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DepED
DEPARTMENT OF EDUCATION

Mathematics

Quarter 1 - Module 19

Characteristics of the Graphs of Quadratic Function

Week 7

Learning Code - M9AL-Ih-13.2

ALTERNATIVE DELIVERY MODE
ADM

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MODULE 19

CHARACTERISTICS OF THE GRAPHS OF QUADRATIC FUNCTIONS

The previous module, you learned to graph quadratic functions. You were given opportunities to explore the basic characteristics of parabola. In this module, you will find out all the characteristics of the graphs of quadratic functions.

WHAT I NEED TO KNOW

LEARNING COMPETENCY

The learners will be able to:

- determine the following (a) domain (b) range (c) intercepts (d)axis of symmetry (e)vertex (f)direction of the opening of the parabola from the given graph of the equation. **M9AL-Ih-13.2**

WHAT I KNOW

Find out how much you already know about the characteristics of the graphs of quadratic functions. Write the letter that you think is the correct answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. What is the graph of a quadratic function?
   A) Circle B) Line C) Parabola D) V-Shape
2. It is the turning point of the parabola, which indicates the highest/lowest point of the graph.
   A) Axis of symmetry B) Range C) Vertex D) Zeros
3. What do you call the *x*-coordinates of the points of the parabola that lie on the *x*-axis?
   A) Axis of symmetry B) Range C) Vertex D) Zeros

4. Which of the following is the vertex form of the quadratic function $y = ax^2 + bx + c$ with vertex at (h, k)?
   A) $y = k(x-h)^2 + a$ B) $y = h(x-a)^2 + k$
   C) $y = a(x-k)^2 + h$ D) $y = a(x-h)^2 + k$

5. What is the vertex form of the equation $y = x^2 + 4x + 7$?

   A) $y = (x-2)^2 - 3$ B) $y = (x-2)^2 + 3$

   C) $y = (x+2)^2 - 3$ D) $y = (x+2)^2 + 3$

6. What is the vertex of the parabola having the equation $y = (x-1)^2 + 3$?

   A) (3, –1) B) (–1, 3) C) (1, 3) D) (3, 1)

7. What is the maximum value of the graph of the quadratic function $y = -x^2 + 4x$?

   A) – 4 B) – 2 C) 2 D) 4

8. What is the *y*-intercept of the parabola defined by the equation $y = (x+2)^2$?

   A) (0,– 4) B) (0, – 2) C) (0, 2) D) (0, 4)

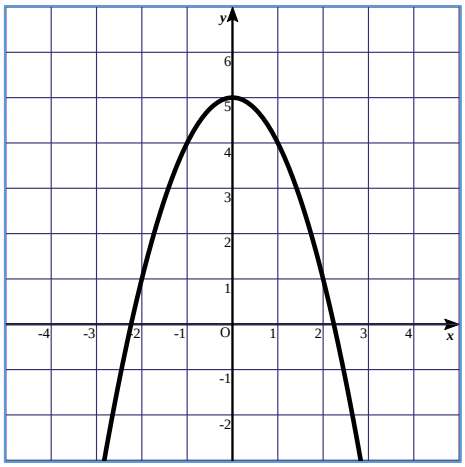
9. Describe the graph of $f(x) = x^2 + 6x - 24$.

   A) Opens to the right B) Opens to the left

   C) Opens upward D) Opens downward

For items 10-14, *refer to the graph at the right.*

10. Which of the following could be the equation of the parabola?



   A) $y = -x^2 + 5$

   B) $y = \frac{x^2}{2} + 5$

   C) $y = -\frac{x^2}{2} + 5$

   D) $y = x^2 + 5$

11. What is the vertex of the parabola?

   A) $(-5,0)$ B) $(0,-5)$ C) $(0,5)$ D) $(5,0)$

12. What is the axis of symmetry of the graph?

   A) $x = 0$ B) $y = 0$ C) $x = 5$ D) $y = 5$

13. What is the range of the parabola?

   A) $(-\infty,-5]$ B) $(-\infty,+\infty)$ C) $(-\infty,5]$ D) $[5,+\infty)$

14. Determine the y- intercept of the graph?

   A) (0, –5) B) (0, 5) C) (0, –3) D) (0, 3)

15. Which of the following describes the parabola having the equation $y = x^2 + 3$?

   A) It lies entirely above the x-axis.

   B) Its vertex lies on the axis.

   C) It lies entirely below the axis.

   D) It intersects the *x*-axis at two distinct points.

WHAT'S IN

Communication, and Critical Thinking

Let's start by a review of graphing quadratic function.

Transform each quadratic function in the form of $y = a(x - h)^2 + k$, then graph using vertex and axis of symmetry.

