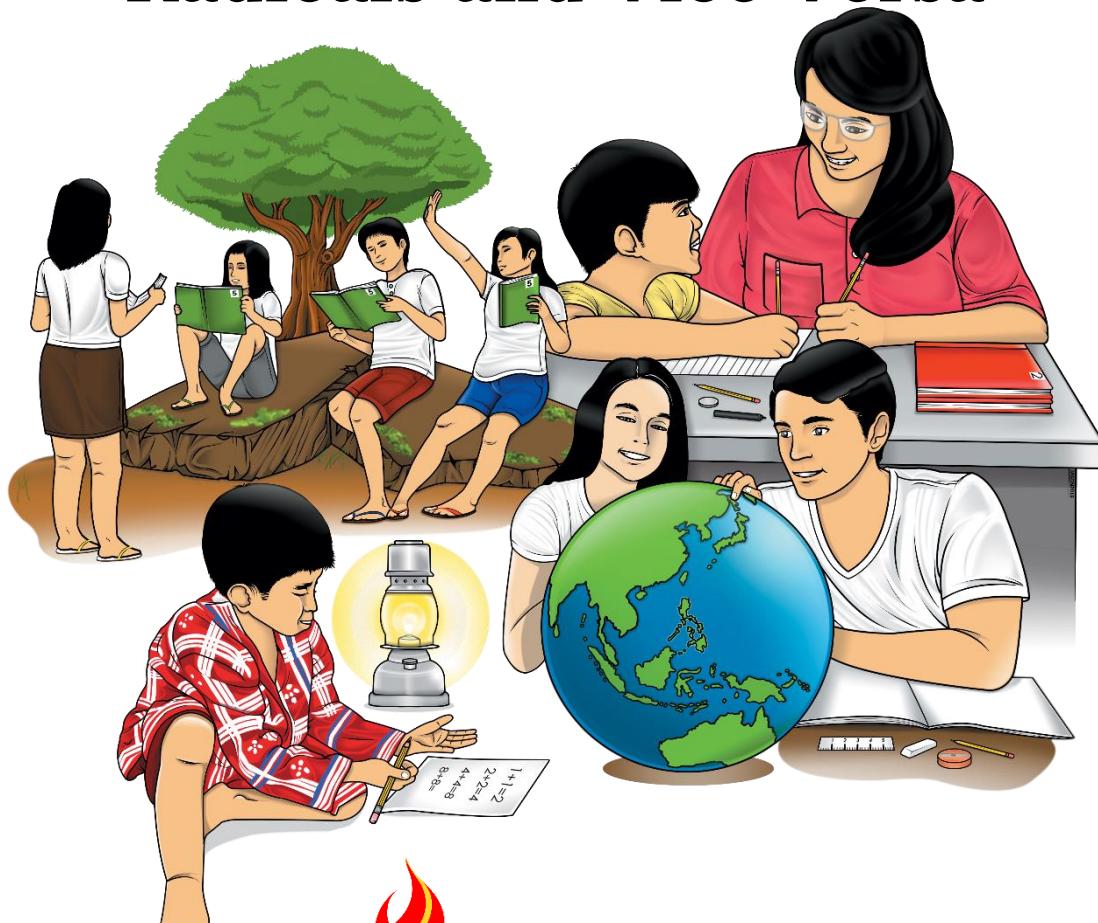


Mathematics

Quarter 2 - Module 6

Writes Expressions with Rational Exponents as Radicals and Vice Versa



Mathematics - Grade 9

Alternative Delivery Mode

**Quarter 2 - Module 6: Writes Expressions with Rational Exponents as Radicals
and Vice Versa**

First Edition, 2020

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Secretary: Leonor Magtolis Briones
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Development Team of the Module

Writer: Elizabeth C. Depositar

Editors: Andrea R. Intal
Juvy G. Delos Santos
Rolando A. Ibayan

Validators: Remylinda T. Soriano
Angelita Z. Modesto
George B. Borromeo

Illustrator: Writer

Layout Artist: Writer

Management Team:

