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Mathematics

Quarter 1 - Module 23

Application of Quadratic Function

Week 9

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MODULE 23

APPLICATION OF QUADRATIC FUNCTION

At the start of this chapter, you were given the opportunity to recognize models of quadratic functions. Indeed, a quadratic function can be seen in many different fields like physics, industry, business, and alike. You have gone through the different modules; you learned the concepts associated with a quadratic function. In this module, you will apply the concepts of a quadratic function in solving real-life situations.

WHAT I NEED TO KNOW

LEARNING COMPETENCY/IES

The learners will be able to:

- Solve problems involving quadratic functions. **M9AL-Ii-j-2**

WHAT I KNOW

Find out how much you already know about the application of quadratic function. Write the letter that you think is the best answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

For question numbers 1-3: From a 96-foot building, an object is thrown straight up into the air then follows a trajectory. The height S(t) of the ball above the building after t seconds is given by the function $S(t) = 80t - 16t^2$.

1. What maximum height will the object reach?
   a. 50 ft. c. 100 ft.
   b. 80 ft. d. 200 ft.
2. How long will it take the object to reach the maximum height?
   a. 1.5 seconds c. 3 seconds
   b. 2.5 seconds d. 2 seconds
3. Find the time at which the object is in the ground.
   a. 3 seconds c. 10 seconds
   b. 6 seconds d. 5 seconds

4. A farmer has 120 m of fencing. He wants to put a fence around three sides of a rectangular plot of land, with the side of a barn forming the fourth side. What dimensions give this area?
   a. 30 m by 60 m
   b. 20 m by 30 m
   c. 20 m by 40 m
   d. 40 m by 60 m

For numbers 5-7: A projectile is launched from a point above the ground. The height at ground level is given by the equation $h = -3t^2 + 24t$, where h is the height in meters and t is the time in seconds.

5. What is the maximum height it can be reach?
   a. 40 meters
   b. 35 meters
   c. 24 meters
   d. 48 meters
6. How long will it take to reach the maximum height?
   a. 2 secs.
   b. 4 secs.
   c. 3 secs.
   d. 5 secs.
7. How many seconds after the launch will the projectile hit the ground?
   a. 8 secs.
   b. 6 secs.
   c. 10 secs.
   d. 4 secs.

For numbers 8-10: An object is thrown vertically upward with a velocity of 95 m/sec. The distance S(t) above the ground after t seconds is given by the formula $S(t) = 96t - 5t^2$.

8. How high will it be at the end of 3 seconds?
   a. 143 m
   b. 220 m
   c. 243 m
   d. 200 m
9. How much time will it take the object to be 172 m above the ground after thrown vertically?
   a. 3 secs
   b. 2.5 secs.
   c. 2 secs
   d. 4 secs
10. How long will it take the object to reach the ground?
    a. 20 secs.
    b. 21.2 secs.
    c. 19.2 secs.
    d. 20.5 secs.

WHAT'S IN

Communication, and Critical Thinking

It is easier to understand this module if you know the concepts associated with quadratic functions particularly the maximum point and the minimum point of the graph. Most real-life problems involve concepts of maxima-minima, as they call it. Consider the example below

Example: $\boldsymbol{f(x) = -2x^2 + 10x - 7}$

$= -2(x^2 - 5x) - 7$	*Factor the coefficient*
$= -2\left(x^2 - 5x + \frac{25}{4}\right) + \frac{25}{4} - 7$	$(\frac{5}{2})^2 = \frac{25}{4}$: *add and subtract* $\frac{25}{4}$
$= \boldsymbol{-2\left(x - \frac{5}{2}\right)^2 + \frac{11}{2}}$	*Factor* $\left(x^2 - 5x + \frac{25}{4}\right)$

The vertex of the graph of this function is at $\left(\frac{5}{2}, \frac{11}{2}\right)$. The value of k or the y-coordinate of the vertex gives the maximum or minimum value depending on the opening of the graph. When the graph opens upward, then the value of k is a minimum. When the graph opens downward, then the value of k is a maximum.

Therefore, the equation has a maximum value of $\frac{11}{2}$ at $x = \frac{5}{2}$.

Determine the maximum/minimum value of each function at the correct value of *x*.

1. $y = -x^2 + 2x + 5$
2. $y = 2x^2 + 8x - 7$
3. $y = 4 + x^2$
4. $f(x) = x^2 - 6x + 4$
5. $f(x) = x^2 - 4x + 1$
6. $y = -x^2 - 2x + 3$
7. $y = 2x^2 + 3x + 4$
8. $y = 3x^2 - 6x + 1$
9. $y + x^2 = 8x - 12$
10. $y + 2x^2 = 5 - 6x$

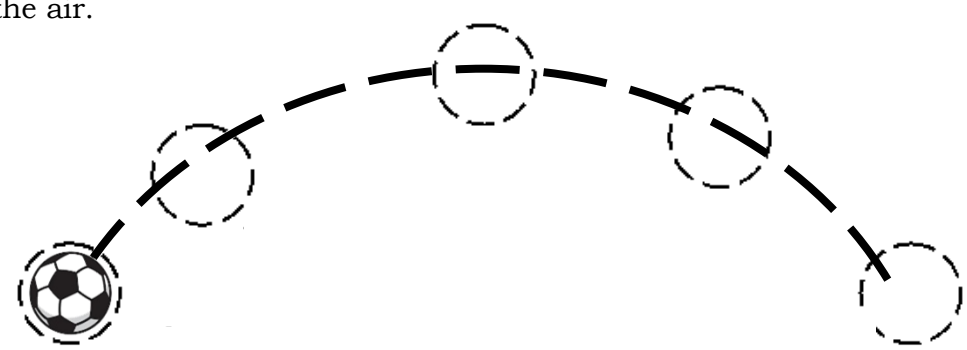
How did you find the activity? Were you able to recall how to determine if a quadratic function has a minimum or maximum value? Were you able to solve for the maximum or minimum value?

WHAT'S NEW

Communication

Applications of a quadratic function can be seen in many different fields like sports, physics, industry, engineering, architecture, business, and a lot more. Consider the situation below.

A ball was thrown in the air above the ground and the height is given by the equation $h = -3t^2 + 12t$, in meters, where t is the time in seconds that the ball has been in the air.



1. What is the maximum height the ball can reach?
2. How long will it take for the ball to reach the maximum height?

WHAT IS IT

Communication, Critical Thinking, and Collaboration

Let's analyze!

The projection made by the ball is defined by the function $h = -3t^2 + 12t$, which is quadratic. Let us transform the equation in standard form or vertex form,

$h = -3t^2 + 12t$	*Write the equation*
$h = -3(t^2 - 4t)$	*Factor out a*
$h = -3(t^2 - 4t + 4) + 12$	*Complete the square*
$h = -3(t - 2)^2 + 12$	*Simplify*

The vertex (h, k) is at (2, 12). If the value of k gives either the maximum or the minimum value, then the maximum height our ball could reach is 12m. It reaches the maximum height after 2 seconds, which is the value of h.

An alternate solution is the used of vertex formula $V = (\frac{-b}{2a}, \frac{4ac-b^2}{4a})$

First, identify the value of a, b, and c in the given equation. Then, substitute in the given formula.

$$a = -3, \quad b = 12, \quad c = 0$$

Time to reach the maximum height: $h = \frac{-b}{2a} = \frac{-12}{2(-3)} = \frac{-12}{-6} = 2$ seconds

Maximum height in 2 seconds: $k = \frac{4ac-b^2}{4a} = \frac{4(-3)(0)-(12)^2}{4(-3)} = \frac{0-144}{-12} = 12\ m$

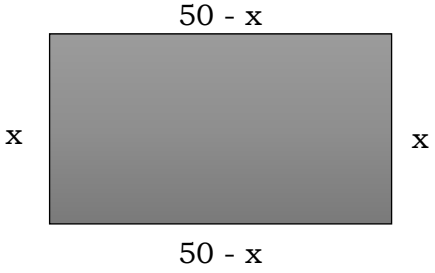
Consider the next examples to understand how to use the concept of a quadratic function in real-life situations.

Example 1. A rectangular garden will be enclosed by 100m of fencing materials. Find the greatest possible area that the materials can enclose?

Solution.

The problem asks for the maximum area that can be enclosed using 100m of fencing materials.

Let x = length of one side of the rectangular garden, then



$\frac{100-2x}{2}$ or 50 – x = the length of the other side adjust to it

Express the area (A) as function of x,

$A = x(50 - x)$

$A = -x^2 + 50x$

Here a = -1, thus the graph opens downward and the vertex gives the maximum value at a given value of x.

To find the vertex, you can choose between completing the square or using the vertex formula. By completing the squares,

$A = -x^2 + 50x$

$= -(x^2 - 50x)$

$= -(x^2 - 50x + 625) + 625$

$= -(x - 25)^2 + 625$

The vertex is at (25, 625). It means that the maximum area that can be enclosed from 100m of fencing materials is $625m^2$.

The length of one side is x = 25m
The length of the other side 50 – x, or 25m

It indicates that the largest area for 100m fencing materials can be given by a square of side 25m.

Example 2: A small shoe factory produces 100 pairs of shoes a day at a profit of Php360.00 per pair of shoes. If a more expensive machine will be used, production can be increased but the profit per pair of shoes diminishes by P2.00 for each additional pair produced. How many additional pairs of shoes should be made in order to maximize the daily profits? How much will the total profit be?

Solution:

The problem asked for the number of pairs of shoes in excess of 100 that can be produced to reach the maximum profit and how much this profit will be.

Let x = number of pair of shoes in excess of 100
(360 – 2x) = profit per pair of shoes
Express the total profit P as function of x.

P = (total number of pair of shoes) (profit per pair of shoes)
P = (100 + x) (360 – 2x)
P = 36,000 +160x – $2x^2$
P = $-2x^2$ + 160x + 26,000

Using the vertex formula with a = -2, b = 160, and c = 36,000, we have;

h = $\frac{-b}{2a} = \frac{-160}{2(-2)} = 40$

k = $\frac{4ac - b^2}{4a} = \frac{4(-2)(36{,}000) - 160^2}{4(-2)}$ = 39,200

Thus, the factory needs to produce an additional 40 pairs of shoes daily to reach the maximum profit of P39,200 a day.

How did you find the given examples? Did you understand how to apply quadratic function in real-life situations? If not, go back to those parts that you find challenging and study further.

WHAT'S MORE

Critical Thinking, Communication and Collaboration

Solve the following problems.

1. Terence is thrown from a point above the ground. The height at ground level is given by the equation h = $-4t^2 + 24t$, where h is the height in meters and t is the time in seconds. What is the maximum height he can reach? How long he will take to reach the maximum height?

2. A ball is thrown from a point above the ground. The height at ground level is given by the equation h = $-3t^2 + 24t$, where h is the height in meters and t is the time in seconds. What is the maximum height the ball can reach? How long will it take to reach the maximum height?

3. A rectangular lot is to be enclosed with 160m of fencing materials. Find the maximum area of such enclosure.

4. If the sum of two numbers is 30. Find their maximum product.

5. A transportation services ask for P60.00 fare for 20 or fewer passengers. In excess of 20, the fare is decreased by P2.00 per person for everyone. How many passengers will produce the maximum profit and how much is the profit?

How did you find the activity? Did you solve all the problems? If not, in which part did you find it challenging? How did you cope up with it?

WHAT I HAVE LEARNED

To solve problems involving quadratic function, particularly maximum or minimum value;

1. Represent the unknowns using a single variable
2. Write the equation of quadratic function in a general form.
3. Choose the appropriate method for the problem
   a. Complete the square and find (h, k)
   b. Use the vertex formula $V = (\frac{-b}{2a}, \frac{4ac-b^2}{4a})$
4. The value of k gives the maximum or minimum value of the function at the given value h.

