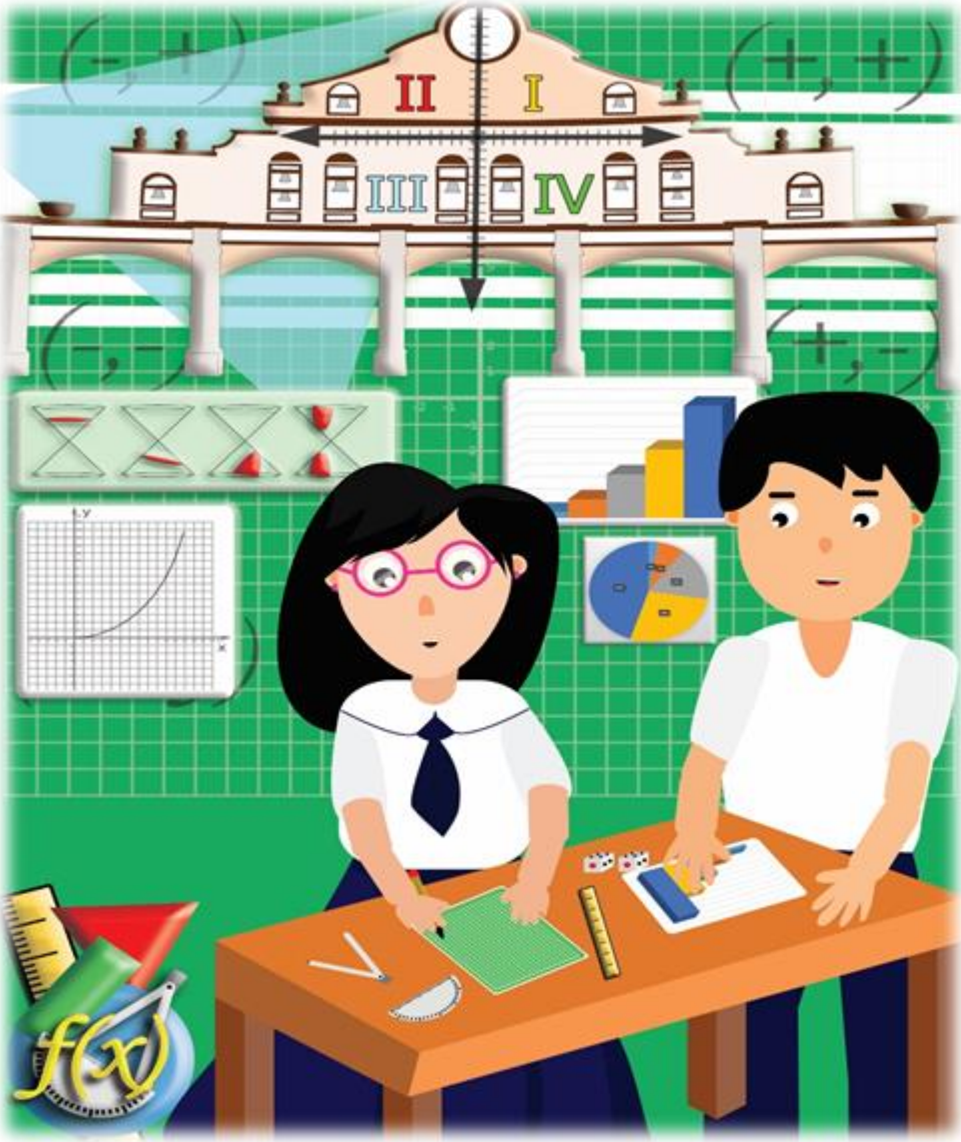
Senior High School

Department of Education
National Capital Region
SCHOOLS DIVISION OFFICE
MARIKINA CITY

GENERAL MATHEMATICS

Quarter 2: Module 8

Validity and Falsity of Real-Life Arguments



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City of Good Character
DISCIPLINE • GOOD TASTE • EXCELLENCE

What I Need to Know

Hello Grade 11 learners! In this module, you will learn how to:

Illustrate different types of tautologies and fallacies **M11GM-IIi-1**;

Determine the validity of categorical syllogisms **M11GM-IIi-2;** and,

Establish the validity and falsity of real-life arguments using logical propositions and fallacies **M11GM-IIi-3**.

This module is divided into three (3) lessons:

Lesson 1: Tautology and Contradiction
Lesson 2: Validity of Categorical Syllogism
Lesson 3: Validity and Falsity of Real-Life Arguments

You can say that you have understood the lesson in this module if you can already:
1. differentiate tautology from contradiction;
2. determine the validity of categorical syllogisms; and
3. establish the validity and falsity of real-life arguments using logical propositions and fallacies.