1. $y = -(x^2 - 4x) + 1$
2. $2x^2 + 6x = 3 - y$
3. $4x^2 - y = 12x - 7$
4. $2 - x^2 = y - 3x$

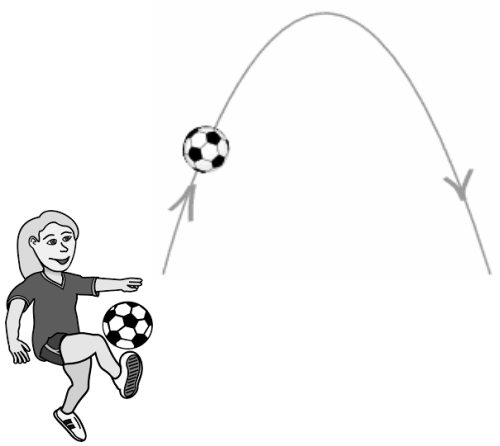
WHAT'S NEW

Communication, and Collaboration

Kick that Ball!

Mara kicked a ball into the air. The height y in feet of the ball after x seconds can be modeled by the quadratic function

$y = -16x^2 + 32x$.



The arc formed by the ball is a curve called parabola.

Can you graph actual parabola formed by the given function above and answer the questions below?

1. What is the opening of the parabola?
2. What are the coordinates of the turning point of the parabola?
3. What is the equation of the line that divides the parabola into two equal parts?
4. At what point/s does the parabola intersect/s the x-axis?
5. At what point/s does the parabola intersect/s the y-axis?
6. For what values of x satisfy the given function based on the graph?
7. For what values of y satisfy the given function based on the graph?

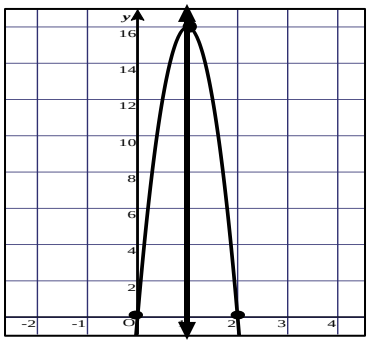
How did you find the activity? Were you able to answer all the questions based on the graph? If not, let us investigate further and

WHAT IS IT

Communication, Critical Thinking, and Collaboration

Let's investigate!

Shown on the right is the graph of the quadratic function defined by



$$y = -16x^2 + 32x$$

Notice that the parabola opens downward. In some cases parabola opens upward. In the quadratic function $y = ax^2 + bx + c$, if the value of $a > 0$, then parabola opens upward and has a minimum point. If $a < 0$ the parabola opens downward and has a maximum point.

The turning point of a parabola is called its **vertex** with coordinates at (h, k). In this case, the vertex is at (1, 16).

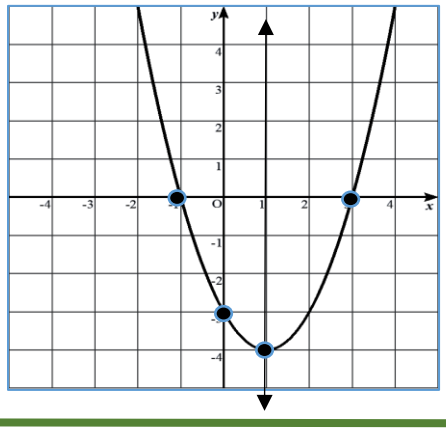
There is a line called the **axis of symmetry** which divides the graph into two parts such that one half of the graph is a reflection of the other half. In this graph, the equation of the axis of symmetry is x = 1. If the quadratic function is expressed in the form of $y = a(x-h)^2 + k$ the vertex is the point (h,k) . The line $x = h$ is the axis of symmetry and k is the minimum or maximum value of the function.

The graph intersect the x-axis at (0, 0) and (2,0). These are called **x-intercepts**. These are the points at which the graph of an equation crosses the x-axis. While, the graph intersects the y-axis at (0, 0). This is called the **y-intercept**. It is a point which the graph of an equation crosses the y-axis.

The **domain** of a quadratic function is the set of real numbers. The **range** depends on whether the parabola opens upward or downward. If it opens upward, then the range is the set {y/ y≥k}; if it opens downward then the range is the set {y/ y≤k}. In this graph, the range is {y /y ≤ 16}, expressed as Set Notation, or [16, +∞), expressed as Group Notation.

To fully understand the characteristics of the graph of a quadratic function, study the following examples.

Example 1. Determine the vertex, direction of the opening of the parabola, axis of symmetry, domain, range and intercepts given the graph.



Solution:

Vertex: (1, -4)

Opening of the parabola: Upward

Vertex is a <u>minimum</u> point.

