = \frac{1}{2}$	$\csc 30^\circ = \frac{1}{\sin 30^\circ} = 2$
$\cos 30^\circ = \frac{ AD }{ AB } = \frac{\frac{\sqrt{3}}{2}a}{a} = \frac{\sqrt{3}}{2}$	$\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{2\sqrt{3}}{3}$
$\tan 30^\circ = \frac{ BD }{ AD } = \frac{\frac{a}{2}}{\frac{\sqrt{3}}{2}a} = \frac{\sqrt{3}}{3}$	$\cot 30^\circ = \frac{1}{\tan 30^\circ} = \sqrt{3}$

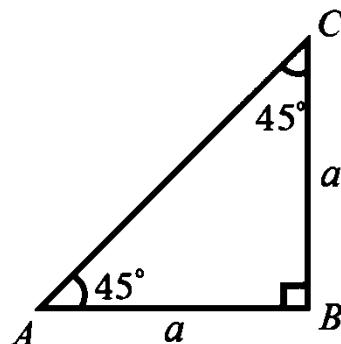
In right triangle ADB,  $m\angle ABD = 60^\circ$ . So, we can determine the values of the trigonometric functions of  $60^\circ$ .

$\sin 60^\circ = \frac{ AD }{ AB } = \frac{\frac{\sqrt{3}}{2}a}{a} = \frac{\sqrt{3}}{2}$	$\csc 60^\circ = \frac{1}{\sin 60^\circ} = \frac{2\sqrt{3}}{3}$
$\cos 60^\circ = \frac{ BD }{ AB } = \frac{\frac{a}{2}}{a} = \frac{1}{2}$	$\sec 60^\circ = \frac{1}{\cos 60^\circ} = 2$
$\tan 60^\circ = \frac{ AD }{ BD } = \frac{\frac{\sqrt{3}}{2}a}{\frac{a}{2}} = \sqrt{3}$	$\cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{\sqrt{3}}{3}$

- Values of the Trigonometric Functions of  $45^\circ$**

If the measure of an acute angle of a right triangle is  $45^\circ$ , then the other acute angle also measures  $45^\circ$ . So the triangle is an isosceles right triangle.

Let us consider the triangle ABC with  $m\angle B = 90^\circ$  and  $m\angle A = m\angle C = 45^\circ$



It can be seen in the triangle above that  $|AB| = |BC|$ . Let  $|AB| = |BC| = a$ . The length of the hypotenuse  $\overline{AC}$  can be obtained using the Pythagorean formula,

$$|AC|^2 = |AB|^2 + |BC|^2$$

$$|AC|^2 = a^2 + a^2$$

$$|AC|^2 = 2a^2$$

$$|AC| = a\sqrt{2}$$

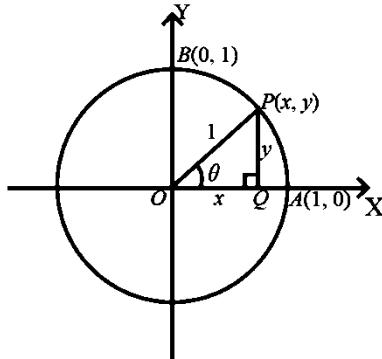
Hence, we can find the values of the trigonometric functions of  $45^\circ$  from the right triangle ABC.

$\sin 45^\circ = \frac{ BC }{ AC } = \frac{a}{a\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\csc 45^\circ = \frac{1}{\sin 45^\circ} = \sqrt{2}$
$\cos 45^\circ = \frac{ AB }{ AC } = \frac{a}{a\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\sec 45^\circ = \frac{1}{\cos 45^\circ} = \sqrt{2}$
$\tan 45^\circ = \frac{ BC }{ AB } = \frac{a}{a} = 1$	$\cot 45^\circ = \frac{1}{\tan 45^\circ} = 1$

- Values of the Trigonometric Functions of Quadrantal Angles  $0^\circ$  and  $90^\circ$**

Consider the figure given below which shows a circle of radius 1 unit centered at the origin.

Let P be a point on the circle in the first quadrant with coordinates  $(x, y)$ .



We draw a perpendicular  $\overline{PQ}$  from P to the x-axis in order to form the right triangle OQP.

Let  $m\angle POQ = \theta$ , then

$$\sin \theta = \frac{|PQ|}{|OP|} = \frac{y}{1} = y \text{ (y - coordinate of P)}$$

$$\cos \theta = \frac{|OQ|}{|OP|} = \frac{x}{1} = x \text{ (x - coordinate of P)}$$

$$\tan \theta = \frac{|PQ|}{|OQ|} = \frac{y}{x}$$

If  $\overline{OP}$  coincides with  $\overline{OA}$ , then angle  $\theta = 0^\circ$ . Since, the coordinates of A are (1, 0), we have

$\sin 0^\circ = \frac{0}{1} = 0$ ( <i>y - coordinate of A</i> )	$\csc 0^\circ = \frac{1}{0}$ ( <i>undefined</i> )
$\cos 0^\circ = \frac{1}{1} = 1$ ( <i>x - coordinate of A</i> )	$\sec 0^\circ = \frac{1}{1} = 1$
$\tan 0^\circ = \frac{\sin 0^\circ}{\cos 0^\circ} = \frac{0}{1} = 0$	$\cot 0^\circ = \frac{1}{0}$ ( <i>undefined</i> )

If  $\overline{OP}$  coincides with  $\overline{OB}$ , then angle  $\theta = 90^\circ$ . Since, the coordinates of B are (0,1), we have

$\sin 90^\circ = \frac{1}{1} = 1$ ( <i>y - coordinate of B</i> )	$\csc 90^\circ = \frac{1}{1} = 1$
$\cos 90^\circ = \frac{0}{1} = 0$ ( <i>x - coordinate of B</i> )	$\sec 90^\circ = \frac{1}{0}$ ( <i>undefined</i> )
$\tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0}$ ( <i>undefined</i> )	$\cot 90^\circ = \frac{0}{1} = 0$



## What is It

- I. Complete the table below that summarizes the values of the trigonometric functions of the angles  $30^\circ$ ,  $45^\circ$  and  $60^\circ$ .

Values of Trigonometric Functions of Special Angles						
$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
$30^\circ$	1.	2.	3.	4.	5.	6.
$45^\circ$	7.	8.	9.	10.	11.	12.
$60^\circ$	13.	14.	15.	16.	17.	18.

II. Evaluate each of the following trigonometric expressions. Choose the answers from the choices in the box below.

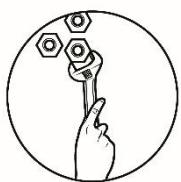
1. $\csc 30^\circ$	6. $2(\cos 30^\circ)^2 + 3(\sin 30^\circ)^2$
2. $\tan 60^\circ$	7. $\sin 45^\circ(\tan 45^\circ - \cos 60^\circ)$
3. $\csc 60^\circ$	8. $(\sin 60^\circ)(\cos 30^\circ) + \tan 45^\circ$
4. $(\sin 45^\circ)^2$	9. $(\sin 30^\circ)(\tan 45^\circ) + (\tan 30^\circ)(\sin 60^\circ)$
5. $\sec 60^\circ + \tan 30^\circ$	10. $(\csc 30^\circ)(\csc 45^\circ) - (\csc 45^\circ)(\cos 45^\circ)$

$\frac{9}{4}$	$\frac{7}{4}$	$\frac{2\sqrt{3}}{3}$	$2 + \frac{\sqrt{3}}{3}$	$\frac{\sqrt{2}}{4}$
$\frac{1}{2}$	1	2	$\sqrt{3}$	$2\sqrt{2} - 1$



## ***What I Have Learned***

- In  $45^\circ - 45^\circ - 90^\circ$  Triangle Theorem: The hypotenuse is  $\sqrt{2}$  times as long as each leg.
- In  $30^\circ - 60^\circ - 90^\circ$  Triangle Theorem: The hypotenuse is twice as long as the shorter leg and the longer leg is  $\sqrt{3}$  times as long as the shorter leg.



## What I Can Do

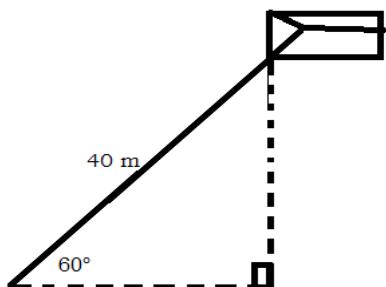
I. Answer the following.

