9

Mathematics

Quarter 1 - Module 13

Problems Involving Quadratic Inequalities

Week 5

Learning Code - M9AL-If-9

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ADM

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MODULE 13

PROBLEMS INVOLVING QUADRATIC INEQUALITIES

In the previous lessons, you have learned about solving quadratic inequalities. In this module, you will be given opportunities to solve real-life problems involving quadratic inequalities. As you go through performing the activities, you will realize how quadratic inequalities will help you apply the concept in your own problems and make decisions to situations related to it.

WHAT I NEED TO KNOW

LEARNING COMPETENCIES

The learners will be able to:

- solve real-life problems involving quadratic inequalities. **M9AL-If-9**

WHAT I KNOW

Let us find out how much you already know about solving real-life problems using quadratic inequality.

Direction: Write the letter which you think is the best answer to each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

For numbers 1 – 4

Rosalia owns a chocolate vending machine, which is a machine that picks a chocolate out of an assortment in a random fashion. Rosalia controls the probability in which each chocolate is picked.

She is running out of "Choco Hany" chocolate candy so she wants to program its probability of getting a different candy twice in a row greater than 2 ¼ times the probability of getting 'Choco Hany in one try". (Remember that a probability must be a number between 0 and 1)

1) Using p to represent the probability of getting 'Choco Hany" in one try, the inequality that models the problem is ______________.

A. $(1 - p)^2 > 2\frac{1}{4}p$ C. $2 - 2p + p^2 < 2\frac{1}{4}p$

B. $p^2 - 1 > 2\frac{1}{4}p$ D. $(1 - p)^2 \geq 2\frac{1}{4}p$

2) Which of the following is the same as (1 – p) ^{2} > $2\frac{1}{4}p$

   A. 1 – 2p + p^2 > 3.25p　　C. 1 – 2p + p^2 > 2.25p

   B. 1 – p^2 > 2.25p　　D. 1 – p^2 > 2.25p

3) Which of the following is equivalent to $p^2 - \frac{9}{4}p + 1 > 0$?

   A. (p-1) (p+4) > 0　B. (p - $\frac{1}{4}$) (p- 4) >0　C. (p + $\frac{1}{4}$) (p- 4) >0　D. (p - $\frac{1}{4}$) (p- 4) >0

4) The probability of getting Choco Hany in one try must be less than _____.

   A. 0.25　B. 0.5　C. 1　D. 4

For numbers 5 – 10

Patricia's rectangular flower garden fence has to be replaced by a new one. She bought 40 feet of metal fencing material to replace the fence on the three sides of her garden. A tall wooden fence which serves as the fourth side was not to be replaced and remains as it is.

side not to be replaced

5) If x is the width of the garden in feet, represent the length of the garden (opposite of the side not to be replaced) in terms of x.

   A. 20 – 2x　B. 40 – 2x　C. 30 - x　D. 40 – x

6) Write equation of the area of the garden (A) in terms of x.

   A. A = 20 – 2x　B. A = 40 + 2x^2　C. A = 20x – 2x^2　D. A = 40x – 2x^2

7) What inequality models "the width of the garden fence will give an area of at least 150 square feet"?

   A. 150 ≤ -2x^2+ 40x　B. 150 ≥ -2x^2+ 40x

   C. 150 ≤ 40x^2 - 2x　D. 150 ≥ 40x^2- 2x

8) What measure for the width will give an area of at least 150 square feet?

   A. 5≤ x ≤ 15　B. 5≥ x ≥ 15　C. 5 ≥ x ≤ 15　D. 5≤ x ≥ 15

9) What inequality models "the width of the garden fence will give an area of at most 200 square feet"?

   A. $40x^2 - 2x \leq 200$　C. $40x^2 - 2x \geq 200$

   B. $-2x^2 + 40x \leq 200$　D. $-2x^2 + 40x \geq 200$

10) What measures for the width will give an area of at least 200 square feet?

   A. 185.86≤ x ≤ 214.14　C. 185.85 ≥ x ≤ 214.14

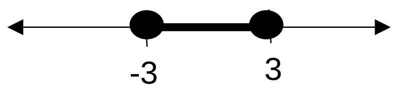
   B. 185.85≥ x ≥ 214.14　D. 185. 86≤ x ≥ 214. 14

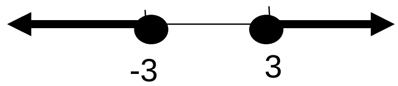
WHAT'S IN

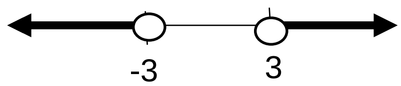
Communication and Critical Thinking

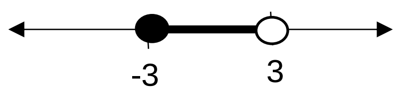
Let us recall a concept you learned from the previous lesson.