Dennis M. Mendoza,
Maria Magdalena M. Lim
Aida H. Rondilla
Lucky S. Carpio,

Printed in the Philippines by _____

Department of Education – NCR

Office Address : Misamis St., Bago Bantay, Quezon City

Telephone : (02) 926-2213, (02) 929-4330, (02) 920-1490

E-mail Address: : ncr@deped.gov.ph

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Mathematics
Quarter 2 - Module 6
Writes Expressions
with Rational Exponents as
Radicals and Vice Versa

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Lesson 1

Writes Expressions with Rational Exponents as Radicals and Vice Versa

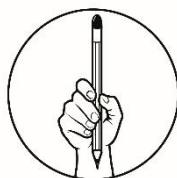
You have learned how to use exponents to express powers of numbers. In this module, you will learn to write expressions with rational exponents as radicals by applying the laws of exponents.



What I Need to Know

LEARNING COMPETENCY

The learners will be able to write expressions with rational exponents as radicals and vice versa. (**M9AL-IIf-1**)



What I Know

Find out how much you already know about the module. Choose the letter of the best answer. Assume all variables are positive. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. Express $(2x)^{\frac{1}{2}}$ in radical form.
A. $2\sqrt{x}$
B. $\sqrt{2x}$
C. $x\sqrt{2}$
D. $\sqrt{2x^{\frac{1}{2}}}$
2. What is the index of the expression $(5a)^{\frac{2}{3}}$ when written in radical form?
A. 5
B. 2
C. a
D. 3
3. Which of the following is the equivalent of $(\sqrt{2xy})^3$ in expression with rational exponent?
A. $(2xy)^{\frac{2}{3}}$
B. $(2xy)^{\frac{3}{2}}$
C. $2(xy)^{\frac{3}{2}}$
D. $2xy$
4. Which of the following is the equivalent of $\sqrt[3]{4}$ in expression with rational exponent?
A. $4^{\frac{1}{3}}$
B. 3^4
C. 4^3
D. $3^{\frac{1}{4}}$
5. Which of the following is a radical expression?
A. $\sqrt[3]{2a}$
B. $ax^2 + by = c$
C. $x + y = C$
D. none

6. In the expression $(2x)^{\frac{1}{2}}$, what is the radicand?
- $2\sqrt{x}$
 - $x\sqrt{2}$
 - $\sqrt{2x}$
 - $2x$
7. Identify the index of the expression $(2xy)^{\frac{3}{5}}$ when written in radical form.
- 5
 - 3
 - xy
 - 2
8. Which of the following is the equivalent of $(3\sqrt{2xy})^3$ in expression with rational exponent?
- $3(2xy)^{\frac{2}{3}}$
 - $(2xy)^{\frac{3}{2}}$
 - $27(2xy)^{\frac{3}{2}}$
 - $2xy$
9. Which of the following is the equivalent of $4^{1/3}$ in radical form?
- $\sqrt[3]{4}$
 - $\sqrt{4}$
 - $\sqrt[4]{3}$
 - $\sqrt{3}$
10. Which of the following is the equivalent of $\sqrt[3]{5}$ in expression with rational exponent?
- $5^{\frac{1}{3}}$
 - 5^4
 - 4^3
 - $3^{\frac{1}{4}}$



What's In

Let us recall how to multiply expressions with rational exponent using laws of exponents.

Study the examples below:

$$1. \ x^2 \cdot x^{\frac{1}{3}} = x^{2+\frac{1}{3}} = x^{\frac{7}{3}} \quad 2. \ a^{\frac{1}{2}} \cdot a^{\frac{3}{2}} = a^{\frac{4}{2}} = a^2 \quad 3. \left(b^{\frac{3}{4}}\right)^2 = b^{\frac{3}{2}}$$

Try this!