Axis of symmetry: x = 1

Domain: { x/ x ∈ ℝ }

Range: { y / y ≥ -4 }

Intercepts: x – intercepts (-1, 0) and (3, 0)

y – intercept (0, -3)

Example 2. Determine the vertex, direction of the opening of the parabola, axis of symmetry, domain, range and intercepts given the graph.

Solution:

Vertex: (-1, 4)

Opening of the parabola: Downward

Vertex is a maximum point.

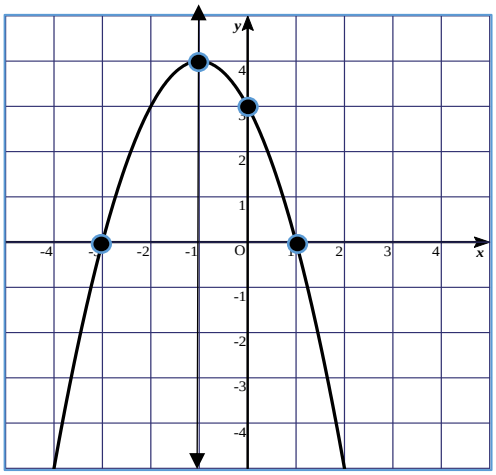
Axis of symmetry: x = -1

Domain: { x/ x ∈ ℝ }

Range: { y / y ≤ 4 }

Intercepts: x – intercepts (1, 0) and (-3, 0)

y – intercept (0, 3)



Notice how vertex plays an important role in determining the characteristics of the graph of a quadratic function. Thus, even without the graph and you know the vertex, you can identify the direction of the opening of the parabola, axis of symmetry, domain, range and intercepts given the graph. Study the next example.

Example 3. Determine the vertex, direction of the opening of the parabola, axis of symmetry, domain, range and intercepts of the quadratic function

$$\boldsymbol{y = -(x+2)^2 - 1}$$

Solution

a. Vertex : (h, k) of the form $y = a(x-h)^2 + k$ is (-2, -1)
b. Direction of the opening of the parabola : since $a < 0$, then it opens downward
c. Axis of symmetry :
   Since x = -2, then the equation of the axis of symmetry is x= 2
d. Domain : the domain of quadratic function is all real numbers
e. Range : since it opens downward then the range is $\{y: y \leq k\}, \{y: y \leq 7\}$ or [7, -∞)
f. Intercepts :
   x – intercepts, let y be zero then compute for the value of x
   The x-intercepts are at (3, 0) and (1, 0)
   y – intercept, let x be zero then compute for the value of y
   The y-intercept is at (0, 3).

Note: (if the equation in the form of $y = ax^2 + bx + c$, then y – intercept is at (0, c).

WHAT'S MORE

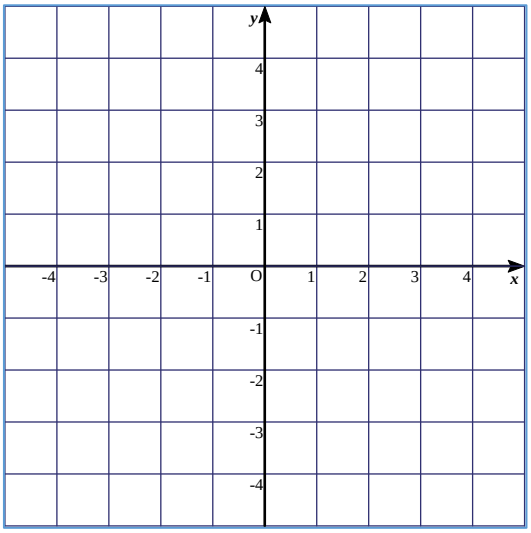
Critical Thinking, Communication and Collaboration

Draw and Describe My Graph!

Sketch the graph of each quadratic function and identify the vertex, direction of the opening of the parabola, domain, range and intercepts given the equation. State whether the vertex is a minimum or a maximum point and write the equation of the axis of symmetry. (Hint: Convert the quadratic function in standard form.)