What I Know

Choose the letter that corresponds to the exact answer. Write your answer on another sheet of paper.

1. A proposition that is always TRUE.
   A. Contradiction C. Contingency
   B. Tautology D. Cannot be determined
2. "You will graduate this year and you will not graduate this year." The statement is an example of a ______________.
   A. Contradiction C. Contingency
   B. Tautology D. Cannot be determined/undefined
3. A proposition that is always FALSE is called _________..
   A. Contradiction C. Contingency
   B. Tautology D. Cannot be determined
4. The statement *You are sleeping, or you are not sleeping* is a kind of __?
   A. Contradiction C. Contingency
   B. Tautology D. Cannot be determined

For numbers 5-7, use the following choices to test the validity of the given categorical syllogism.

A. Valid	B. Invalid	C. Legal	D. Illegal

5. Not all boiling water is hot.
   Chili sauce is hot.
   Therefore, chili sauce is not a boiling water.

6. All cats like to sleep.
   Some domesticated animals do not like to sleep.
   Therefore, some domesticated animals are not cats.

7. Some beauty queens are tall.
   Ms. Pia Wurtzbach is a beauty queen
   Therefore, Ms. Pia Wurtzbach is tall.

For numbers 8-10, use the following choices to test the validity of the following arguments.

A. Valid	B. Fallacy	C. Legal	D. Illegal

8. If pigs die, then the African Swine Fever is not yet over. Pigs die. Thus, African Swine fever is not yet over.
9. If you want to be an engineer, then you must take the STEM strand. You do not want to be an engineer. Thus, you must not take the STEM strand.
10. If you play in the rain, you will get sick. You did not get sick. Therefore, you did not play in the rain.

LESSON 1: Tautology and Contradiction

What's In

Complete the truth table for the conjunction and disjunction of the propositions p and q. Answer the questions that follows.

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	
p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

1. When is the conjunction of the propositions p and q true?
2. When is the disjunction of the propositions p and q false?

What's New

Consider the following statements:

- ❖ Chona will pass the General Mathematics Class this year or Chona will not pass the General Mathematics Class this year.
- ❖ Chona will pass the General Mathematics Class this year and Chona will not pass the General Mathematics Class this year.

1. Change the following statement in symbols if we let:
   p: Chona will pass the General Mathematics class this year.
2. Which statement do you think is TRUE? Why?
3. Which statement do you think is FALSE? Why?

What is It

The first sentence *Chona will pass the General Mathematics class this year or Chona will not pass the General Mathematics class this year* in What's New can be transformed in symbols as (p v ~p) whose truth table is:

p	~p	(p v ~p)
T	F	T
F	T	T

While the second sentence *Chona will pass the General Mathematics class this year and Chona will not pass the General Mathematics class this year* can be transformed in symbols as (p ^ ~p) whose truth table is:

p	~p	(p v ~p)
T	F	F
F	T	F

Based on the truth table, the first statement is a **Tautology** and the second statement is a **Contradiction**.

Definition:

A proposition that is always true is called a **tautology**, while a proposition that is always false is called a **contradiction.**

Examples:

Determine if the following propositions is a tautology, a contradiction or neither using a truth table.

1. $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
2. $(p \vee q) \wedge ((\sim p) \wedge (\sim q))$
3. $\sim ((r \wedge p) \vee \sim q)) \leftrightarrow (p \rightarrow q)$

Solution:

1. $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r))$	$p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

The proposition $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ is a tautology.

2. $(p \vee q) \wedge ((\sim p) \wedge (\sim q))$

p	q	$p \vee q$	$\sim p$	$\sim q$	$(\sim p) \wedge (\sim q)$	$(p \vee q) \wedge ((\sim p) \wedge (\sim q))$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	F

The proposition $(p \vee q) \wedge ((\sim p) \wedge (\sim q))$ is a contradiction.

3. $\sim ((r \wedge p) \vee \sim q)) \leftrightarrow (p \rightarrow q)$

p	q	r	$r \wedge p$	$\sim q$	$(r \wedge p) \vee \sim q$	$\sim((r \wedge p) \vee \sim q)$	$p \rightarrow q$	$\sim ((r \wedge p) \vee \sim q)) \leftrightarrow (p \rightarrow q)$
T	T	T	T	F	T	F	T	F
T	T	F	F	F	F	T	T	T
T	F	T	T	T	T	F	F	T
T	F	F	F	T	T	F	F	T
F	T	T	F	F	F	T	T	T
F	T	F	F	F	F	T	T	T
F	F	T	F	T	T	F	T	F
F	F	F	F	T	T	F	T	F

The proposition $\sim ((r \wedge p) \vee \sim q)) \leftrightarrow (p \rightarrow q)$ is neither a tautology nor a contradiction.