Perform the indicated operation and simplify, if needed.

| | |
|--|--|
| 1. $a^{\frac{2}{3}} \cdot a^{\frac{3}{2}}$ | 4. $(4xy^2)^{\frac{1}{2}}$ |
| 2. $b^0 \cdot b^{\frac{1}{5}}$ | 5. $\left(m^{\frac{3}{4}} n\right)^{-2}$ |
| 3. $\left(y^{\frac{1}{3}}\right)^2$ | |

Evaluate.

| | |
|-------------------|--------------------|
| 1. $\sqrt{25}$ | 4. $\sqrt[4]{16}$ |
| 2. $\sqrt[4]{49}$ | 5. $\sqrt[4]{625}$ |

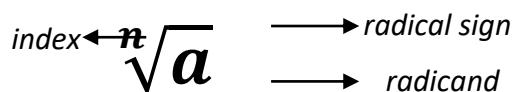


What's New

WRITING EXPRESSIONS WITH RATIONAL EXPONENTS AS RADICALS AND VICE VERSA

In the previous lesson, expressions with rational exponents (fractional exponents) are expressions with exponents that are rational. Radicals and expressions with rational exponents are alternate ways of expressing the same thing.

A radical expression is an algebraic expression written in the form $\sqrt[n]{a}$. The symbol $\sqrt{}$ is called the *radical sign*, a is the *radicand* and n is the *index*.



Convert Expressions with Rational Exponent into Radicals

If we apply the laws of exponents, we can see there are two possible ways to change an expression with fractional exponent into a radical expression.

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (\sqrt[n]{a})^m \text{ or } a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$$

All RULES that apply to whole number exponents, also apply to fractional exponents.

Examples:

$$1. 7^{\frac{2}{3}} = (\sqrt[3]{7})^2 = \sqrt[3]{7^2}$$

In the expression with rational (fractional) exponent as $\frac{2}{3}$, the denominator 3 of the rational exponent determines the index and the numerator 2 as exponent of the radicand 7 or radical.

$$2. (xy)^{\frac{2}{3}} = \sqrt[3]{(xy)^2} \\ = \sqrt[3]{x^2y^2}$$

The exponent applies to x and y because of the parentheses.

$$3. 8x^{\frac{2}{5}} = 8\sqrt[5]{x^2}$$

The exponent applies only to the variable x.

$$4. x^{-\frac{2}{3}} = \frac{1}{x^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{x})^2} \text{ or } \frac{1}{\sqrt[3]{x^2}}$$

$$5. (a^2 + b^3)^{\frac{1}{2}} = \sqrt{a^2 + b^3}$$

Convert Radicals to Expressions with Rational Exponent

Any radical expressions in the form $\sqrt[n]{a^m}$ can be written as an expression with rational exponent in the form $a^{\frac{m}{n}}$. The index n of the radical will become the denominator and the exponent m of the radicand will become the numerator of the rational exponent.

1. $\sqrt[3]{b} = b^{\frac{1}{3}}$
2. $\sqrt[3]{2^4} = 2^{\frac{4}{3}}$
3. $\sqrt[4]{2x} = (2x)^{\frac{1}{4}}$
4. $2(\sqrt[3]{y^4}) = 2y^{\frac{4}{3}}$

The index is 3 and the exponent is 1.
The index is 3 and the exponent is 4.
The index is 4 and the exponent is 1.
The index is 3 and the exponent is 4, the fractional exponent is only for y.

Try this!

A. Express the following in radical form.

1. $5^{\frac{3}{4}}$ _____

4. $b^{\frac{7}{4}}$ _____

2. $(3xy)^{\frac{2}{3}}$ _____

5. $(x+y)^{\frac{1}{3}}$ _____

3. $6a^{\frac{3}{5}}$ _____

B. Express the following with rational exponent.

1. $\sqrt[8]{ab}$ _____

3. $\sqrt[5]{m^7}$ _____

2. $(\sqrt{4})^9$ _____

4. $(\sqrt[9]{23})^7$ _____

5. $\sqrt[3]{2^5}$ _____



What is It

(Communication and Critical Thinking)

WRITING EXPRESSIONS WITH RATIONAL EXPONENTS AS RADICALS AND VICE VERSA

Example1:

$$\begin{aligned}4^{\frac{1}{2}} &= (\sqrt{4})^1 \\&= \sqrt{4^1} \\&= \sqrt{4}\end{aligned}$$