Now that you are equipped with knowledge on applying quadratic function in solving real-life problems, it's about time to find out what you can do.

WHAT I CAN DO

Critical Thinking

Solve the following problems.

1. A ball was thrown upward from a height of 6 feet with an initial velocity of 32 feet per second. The height S at any given time t is given by $\boldsymbol{s(t) = -16t^2 + 32t + 6}$, where s(t) is measured in feet and t in seconds.
   a. Find the maximum height the ball reached before it begins falling.
   b. Find the time when the maximum height is reached.

2. A projectile is launched from a point above the ground. The height at ground level is given by the equation $\boldsymbol{h = -3t^2 + 24t}$, where h is the height in meters and t is the time in seconds.
   a. What is the maximum height it can be reached?
   b. How long will it take to reach the maximum height?

3. A rectangle has a perimeter of 40m. What could be the maximum area of the rectangle?

4. The difference of two numbers is 18. What are numbers so that their product is a minimum?

5. Mang Toti has a rectangular lot with one side against the wall. If he has 160m of fencing materials, what could be the maximum area of the lot and what are the dimensions of the lot?

6. A jeepney service transporting passengers to a certain place charges P20.00 and carries 600 passengers per day. If they will increase the fare by P2.00, they will lose 30 passengers. Find the fare that will give the maximum income.

7. A farmer has 120 m of fencing materials. He wants to put a fence around three sides of a rectangular plot of land, with the side of a barn forming the fourth side. Find the maximum area of the farmer can enclose. What dimensions give the area?

Congratulations for reaching this far! You are now ready to take the assessment test. Good luck!

ASSESSMENT

Read each item carefully. Identify the choice that best completes the statement or answers the question

For numbers 1-3: A ball is launched upward at 14 m/s from a platform 30 m high. The height of the ball is given by the function $h(t) = -4.9t^2 + 14t + 30$.

1. Find the maximum height the ball reaches.
   a. 50 m c. 80 m
   b. 20 m d. 40 m
2. How long will it take the ball to reach the maximum height?
   a. 2.2 secs. c. 2 secs.
   b. 1. 81 secs. d. 1.43 secs.
3. How long will it take the ball to reach the ground?
   a. 3.21 secs. c. 2.23 secs.
   b. 4.29 secs. d. 2.1 secs.

For numbers 4-6: Elaine shoots an arrow upward at a speed of 32 feet per second from a bridge that is 28 feet high. The height of the arrow is given by the function $h(t) = -16t^2 + 32t + 28$, where t is the time in seconds.

4. How long does it take the arrow to reach its maximum height?
   a. 1 sec. c. 2 secs.
   b. 1.5 secs. d. 3 secs.

5. What is the maximum height that the arrow reaches?
   a. 24ft. c. 44ft.
   b. 30ft. d. 32ft.

6. How long would it take before the arrow reached the ground?
   a. 3.5 secs. c. 3.9 secs.
   b. 2.7 secs. d. 2.5 secs.

7. The height in meters of a projectile after t seconds is given by $h(t) = 160t - 80t^2$. Find the maximum height that can be reached by the projectile.
   a. 40 m c. 30 m
   b. 80 m d. 90 m

8. A person standing close to the edge on the top of an 80-foot tower throws a ball with an initial speed of 64 feet per second. After t seconds, the height of the ball above the ground is $s(t) = -16t^2 + 64t + 80$. After how many seconds will the ball reach its maximum height?
   a. 2 secs c. 4 secs.
   b. 3 secs d. 5 secs.

9. What are the dimensions of the largest area of a pen that can be closed by a 64-m fence?
   a. 16 m by 16 m c. 12 m by 12 m
   b. 20 m by 20 m d. 10 m by 10 m

10. A farmer has 120 m of fencing. He wants to put a fence around three sides of a rectangular plot of land, with the side of a barn forming the fourth side. What dimensions give this area?
    a. 30 m by 60 m c. 20 m by 40 m
    b. 20 m by 30 m d. 40 m by 60

E-Search

You may also check the following link for your reference and further learnings on determining quadratic function given table of values.

- https://www.youtube.com/watch?v=jLzkaJk0iZ0
- https://www.youtube.com/watch?v=vAPPYoBV2Ow
- https://www.youtube.com/watch?v=OXViZtD2BTE
- https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratic-forms-features/e/rewriting-expressions-to-reveal-information

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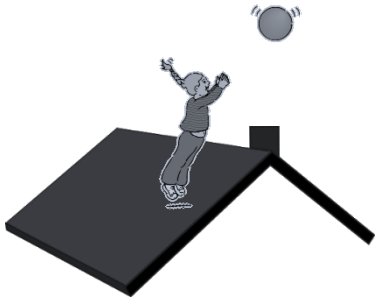
Dolciani, Mary P., Algebra and Trigonometry: Structure and Method, Houghton Mifflin Company. Boston
Mathematics Learners Materials in Grade 9
McKeague, Charles P., Elementary and Intermediate Algebra: A combined Course, Saunders College Publishing, USA

https://www.shmoop.com/functions/quadratic-functions-examples.html

Illustrations:
https://www.freepik.com/free-vector/woman-with-long-hair-teaching-online_7707557.htm
https://www.freepik.com/free-vector/kids-having-online-lessons_7560046.htm
https://www.freepik.com/free-vector/illustration-with-kids-taking-lessons-online-design_7574030.htm

PROBLEM – BASED WORKSHEET

Ericka is standing on the roof of a building. She tosses a ball into the air so that her friend Diana, who is standing on the ground, can catch it. The function $y = -16x^2 + 32x + 80$ models the height of the ball in feet, where x is the time in seconds.



Let's Analyze!

1. Diana lets the ball hit the sidewalk. Determine the total time the ball is in the air until it hits the ground.
2. What is the maximum height the ball could reach? After how many seconds?
3. Is the ball ever at a height of 100 ft? Justify your answer
4. How many times is the ball at a height of exactly 92 ft?
5. Ericka consider the equation $-16x^2 + 32x + 80 = 10$. What does this equation mean?

ANSWER KEY

WHAT I KNOW

1. C	3. D	5. D	7. A	9. C
2. B	4. A	6. B	8. C	10. C

WHAT'S IN

1. Maximum Value of 6 at x=1
2. Minimum Value of -2 at x= -15
3. Minimum Value of 4 at x=0
4. Minimum Value of -5 at x= 3
5. Minimum Value of -3 at x = 2
6. Minimum Value of 4 at x = -1
7. Maximum Value of 23/8 at x = -3/4
8. Minimum Value of -2 at x = 1
9. Maximum Value of 4 at x = 4
10. Maximum Value of 19/2 at x = -3/2

WHAT'S MORE

1. The maximum height is 36m and Terrence reached it after 3seconds.
2. The maximum height the ball could reach is 48m after 4 seconds.
3. The maximum area is 1,600m^2 at dimension 40m by 40m.
4. The maximum product is 225 and the numbers are both 15.
5. The maximum profit is P1,250.00 for additional 5 passengers.

WHAT I CAN DO

1. a. The maximum height the ball could reach is 22m.
   b. The maximum height can be reached after 1 second.
2. a. The maximum height the projectile could reach is 48m.
   b. The maximum height can be reached after 4 second.
3. The maximum area is 100m^2.
4. The minimum product is -81.
5. The maximum area is 3,200m^2 for dimensions of 40m by 60m
6. The maximum income can be reached by increasing the fare by P5.00 for a total fare of P25.00.
7. The maximum area is 1,800m^2 at dimensions 30m by 60m.

ASSESSMENT

1. D	3. B	5. C	7. B	9. A
2. D	4. A	6. B	8. A	10.A

PROBLEM – BASED WORKSHEET

1. 3.45 seconds
2. 96ft after 1 second
3. No, because the maximum height the ball could reach is 96ft.
4. 2 times
5. The time when the ball is at a height of 10 ft.