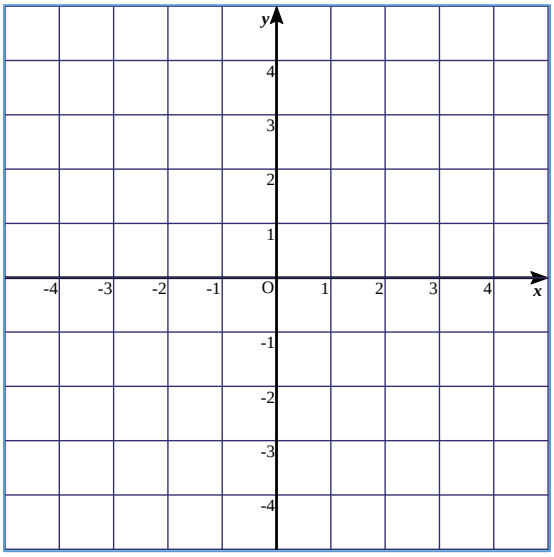
1. $y = (x+1)^2 - 4$

Vertex: _____
Opening of the graph: _____
Vertex is a _____ point.
Axis of symmetry: _____
Domain: _____
Range : _____
x – intercept/s: _____
y – intercept/s: _____



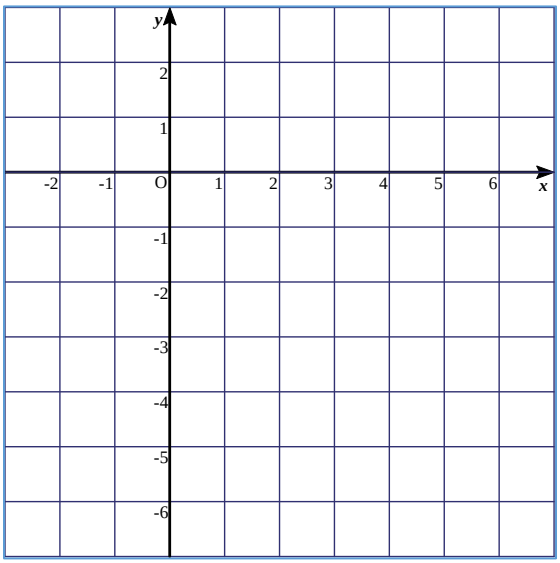
2. $y = -(2x^2 - 4x - 1)$

Vertex: _____
Opening of the graph: _____
Vertex is a _____ point.
Axis of symmetry: _____
Domain: _____
Range : _____
x – intercept/s : _____
y – intercept/s : _____



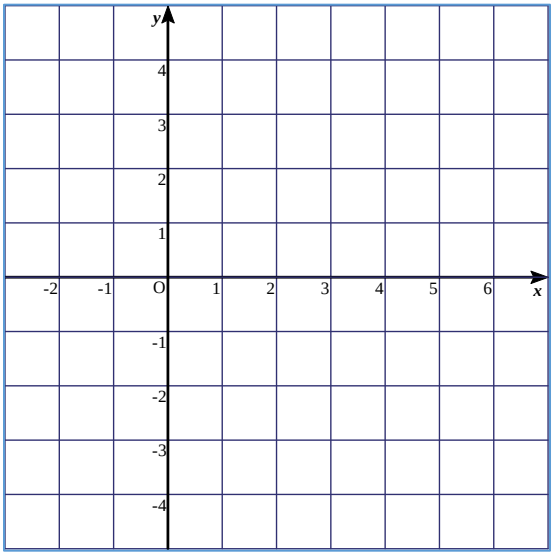
3. $2x^2 + y = 8x - 6$

Vertex: _____
Opening of the graph: _____
Vertex is a _____ point.
Axis of symmetry: _____
Domain: _____
Range : _____
x – intercept/s : _____
y – intercept/s : _____



4. $y - 6x - 1 = -3x^2$

Vertex: _____
Opening of the graph: _____
Vertex is a _____ point.
Axis of symmetry: _____
Domain: _____
Range : _____
x – intercept/s : _____
y – intercept/s : _____



How did you find the activity? Were you able to graph all the quadratic functions? Were you able to identify the characteristics of the parabola? If not, in which part did you find challenging? How did you cope up with it?

WHAT I HAVE LEARNED

Characteristics of the graph of quadratic function

The graph of a quadratic is a smooth curve called parabola.

- **Vertex** – the minimum or the maximum point on the quadratic function. If the quadratic function is expressed in the form of $y = a(x-h)^2 + k$ the vertex is the point (h, k). If it is in the form of $y = ax^2 + bx + c$ use the formula $h = -\frac{b}{2a}$ and $k = f(h)$.
- **Domain** – The domain of a quadratic function is the set of real numbers
- **Range** – The range depends on whether the parabola opens upward or downward. If it opens upward, then the range is the set $\{y: y \geq k\}$; if it opens downward then the range is the set $\{y: y \leq k\}$.
- **Axis of symmetry** – divides the graph into two parts such that one half of the graph is a reflection of the other half. The equation of the line is $x = h$.
- **Direction of the opening of the parabola** – If the value of > 0 , then parabola opens upward and has a minimum point. If < 0 , then parabola opens downward and has a maximum point.
- **Intercepts –** x – intercept (let y be zero then solve for x) y – intercept (y = c)

WHAT I CAN DO

Critical Thinking

From the graph of the following quadratic functions in module 18, determine their vertex, direction of the opening of the parabola, axis of symmetry, domain, range and y - intercept. (Hint: Convert the quadratic function in standard form.)