Example2:

$$\begin{aligned}27^{\frac{1}{3}} &= (\sqrt[3]{27})^1 \\&= \sqrt[3]{27^1} \\&= \sqrt[3]{27}\end{aligned}$$

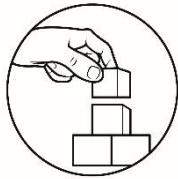
Example3:

$$\begin{aligned}x^{\frac{3}{4}} &= (\sqrt[4]{x})^3 \\&= \sqrt[4]{x^3}\end{aligned}$$

Conclusion:

The _____ of the fractional exponent becomes the _____ of the radical. The _____ of the fractional exponent becomes the _____ of the expression under the radical symbol OR the exponent of the entire radical.

Your own example:



What's More

Activity 1: MATCH-UP

Direction: Match the cards at the top to their answers at the bottom by transforming the given radical form to exponential form and/or exponential form to radical form. Record your answers in the table at the right.

① $5^{\frac{1}{2}}$

② $2m^{\frac{1}{4}}$

③ $(3m)^{\frac{2}{3}}$

④ $(5m)^{\frac{1}{2}}$

⑤ $m^{\frac{7}{2}}$

⑥ $-m^{\frac{1}{4}}$

⑦ $(5m)^{\frac{2}{3}}$

⑧ $(3m^4n)^{\frac{2}{3}}$

⑨ $(3m^4n^2)^{\frac{3}{2}}$

⑩ $(2mn)^{\frac{2}{5}}$

A $\sqrt{5m}$

B $\sqrt{5}$

C $\sqrt{m^7}$

D $\sqrt{27m^{12}n^6}$

E $\sqrt[3]{9m^2}$

F $\sqrt[3]{25m^2}$

G $\sqrt[3]{3m^8n^2}$

H $\sqrt[3]{9m^8n^2}$

I $\sqrt[5]{2}$

J $3\sqrt[4]{m}$

K $-\sqrt[4]{m}$

L $\sqrt[4]{2m}$

M $2\sqrt[4]{m}$

N $\sqrt[4]{-m}$

O $\sqrt[5]{4m^2n^2}$

| | |
|---|--|
| ① | |
| ② | |
| ③ | |
| ④ | |
| ⑤ | |
| ⑥ | |
| ⑦ | |
| ⑧ | |
| ⑨ | |
| ⑩ | |

Activity 2: TRY THIS OUT**Test Yourself!**

- A.** Write each of the following in radical form. Express your answer in simplified form.

1. $50^{\frac{1}{8}}$ _____

5. $78^{\frac{4}{9}}$ _____

9. $12m^{\frac{3}{5}}$ _____

2. $16^{\frac{2}{3}}$ _____

6. $x^{\frac{1}{2}}y^{\frac{3}{2}}$ _____

10. $(a+b)^{\frac{3}{5}}$ _____

3. $89^{\frac{3}{2}}$ _____

7. $mp^{\frac{1}{2}}$ _____

4. $35^{\frac{7}{4}}$ _____

8. $2a^{\frac{1}{5}}$ _____

- B.** Write each of the following in rational form:

11. $\sqrt[8]{93^5}$ _____

15. $\sqrt[3]{7^5}$ _____

19. $(\sqrt[3]{3b})^2$ _____

12. $(\sqrt{31})^9$ _____

16. $(\sqrt{2ab})^3$ _____

20. $\sqrt[3]{x+2}$ _____

13. $\sqrt[5]{22^7}$ _____

17. $\left(\sqrt[3]{x^2y^3z^4}\right)^2$ _____

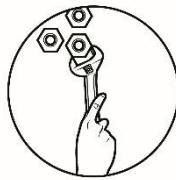
14. $(\sqrt[9]{63})^8$ _____

18. $(\sqrt[7]{5ab^2})^5$ _____

**What I Have Learned****REMEMBER**

If m and n are positive integers that are relatively prime, and let a be a real number such that $\sqrt[n]{a}$ defined in real numbers, then

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m = (a^{\frac{1}{n}})^m$$



What I Can Do

(Communication, Creative and Critical Thinking)

CHECKING YOUR UNDERSTANDING

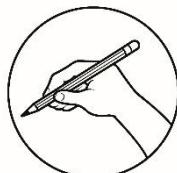
- A. The approximate number of Calories C that an animal needs each day is given by $C = 72m^{\frac{3}{4}}$, where m is the animal's mass in kilograms. Find the number of Calories that a 625 kg bear needs each day. To solve this, evaluate the equation when m = 625.

Solution:



- B. A particular panda consumes 1944 Calories each day. How much does this panda weigh?

Solution:

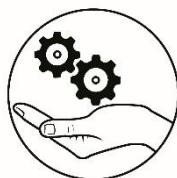


Assessment

Read each item carefully. Choose the letter of the best answer. Assume that all variables represent positive real numbers.

- What is the index of the expression $(4a)^{\frac{3}{7}}$ when written in radical form?
A. 4 B. 3 C. 7 D. $4a$
- Which of the following is the equivalent of $2\sqrt{x^3}$ in expression with rational exponent?
A. $2x^{\frac{1}{2}}$ B. $(2x)^{\frac{3}{2}}$ C. $2x^{\frac{3}{2}}$ D. $2x^{\frac{2}{3}}$
- Which expression is the same as $(2x)^{\frac{1}{4}}$?
A. $\sqrt[4]{2x}$ B. $2\sqrt[4]{x}$ C. $2x^{\frac{1}{4}}$ D. $\frac{1}{2x}$
- Which of the following is the equivalent radical expression of $(a)^{\frac{4}{5}}$?
A. $\sqrt[5]{a}$ B. $\sqrt[5]{a^4}$ C. $\sqrt[4]{5}$ D. $\sqrt[4]{a^5}$
- Find the value of C when m = 27 and $C = 72m^{\frac{2}{3}}$.
A. 468 B. 486 C. 646 D. 648

6. Which of the following is a radical expression?
- A. $x + y = C$ B. $\sqrt[5]{3x^2}$ C. $ax^2 + by = c$ D. none
7. The expression $5a^{\frac{3}{2}}$ in radical form is ____.
- A. $5\sqrt{a}$ B. $\sqrt{2x}$ C. $a\sqrt{25}$ D. $5\sqrt{a^3}$
8. What is the index of the expression $(8x)^{\frac{1}{4}}$ when written in radical form?
- A. 8 B. 1 C. 4 D. $8x$
9. Which of the following is the equivalent of $7\sqrt{y^3}$ in expression with rational exponent?
- A. $7y^{\frac{2}{3}}$ B. $(7y)^{\frac{3}{2}}$ C. $y^{\frac{3}{2}}$ D. $7y^{\frac{3}{2}}$
10. Which of the following is the equivalent of $9\sqrt[3]{ab}$ in expression with rational exponent?
- A. $9ab^3$ B. $(9ab)^{\frac{1}{3}}$ C. $9a^{\frac{1}{3}}b^{\frac{1}{3}}$ D. $(ab)^{\frac{1}{3}}$



Additional Activities

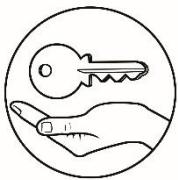
REFLECTIVE LEARNING

In today's world, there are many luxuries that are supposed to make life easier. At times they may, but other times they make things more complicated. As a student, how will you apply what you learned about simplifying expressions with zero, negative integral and rational exponent in today's situation? Do you believe in simple living? What does it mean to live simple? Is it good to live a simple life especially in this time of pandemic?

E-Search

To further explore the concept learned today and if it's possible to connect the internet, you may visit the following links:

<https://www.youtube.com/watch?v=0gicD4STzpg>
<https://www.youtube.com/watch?v=TrJUOKLKlsU>
<https://www.youtube.com/watch?v=w79rpKCKlFw>