What's More

Complete the truth table of the proposition ((p → r) ^ (q → r)) → ((p v q) → r) with the correct truth values of the proposition and determine whether it is a tautology, a contradiction or neither.

p	q	r	p → r	q → r	(p → r) ^ (q → r)	p v q	(p v q) → r	((p → r) ^ (q → r)) → ((p v q) → r)
T	T	T	T	T		T		
T	T	F	F	F		T		
T	F	T	T	T		T		
T	F	F	F	T		T		
F	T	T	T	T		T		
F	T	F	T	T		T		
F	F	T	T	T		F		
F	F	F	T	T		F		

What I Have Learned

Write the term/expression that will complete the statements.

1. A proposition that is always true is called a ____________________.
2. A proposition that is always false is called a ____________________.

What I Can Do

Construct a truth table for the proposition **((p v q) ^ r)** → **((p ^ r) v (q ^ r))** and determine whether it is a tautology, a contradiction or neither.

p	q	r	p v q	(p v q) ^ r	p ^ r	q ^ r	(p ^ r) v (q ^ r)	((p v q) ^ r) → ((p ^ r) v (q ^ r))
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

Rubrics for Scoring

Score	Description
15 points	100% complete solutions and correct answers are present in the submitted output.
10 points	75% of the solutions are correct but answer is incorrect.
5 points	50% 75% of the solutions are correct but answer is incorrect.
No point earned	No output was submitted.

Assessment

Complete the truth table by writing the correct truth values for each column for the proposition $\mathbf{(\sim(p \rightarrow q) \wedge (\sim p)) \leftrightarrow (q \rightarrow r)}$ and determine whether it is a tautology, a contradiction or neither.

p	q	r	p → q	~ (p → q)	~p	~ (p → q) ^ (~ p)	q → r	(~(p → q) ^ (~p)) ↔ (q → r)
T	T	T	T		F		T	
T	T	F	T		F		F	
T	F	T	F		F		T	
T	F	F	F		F		T	
F	T	T	T		T		T	
F	T	F	T		T		F	
F	F	T	T		T		T	
F	F	F	T		T		T	

Additional Activities

Construct a truth table for the proposition: $\mathbf{((p \vee r) \rightarrow (q \vee r)) \leftrightarrow ((p \vee q) \wedge \sim r)}$ and determine whether it is a tautology, a contradiction or neither.

p	q	r	p v r	q v r	(p v r) → (q v r)	p v q	~r	(p v q) ^ ~r)	((p v r) → (q v r)) ↔ ((p v q) ^ ~r)
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

LESSON 2: Validity of Categorical Syllogism

What's In

Complete the table by writing the correct truth values for the proposition **$((p \rightarrow q) \wedge (\sim q)) \rightarrow \sim p$** and determine whether it is a tautology, a contradiction or neither.

p	q	$p \rightarrow q$	$\sim q$	$(p \rightarrow q) \wedge \sim q$	$\sim p$	$((p \rightarrow q) \wedge (\sim q)) \rightarrow \sim p$
T	T	T	F		F	
T	F	F	T		F	
F	T	T	F		T	
F	F	T	T		T	

1. When is a proposition a tautology?
2. When is a proposition a contradiction?

What's New

Consider the statement below and answer the questions that follow.

Stars are huge celestial bodies.
Liza Soberano is a star.
Therefore, Liza Soberano is a huge celestial body.

1. What is the meaning of the word "star" in the first sentence?
2. What is the meaning of the word "star" in the second sentence?
3. Do you agree in the conclusion that Liza Soberano is a huge celestial body?
4. Do you consider the statement a valid statement?

What is It

The statement in What's New is an example of a categorical syllogism.

Definition:

Syllogism: A deductive argument composed of exactly two premises and one conclusion.

Categorical Syllogism: A deductive argument composed of three categorical propositions/statements, one of which serves as the conclusion of the argument and the other two of which serve as the major and minor premises, respectively.

Major term: It is the predicate term of the conclusion.

Minor term: It is the subject term of the conclusion.

Middle term: It is the repeated term in the premises which is not in the conclusion.

Major Premise: It is the premise containing the major term.

Minor Premise: It is the premise containing the minor term.

Fallacy: It is an invalid syllogism.

Note:

- ❖ The predicate of an affirmative proposition is particular.
- ❖ The predicate of a negative proposition is universal.
- ❖ The statement of the form "All are" indicates a positive universal statement.
- ❖ The statement of the form "No, none or Not" indicates a negative universal statement.
- ❖ The statement of the form "some are" or "some are not" is called an existential statement.