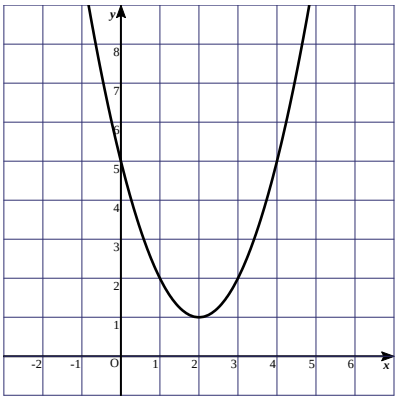
1. $y = (x + 2)^2 - 3$
2. $y = -(x - 3)^2 + 2$
3. $y = 2x^2 - 8x + 3$
4. $y = -x^2 + 2x + 1$
5. $y + 3x^2 = 6x - 1$
6. $y = -3x^2 - 6x + 4$
7. $y = 2x^2 - 12x + 9$
8. $2x^2 - 16x = y - 33$
9. $y - 3x^2 + 6x = 5$
10. $y - 4 = 3x^2 + 12x$

ASSESSMENT

Read each item carefully. Identify the choice that correctly completes the statement or answers the question

1. Which of the following describes the parabola having the equation $y = -x^2 - 3$?
   A) It lies entirely above the x-axis.
   B) Its vertex lies on the axis.
   C) It lies entirely below the axis.
   D) It intersects the ***x***-axis at two distinct points.

For items 2-5, refer to the graph at the right.



2. Which of the following could be the equation of the parabola?
   A) $y = (x-2)^2 + 1$ B) $y = (x+1)^2 + 2$
   C) $y = (x+2)^2 + 1$ D) $y = (x-1)^2 + 2$

3. Determine the y- intercept of the graph?
   A) $(0,-5)$ B) $(0,5)$
   C) $(0,-3)$ D) $(0,3)$

4. What is the range of the parabola?
   A) $(-\infty,-1]$ B) $(-\infty,+\infty)$
   C) $(-\infty,1]$ D) $[1,+\infty)$

5. What is the axis of symmetry of the graph?
   A) $x = 2$ B) $y = 2$ C) $x = 1$ D) $y = 1$

6. What is the vertex of the parabola?
   A) $(-2,-1)$ B) $(2,1)$ C) $(-2,1)$ D) $(2,-1)$

7. Describe the graph of $f(x) = -x^2 + 6x - 24$.
   A) Opens to the right B) Opens to the left
   C) Opens upward D) Opens downward

8. What is the y-intercept of the parabola defined by the equation $y = (x+2)^2$?
   A) (0,– 4) B) (0, – 2) C) (0, 2) D) (0, 4)

9. What is the minimum value of the graph of the quadratic function $y = x^2 + 4x$?
   A) – 4 B) – 2 C) 2 D) 4

10. What is the vertex of the parabola having the equation $y = (x-3)^2 + 1$?
    A) (3, –1) B) (–1, 3) C) (1, 3) D) (3, 1)

ADDITIONAL ACTIVITIES

Communication, Critical Thinking, Creativity and Character Building

The General Projectile Function

If an object is thrown straight up from the top of a building h feet tall with an initial velocity of v feet per second, then the height of the object as a **function** of time can be modeled by the quadratic **function** $h(t) = -16t^2 + vt + h$, where h(t) is the height of the object (in feet) t seconds after it is thrown. This is the general projectile function.

If Mara hit the ball at 3 ft. above the ground with an initial velocity of 32 ft/sec., the path of the ball is given by the function $h(t) = -16t^2 + 32t + 3$.

If Clara kicked a ball at 4ft above the ground with an initial level of velocity of 16ft/sec, then the path of the ball is given by the function $h(t) = -16t^2 + 16t + 4$.

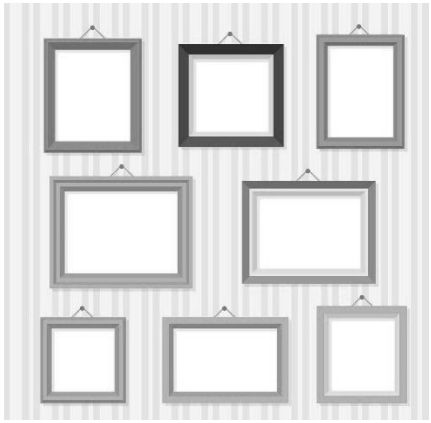
Your Task!

Without graphing determine the vertex, direction of the opening of the parabola, axis of symmetry, domain, range and y – intercept of each of the general projectile function given above.

PROBLEM – BASED WORKSHEET

The Frame Designer

Jeanna is assigned to design a rectangular mini frame of perimeter 12 inches.



Let's Analyze!

1. Regardless of dimensions as long as the perimeter is 12, complete the table for the width, length and Area.

width (w)	1	2	3	4	5	w
length						
Area						

2. Write a function A(w) for the area of the frame in terms of the width.

3. Graph the function A(w). Label each axis.

4. Determine the following:
   a. Domain
   b. Range
   c. Vertex
   d. Axis of symmetry
   e. Direction of the opening of the parabola

5. Determine the maximum area of the frame that Jeanna can design. What are the dimensions of this screen?

E-Search

You may also check the following link for your reference and further learnings on the characteristics of the graphs of quadratic function.

- https://www.khanacademy.org/math/algebra-home/alg-functions/alg-determining-the-range-of-a-function/v/domain-and-range-of-a-function-given-a-formula
- https://www.youtube.com/watch?v=V2udel2WylU
- https://www.youtube.com/watch?v=VdVPg04t_6w

REFERENCES

http://www.mesacc.edu/~scotz47781/mat120/notes/projectile_motion/projectile_motion.html#:~:text=Projectile%20Motion,seconds%20after%20it%20is%20thrown.
https://www.slideshare.net/paolodagaojes/9-math-lm-u1m2v10
https://www.slideshare.net/dionesioable/module-2-quadratic-functions
https://sciencing.com/everyday-examples-situations-apply-quadratic-equations-10200.html

Illustrations:
https://www.freepik.com/free-vector/woman-with-long-hair-teaching-online_7707557.htm
https://www.freepik.com/free-vector/kids-having-online-lessons_7560046.htm

ADDITIONAL ACTIVITIES

For $h(t) = -16t^2 + 32t + 3$
or $h(t) = -16(t - 1)^2 + 19$

Vertex: (1, 19)

Opening of the graph: Downward

Axis of symmetry: x = 1

Domain: $\{x/\ x \in \mathbb{R}\}$

Range: $\{y/y \leq 19\}$

y – intercept: (0, 3)

For $h(t) = -16t^2 + 16t + 4$
or $h(t) = -16(t - 1)^2 + 8$

Vertex: $\left(\frac{1}{2}, 8\right)$

Opening of the graph: Downward

Axis of symmetry: x = 1/2

Domain: $\{x/\ x \in \mathbb{R}\}$

Range: $\{y/y \leq 8\}$

y – intercept: (0, 4)

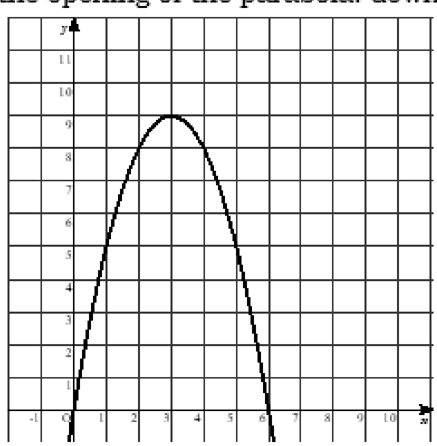
PROBLEM – BASED WORKSHEET

1.

width (w)	1	2	3	4	5	w
length	5	4	3	2	1	6 - w
Area	5	8	9	8	5	w(6 – w)

2. $A(w) = -w^2 - 6w$

3. Graph the function A(w). Label each axis.
4. Determine the following:
   a. Domain: $x \in \mathbb{R}$
   b. Range: $y \leq 9$
   c. Vertex (3, 9)
   d. Axis of symmetry: x = 3
   e. Direction of the opening of the parabola: downward



5. The maximum area of the frame that Jeanna can design is 9 square inches with dimensions 3 inches by 3 inches.

5. $y = -3(x + 3)^2 + 26$

   Vertex: (-3, 26)

   Opening of the graph: Downward

   Axis of symmetry: x = -3

   Domain: $\{x / x \in \mathbb{R}\}$

   Range: $\{y / y \leq 26\}$

   y – intercept: (0, -1)

6. $Y = -3(x + 1)^2 + 7$

   Vertex: (-1, 7)

   Opening of the graph: Downward

   Axis of symmetry: x = -1

   Domain: $\{x / x \in \mathbb{R}\}$

   Range: $\{y / y \leq 7\}$

   y – intercept: (0, 4)

7. $y = 2(x - 3)^2 + 27$

   Vertex: (3, 27)

   Opening of the graph: Upward

   Axis of symmetry: x = 3

   Domain: $\{x / x \in \mathbb{R}\}$

   Range: $\{y / y \geq 27\}$

   y – intercept: (0, 9)

8. $y = 2(x - 4)^2 + 1$

   Vertex: (-4, 1)

   Opening of the graph: Upward

   Axis of symmetry: x = -4

   Domain: $\{x / x \in \mathbb{R}\}$

   Range: $\{y / y \geq 1\}$

   y – intercept: (0, 33)

9. $y = 3(x - 1)^2 + 2$

   Vertex: (1, 2)

   Opening of the graph: Upward

   Axis of symmetry: x = 1

   Domain: $\{x / x \in \mathbb{R}\}$

   Range: $\{y / y \geq 2\}$

   y – intercept: (0, 5)

10. $y = 3(x + 2)^2 - 8$

   Vertex: (-2, -8)

   Opening of the graph: Upward

   Axis of symmetry: x = -2

   Domain: $\{x / x \in \mathbb{R}\}$

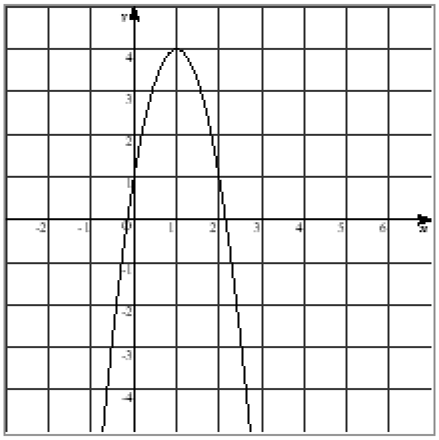
   Range: $\{y / y \geq -8\}$

   y – intercept: (0, 4)

ASSESSMENT

1. C
2. D
3. A
4. B
5. D
6. A
7. C
8. D
9. D
10. C

4. $y - 6x - 1 = -3x^2$ or $y = -3(x-1)^2 + 4$



Vertex: (1, 4)

Opening of the graph: Downward

Vertex is a <u>maximum</u> point

Axis of symmetry: x = 1

Domain: $\{x / x \in \mathbb{R}\}$

Range: $\{y/y \leq 4\}$

WHAT I CAN DO

1. $y = (x + 2)^2 - 3$

Vertex: (-2, -3)

Opening of the graph: Upward

Axis of symmetry: x = -2

Domain: $\{x / x \in \mathbb{R}\}$

Range: $\{y/y \geq -3\}$

y – intercept: (0, 1)

2. $y = -(x - 3)^2 + 2$

Vertex: (3, 2)

Opening of the graph: Downward

Axis of symmetry: x = 3

Domain: $\{x / x \in \mathbb{R}\}$

Range: $\{y/y \leq 2\}$

y – intercept: (0, -7)

3. $y = (x - 2)^2 - 5$

Vertex: (2, -5)

Opening of the graph: Upward

Axis of symmetry: x = 2

Domain: $\{x / x \in \mathbb{R}\}$

Range: $\{y/y \geq -5\}$

y – intercept: (0, -1)

4. $y = -(x-1)^2 + 2$

Vertex: (1, 2)

Opening of the graph: Downward

Axis of symmetry: x = 1

Domain: $\{x / x \in \mathbb{R}\}$

Range: $\{y/y \leq 2\}$

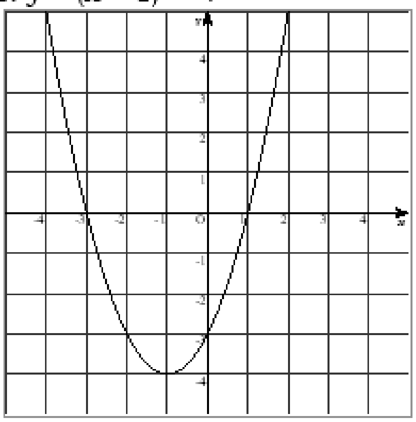
y – intercept: (0, 1)

WHAT'S NEW

1. downward
2. (1, 16)
3. x = 1
4. (0,0) and (2,1)
5. (0, 0)
6. set of Real numbers or $\{x/ \ x \in \mathbb{R}\}$
7. $\{y/y \leq 16\}$

WHAT'S MORE

1. $y = (x + 1)^2 - 4$



Vertex: (-1, -4)

Opening of the graph: Upward

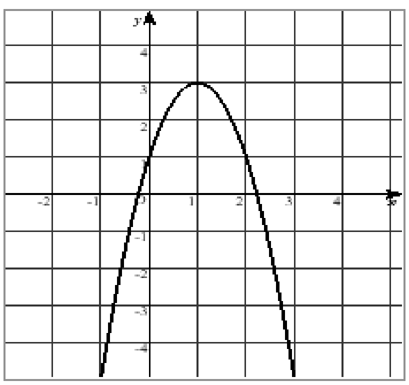
Vertex is a <u>minimum</u> point

Axis of symmetry: x = -1

Domain: $\{x/ \ x \in \mathbb{R}\}$

Range: $\{y/y \geq -4\}$

2. $y = -(2x^2 - 4x - 1)$ or $y = -2(x - 1)^2 + 3$



Vertex: (1, 3)

Opening of the graph: Downward

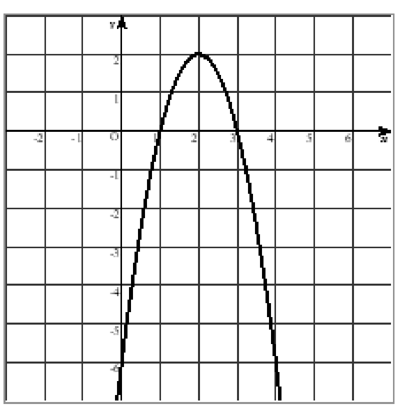
Vertex is a <u>maximum</u> point

Axis of symmetry: x = 2

Domain: $\{x/ \ x \in \mathbb{R}\}$

Range: $\{y/y \leq 3\}$

3. $y + 2x^2 = 8x - 6$ or $y = -2(x - 2)^2 + 2$



Vertex: (2, 2)

Opening of the graph: Downward

Vertex is a <u>maximum</u> point

Axis of symmetry: x = 2

Domain: $\{x/ \ x \in \mathbb{R}\}$

Range: $\{y/y \leq 2\}$

ANSWER KEY

WHAT I KNOW

1. C	6. C	11. C
2. C	7. D	12. A
3. D	8. D	13. C
4. D	9. C	14. B
5. D	10. A	15. A

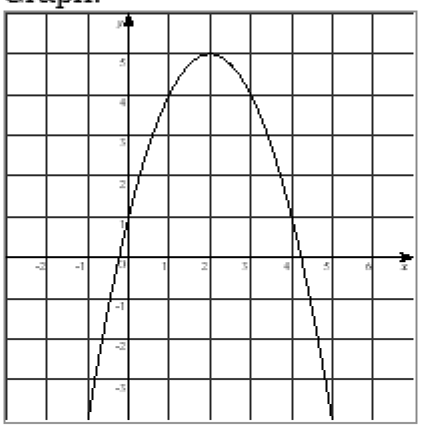
WHAT'S IN

1. $y = -(x - 2)^2 + 5$

Vertex: (2, 5)

Axis of symmetry: x = 2

Graph:

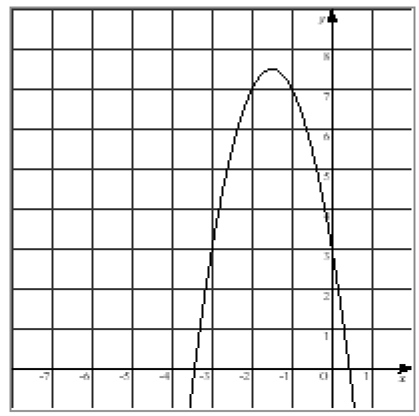


2. $y = -2\left(x + \frac{3}{2}\right)^2 + \frac{15}{2}$

Vertex: $\left(-\frac{3}{2}, \frac{15}{2}\right)$

Axis of symmetry: $x = -\frac{3}{2}$

Graph:

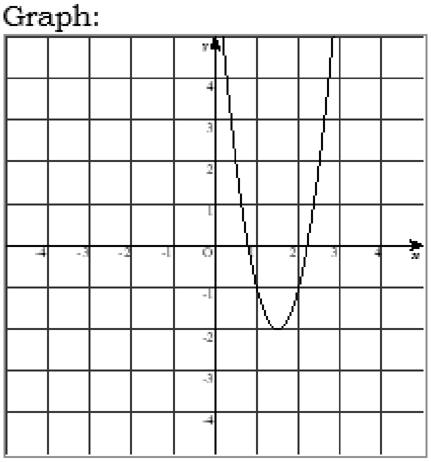


3. $y = 4\left(x - \frac{3}{2}\right)^2 - 2$

Vertex: $\left(\frac{3}{2}, -2\right)$

Axis of symmetry: $x = \frac{3}{2}$

Graph:



4. $y = -\left(x - \frac{3}{2}\right)^2 + \frac{17}{4}$

Vertex: $\left(\frac{3}{2}, \frac{17}{4}\right)$

Axis of symmetry: $x = \frac{3}{2}$

Graph: