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DepED
DEPARTMENT OF EDUCATION

Mathematics

Quarter 1 - Module 22

Equation of Quadratic Function Given Graphs and Zeros

Week 9

Learning Code - M9AL-Ij-15.2

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ADM

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MODULE 22

EQUATION OF QUADRATIC FUNCTION GIVEN GRAPHS AND ZEROS

In the previous module, you learned how to determine the equation of a quadratic function from several points given on its table of values. What if you are given graph or zeroes of the quadratic function, can you give the equation? In this module, you will learn how to derive the equation of quadratic function given its graph or zeros. As you go through this module, you will be given opportunities to decide on what is the best method to use to find the equation of quadratic function.

WHAT I NEED TO KNOW

LEARNING COMPETENCY/IES

The learners will be able to:

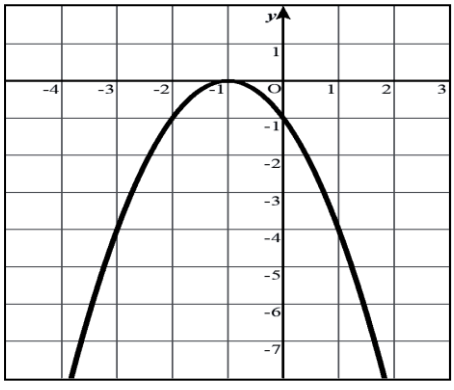
- Determine the equation of a quadratic function given a graph or zeros of the function. **M9AL-Ij-15.2**

WHAT I KNOW

Find out how much you already know about the equation of quadratic function given graphs and zeros. Write the letter that you think is the best answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. What do you call the graph of the quadratic function?
   A) Circle B) Hyperbola C) Line D) Parabola
2. What are the zeros of the quadratic function $= x^2 - 2x - 63$?
   A) -9 & -7 B) -9 & 7 C) -7 & 9 D) 7 & 9
3. Which quadratic function has no zeros?
   A) $y = x^2 - 7$ C) $y = -x^2 + 7$
   B) $y = -x^2 - 7$ D) $y = x^2 + 7x$

For items 4 – 7, refer to the graph at the right.



4. What is the vertex of the graph?
   A) $(0, -1)$
   B) $(0, 1)$
   C) $(-1.\ 0)$
   D) $(1, 0)$

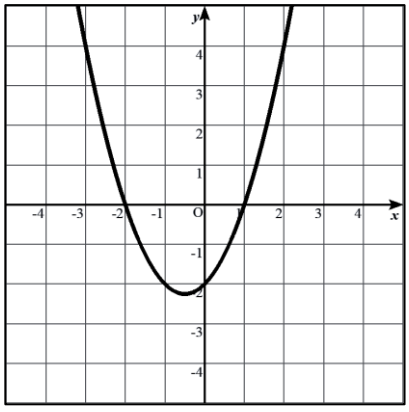
5. What is the zero(s) of the graph?
   A) – 1 B) 0 C) 0 and – 1 D) None
6. What is the y-intercept of the graph?
   A) – 1 B) 0 C) 1 D) None
7. Which equation define the given graph?
   A) $y = -x^2 - 2x - 1$ C) $y = -x^2 + 2x - 1$
   B) $y = -x^2 - 2x + 1$ D) $y = -x^2 + 2x + 1$
8. What quadratic function that passes the point (1, 1) and to its only zero of 2?
   A) $y = x^2 - 4x - 4$ C) $y = x^2 - 4x + 4$
   B) $y = x^2 + 4x - 4$ D) $y = x^2 + 4x + 4$
9. Find the equation of the quadratic function that doesn't cross the x-axis and passes through the point (1, 7).
   A) $y = x^2 + 5$ C) $y = x^2 - 5$
   B) $y = 2x^2 + 5$ D) $y = 2x^2 - 5$
10. What quadratic function contain the points (–1, 0), (1, 0) and (2, 3)?
    A) $y = -x^2 - 1$ C) $y = x^2 - 1$
    B) $y = -x^2 + 1$ D) $y = x^2 + 1$

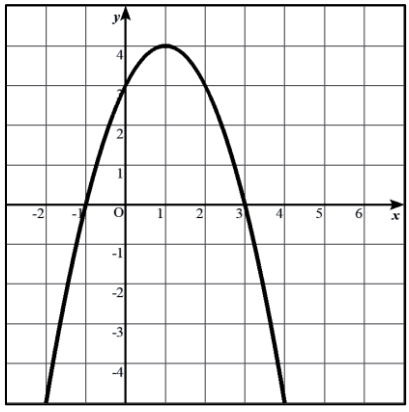
WHAT'S IN

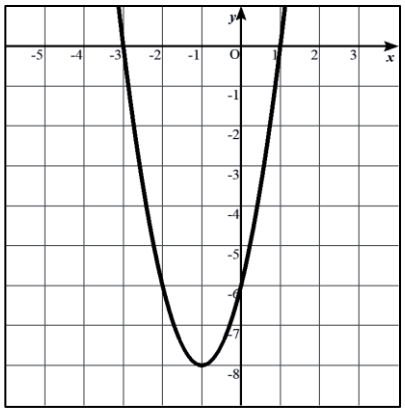
Communication, and Critical Thinking

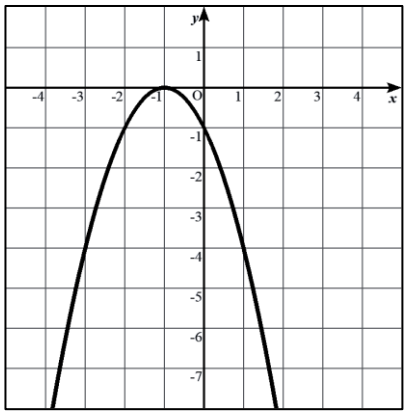
It is easier to understand this module if you know how to find the coordinates of points, particularly the zeroes, on a graph of a quadratic function. Recall that zeros of the quadratic function are actually the x-intercepts of the parabola that corresponds to it.

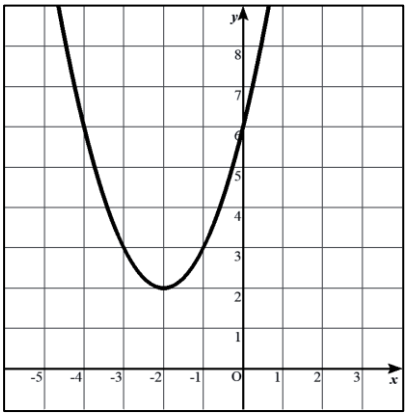
Determine the zeroes of the quadratic function whose graphs are given below:

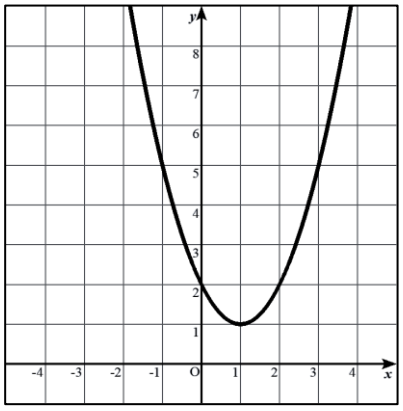
1. 

2. 

3. 

4. 

5. 

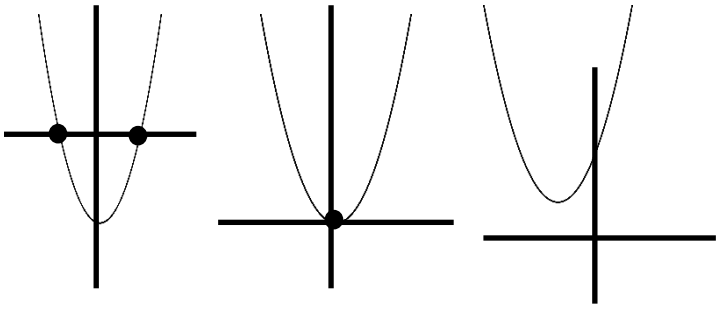
6. 

How did you find the activity? Were you able to spot the zeroes of the functions? Can you give other points on the parabola aside from the zeroes?

WHAT'S NEW

Communication

The graph of a quadratic function is a smooth curve called a parabola. A parabola may intersect the x–axis at two points, one point or no point at all.



two points one point no point

The x-coordinates of the points of intersection of the parabola and the x-axis are the zeroes of the quadratic function if it exists. Graphically, these are the x-

intercepts of the graph of a quadratic function. An x-intercept is a point on the graph that intersects the x-axis.

How will the points on the parabola, particularly the x-intercepts or the zeroes of the function, help in determining the equation of the quadratic function represented by the parabola?

WHAT IS IT

Communication, Critical Thinking, and Collaboration

Deriving Quadratic Function Given Its Graph

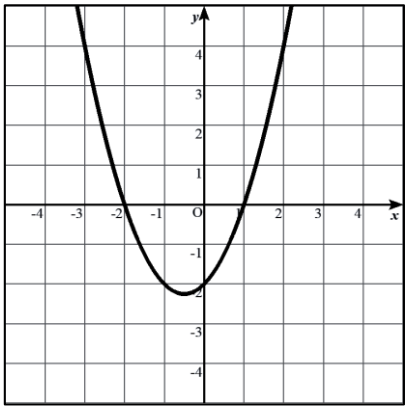
If the graphs representing quadratic function $y = ax^2 + bx + c$ is given, the equation representing the graph can be determined given any of the following conditions:

a. Three points on the parabola none of which is an x-intercepts nor a vertex.
b. The x-intercepts and a point on the parabola for graphs where the coordinates of the vertex are not integers.
c. A vertex and a point on the parabola

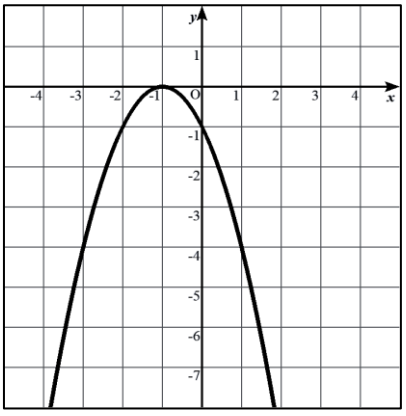
Take note that it is more convenient if the points to consider are integral values. Study the examples below.

Example 1: Determine the quadratic function $y = ax^2 + bx + c$ represented by the given graphs.

a.



b.

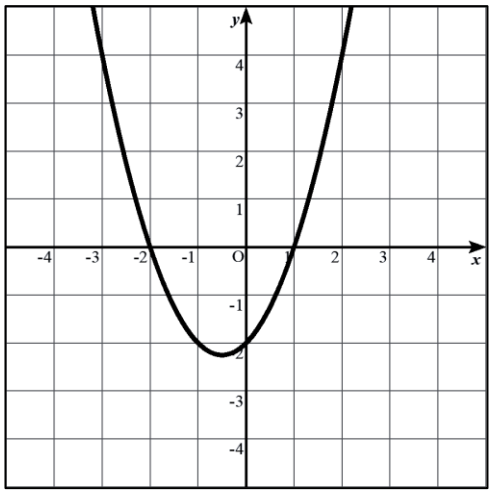


Solution:

a. In the graph, the most convenient points to considers are the x-intercepts, (-2, 0) and (1,0) and the y-intercept (0, -2)

Given three points on the graph, you can find the equation of quadratic function creating systems of equation in three unknowns. However, if the x-intercepts are given, you can have an alternate solution:

If x = -2 and x = 1 are the x-intercepts, then (x + 2) and (x – 1) are factors of the equation of quadratic function. Thus,



$$y = \mathbf{a}\,(x + 2)\,(x - 1)$$

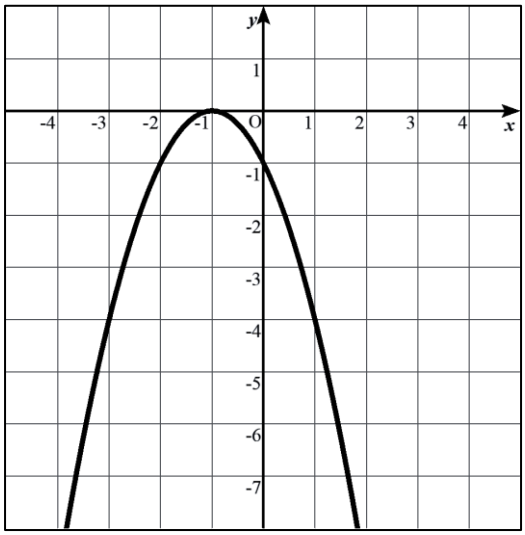
Take note that the value of ***a*** is not always 1 and the third point on the graph can help solve for the value of ***a***. Substitute (0, -2) to the equation,

$$-2 = a\,(0 + 2)\,(0 - 1)$$
$$-2 = -2a$$
$$a = 1$$

Substitute the obtained value to the equation and simplify,

$$y = 1(x + 2)\,(x - 1)$$
$$\mathbf{y = x^2 + x - 2}$$

b. In this graph, the most convenient points to consider are the vertex (-1, 0), the y-intercept (0, -1) and any of the points (-3, -4) or (1, -4).



However, if the vertex is given, you only need one of the point mentioned above.

Substitute the coordinates of the vertex (-1, 0) in the vertex form of the equation of quadratic function,

$$y = \mathbf{a}\,(x+1)^2 + 0$$

Find the value of ***a*** by choosing any of the points (0, -1), (-3, -4) or (1, -4), then substitute:

$$-1 = a\,(0 + 1)^2$$
$$-1 = a$$

Substitute a = -1 in the equation, then transform it into $y = ax^2 + bx + c$;

$$y = -1\,(x + 1)^2 \quad \rightarrow \quad y = -x^2 - 2x - 1$$

Deriving Quadratic Function Given Its Zeroes

If the zeroes are given, then the equation of the quadratic function can be determined. Recall that zeroes, x_1 and x_2, of the function are the roots, of the equivalent quadratic equation,

$$y = (x - x_1)\,(x - x_2) \quad \rightarrow \quad 0 = (x - x_1)\,(x - x_2)$$

Example 2: Determine the quadratic function given the following zeroes:

a. 5, -3 b. 4, $\frac{2}{3}$ c. $1 + \sqrt{3}$

Solution:

a. If 5 and -3 are zeroes of quadratic function $y = ax^2 + bx + c$, then $(x - 5)$ and $[x - (-3)]$ are factors of y,

$y = (x - 5)(x + 3) \quad \rightarrow y = x^2 - 2x - 15$

b. If 4 and $\frac{2}{3}$ are zeroes of quadratic function $y = ax^2 + bx + c$, then $(x - 4)$ and $\left(x - \frac{2}{3}\right)$ are factors of y,

$y = (x - 4)\left(x - \frac{2}{3}\right)$

$y = (x - 4)(3x - 2)$

$y = 3x^2 - 14x + 8$

c. If $1 + \sqrt{3}$ is a zero of quadratic function $y = ax^2 + bx + c$, then its conjugate $1 - \sqrt{3}$ is also a zero of the function, thus $[x + (1 + \sqrt{3})]$ and $[x - (1 - \sqrt{3})]$ are factors of y,

$y = [x - (1 + \sqrt{3})][x - (1 - \sqrt{3})]$

$y = [(x - 1) - \sqrt{3}][(x - 1) + \sqrt{3}]$

$y = (x - 1)^2 - \sqrt{3}^2$

$y = x^2 - 2x + 1 - 3$

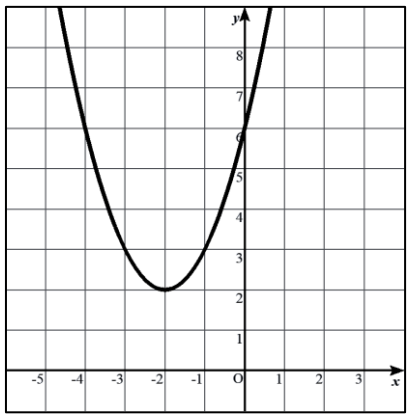
$y = x^2 - 2x - 2$

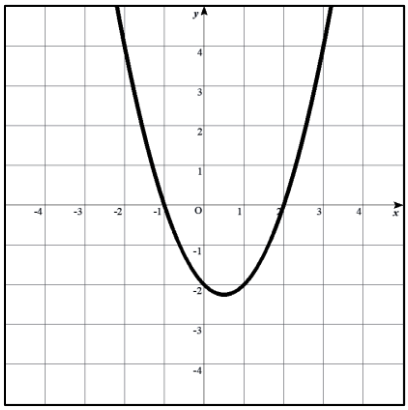
How did you find the given examples? Were you able to understand how to derive equations given the graph or zeros of a quadratic function? If not, go back to those parts that you find challenging and study further.

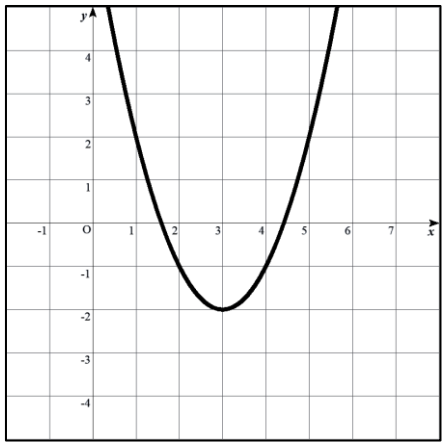
Critical Thinking, Communication and Collaboration

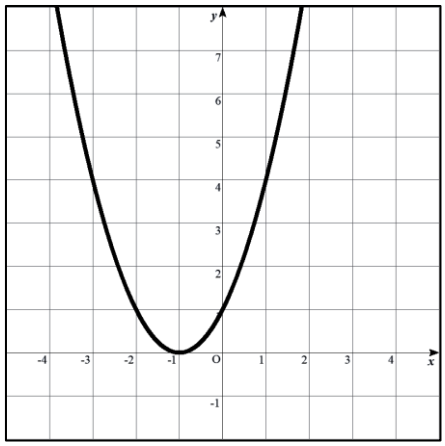
WHAT'S MORE

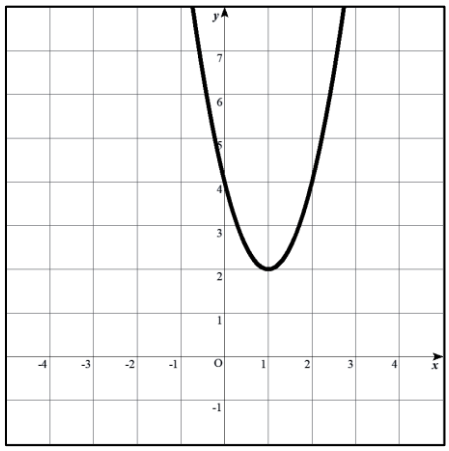
A. Determine the quadratic function $y = ax^2 + bx + c$ represented by the given graphs

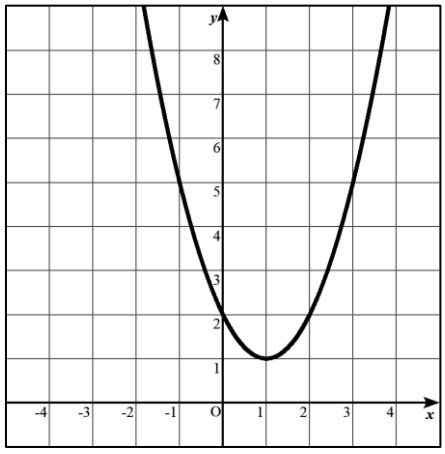
1. 

2. 

3. 

4. 

5. 

6. 

B. Determine the quadratic function given the following zeroes:

1. 2, -3
2. 5, 7
3. -1, -4
4. 6, -5
5. 8, 1
6. $\frac{1}{2}, 2$
7. $5, -\frac{1}{3}$
8. $\frac{1}{4}, \frac{3}{4}$
9. $\frac{3}{5}, -\frac{1}{5}$
10. $-\frac{3}{4}, \frac{4}{3}$
11. $\pm\sqrt{3}$
12. $2 - \sqrt{3}$
13. $1 + \sqrt{5}$
14. $-3 + \sqrt{2}$
15. $-4 - \sqrt{7}$

How did you find the activity? Did you get all the equations of quadratic functions from the graphs and zeros? If not, in which part did you find challenging? How did you cope up with it?

WHAT I HAVE LEARNED

1. If the graphs representing quadratic function $y = ax^2 + bx + c$ is given, the equation representing the graph can be determined given any of the following conditions:
   a. Three points on the parabola, not x-intercepts nor vertex
   b. The x-intercepts and a point on the parabola, if coordinates a vertex are not integral
   c. A vertex and a point of the parabola
2. The zeroes, x_1 and x_2, of the function are the roots, of the equivalent quadratic equation. That is,

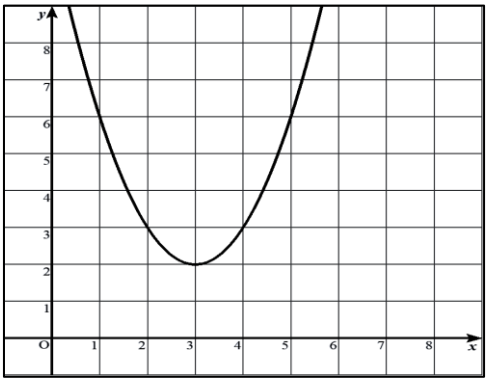
$$y = (x - x_1)(x - x_2) \quad \rightarrow \quad 0 = (x - x_1)(x - x_2)$$

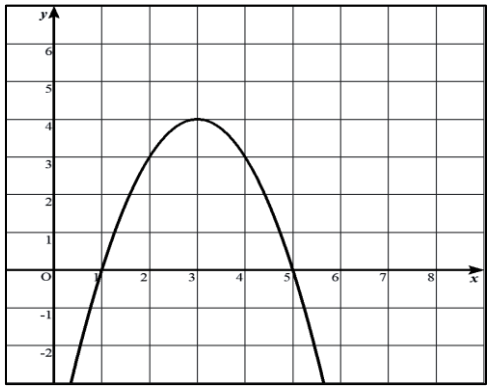
Now that you are equipped with knowledge on determining the equation of quadratic function given graphs and zeros, it's about time to find out what you can do.

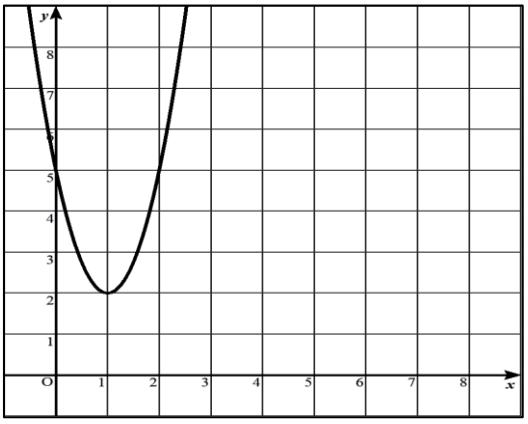
WHAT I CAN DO

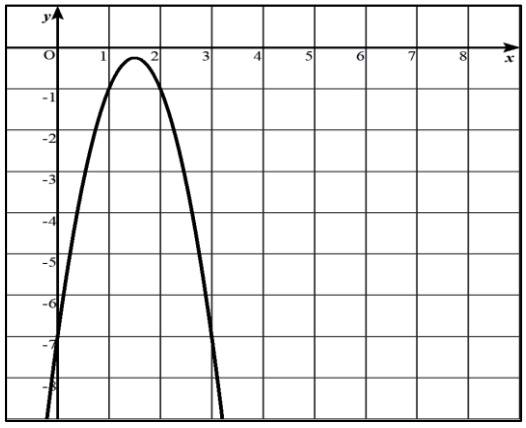
Critical Thinking

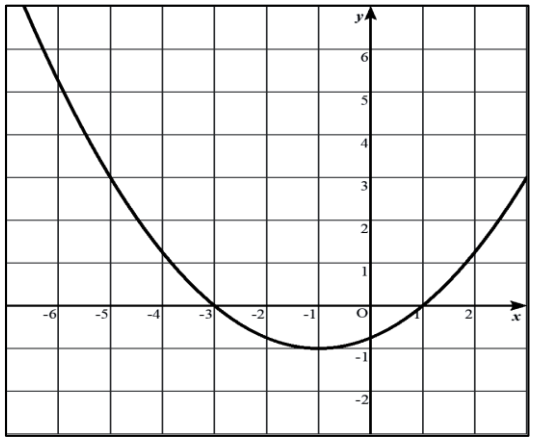
A. Determine the quadratic function $y = ax^2 + bx + c$ represented by the given graphs

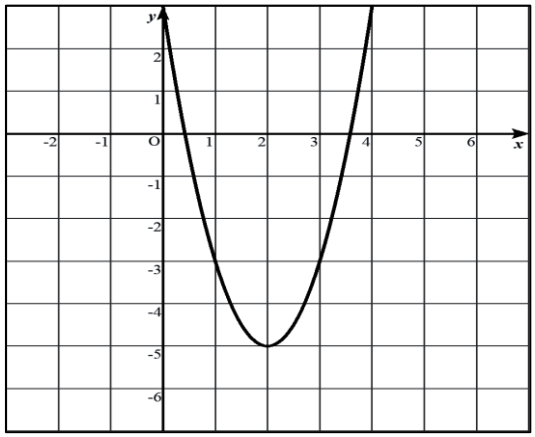
1. 

2. 

3. 

4. 

5. 

6. 

B. Determine the quadratic function given the following zeroes:

1. 5, 8
2. -2, 7
3. 11, -4
4. $\frac{3}{2}, 1$
5. $\frac{1}{2}, \frac{3}{2}$
6. $\frac{2}{3}, -\frac{1}{6}$
7. $\pm\sqrt{5}$
8. $1 \pm \sqrt{2}$
9. $-2 \pm 3\sqrt{3}$
10. $\frac{3\pm\sqrt{17}}{4}$

ASSESSMENT

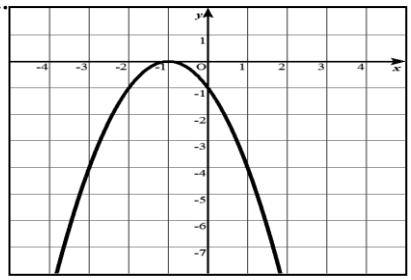
Read each item carefully. Identify the choice that best completes the statement or answers the question

1. What do you call the graph of the quadratic function?
   A) Circle B) Hyperbola C) Line D) Parabola

2. What are the zeros of the quadratic function $= 2x^2 - 7x + 3$?
   A) -3 & $-\frac{1}{2}$ B) -3 & $\frac{1}{2}$ C) $-\frac{1}{2}$ & 3 D) $\frac{1}{2}$ & 3

3. Which quadratic function has no zeros?
   A) $y = x^2 - 4$ C) $y = -x^2 + 4$
   B) $y = -x^2 - 4$ D) $y = x^2 + 4x$

For items 4 – 7, refer to the graph at the right.



4. What is the vertex of the graph?
   A) (0, −1)
   B) (0, 1)
   C) (– 1. 0)
   D) (1, 0)

5. How many zero(s) does the graph have?
   A) 0 B) 1 C) 2 D) None

6. What is the range of the graph?
   A) $(-\infty, 0)$ B) $(-\infty, 0]$ C) $(0, \infty)$ D) $[0, \infty)$

7. Which equation define the given graph?
   A) $y = -x^2 - 2x - 1$ C) $y = -x^2 + 2x - 1$
   B) $y = -x^2 - 2x + 1$ D) $y = -x^2 + 2x + 1$

8. What quadratic function that passes the point (2, 1) and to its only zero of 3?
   A) $y = x^2 - 6x - 9$ C) $y = x^2 - 6x + 9$
   B) $y = x^2 + 6x - 9$ D) $y = x^2 + 6x + 9$

9. Which of the following equations of the quadratic function that doesn't cross the x-axis and passes through the point (1, 4).
   A) $y = x^2 - 2x + 5$ C) $y = 3x^2 - 5x$
   B) $y = 2x^2 - 3x + 1$ D) $y = x^2 + 2x - 5$

10. What quadratic function contain the points (–5, 0), (0, 0) and (2, 14)?
    A) $y = -x^2 - 5x$ C) $y = x^2 - 5x$
    B) $y = -x^2 + 5x$ D) $y = x^2 + 5x$

ADDITIONAL ACTIVITIES

Communication, Critical Thinking, Creativity and Character Building

Activity 1: Parabola Around Us

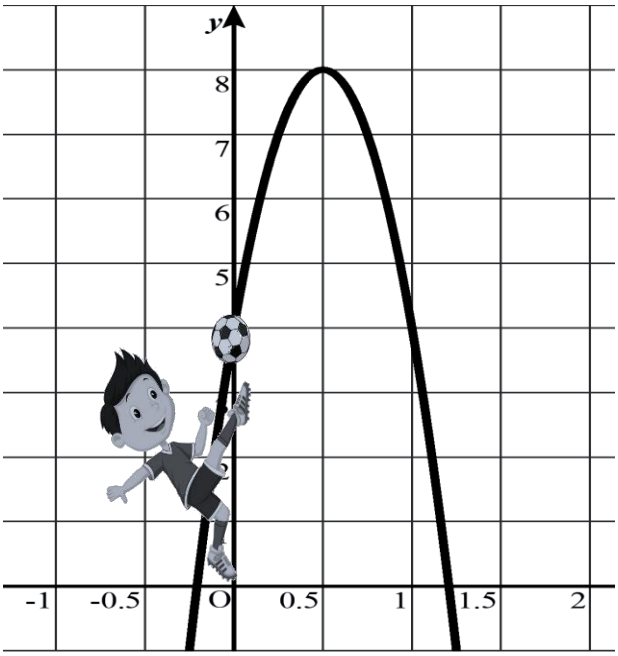
There are a lot of objects around us that looks like a parabola like the architectural design of Mactan Cebu international airport.

For this activity, you will explore the rich arts and architectures of the Philippines that have parabolic design.

1. Find an example of a real parabolic curve found in the Philippines
2. Research about its origin and its actual size.
3. Using any graphing applications (android apps, internet app, computer software, and a like), insert the picture into the rectangular plane and adjust the scale to its actual size.
4. Insert a point for the vertex of your picture and determine its coordinates.
5. Locate for additional points that your graph pass through. Determine the coordinates of this points.
6. Using these points, find the equation in general form of the parabola formed by your picture.

PROBLEM – BASED WORKSHEET

A soccer player passes the ball to a teammate, and the teammate kicks the ball. The graph displays the relation of the height in feet of the ball, in feet, *t* seconds after it is kicked.



Let's Analyze!

1. What is the maximum height that the ball could reach?
2. After how many seconds did the ball reached the maximum height?
3. Determine the function h(t) from the given graph.

E-Search

You may also check the following link for your reference and further learnings on determining quadratic function given table of values.

- https://www.youtube.com/watch?v=jLzkaJk0iZ0
- https://www.youtube.com/watch?v=vAPPYoBV2Ow
- https://www.youtube.com/watch?v=OXViZtD2BTE
- https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratic-forms-features/e/rewriting-expressions-to-reveal-information

REFERENCES

MacKeague, Charles, Intermediate Algebra, Concepts and Graphs. Saunders College Publishing, USA
Mathematics 9 Learner's Material, Department of Education
Ogena, Ester, et. al. Our Math Grade 9. Mc Graw Hill, Vibal Group. Inc
https://www.radford.edu/rumath-smpdc/Performance/src/Leslie%20Cumbow%20-%20Discovering%20Quadratics.pdf

Illustrations:

https://www.freepik.com/free-vector/woman-with-long-hair-teaching-online_7707557.htm
https://www.freepik.com/free-vector/kids-having-online-lessons_7560046.htm
https://www.freepik.com/free-vector/illustration-with-kids-taking-lessons-online-design_7574030.htm
http://www.usccg.com/tag/quality-control/
http://www.erpbusinessschool.lk/programmes/professional-diploma-in-enterprise-resource-planning/careers_chemd_guygirllab_0/
https://www.dezeen.com/2018/12/03/mactan-cebu-international-airport-philippines-integrated-design-associates/

ANSWER KEY

WHAT I KNOW

1. D
2. C
3. B
4. C
5. A
6. A
7. A
8. C
9. B
10. C

WHAT'S IN

1. -2, 1
2. -1, 3
3. -3, 1
4. -1
5. none
6. none

WHAT'S MORE

A.
1. $y = x^2 + 4x + 6$
2. $y = x^2 - x - 2$
3. $y = x^2 - 6x + 7$
4. $y = x^2 + 2x + 1$
5. $y = 2x^2 - 4x + 4$
6. $y = x^2 - 2x + 2$

B.
1. $y = x^2 + x - 6$
2. $y = x^2 - 12x + 35$
3. $y = x^2 + 5x + 4$
4. $y = x^2 - x - 30$
5. $y = x^2 - 9x + 8$
6. $y = 2x^2 - 5x + 2$
7. $y = 3x^2 - 14x - 5$
8. $y = 16x^2 - 16x + 3$
9. $y = 25x^2 - 10x - 3$
10. $y = 12x^2 - 7x - 12$
11. $y = x^2 - 3$
12. $y = x^2 - 4x + 1$
13. $y = x^2 - 2x - 4$
14. $y = x^2 + 6x + 7$
15. $y = x^2 + 8x + 9$

WHAT I CAN DO

A.
1. $y = x^2 - 6x + 11$
2. $y = -x^2 + 6x - 5$
3. $y = 3x^2 - 6x + 5$
4. $y = -3x^2 + 9x - 7$
5. $y = \frac{1}{4}x^2 + \frac{1}{2}x - \frac{3}{4}$
6. $y = 2x^2 - 8x + 3$

B.
1. $y = x^2 - 13x + 40$
2. $y = x^2 - 5x - 14$
3. $y = x^2 - 7x - 44$
4. $y = 2x^2 - 5x + 3$
5. $y = 4x^2 - 8x + 3$
6. $y = 18x^2 - 9x - 2$
7. $y = x^2 - 5$
8. $y = x^2 + 2x - 1$
9. $y = x^2 - 4x - 23$
10. $y = 2x^2 - 3x - 1$

ASSESSMENT

1. D
2. D
3. B
4. C
5. B
6. B
7. A
8. C
9. A
10. D

ADDITIONAL ACTIVITIES

Use this rubric for checking your output.

RUBRIC FOR ACTIVITY				
#	Element	0	1	2
1	Picture is of a parabola	No picture is turned in	N/A	Picture is of a Parabola
2	Dimensions of parabola are noted.	No dimensions given	Dimensions given incorrectly	
3	Picture is inserted into Geogebra correctly	No picture inserted	N/A	Picture inserted correctly
4	Vertex and second point are identified correctly	Points are not correctly identified	One point is correctly identified	Both points are correctly identified
5	Equation in vertex form is computed correctly	No equation is computed	Equation is computed with error	Equation is done correctly

PROBLEM – BASED WORKSHEET

1. 8 ft
2. after 0.5 second of ½ second
3. $h(t) = -16t^2 + 16t + 4$