1. What angles in trigonometry are referred to as special angles?
2. Write the formulas for sin, cos, and tangent for special angles.
3. Answer the letters to complete the table.

angle ratio	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin$	0	<b>A.</b>	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	<b>B.</b>
$\cos$	1	$\frac{\sqrt{3}}{2}$	<b>C.</b>	$\frac{1}{2}$	0
$\tan \theta$	<b>D.</b>	$\frac{\sqrt{3}}{3}$	1	<b>E.</b>	UNDEFINED
$\csc \theta$	UNDEFINED	2	<b>F.</b>	<b>G.</b>	1
$\sec \theta$	1	<b>H.</b>	$\sqrt{2}$	2	<b>I.</b>
$\cot \theta$	<b>J.</b>	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

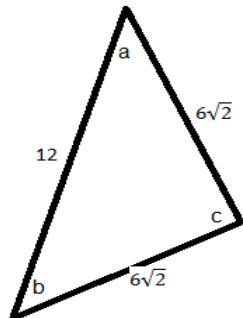
## II. PROBLEM SOLVING INVOLVING SPECIAL RIGHT TRIANGLES.

1. A person is flying a kite using 40 meters of string as shown in the diagram. What is the approximate measure of the height,  $h$ , of the kite? Use a calculator, and show your calculation below. Round the height to the nearest tenth of a meter.

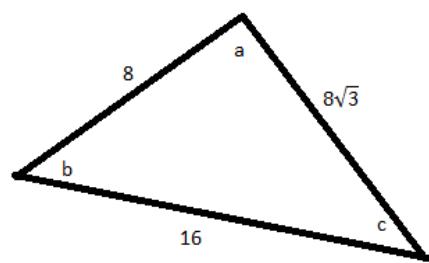


Determine the angle measures for items 2 and 3.

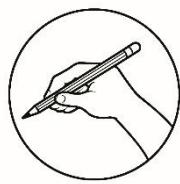
2.  $a = \underline{\hspace{2cm}}$ ,  $b = \underline{\hspace{2cm}}$ ,  $c = \underline{\hspace{2cm}}$



3.  $a = \underline{\hspace{2cm}}$ ,  $b = \underline{\hspace{2cm}}$ ,  $c = \underline{\hspace{2cm}}$



4. Sketch a  $30^\circ - 60^\circ - 90^\circ$  triangle below. Indicate the angle measures. The shorter leg measures  $\frac{1}{2}$  cm. Find the lengths of the other two sides and indicate these measures in the triangle.
5. Sketch a  $45^\circ - 45^\circ - 90^\circ$  triangle below. Indicate the angle measures. The hypotenuse measures 1 cm. Find the length of each leg and indicate the measures in the triangles.



## Assessment

I. Answer each of the following items. Write the letter that corresponds to the correct answer.

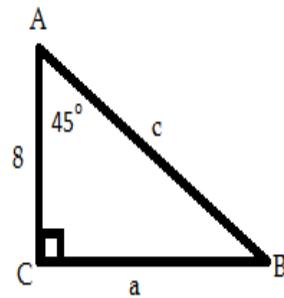
For items 1 – 3, refer to  $\triangle ACB$  at the right.

1. If  $m\angle A = 45^\circ$ , what is the measure of  $\angle B$ ?

- a.  $30^\circ$       b.  $45^\circ$       c.  $60^\circ$       d.  $90^\circ$

2. What is the length of  $\overline{CB}$  (a)?

- a.  $8\sqrt{2}$       b. 8      c.  $4\sqrt{2}$       d. 4



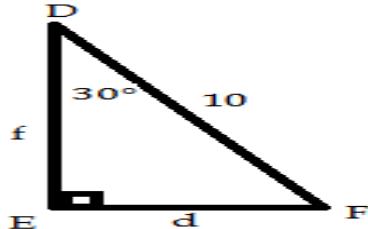
3. What is the length of  $\overline{AB}$  (c)?

- a.  $8\sqrt{2}$       b. 8      c.  $4\sqrt{2}$       d. 4

For items 4 – 6, refer to  $\triangle DEF$  at the right.

4. If  $m\angle D = 30^\circ$ , what is the measure of  $\angle F$ ?

- a.  $90^\circ$       b.  $60^\circ$       c.  $45^\circ$       d.  $30^\circ$



5. What is the length of  $\overline{EF}$  (d)?

- a.  $10\sqrt{3}$       b. 10      c.  $5\sqrt{3}$       d. 5

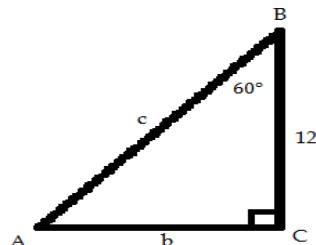
6. What is the length of  $\overline{DE}$  (f)?

- a. 5      b.  $5\sqrt{3}$       c. 10      d.  $10\sqrt{3}$

For items 7 – 9, refer to  $\triangle ABC$  at the right.

7. If  $m\angle B = 60^\circ$ , what is the measure of  $\angle A$ ?

- a.  $30^\circ$       b.  $45^\circ$       c.  $60^\circ$       d.  $90^\circ$



8. What is the length of  $\overline{AB}$  (c)?

- a. 12      b.  $12\sqrt{3}$       c. 24      d.  $24\sqrt{3}$

9. What is the length of  $\overline{AC}$  (b)?

- a. 12      b.  $12\sqrt{3}$       c. 24      d.  $24\sqrt{3}$

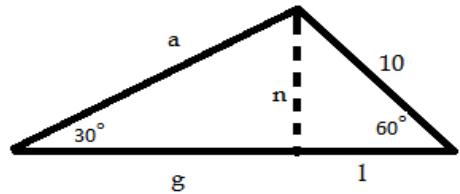
For items 10 – 13, refer to the figure at the right.

10. What is the value of  $l$ ?

- a. 5      b.  $5\sqrt{3}$       c. 15      d.  $10\sqrt{3}$

11. What is the value of  $n$ ?

- a. 5      b.  $5\sqrt{3}$       c. 15      d.  $10\sqrt{3}$



12. What is the value of  $a$ ?

- a. 5      b.  $5\sqrt{3}$       c. 15      d.  $10\sqrt{3}$

13. What is the value of  $g$ ?

- a. 5      b.  $5\sqrt{3}$       c. 15      d.  $10\sqrt{3}$

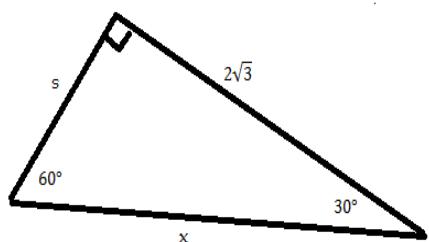
For items 14 – 15, refer to the figure at the right.

14. What is the value of  $x$ ?

- a. 2      b.  $2\sqrt{3}$       c. 4      d.  $4\sqrt{3}$

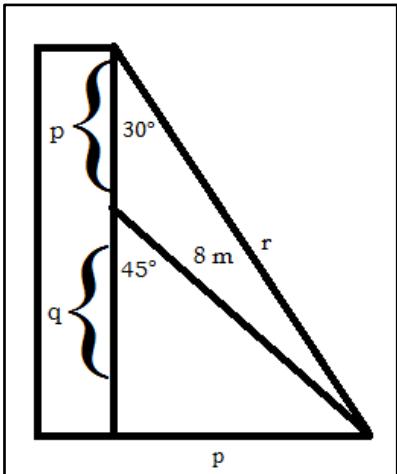
15. What is the value of  $s$ ?

- a. 2      b.  $2\sqrt{3}$       c. 4      d.  $4\sqrt{3}$





## Additional Activities



### House Repair Case:

Heavy wind brought by "Hanging Habagat" damaged a house. Workers placed an 8-m brace against its wall at a  $45^\circ$  angle. Then, at the same spot on the ground, they placed a longer brace that makes a  $30^\circ$  angle with the wall of the house. Show the complete solution for each question below.

- How long is the longer brace? Round off your answer to the nearest tenth of a meter.
- About how much higher on the wall of the house does the longer brace reach than the shorter brace?

### A. Exploration

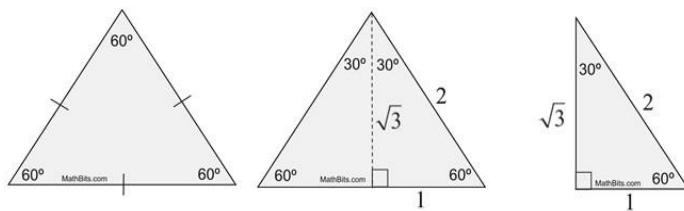
In the next module, we will apply your understanding of the lesson to real – life situations.

## PROBLEM – BASED LEARNING WORKSHEET

Special Right Triangle:  $30^\circ - 60^\circ - 90^\circ$

There are two "special" right triangles that will continually appear throughout your study of mathematics: the  $30^\circ-60^\circ-90^\circ$  triangle and the  $45^\circ-45^\circ-90^\circ$  triangle. The special nature of these triangles is their ability to yield exact answers instead of decimal approximations when dealing with the values of trigonometric functions

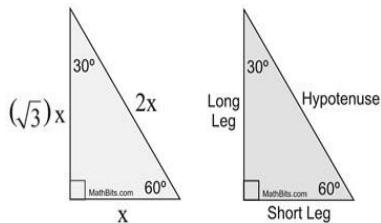
Congruent  $30^\circ-60^\circ-90^\circ$  triangles are formed when an altitude is drawn in an equilateral triangle. Remember that the altitude of an equilateral triangle bisects the angle and is the perpendicular bisector of the side opposite the angle. If the side of the equilateral triangle is set to a length of 2 units, the Pythagorean formula can be used to find the length of the altitude which is  $\sqrt{3}$  units.



Once the lengths of the sides of the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle are established, a series of relationships (patterns) can be identified between or among the sides of the triangle. **ALL**  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles possess these same patterns. These relationships will be referred to as "short cut formulas" that can quickly answer questions regarding side lengths of  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles, without having to apply any other strategies such as the Pythagorean Theorem or trigonometric functions.

Since  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles are similar, their corresponding sides are proportional. As such, we can establish a pattern as to how their sides are related. The following pattern formulas will let you quickly find the lengths of the sides of a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle even there is only ONE given length of a side of the triangle. Remember, these formulas work ONLY in a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle!

Labeling:



Pattern Formulas:  
Short Leg is half of the Hypotenuse.

$$SL = \frac{1}{2} H$$

Long Leg is half the Hypotenuse  $\cdot \sqrt{3}$

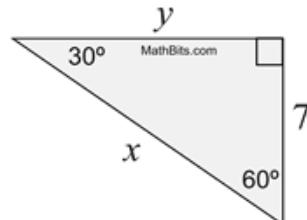
$$LL = \frac{1}{2} H \cdot \sqrt{3}$$

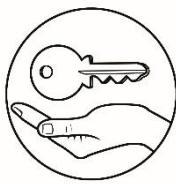
Combining the two formulas above:

$$LL = SL \cdot \sqrt{3}$$

### Let's Analyze

1. How many special right triangles are considered in the study of mathematics?
2. What is formed when the altitude of an equilateral triangle is drawn?
3. Describe the altitude of an equilateral triangle.
4. In a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle, state the relationship between the
  - a. hypotenuse and the short leg.
  - b. short leg and the long leg.
  - c. hypotenuse and the long leg.
5. In the figure at the right, find the value of  $x$  and  $y$ .





## Answer Key

I.		What's In:		What I Know:		What's New:		What is it:		II.	
1. A	6. A	11. A	A.	1. 0.7660	1. 32°	B.	C.	A. 1. 45°	B. 1. 60°	C. 1. 45°	3. 90°
2. B	7. D	12. C		2. 0.9962	2. 60°			2. 45°	2. 30°	2. 45°	3. 90°
3. A	8. D	13. D		3. 0.7002	3. 61°			3. 13	3. 13	3. 13	3. 90°
4. C	9. B	14. B		4. 0.7071	4. 15°			4. 8	4. 15°	4. 8	5. 10
5. D	10. B	15. C		5. 2.4751	5. 36°			5. 10	5. 36°	5. 10	6. $\frac{9}{4}$
What I Can Do:		What I Know:		What's In:		What's New:		What is it:		II.	
1. $30^\circ, 45^\circ$ and $60^\circ$ angles	2. $\sin \theta = \frac{\text{bottom fingers}}{2}; \cos \theta = \frac{\text{top fingers}}{2}; \tan \theta = \frac{\text{top fingers}}{\text{bottom fingers}}$	3. $A. \frac{1}{2}$	F. $\sqrt{2}$	4. $\frac{\sqrt{3}}{2}$		5. $34.6 \text{ m}$	6. $a = 45^\circ, b = 45^\circ, c = 90^\circ$	7. $\frac{\sqrt{2}}{2}$	8. $\frac{1}{2}$	9. $\frac{1}{2}$	10. $2\sqrt{2} - 1$
1. $30^\circ, 45^\circ$ and $60^\circ$ angles	2. $\sin \theta = \frac{\text{bottom fingers}}{2}; \cos \theta = \frac{\text{top fingers}}{2}; \tan \theta = \frac{\text{top fingers}}{\text{bottom fingers}}$	3. $A. \frac{1}{2}$	F. $\sqrt{2}$	4. $\frac{\sqrt{3}}{2}$		5. $34.6 \text{ m}$	6. $a = 45^\circ, b = 45^\circ, c = 90^\circ$	7. $\frac{\sqrt{2}}{2}$	8. $\frac{1}{2}$	9. $\frac{1}{2}$	10. $2\sqrt{2} - 1$

I.		What I Can Do:		What I Know:		What's In:		What's New:		II.	
1. $30^\circ, 45^\circ$ and $60^\circ$ angles	2. $\sin \theta = \frac{\text{bottom fingers}}{2}; \cos \theta = \frac{\text{top fingers}}{2}; \tan \theta = \frac{\text{top fingers}}{\text{bottom fingers}}$	3. $A. \frac{1}{2}$	F. $\sqrt{2}$	4. $\frac{\sqrt{3}}{2}$		5. $34.6 \text{ m}$	6. $a = 45^\circ, b = 45^\circ, c = 90^\circ$	7. $\frac{\sqrt{2}}{2}$	8. $\frac{1}{2}$	9. $\frac{1}{2}$	10. $2\sqrt{2} - 1$
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1. $30^\circ, 45^\circ$ and $60^\circ$ angles	2. $\sin \theta = \frac{\text{bottom fingers}}{2}; \cos \theta = \frac{\text{top fingers}}{2}; \tan \theta = \frac{\text{top fingers}}{\text{bottom fingers}}$	3. $A. \frac{1}{2}$	F. $\sqrt{2}$	4. $\frac{\sqrt{3}}{2}$		5. $34.6 \text{ m}$	6. $a = 45^\circ, b = 45^\circ, c = 90^\circ$	7. $\frac{\sqrt{2}}{2}$	8. $\frac{1}{2}$	9. $\frac{1}{2}$	10. $2\sqrt{2} - 1$

**Assessment:****Additional Activities:****Problem - Based Learning Worksheet:**

1. Two special right triangles  
2. Two Congruent  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles  
3. The altitude of an equilateral triangle bisects the angle and is the perpendicular bisector of the side opposite the angle.  
4. a.  $SL = \frac{1}{2}H$    b.  $LL = SL\sqrt{3}$    c.  $LL = \frac{1}{2}H\sqrt{3}$   
5.  $x = 14$  and  $y = 7\sqrt{3}$

## **References**

To further explore the concept learned today and if it possible to connect the internet, you may visit the following links:

<https://youtu.be/-H6LXoxqi3c>

<https://youtu.be/-wWsCkbOzsc>

<https://youtu.be/nVTtSE5nv7c>

<https://youtu.be/hftTj9RfuxM>

Baccay, Esperanza, Reyes (2014) Exploring Mathematics 9 (K to 12 Enhanced Basic Education Curriculum) Sunshine Interlinks Publishing House, Inc Quezon City, Philippines

Bernabe, Dilao, Quiming, Garces, Oracion (2014) Our World of Math 9 (K to 12) Vibal Group, Inc Quezon City, Philippines

Merden, Leonides et al (2014) Mathematics Learner's Material 9 (Module 7 Triangle Trigonometry) Department of Education Pasig City, Philippines

Department of Education, Mathematics Learner's Material 9

<https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-special-right-triangles/a/special-right-triangles-review>

<https://SteveHinds-30-60-90Triangleproblems.compressed.pdf>

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