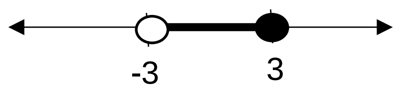
A. Write the interval notation of the indicated region:

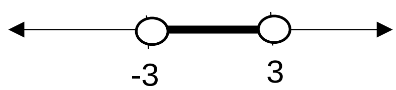
1. 

2. 

3. 

4. 

5. 

6. 

7. 

B. Let us review how to solve quadratic inequality

Solve the quadratic inequality $x^2 – 4x > -3$

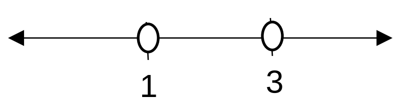
Step 1. Find all the zeroes of

$x^2 – 4x = - 3$

$x^2 – 4x + 3 = 0$

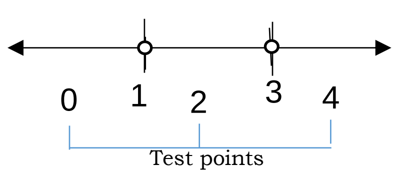
$(x -3) (x – 1) = 0$, $x = 3$ and $x = 1$

Step 2. Plot the zeroes on the number line with open or hollow points.



We choose open points. Why do you think so?

Step 3. Choose a test value in each interval.

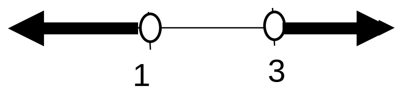


$x = 0$	$0^2 – 4(0) > -3$	$0 > - 3$	TRUE

x = 2	$(2)^2 - 4(2) > -3$	$-4 > -3$	FALSE
x = 4	$(4)^2 - 4(4) > -3$	$0 > -3$	TRUE

Hence the solutions lie to the left of 1 or to the right of 3

Step 4: Graph and state the solution



The solution is $(-\infty, 1) \cup (3, \infty)$ or

$\{x/x$ is a real, $x < 1$ or $x > 3\}$

(read as "all x such that x is a real number less than 1 or greater than 3)

Now try this!

Find the solution of each quadratic inequality.

1. $x^2 + 18 \geq 9x$
2. $2x^2 + x - 6 \leq 0$
3. $x^2 - 4x + 1 > 6$
4. $x^2 - 6x + 8 \geq 0$
5. $2x^2 + 1 < -3x$

How did you find the activity? Do you think your knowledge of the concept of inequality will help you in solving real-life problems?

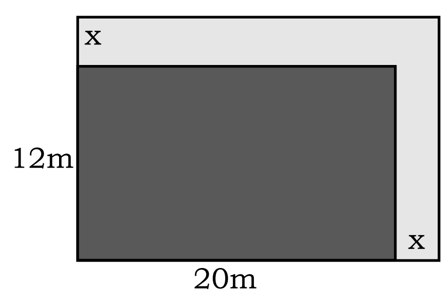
WHAT'S NEW

Communication, Critical Thinking, and Collaboration

Were you able to solve for the solutions of the given quadratic inequalities? What difficulties did you encounter? How did you cope up with these difficulties?

Now that you have recalled how to solve quadratic inequalities, you can proceed to applying the concept in real-life problems. There are a lot of situations that can be represented by quadratic inequalities. Consider the situation below:

Rod is preparing to expand his lot for a car wash business. The lot measures 12m by 20m and he wants to expand the size by adding an equal distance to two of its sides as shown. However, the new lot with expansion should not exceed 345 m^2, what range of distances in meters can Rod add in his lot?



1. How would you represent the length and the width of the new rectangle that will be formed?
2. What mathematical sentence would represent the given situation?
3. What do you think is the range of distances that Rod can be added so that the area will not exceed 345m^2?

