Department of Education
National Capital Region
**SCHOOLS DIVISION OFFICE
MARIKINA CITY**

STATISTICS AND PROBABILITY

Quarter 3: Module 5
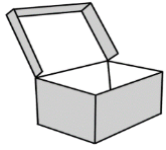## SAMPLING DISTRIBUTION
OF THE SAMPLE MEANS

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What I Need to Know

Hello Grade 11 learners! In this module, you will learn how to:

Illustrate random sampling **M11/12SP-IIId-2,**
Distinguish between parameter and statistic **M11/12SP-IIId-3** and
Identify sampling distributions of statistics (sample mean). **M11/12SP-IIId-4**.

You can say that you have understood the lesson in this module if you can already:
1. Explain random sampling,
2. Differentiate a parameter from statistic,
3. Identify sampling distributions of statistics (sample mean) and
4. Construct the sampling distribution of the sample means.

What I Know

Read each item carefully then choose the letter that corresponds to the correct answer.

1. It is the total number of elements from which the data under consideration shall be obtained.

    A. parameter   C. sample

    B. population   D. statistic

<u>For numbers 2 and 3, refer to the choices in item no. 1.</u>

2. It is used to describe the characteristic of a population.

3. It is used to label the characteristics of a population.

4. It is a process where each member of the population has an equal chance of being selected to form a sample.

    A. population parameter   C. sampling error

    B. random sampling   D. statistic

5. A population has the following scores 32, 21, 48, 22 and 26. Using sample size of 2, how many possible samples will be selected from the population?

A. 10   B. 9   C. 8   D. 7

For number 6 to 10, refer to item number 5 problem. Based on the data given, construct the sampling distribution of the sample means before answering item nos. 6 to 10.

6. What is the lowest sample mean in the sampling distribution of the sample means?

     A. 21   B. 21.33   C. 21.5   D. 21.67

7. What is the highest sample mean in the sampling distribution of the sample means?

     A. 40   B. 40.25   C. 40.5   D. 41

8. The sum of all the frequencies is _____.

     A. 6   B. 8   C. 10   D. 12

9. Which of the following statement is true on the constructed sampling distribution of the sample means?

     A. The sample mean with the highest $P(\bar{x})$ is 40.

     B. Only one sample mean got a $P(\bar{x})$ value which is higher than 0.1.

     C. Each sample mean got an equal value of $P(\bar{x})$.

     D. None of the above.

10. The mean of the population is 29.8. How many sample means is less than the population mean?

     A. 6   B. 5   C. 4   D. 3

What's In

SCENARIO: Imagine that you are the General Mathematics teacher of Concepcion Integrated School and you are handling 240 Grade 11 students. The school principal asked you to choose fifteen (15) students to represent a special distant learning seminar for a week.

Answer the following:

1. How are you going to choose 15 students from the entire Grade 11 fairly?

2. Can you cite two or more ways of choosing a group of 15 students without being biased? Explain.

3. Do you think it will be best if you will just decide who will be the 15 students that will represent the school?

What's New

On December 31, 2020, different news organizations heralded the result of the Pulse Asia survey on winnable presidential contenders for 2022 election.

According to Llanesca Panti of GMA News Online, the survey was conducted on 2,400 respondents from November 23 to December 2, 2020.

COMELEC data shows that there were 61, 843, 750 registered voters on May 2019 elections.

2, 400

RESPONDENTS

On behalf of the 61, 843, 750 registered voters.

Collecting data from a predefined group of respondents to gain information in various topics can be conducted through surveys. But choosing respondents should be done both scientifically and methodologically to secure the validity of the result. The respondents must be the representative of the group defined by the survey. In the case of the Pulse Asia result, is it possible that a group of not merely one percent (1%) of the total registered voting population may represent the views or opinions of the entire college of voters?
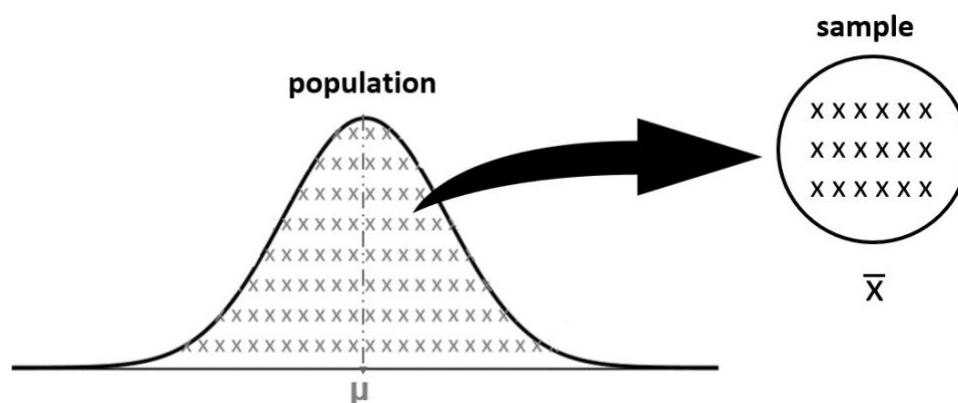
To understand this, we should learn the concepts of population and sample. How a sample will be a representative of a population?

What is It

RANDOM SAMPLING

Population is the total number of elements from which the data under consideration shall be obtained.

Random sampling is one of the simplest forms of collecting data from the population. It is a process where each member of the population has an equal chance of being selected to form a sample.

Let say for example, that the total number of the secondary teachers in Concepcion Integrated School is 120. The principal wanted to choose 30 teachers to answer a survey regarding their perceptions on the effects of the pandemic on the student's engagement on the learning process in the new normal and to conduct a survey, a sample group of 30 teachers is selected to do the survey. In this case, the population is the total number of employees in the company and the sample group of 30 employees is the sample. Each member of the workforce has an equal opportunity of being chosen because all the employees which were chosen to be part of the survey were selected randomly.

Knowing the nature of the population will help you decide which of its characteristics you wanted to consider or focus to have a suitable sample that will produce accurate data.
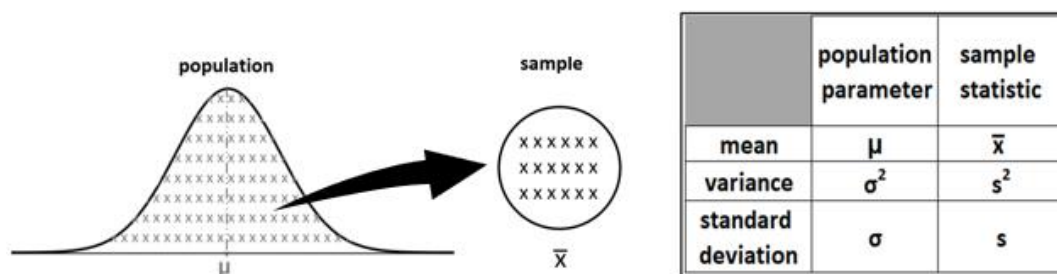
PARAMETER AND STATISTIC

A population and a sample are described by their respective characteristic. These are features that will help us to analyze or evaluate the data which they represent.

Parameter is used to describe the characteristic of a population.

Statistic is used to label the characteristics of a population.

A population parameter has an unbiased estimate from a sample statistic given that the sample is a representative of a population.
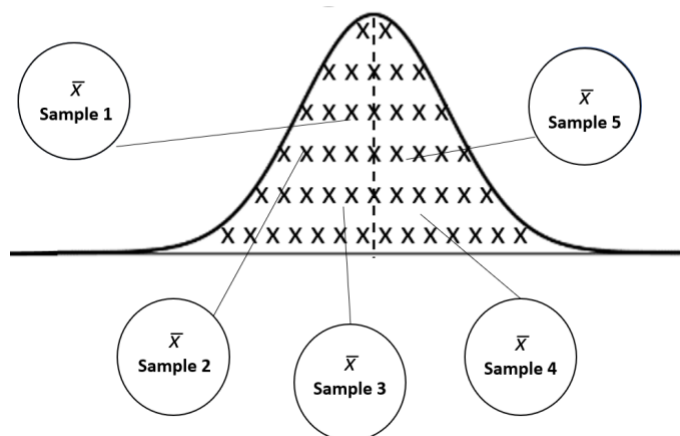
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	population parameter	sample statistic
mean	μ	$\bar{x}$
variance	σ^2	s^2
standard deviation	σ	s

The mean, variance and standard deviation of a population are parameters denoted by **μ**, **σ^2** and **σ**, respectively; while a sample are statistics denoted by $\bar{x}$, **s^2** and **s**, respectively.

IDENTIFYING SAMPLING DISTRIBUTIONS OF STATISTICS (SAMPLE MEAN)

The elements of the sample are chosen from the population by certain form of random process. This is the idea of random sampling. As mentioned previously, if the sample is a representative of the population, then the sample mean is an unbiased estimate of the population mean. But what if, by any chance, that the population mean is not known or may not be easy to determine?

Consider a population of size N, and samples with size n.

In this case, we will estimate a population mean by taking samples. These samples will yield sample means. It is expected to have different values of the means for every sample. Consequently, some sample means will be less than, equal to or greater than the population mean **μ**.

The sample means will form a frequency distribution. Hence, we can form probability distribution which is called the ***sampling distribution of the sample means***.

In this topic, it is important that we determine the number of possible samples which can be drawn from the population. These samples will be the composition of the sampling distribution of the sample means.

Example 1: The semestral grades of the top 5 Grade 11 students of Calumpang National High School in General Mathematics are as follows: 93, 94, 95, 97 and 98.

Samples of size 3 are drawn at random from this population. Determine the number of possible samples, list the possible samples from the population, construct the sampling distribution of the sample means and illustrate the sampling distribution of the sample means using a histogram.

Answer:

➢ Number of possible samples can be determine using the combination formula.

$$C(N,n) = \frac{N!}{n!(N-n)!}$$

Where:

$C(N,n)$ is the number of possible samples which can be drawn from the population

N is the population size

n is the sample size

Since there are 5 elements in the population, N = 5. Samples of size 3 will be drawn at random, n = 3. Using the combination formula,

$$C(5,3) = \frac{5!}{3!(5-3)!} = 10$$

therefore, there will be 10 possible samples from the population.

➢ List all possible samples of size 3 from the population (93, 94, 95, 97 and 98) and compute the mean of each sample.

SAMPLE			Mean	
93	94	95	94.00	← (93 + 94 + 95) / 3
93	94	97	94.67	← (93 + 94 + 97) / 3
93	94	98	95.00	
93	95	97	95.00	
93	95	98	95.33	
93	97	98	96.00	
94	95	97	95.33	
94	95	98	95.67	
94	97	98	96.33	
95	97	98	96.67	

10 possible samples of size 3

➢ Construct the sampling distribution of the sample means.

SAMPLING DISTRIBUTION OF SAMPLE MEANS

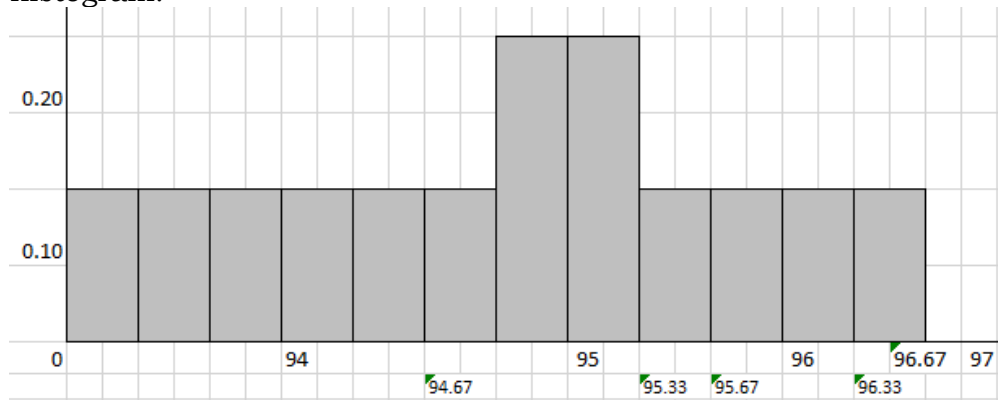| Sample Mean	Frequency	Probability P(x̄)
94.00	1	1/10 = 0.10
94.67	1	1/10 = 0.10
95.00	2	2/10 = 0.20
95.33	2	2/10 = 0.20
95.67	1	1/10 = 0.10
96.00	1	1/10 = 0.10
96.33	1	1/10 = 0.10
96.67	1	1/10 = 0.10
	10	1.00

In listing the sample means, the values will be listed only once. For example, 95.00 and 95.33 appeared twice, thus the occurrence will be reflected on the frequency.

➤ The sampling distribution of the sample means can be illustrated using a histogram.

The mean of the population is 95.40 (to be discussed in the next module). Observe that the samples are either less than or greater than 95.40. The difference between each sample and the population mean is called **sampling error**. It is the error emanating from the process of making sampling. This error will be reduced by increasing the sample size.

Example 2: The population consists of the 5 barangays in Marikina City with the highest number of active cases on January 26, 2021 (Source: PIO Marikina).

BARANGAY	ACTICE CASES
STO NINO	48
SAN ROQUE	32
CONCEPCION UNO	26
KALUMPANG	22
CONCEPCION DOS	21

Construct the sampling distribution of the sample means using samples of size 2.

Solution:

➢ Using the combination formula; there will be 10 possible samples.

$$C(N,n) = \frac{N!}{n!(N-n)!}$$

$$C(5,2) = \frac{5!}{2!(5-2)!} = 10$$

➢ Possible samples of size 2.

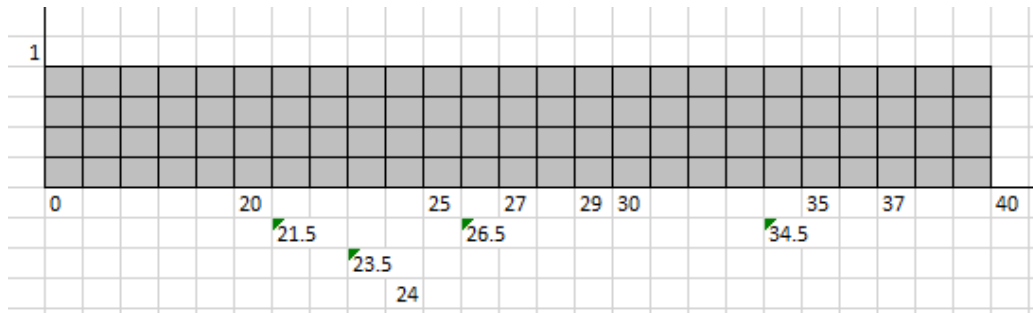| SAMPLES		Mean
48	32	40
48	26	37
48	22	35
48	21	34.5
32	26	29
32	22	27
32	21	26.5
26	22	24
26	21	23.5
22	21	21.5

➢ Sampling distribution of sample means.

Sample Mean	Frequency	Probability P($\bar{x}$)
21.5	1	0.1
23.5	1	0.1
24	1	0.1
26.5	1	0.1
27	1	0.1
29	1	0.1
34.5	1	0.1
35	1	0.1
37	1	0.1
40	1	0.1
	10	1

➢ Histogram

Example 3: Construct the sampling distribution of the sample means if random samples of size 2 are drawn from a population consisting of 10, 15, 20 and 25.

➢ Using the combination formula, how many possible samples can be drawn from the population? _____
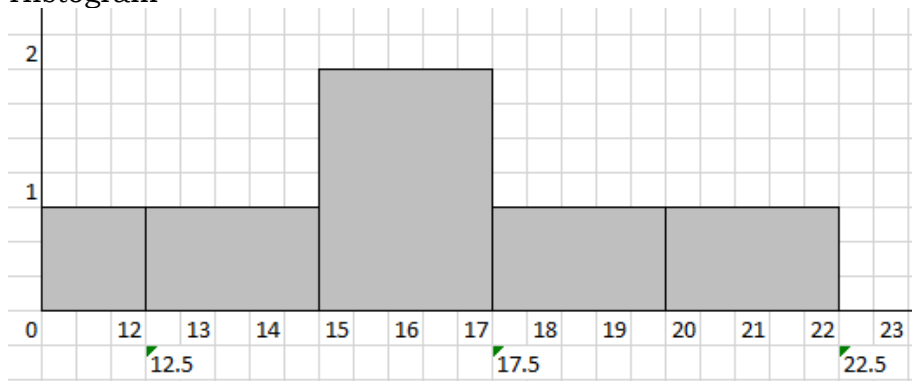
➢ Possible samples of size 2.

SAMPLES		Mean
10	15	12.50
10	20	15.00
10	25	17.50
15	20	17.50
15	25	20.00
20	25	22.50

➢ Sampling distribution of sample means.

Sample Mean	Frequency	Probability P($\bar{x}$)
12.5	1	1/6
15	1	1/6
17.5	2	2/6
20	1	1/6
22.5	1	1/6
	6	1

➢ Histogram

Exercises:

1. Determine the number of possible samples of size 3 from the population with the following sizes:

(a) N = 5   (b) N = 9   (c) N = 15   (d) N = 20   (e) N = 30

2. A population has the following numbers 2, 4, 6, 8, and 10. Samples of size 2 will be drawn from this population.

     (a) List all the possible samples and the corresponding mean.

     (b) Construct the sampling distribution of the sample means.

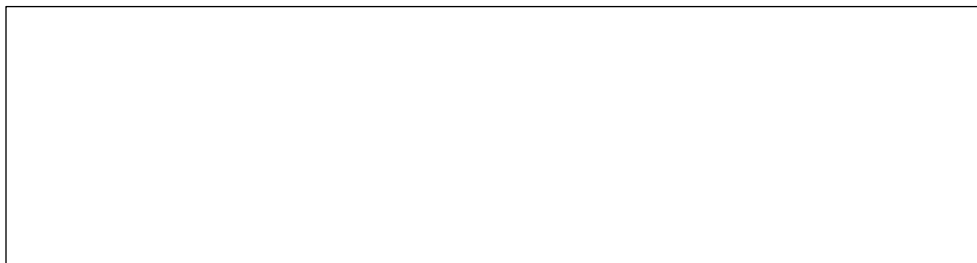| Sample	Mean		SAMPLING DISTRIBUTION OF THE SAMPLE MEANS		
			Sample Mean	Frequency	$P(\bar{x})$

     (c) Draw the histogram of the sampling distribution.

3. A population has the following scores 11, 17, 19, 21, and 25. Samples of size 3 will be drawn from this population.

(a) List all the possible samples and the corresponding mean.

(b) Construct the sampling distribution of the sample means.

Sample	Mean		SAMPLING DISTRIBUTION OF THE SAMPLE MEANS		
			Sample Mean	Frequency	P($\bar{x}$)

(c) Draw the histogram of the sampling distribution.

💡 **What I Have Learned**

Fill in the blanks with the appropriate term/s.

➢ _____ is the total number of elements from which the data under consideration shall be obtained.

➢ _____ is one of the simplest forms of collecting data from the population. It is a process where each member of the population has an equal chance of being selected to form a sample.

➢ _____ is used to describe the characteristic of a population.

➢ _____ is used to label the characteristics of a population.

➢ The sample means will form a frequency distribution. Hence, we can form probability distribution which is called the _____ .

➢ The difference between each sample and the population mean is called _____ .
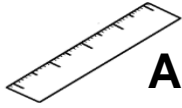
What I Can Do

Task: Ask five (5) of your classmates their weight in kilograms. Construct the sampling distribution of the sample means with random samples of size 2 with the data that you will attained from your classmates.

Rubric:

POINTS	4	3	2	1
Understands the Problem	Identifies special factors that influences the approach before starting the problem	Understands the problem	Understands enough to solve part of the problem or to get part of the solution	Doesn't understand enough to get started or make progress
Uses Information Appropriately	Explains why certain information is essential to the solution	Uses all appropriate information correctly	Uses some appropriate information correctly	Uses inappropriate information
Applies Appropriate Procedures	Explains why procedures are appropriate for the problem	Applies completely appropriate procedures	Applies some appropriate procedures	Applies inappropriate procedures
Uses Representations	Uses a representation that is unusual in its mathematical precision	Uses a representation that clearly depicts the problem	Uses a representation that gives some important information about the problem	Uses a representation that gives little or no significant information about the problem
Answers the Problem	Correct solution of problem and made a general rule about the solution or extended the solution to a more complicated solution	Correct solution	Copying error, computational error, partial answer for problem with multiple answers, no answer statement, answer labeled incorrectly	No answer or wrong answer based upon an inappropriate plan

source: https://www.uen.org/rubric/previewRubric.html?id=13

Assessment

Read each item carefully then choose the letter that corresponds to the correct answer.

1. It is the total number of elements from which the data under consideration shall be obtained.

   A. parameter   C. sample

   B. population   D. statistic

For numbers 2 and 3, refer to the choices in item no. 1.

2. It is used to describe the characteristic of a population.

3. It is used to label the characteristics of a population.

4. It is a process where each member of the population has an equal chance of being selected to form a sample.

   A. population parameter   C. sampling error

   B. random sampling   D. statistic

5. A population has the following scores 32, 21, 48, 22 and 26. Using sample size of 2, how many possible samples will be selected from the population?

   A. 10   B. 9   C. 8   D. 7

For number 6 to 10, refer to the item no. 5 problem. Based on the data given, construct the sampling distribution of the sample means before answering item number 6 to 10.

6. What is the lowest sample mean in the sampling distribution of the sample means?

   A. 21   B. 21.33   C. 21.5   D. 21.67

7. What is the highest sample mean in the sampling distribution of the sample means?

   A. 40   B. 40.25   C. 40.5   D. 41

8. The sum of the frequency value is _____.

   A. 6   B. 8   C. 10   D. 12

9. Which of the following statement is true on the constructed sampling distribution of the sample means.

   A. The sample mean with the highest $P(\bar{x})$ is 40.

   B. Only one sample mean got a $P(\bar{x})$ value which is higher than 0.1.

   C. Each sample mean got an equal value of $P(\bar{x})$.

   D. None of the above.

10. The mean of the population is 29.8. How many sample means is less than the population mean?

   A. 6   B. 5   C. 4   D. 3

Additional Activities

Task: Ask five (5) of your classmates their heights in centimeters. Construct the sampling distribution of the sample means with random samples of size 3 with the data that you will attained from your classmates.

📚 References

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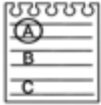
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Answer Key

Possible samples of size 2.

SAMPLE	Mean
2 4	3
2 6	4
2 8	5
2 10	6
4 6	5
4 8	6
4 10	7
6 8	7
6 10	8
8 10	9

Sampling distribution of sample means.

Sample Mean	Frequency	Probability P(x)
3	1	0.1
4	1	0.1
5	2	0.2
6	2	0.2
7	2	0.2
8	1	0.1
9	1	0.1
	10	1

Histogram

Possible samples of size 3.

SAMPLE	Mean
11 17 19	15.67
11 17 21	16.33
11 17 25	17.67
11 19 21	17.00
11 19 25	18.33
11 21 25	19.00
17 19 21	19.00
17 19 25	20.33
17 21 25	21.67
19 21 25	19.00

Sampling distribution of sample means.

Sample Mean	Frequency	Probability P(x)
15.67	1	0.1
16.33	1	0.1
17	1	0.1
17.67	1	0.1
18.33	1	0.1
19	3	0.3
20.63	1	0.1
21.67	1	0.1
	10	1

Histogram

WHAT I KNOW

1	B
2	A
3	D
4	B
5	A
6	C
7	A
8	S
9	C
10	A

WHAT'S MORE

1	A	10
	B	84
	C	455
	D	1140
	E	4060

WHAT I HAVE LEARNED

> POPULATION
> RANDOM SAMPLING
> PARAMETER
> STATISTIC
> SAMPLING DISTRIBUTION OF THE SAMPLE MEANS
> SAMPLING ERROR

ASSESSMENT

1	B
2	A
3	D
4	B
5	A
6	C
7	A
8	S
9	C
10	A

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