1. The categorical syllogism must have only three terms which are the major term, minor term and the middle term. Each term is used consistently (same meaning) throughout the propositions.
2. The premises (major and minor) cannot be both negative.
3. The conclusion must be negative when one of the premises (either the major premise or the minor premise) is negative.
4. The major and the minor terms should only be universal in the conclusion if they are universal in the premises, which means that the form of the predicate term (universal/particular) in the conclusion must agree to the form of the major term (universal/particular) in the major premise.

Same rule applies to the form of the subject (universal/particular) in the conclusion and it must also agree to the form of the minor term (universal/particular) found in the minor premise.

5. One of the middle terms which is found either in the major premise or the minor premise must be universal.
6. If the premises are affirmative, then the conclusion must be affirmative.
7. If one of the premises is negative (indicated by the words no, none, or not), then the conclusion must be negative.
8. If one premise is particular, then the conclusion must be particular.

If the statement commits at least one of the rules, then the statement is invalid or a fallacy.

Examples:

Test the validity of the following categorical syllogism.

1. All cows give milk.
   Some farm animals do not give milk.
   Therefore, some farm animals are not cows.
   Answer: It is a valid statement because none of the rules were violated.

2. Stars are huge celestial bodies.
   Liza Soberano is a star,
   Therefore, Liza Soberano is a huge celestial body.
   Answer: It is an invalid statement because it violates the second rule.
   Although the statement contains only three terms which are star, huge celestial body and Liza Soberano, the word star was used into two different contexts/meanings which makes the statement to contain 4 terms.

3. No Narra trees bear fruits.
   No Yakal trees bear fruits.
   No Yakal trees are Narra trees.
   Answer: It is an invalid statement because it violates rule number 2. The premises cannot be both negative.

4. Filipinos are hospitable.
   Sam is not a Filipino.
   Therefore, Sam is hospitable.
   Answer: It is an invalid statement because it violates rule number 7 which states that if one of the premises is negative, then the conclusion must be negative.

5. All rich people are hardworking.
   Some farmers are hardworking.
   Therefore, all farmers are rich people.

Answer: It is an invalid statement because it violates rule number 8 which states that if one of the premises is particular, the conclusion must also be particular.

6. All superheroes are brave.
   But no villain is a superhero.
   Therefore, no villain is brave.
   Answer: It is an invalid statement because it violates rule number 4 and rule number 5. It violates rule number 5 which states that one of the middle terms is universal wherein from the given statement the middle terms “superhero” are both universal. Moreover, it also violates rule 4 because the predicate (brave) in the conclusion is universal and does not agree to the form of the major term (brave) in the major premise which is particular.

7. All comedians are funny.
   Vice Ganda is a comedian.
   Therefore, Vice Ganda is funny.
   Answer: This is a valid statement because none of the rules were violated.

What's More

Determine whether the following categorical syllogisms is valid or invalid.

1. All cats have whiskers.
   Some cats are carnivores.
   Therefore, some carnivores have whiskers.

2. Some mammals are warm blooded animals.
   All whales are mammals.
   Thus, some whales are warm blooded animals.

3. All dogs can swim.
   Some pets are dogs.
   Thus, some pets cannot swim.

4. Some students sing.
   Some students dance.
   Therefore, some dancers sing.

5. Some rational numbers are integers.
   All integers are real numbers.
   Therefore, all real numbers are rational numbers.

6. All successful businessmen are investors.
   Some politicians are investors.
   Thus, some politicians are successful businessmen.

7. All ballet dancers are graceful.
   Some gymnasts are ballet dancers.
   Therefore, some gymnasts are graceful.

8. All dogs are loyal to their owners.
   Some four-legged animals are dogs.
   Therefore, some four-legged animals are loyal to their owners.

9. Some superheroes have superpowers.
   No villains are superheroes.
   Therefore, some villains have superpowers.

10. Some snakes crawl.
    All snakes are venomous.
    Therefore, some things that crawl are venomous.

What I Have Learned

Write the term/expression that will complete the statements.

1. The predicate term of the conclusion is called _________________.
2. The subject term of the conclusion is called __________________.
3. The premise containing the major term is __________________.
4. The premise containing the minor term is ___________________.
5. An invalid syllogism/argument is called ____________________.

What I Can Do

Test the validity of the following categorical syllogism. If the syllogism is a fallacy, give the rule that it violates that makes the syllogism invalid.

1. Some monkeys are not playful.
   All monkeys are mammals.
   Thus, some mammals are playful.

2. All wrestlers are fearless.
   Some boxers are not fearless.
   Therefore, some boxers are wrestlers.

3. Some numbers are divisible by 2.
All even numbers are divisible by 2.
Therefore, all numbers are even numbers.

4. All soldiers have guns.
Some soldiers are marines.
Hence, some marines have guns.

5. All soldiers are not forest rangers.
Some agents are not soldiers.
Therefore, some agents are not forest rangers.

Rubrics for Scoring

Score	Description
15 points	Output shows complete solutions with correct answers.
10 points	Output submitted has 75% correct solutions with incorrect answer.
5 points	Output submitted has 50% correct solution with incorrect answer.
No point earned	No output at all was submitted.

Assessment

Refer to the choices below to test the validity of each argument. Write the letter of the correct answer on a separate sheet of paper.

A. Valid	B. Fallacy	C. Contingency	D. Cannot be determined

1. Some seashells are tasty.
All seashells cannot swim.
Therefore, some tasty things cannot swim.

2. Some doctors are not neurologists.
Some doctors are not cardiologists.
Therefore, some cardiologists are not neurologists.

3. All Filipinos are people.
Juan Dela Cruz is a person.
Therefore, Juan Dela Cruz is a Filipino.

4. All saints are holy.
   Some Filipinos are holy.
   Therefore, some Filipinos are saints.

5. Some dancers are not singers.
   All singers are celebrities.
   Some celebrities are dancers.

Additional Activities

Test the validity of the following categorical syllogism. If the syllogism is a fallacy, give the rule that it violates that makes the syllogism invalid.

1. Some dogs are friendly.
   Some dogs are huskies.
   Therefore, some huskies are friendly.
2. Some students are not good in English.
   All students are not good in Math.
   Thus, some who are not good in Math are not good in English.
3. All senators are politicians.
   Some lawmakers are not politicians.
   Therefore, some lawmakers are not senators.
4. All motorcycle riders wear helmets.
   Ding does not wear a helmet.
   Hence, Ding is not a motorcycle rider.
5. All rational numbers are real numbers.
   $\sqrt{2}$ is not a rational number.
   Therefore, $\sqrt{2}$ is a real number.

Lesson 3: Validity and Falsity of Real-Life Arguments

What's In

Write the following statements in symbolic form, given the simple propositions:
p: I am a Marikeño.
q: I am a Mabuting Tao.

1. If I am a Marikeño, then I am a Mabuting Tao.
2. I am a Marikeño or I am a Mabuting Tao.
3. If I am not a Mabuting Tao, then I am not a Marikeño.
4. I am a Marikeño and I am a Mabuting Tao.
5. If I am not a Marikeño, then I am not a Mabuting Tao.

What's New

Your friend says:

If it is raining, then online classes are suspended.
Online classes are not suspended.
Therefore, it is not raining.

1. Do you agree with your friend's opinion?
2. Is it a valid argument?

What is It

To test the validity of an argument, we can either use a truth table or we can apply some common rules of inference and the table of some known fallacies.

Definition:

An **argument** is a compound proposition of the form,

$(p_1, p_2, p_3, \ldots, p_n) \rightarrow q$, where $p_1, p_2, p_3, \ldots, p_n$ are the premises, and q is the conclusion.

Definition:

An **argument** can be written in propositional form as,
$(p_1, p_2, p_3, \ldots, p_n) \rightarrow q$, or it can be written in column or standard form.

$$
\begin{array}{l}
p_1 \\
p_2 \\
p_3 \\
. \\
. \\
. \\
p_n \\
\therefore q
\end{array}
$$

A **valid argument** satisfies the validity condition, that is, the conclusion q is true whenever the premises $p_1, p_2, p_3, \ldots, p_n$ are all true.
By the truth table, for a valid argument, the conditional,
$(p_1 \wedge p_2 \wedge p_3 \wedge \ldots \wedge p_n) \rightarrow q$ is a tautology.
An argument $(p_1 \wedge p_2 \wedge p_3 \wedge \ldots \wedge p_n) \rightarrow q$, which is not valid is called a **fallacy.**
In a fallacy, it is possible for the premises $p_1, p_2, p_3, \ldots, p_n$ to be true, while the conclusion is false.
Moreover, by the truth table, it is a fallacy when the conditional $(p_1 \wedge p_2 \wedge p_3 \wedge \ldots \wedge p_n) \rightarrow q$ is not a tautology.

The table on Rules of Inference display some basic forms of valid arguments in logic which can be used to test the validity of an argument.

Rules of Inference

Let p, q, r be propositions.

	Propositional Form	Standard Form
Modus Ponens	$((p \rightarrow q) \wedge p) \rightarrow q$	$p \rightarrow q$ $\underline{p}$ $\therefore q$
Modus Tolens	$((p \rightarrow q) \wedge (\sim q)) \rightarrow \sim p$	$p \rightarrow q$ $\underline{\sim q}$ $\therefore \sim p$
Transitive Reasoning or Hypothetical Syllogism	$((p \rightarrow q) \wedge (q \rightarrow r))$ $\rightarrow (p \rightarrow r)$	$p \rightarrow q$ $\underline{q \rightarrow r}$ $\therefore p \rightarrow r$ $\sim r \rightarrow \sim p$
Disjunctive Syllogism	$((p \vee q) \wedge (\sim p)) \rightarrow q$ or $((p \vee q) \wedge (\sim q)) \rightarrow p$	$p \vee q$ $\underline{\sim p}$ $\therefore q$ $p \vee q$ $\underline{\sim q}$ $\therefore p$

On the other hand, the Table of Fallacies can be used to test that an argument is invalid or a fallacy.

Table of Fallacies

Let p, q, and r be propositions.

	Propositional Form	Standard Form
Fallacy of the Converse	$((p \rightarrow q) \wedge q) \rightarrow p$	$p \rightarrow q$ $\underline{q}$ $\therefore p$
Fallacy of the Inverse	$((p \rightarrow q) \wedge (\sim p)) \rightarrow \sim q$	$p \rightarrow q$ $\underline{\sim p}$ $\therefore \sim q$
Misuse of Disjunctive Reasoning	$((p \vee q) \wedge (p)) \rightarrow \sim q$ or $((p \vee q) \wedge (q)) \rightarrow \sim p$	$p \vee q$ $\underline{p}$ $\therefore \sim q$ $p \vee q$ $\underline{q}$ $\therefore \sim p$
Misuse of Transitive Reasoning	$((p \rightarrow q) \wedge (q \rightarrow r))$ $\rightarrow (p \rightarrow r)$	$p \rightarrow q$ $\underline{q \rightarrow r}$ $\therefore r \rightarrow p$ $\sim p \rightarrow \sim r$

The argument in What's New:
If it is raining, then online classes are suspended.
Online classes are not suspended.
Therefore, it is not raining.

This argument can be stated into two simple propositions as:
p: It is raining.
q: Online classes are suspended.
In symbolic form:
$((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$

- To determine the validity or falsity of this argument by the truth table:

p	q	$p \rightarrow q$	$\sim q$	$(p \rightarrow q) \wedge \sim q$	$\sim p$	$((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

The argument is valid because the truth table is a tautology.
The statement $((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$ is valid by *modus tolens.*

Examples:
Decide if the following arguments are valid or a fallacy either by the truth table or by the rules of inference and by the table of fallacies.

1. If I read my module carefully, I will understand how to do the exercises. I understand how to do the exercises. Therefore, I read my module carefully.

Solution:
Let
p: I read my module carefully.
q: I will understand how to do the exercises.
In symbolic form:
$((p \rightarrow q) \wedge q) \rightarrow p$
By the truth table:

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$((p \rightarrow q) \wedge q) \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

The statement is invalid since the truth table is not a tautology.
The statement is invalid by the fallacy of the converse.

2. If you like dogs, then you love animals. If you love animals, then you like cats. Therefore, If you like cats, then you like dogs.

Solution:

Let

p: You like dogs.

q: You love animals.

r: You like cats.

In symbolic form:

$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (r \rightarrow p)$

To test the validity by the truth table:

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$r \rightarrow p$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (r \rightarrow p)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	F	T	T
T	F	F	F	F	F	T	T
F	T	T	T	T	T	F	F
F	T	F	T	F	F	T	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

- ❖ The argument is invalid or a fallacy since the truth table is not a tautology.
- ❖ The statement is invalid by the misuse of transitive reasoning.

3. If you like dogs, then you love animals. If you don't like cats, then you don't love animals. Therefore, if you like dogs then you like cats.

Solution:

Let,
p: You like dogs.
q: You love animals.
r: You like cats.
In symbolic form:
If you like dogs, then you love animals: $p \rightarrow q$.
If you don't like cats, then you don't love animals: $\sim r \rightarrow \sim q$ which is logically equivalent to $q \rightarrow r$.
If you like dogs, then you like cats: $p \rightarrow r$
The argument in symbolic form is:
$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
To test the validity of the argument by the truth table:

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

- ❖ The statement is valid since the truth table is a tautology.
- ❖ The statement is valid by transitive reasoning or hypothetical syllogism.

What's More

Test the validity of the following arguments by the truth table.

1. If my parents go to the mall, they will buy me a new cellphone. My parents go to the mall. Therefore, they will buy me a new cellphone.

p	q				
T	T				
T	F				
F	T				
F	F				

2. If you like dogs, then you love animals. If you love animals, then you like cats. Therefore, if you don't like cats, then you don't like dogs.

p	q	r							
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

3. If Juan plays online games all day, then Juan will not do his module. Juan did not play online games all day. Therefore, Juan will do his module.

p	q				
T	T				
T	F				
F	T				
F	F				

4. If the movie is boring, the moviegoers will fall asleep. The moviegoers fell asleep. Therefore, the movie is boring.

p	q				
T	T				
T	F				
F	T				
F	F				

What I Have Learned

Write the term/expression that will complete the statements.

1. An argument is valid if the truth table results to a ____________________.
2. An invalid argument is called a ______________________________.
3. A truth table is a tautology if the whole proposition contains all _____.

What I Can Do

Test the validity of the following arguments. State the rule of inference or the fallacy used.

1. If we plant trees, then there is a sufficient supply of oxygen in the air. There is a sufficient supply of oxygen in the air. Therefore, we plant trees.

2. If I do not water the plants regularly, then the plants will die. The plants died. Thus, I did not water the plants regularly.

3. If it is raining, then internet signal is slow. If the internet signal is slow, then online classes are canceled. Therefore, if online classes are not canceled, then it is not raining.

4. If I do not obey my parents, then my parents get mad at me. My parents get mad at me. Hence, I do not obey my parents.

5. If you are not good in Math, then you are not good in Science. You are good in Science. Therefore, you are good in Math.

Rubrics for Scoring

Score	Description
15 points	Output shows complete solutions with correct answers.
10 points	Output submitted has 75% correct solutions with incorrect answer.
5 points	Output submitted has 50% correct solution with incorrect answer.
No point earned	No output at all was submitted.

Assessment

From the four (4) choices below, test the validity of the following arguments. Write the letter of the correct answer on a clean sheet of paper.

A. Valid	B. Fallacy	C. Legal	D. Illegal

1. If you are kind, then you have many friends. You are not kind. Hence, you do not have many friends.

2. If you plant trees, then air pollution decreases. Air pollution does not decrease. Thus, you did not plant trees.

3. If I eat, then I wash the dishes. I don't eat. Therefore, I did not wash the dishes.

4. If you want to put up a bakeshop, then you should study Bread and Pastry Production. If you study bread and pastry production, then you must enroll in Malanday National High School. Therefore, If you don't want to put up a bakeshop, then you are not enrolled in Malanday National High School.

5. If a number is prime, then its factors are 1 and itself. The factors are not 1 and itself. Therefore, the number is not prime.

Additional Activities

Test the validity of the following arguments either by truth table or by rules of inference and fallacies as indicated in each item.

1. If a number is even, then it is divisible by 2. Five is not even. Therefore 5 is not divisible by 2. (Test the validity by the rule of inference or by fallacy.)

2. If I pass the interview, then I am hired. I am hired. Therefore, I passed the interview. (Test the validity by truth table.)

p	q			
T	T			
T	F			
F	T			
F	F			

3. If it is sunny, then we will go on a picnic. It is not sunny. Therefore, we will not go on a picnic. (Test the validity by the rule of inference or by fallacy.)

4. If you get your module and study your lessons, then you will pass the examination. (Test the validity by truth table.)

p	q	r		
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

5. If you are active, then you exercise every day. If you exercise every day, then you are healthy. Therefore, if you are not active, then you are not healthy. (Test the validity by the rule of inference or by fallacy.)

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Answer Key

Lesson 1:

What I Know

1. B	2. A	3. A	4. B	5. B
6. B	7. B	8. A	9. B	10 A

What's In

p	q	p ^ q
T	T	T
T	F	F
F	T	F
F	F	F

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

1. The conjunction is TRUE, if the propositions p and q are both TRUE.
2. The disjunction is FALSE, if the propositions p and q are both FALSE.

What's New

1. p v ~p
   p ^ ~p
2. Statement 1 is true because there is only one possibility that will happen at a given time in a given situation which is either Chona will pass or Chona will not pass the General Mathematics class.
3. Statement 2 is FALSE because it is impossible for both situations to happen at the same time.

What's More

p	q	r	p → r	q → r	(p → r) ^ (q → r)	p v q	(p v q) → r	((p → r) ^ (q → r)) → ((p v q) → r)
T	T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F	T
T	F	T	T	T	T	T	T	T
T	F	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	F	F
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	T

The proposition ((p → r) ^ (q → r)) → ((p v q) → r) is neither a tautology nor a contradiction.

What I Have Learned

1. Tautology
2. Contradiction

What I Can Do

p	q	r	p v q	(p v q) ^ r	p ^ r	q ^ r	(p ^ r) v (q ^ r)	((p v q) ^ r) → ((p ^ r) v (q ^ r))
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	F	T	T
T	F	F	T	F	F	F	F	T
F	T	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F	T
F	F	T	F	F	F	F	F	T
F	F	F	F	F	F	F	F	T

The proposition ((p v q) ^ r) → ((p ^ r) v (q ^ r)) is a tautology.

Assessment

p	q	r	p → q	~ (p → q)	~p	~ (p → q) ^ (~ p)	q → r	(~(p → q) ^ (~p)) ↔ (q → r)
T	T	T	T	F	F	F	T	F
T	T	F	T	F	F	F	F	T
T	F	T	F	T	F	F	T	F
T	F	F	F	T	F	F	T	F
F	T	T	T	F	T	F	T	F
F	T	F	T	F	T	F	F	T
F	F	T	T	F	T	F	T	F
F	F	F	T	F	T	F	T	F

The proposition (~(p → q) ^ (~p)) ↔ (q → r) is neither a tautology nor a contradiction.

Additional Activities

p	q	r	p v r	q v r	(p v r) → (q v r)	p v q	~r	(p v q) ^ ~r)	((p v r) → (q v r)) ↔ ((p v q) ^ ~r)
T	T	T	T	T	T	T	F	F	F
T	T	F	T	T	T	T	T	T	T
T	F	T	T	T	T	F	F	F	F
T	F	F	T	F	F	F	T	F	T
F	T	T	T	T	T	F	F	F	F
F	T	F	F	T	T	F	T	F	F
F	F	T	T	T	T	F	F	F	F
F	F	F	F	F	T	F	T	F	F

The proposition ((p v r) → (q v r)) ↔ ((p v q) ^ ~r) is neither a tautology nor a contradiction.

Lesson 2:

What's In

p	q	p → q	~ q	(p → q) ^ ~q	~p	((p → q) ^ (~ q)) → ~p
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

The proposition ((p → q) ^ (~ q)) → ~p is a tautology.

1. A proposition is a tautology if the truth values are all TRUE.
2. A proposition is a contradiction if the truth values are all FALSE.

What's New

1. Heavenly body
2. Actress/Celebrity
3. No, Liza Soberano is a person and not a heavenly body.
4. No, Liza Soberano is human and not a thing heavenly body)

What's More

1. Valid
2. Invalid/Fallacy
3. Invalid/Fallacy
4. Invalid/Fallacy
5. Invalid/Fallacy
6. Invalid/Fallacy
7. Valid
8. Valid
9. Invalid/Fallacy
10. Valid

What I Have Learned

1. Major term
2. Minor term
3. Major premise
4. Minor premise
5. Fallacy

What I Can Do

1. Fallacy, rule number 7
2. Fallacy, rule number 7
3. Fallacy, rule number 8
4. Valid
5. Fallacy, rule number 2

Assessment

1. A 2. B 3. B 4. B 5. B

Additional Activities

1. Invalid/Fallacy
2. Invalid/Fallacy
3. Valid
4. Valid
5. Invalid/Fallacy

Lesson 3

What's In

1. $p \to q$
2. $p \vee q$
3. $\sim p \to \sim q$
4. $p \wedge q$
5. $\sim p \to \sim q$

What's New

1. No or Yes
2. Yes

What's More

1. Valid

p	q	$p \to q$	$(p \to q) \wedge p$	$((p \to q) \wedge p) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

2. Valid

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	~r	~p	$\sim r \to \sim p$	$((p \to q) \wedge (q \to r)) \to (\sim r \to \sim p)$
T	T	T	T	T	T	F	F	T	T
T	T	F	T	F	F	T	F	F	T
T	F	T	F	T	F	F	F	T	T
T	F	F	F	T	F	T	F	F	T
F	T	T	T	T	T	F	T	T	T
F	T	F	T	F	F	T	T	T	T
F	F	T	T	T	T	F	T	T	T
F	F	F	T	T	T	T	T	T	T

3. Fallacy

p	q	$p \to q$	~p	$(p \to q) \wedge \sim p$	~q	$((p \to q) \wedge \sim p) \to \sim q$
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

4. Fallacy

p	q	$p \to q$	$(p \to q) \wedge q$	$((p \to q) \wedge q) \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

What I Have Learned

1. Tautology
2. Fallacy
3. True

What I Can Do

1. Fallacy
2. Fallacy
3. Valid
4. Fallacy
5. Valid

Assessment

1. B 2. A 3. B 4. B 5. A

Additional Activities

1. $((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$ Fallacy of the Inverse
2. Fallacy

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$((p \rightarrow q) \wedge q) \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

3. $((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$ Fallacy of the Inverse.
4. Fallacy

p	q	r	$(p \wedge q)$	$(p \wedge q) \rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

5. $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (\sim p \rightarrow \sim r)$ Fallacy by Misuse of Transitive Reasoning

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