WHAT IS IT

Communication, Critical Thinking, Creativity and Character Building

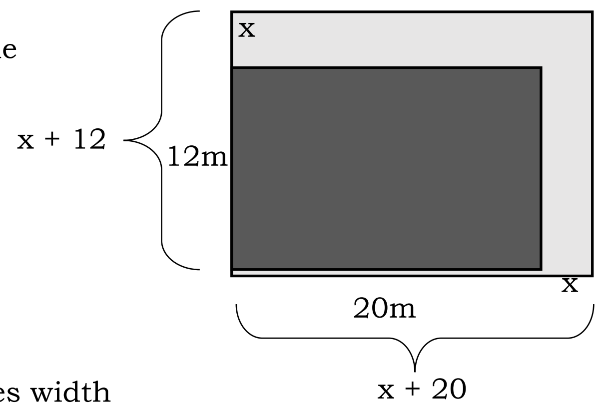
Let's analyze.

The phrases "should not exceed", "less than", "more than", "range of distances", "range of cost", and alike are indicators of inequalities.

In solving problems involving quadratic inequalities, you need to read through the entire problem. Highlight the important information and key words that you need to solve the problem. And then, identify your variables.

If we let x = the additional constant width to be added to each side then the width of the expanded rectangle will be represented as (x + 12)m, and the length will be represented as (x + 20)m.

(Please refer to the figure at the right)



area = length times width

$$area \le 345m^2$$

$$length \cdot width \le 345\, m^2$$

$$(x+12)(x+20) \le 345$$

If the area should not exceed 354m^2, then the inequality that will represent the situation will be:

$$(x + 12)(x + 20) \le 345$$

Solving for the possible values of x,

$$(x + 12)(x + 20) \le 345$$
$$x^2 + 32x + 240 \le 345$$
$$x^2 + 32x - 105 \le 0$$

Then, solve for the roots of the equivalent quadratic equation,

$$x^2 + 32x - 105 = 0$$
$$(x + 35)(x - 3) = 0$$
$$x + 35 = 0 \quad \text{and} \quad x - 3 = 0$$
$$x = -35 \qquad\qquad x = 3$$

Since the value of x refers to length of expansion, it cannot be negative. Thus, the range of distance that Rod can add on both sides is 0m ≤ x ≤ 3m in order not to exceed 345m^2.

Checkpoint:

a. Can he not add extra meters and still satisfy his intention?
b. Can he add 0.5 m on both sides? 1.7 m? 2.9 m? 3.25 m? Why or why not?

WHAT'S MORE

Critical Thinking

Now try to solve the following problems involving quadratic inequalities.

1. An object is launched from ground level directly upward at a rate of 144 feet per second. The equation for the object's height is $y = -16x^2 + 144x$, where x is the time the object is at flight in seconds, and y is the height of the ball in feet. What values of x is the object at or above a height of 288 feet? How long is the object at or above this height?

2. A tennis ball is hit into the air. The height of the ball, $h(t)$, in feet, at time t, in seconds, is modeled by the equation $h(t) = -16t^2 + 64t + 4$. Determine the interval of time when the height of the ball is greater than or equal to 52 feet?

3. Vanessa has 180 feet of fencing that she intends to use to build a rectangular play area for her dog. She wants the play area to enclose at least 1800 square feet. What are the possible integral widths of the play area?

How did you find the problems above? Where you able to solve for the required solution easily or you encountered difficulty in solving it? Which part of the problem-solving process is challenging? What did you do to cope with these challenges?

WHAT I HAVE LEARNED

To solve for problems, read through the entire problem involving quadratic inequality and highlight the important information and key words needed to solve the problem. Then proceed to setting the variables to be used to write the equation or inequality to finally solve the problem.

Remember to take note of the inequality symbols. There are clue words that can help in determining the inequality symbol for a particular problem. For a situation considering "**greater than**", we use the symbol "**>**" and for "**less than**" we use "**<**". If the problem used a clue word "**at least"**, we use the symbol "**≥"** while those situations that asked for **"at most",** we use **"≤".**

WHAT I CAN DO

Critical Thinking

Now that you are equipped with the knowledge of solving real-life problems involving quadratic inequality, work on the following problems independently,

1) A rectangular parking lot must have a perimeter of 440 feet and an area of at least 10,500 square feet. Describe the possible lengths of the parking lot.

2) A small canteen offers special meal package to their customers. The profit P that the canteen will earn at x number of customer is modeled by the function $P(x) = -25x^2 + 1000x - 4375$. How many people are needed for a profit of at least Php5,000?

3) Ana has 40 ft of fencing materials to be used for three sides of a rectangular garden, while a wall serves as her fourth side. What measures of the width will give an area of at least 150 square ft?

4) An object is launched from ground level directly upward at a rate of 48 meters per second. The equation for the object's height is $y = -16x^2 + 48x$. What values of x is the object at or above a height of 32 meters?

ASSESSMENT

Write the letter of the correct answer on your answer sheet. If your answer is not found among the choices, write the correct answer

For number 1 to 3

A consultant advises the owners of a beauty salon that their profit p each month can be modeled by $p(x) = -50x^2 + 3500x - 2500$, where x is the average cost that a customer is charged. What range of costs will bring in a profit of at least 50 000?

1) What quadratic inequality models the problem above?
   A. $50\,000 \leq -50x^2+3500x - 2500$
   C. $50\,000 > -50x^2+3500x - 2500$
   B. $50\,000 < -50x^2+3500x - 2500$
   D. $50\,000 \geq -50x^2+3500x - 2500$

2) Using the quadratic formula, what are the zeros/roots of given quadratic inequality in question number 1.
   A. $5 \pm 35\sqrt{7}$ B. $2 \pm 5\sqrt{7}$ C. $35 \pm 5\sqrt{7}$ D. $2 \pm 35\sqrt{7}$

3) What range of costs will bring in a profit of at least 50 000?
   A. $21.8 < x < 48.2$
   B. $21.8 \leq x \leq 48.2$
   C. $x \leq 21.8$ or $x \geq 48.2$
   D. $21.8 \geq x$ or $x < 48.2$

For number 4 to 5

The human cannonball is an act where a performer is launched through the air. The height of the performer can be modeled by $h(x)= -0.005x^2 + x + 20$, where h is the height in feet and x is the horizontal distance traveled in feet. The circus act is considering a flight path directly over the main tent.

4) If the performer wants at least 5ft of vertical height clearance, how tall can the tent be?
   A. approximately 160 ft
   B. approximately 165 ft
   C. approximately 170 ft
   D. approximately 175 ft
5) How far from the central pole should the cannon be placed?
   A. approximately 90ft
   B. approximately 100ft
   C. approximately 110ft
   D. approximately 120ft

For number 6 to 7

The profit p of a certain barber shop is modeled by the function $y = -50x^2 +3500x -2500$, where x is the average cost that a customer is charged. What range of cost will bring in a profit of at least P50,000?

6) Which inequality statement represents the given problem?
   A. $-50x^2 +3500x -52500 \geq 0$
   B. $-50x^2 +3500x -2500 \leq 0$
   C. $-50x^2 +3500x -52500 \leq 0$
   D. $-50x^2 +3500x -2500 \geq 0$

7) What range of cost will bring in a profit of at least P50,000?
   A. From P0.00 to P21.78 and P48.22 above
   B. Between P21.78 and P48.22
   C. From P0.00 to P48.22
   D. At least P21.78 and at most P48.22

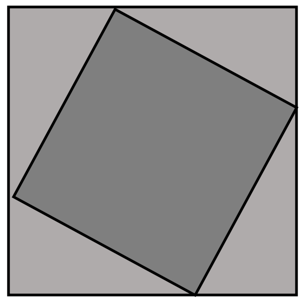
Were you able to answer all the real-life problems? Now that you understand how to apply the concept of quadratic inequality in real-life situations, you are ready to go deeper and create problems of your own.

ADDITIONAL ACTIVITIES

Critical Thinking, Creativity and Character Building

A Tile Over a Tile

Supposed you have two square tiles of different sizes. Place the smaller square tile on top of the larger square tile as shown.



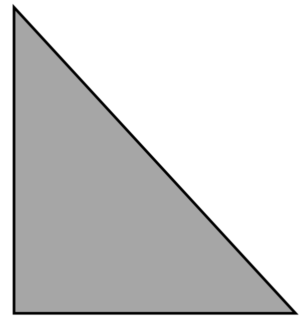
1. How will you describe the sides of the four triangular regions? (Hint: observe the lengths of each side of the triangles, and the type of triangles they are)
2. Suppose the length of the side of the bigger square 30cm and the shorter leg of the triangular region is denoted by x, how will you represent the longer leg?
3. Write a function for the area A of one of the triangular regions in terms of x.
4. If the area of each triangle is at least 40.5cm^2, what quadratic inequality represents the situation?
5. For what values of x satisfy the inequality? Round your answer to the nearest whole numbers.

How did you find the activity? What challenges did you encounter while doing the activity? How did you cope with these challenges? Do you think you can now solve any problem related to quadratic inequalities?

PROBLEM – BASED WORKSHEET

A. Right Triangular Plane

Tony needs to draw a right triangle as part of his school assignment. He wants length of one leg is 7 cm more than the other leg and ensure that the length of the hypotenuse is at least 13 cm.



1. If s represents the length of the shorter leg, what mathematical sentence would solve the given problem?
2. What is the smallest integral length of the longer leg?
3. What could be the range of the shorter leg?

Search

You may also check the following link for your reference and further learnings on problems involving quadratic inequalities.

https://www.mathsisfun.com/algebra/inequality-questions-solving.html
https://www.youtube.com/watch?v=GDppV18XDCs
https://www.youtube.com/watch?v=tB1jkT7LrWE
https://www.youtube.com/watch?v=OYajy-IfgAo
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ANSWER KEY

WHAT I KNOW

1. A	6. D
2. C	7. A
3. B	8. A
4. A	9. B
5. B	10. A

WHAT'S IN

A.
1. $[-3, 3]$
2. $(-\infty, -3] \cup [3, \infty)$
3. $(-\infty, -3) \cup (3, \infty)$
4. $[-3, 3)$
5. $(-3, 3]$
6. $(-3, 3)$
7. $(-\infty, 3)$

B.
1. $(-\infty, 3] \cup [6, \infty)$
2. $[-2, 3/2]$
3. $(-\infty, -1) \cup (5, \infty)$
4. $(-\infty, 2] \cup [4, \infty)$
5. $(-1. -1/2)$

WHAT'S MORE

1. The object is at or above 288 ft from 3 to 6 seconds after its release.
2. $h(t) = -16t^2 + 64t + 4$
   $-16t^2 + 64t + 4 \geq 52$
   $-16t^2 + 64t - 48 \geq 0$
   $16t^2 - 64t + 48 \leq 0$
   $t^2 - 4t + 3 \leq 0$
   $\mathbf{1 \leq t \leq 3}$
3. $Let\ w = width\ of\ the\ fence$
   $2l + 2w = 180$
   $l = \frac{180 - 2w}{2} = 90 - w$
   $w(90 - w) \geq 1800$
   $90w - w^2 \geq 1800$
   $w^2 - 90w + 1800 \leq 0$
   $\mathbf{30 \leq w \leq 60}$
   The possible integral widths of the play area are **30ft x 60ft**
   36ft x 50ft
   40ft x 45ft

WHAT I CAN DO

1. $let\ l = the\ legth\ of\ the\ parking\ lot$
   $2l + 2w = 440$
   $l + w = 220$
   $w = 220 - l$
   $l(220 - l) \geq 10500$
   $220l - l^2 \geq 10500$
   $l^2 - 220l + 10500 \leq 0$
   $\mathbf{70 \leq l \leq 150}$
   The approximate length of the parking lot is at least 70 feet and at most 150 feet
2. To earn a profit of at least Php5,000, the canteen need at least 15 to at most 25 customers.
3. The width should be at least 5ft and at most 15 ft to give an area of at least 150ft^2.
4. The object will be at or above a height of 32 meters between 1second to 2 seconds.

ASSESSMENT

1. A
2. C
3. B
4. B
5. B
6. A
7. D

ADDITIONAL ACTIVITIES

1. The triangles are congruent by SSS or SAS.
2. 30 – x
3. $A = -\frac{1}{2}x^2 + 15x$
4. $-\frac{1}{2}x^2 + 15x - 40.5 \geq 0$
5. At least 3cm but less than 15cm, since x represent the shorter leg.

PROBLEM – BASED WORKSHEET

1. If s is the length of the shorter leg, then, then the longer is s + 7.
Applying the Pythagorean theorem, the mathematical sentence that will help us solve the problem is, $(\mathbf{s})^2 + (\mathbf{s + 7})^2 \geq \mathbf{13^2}$ or $\mathbf{s^2 + (s + 7)^2 \geq 169}$

2. Solving quadratic inequality,

$$s^2 + (s + 7)^2 \geq 169$$
$$s^2 + (s + 7)^2 = 169$$
$$s^2 + s^2 + 14s + 49 = 169$$
$$2s^2 + 14s - 120 = 0$$
$$s^2 + 7s - 60 = 0$$
$$(s + 12)\ (s - 5) = 0$$
$$s = -12 \quad \text{or} \quad \mathbf{s = 5}$$

3. We only consider the **positive value of *s*** since the situation involves measures of length. Thus, the range for the length 0f the **shorter leg** is **at least 5 cm**, and the range of length of the **longer leg must be at least 12 cm**.