Answer Key

| | | | | | | | | | |
|------------------|------------------|-------------------|-------------------|-------------------|------------------------------|--------------------------------|--------------------------------|------------------------|-------------------------------|
| 1. $\frac{5}{6}$ | 2. $\frac{3}{4}$ | 3. $\frac{22}{3}$ | 4. $\frac{63}{4}$ | 5. $\frac{76}{3}$ | 6. $\frac{(2ab)^2}{(x+2)^3}$ | 7. $\frac{xy^2z^3}{x^2y^3z^2}$ | 8. $\frac{(5ab^2)^3}{(x^2)^4}$ | 9. $\frac{9}{(x+2)^2}$ | 10. $\frac{(x+2)^2}{(2ab)^3}$ |
|------------------|------------------|-------------------|-------------------|-------------------|------------------------------|--------------------------------|--------------------------------|------------------------|-------------------------------|

| | | | | | | | | | |
|------------------------|-------------------|----------------------|--------------------|--------------------|-------------------|--------------------|--------------------|------------------------|----------------------------|
| A. $\sqrt[3]{8x^2y^3}$ | B. $\sqrt[4]{16}$ | C. $(\sqrt[3]{5})^2$ | D. $\sqrt[3]{m^6}$ | E. $\sqrt[4]{m^8}$ | F. $\sqrt[3]{27}$ | G. $\sqrt[3]{m^6}$ | H. $\sqrt[4]{m^8}$ | I. $\sqrt[3]{8x^2y^3}$ | J. $\sqrt[3]{(8x^2y^3)^2}$ |
|------------------------|-------------------|----------------------|--------------------|--------------------|-------------------|--------------------|--------------------|------------------------|----------------------------|

A. ACTIVITY 2: TRY IT OUT

1. B 2. M 3. E 4. A 5. O 6. K 7. P 8. G, H 9. D 10. O

B. WHAT'S MORE

denominator = $\sqrt[3]{16}$
 numerator = $\sqrt[3]{(4\sqrt[3]{2})^2}$
 denominator = $\sqrt[3]{(4\sqrt[3]{2})^2}$
 numerator = $(\sqrt[3]{8 \cdot 2})^2$
 denominator = $(\sqrt[3]{8 \cdot 2})^2$

Solution for Activity 2: Try It Out

1. $(ab)^3$ 2. 2^9 3. m^6 4. 2^{31} 5. $\sqrt[3]{x+y}$

C. WHAT IS IT

1. $\sqrt[3]{5}$ 2. $\sqrt[3]{(3xy)^2}$ 3. $6\sqrt[3]{a^2}$ 4. $\sqrt[3]{b^2}$ 5. $\sqrt[3]{x+y}$

D. WHAT'S NEW

1. S 2. T 3. Z 4. Z 5. M
 1. $\frac{m}{n}$ 2. $\frac{6}{7}$ 3. $\frac{y}{x}$ 4. $\frac{2\sqrt[3]{x} + 3\sqrt[3]{y}}{2\sqrt[3]{y}}$ 5. $\frac{1}{m+n}$

E. WHAT'S IN

1. B 2. D 3. E 4. A 5. A 6. D 7. A 8. C 9. A 10. A

F. WHAT I KNOW

ASSESSMENT

5. D
4. B
3. A
2. C
1. C

6. B
7. D
8. C
9. D
10. C

| | |
|---|---|
| <p>A. Given $m = 625 \text{ g}$</p> <p>B. Given $C = 1944 \text{ calories}$</p> <p>For the Calories given by $C = 72m^{\frac{3}{4}}$</p> <p>By substitution, $C = 72(625)^{\frac{3}{4}} = 72(\sqrt[4]{625})^3$</p> <p>$C = 72(5)^3 = 72(125)$</p> <p>$C = 9000 \text{ calories}$</p> | <p>$m = 81$</p> <p>$m^{\frac{3}{4}} = 3$</p> <p>$3^{\frac{3}{4}} = (m^{\frac{3}{4}})^{\frac{1}{3}}$</p> <p>$27 = m^{\frac{3}{4}}$</p> <p>$27^{\frac{4}{3}} = m^3$</p> <p>$m = 81$</p> |
|---|---|

WHAT I CAN DO

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- <https://www.mesacc.edu/~scotz47781/mat120/notes/exponents/review/review.html>
- <https://classroomclipart.com/clipart/Clipart/Mathematics.htm>

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph