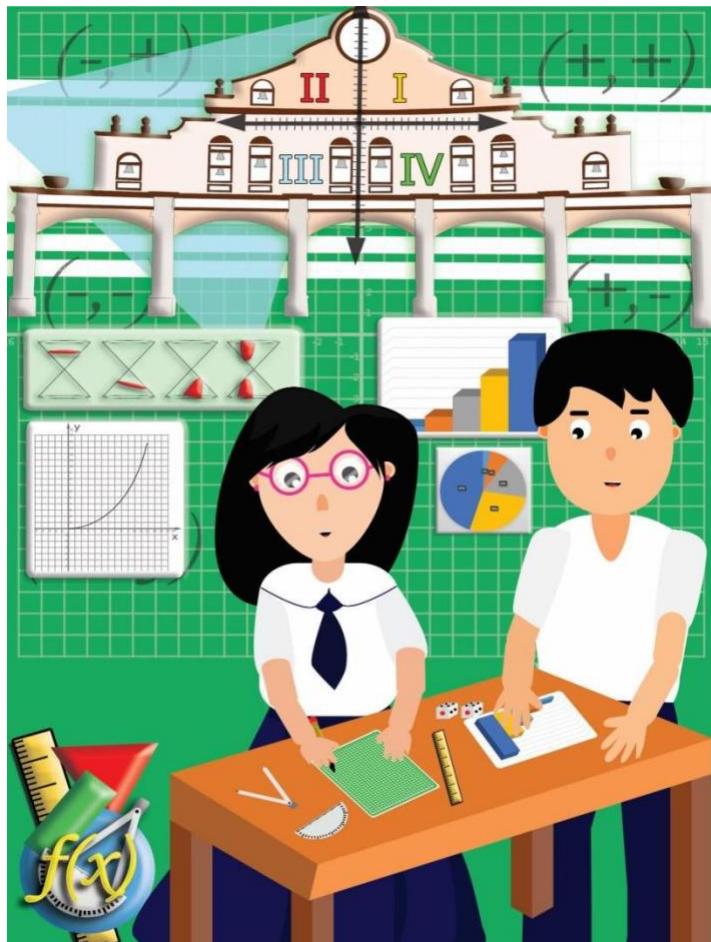


STATISTICS AND PROBABILITY

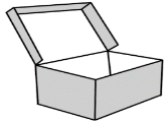
Third Quarter : Module 1

Random Variables and Probability Distribution



Writer: Jennifer Mesias (SEHS)

Cover Illustrator: Joel J. Estudillo (SNNHS)



What I Need to Know

Hello Grade 11 learners! In this module, you will learn how to

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Illustrate a random variable (discrete and continuous) M11/12SP-LLLa-1,
Distinguish between a discrete and a continuous random variable M11/12SP-LLLa-2,
Find the possible values of a random variable M11/12SP-LLLa-3 and
Illustrate a probability distribution for a discrete random variable and its properties M11/12SP-LLLa-4.

You can say that you have understood the lesson in this module if you can already:

1. Illustrate a random variable (discrete and continuous),
2. Define random variables,
3. Distinguish discrete and continuous random variable,
4. Find the possible value of a random variable and,
5. Construct a probability distribution for a discrete random variable and state its properties.



What I Know

Read each question carefully then write the letter that corresponds to your answer.

1. Which of the following is **NOT** a property of a random variable?
 - A. A random variable represent numerical outcomes for different situations or events.
 - B. A random variable can be discrete or continuous.
 - C. A random variable is a function of events.
 - D. Elements in random variable use an upper case letter.
2. Which of the following is **NOT** a random variable?
 - A. The heights of randomly selected buildings in Makati
 - B. The suit of a card randomly selected from a 52-card deck
 - C. The number of children in randomly selected household in Marikina
 - D. The amount of money won (or lost) by the next person to walk out of a casino
3. A random variable is _____.
 - A. list of possible outcomes of an experiment
 - B. any event in which outcomes are equally likely
 - C. any number that changes in a predictable way in the long run
 - D. a variable whose value is a numerical outcome associated with a random experiment



4. Which of these are the two types of random variable?
 - A. Discrete and continuous
 - B. Nominal and ordinal
 - C. Quantitative and qualitative
 - D. Ratio and interval
5. It is a type of a variable that can be counted.
 - A. discrete
 - B. qualitative
 - C. Continuous
 - D. Quantitative
6. Which variable can assume an infinite number of values?
 - A. discrete
 - B. qualitative
 - C. Continuous
 - D. quantitative
7. Which of these is NOT a continuous variable?
 - A. A person's height each year
 - B. The volume of water in a tank
 - C. Dosage of medicine
 - D. Family size
8. Which one of the following is a discrete data?
 - A. Ana is 150 cm tall.
 - B. Ana ran 100 meters in 15 seconds.
 - C. Ana weighs 50 kg.
 - D. Ana has two brothers and one sister.
9. Five coins are tossed simultaneously and the number of heads are recorded. Let x be the number of head appeared. What is x ?
 - A. random variable
 - B. qualitative variable
 - C. Variable
 - d. Probability variable
10. You have a bag containing 3 red chips and 6 white chips and you draw 4 chips. Let random variable Y be the number of red chips drawn from the bag out of 4 draws without replacement. Find the values of the random variable Y .
 - A. $Y = \{1, 2, 3, 4\}$
 - B. $Y = \{0, 1, 2, 3, 4\}$
 - C. $Y = \{0, 1, 2, 3\}$
 - D. $Y = \{1, 2, 3, 4, 5\}$
11. If two coins are tossed, which is not a possible value of the random variable for the number of heads?
 - A. 0
 - B. 1
 - C. 2
 - D. 3
12. Let D represents the defective computer and N represents the non-defective computer. If we let Z be the random variable representing the number of defective computer, what is the value of Z if three computers are tested at random?
 - A. $Z = \{1, 2, 3, 4\}$
 - B. $Z = \{0, 1, 2, 3, 4\}$
 - C. $Z = \{0, 1, 2, 3\}$
 - D. $Z = \{1, 2, 3, 4, 5\}$
- 13-15: The table below shows the probability of answering True on a 3 question true-false quiz.

Sample Space	Number of True
TTT	3
TTF	2
TFT	2
TFF	1
FTT	2
FTF	1
FFT	1
FFF	0

13. What is the probability of getting one True?



- A. $\frac{1}{8}$ B. $\frac{3}{8}$ C. $\frac{5}{8}$ D. $\frac{7}{8}$
 14. What is the probability of getting three True?
 A. $\frac{1}{8}$ B. $\frac{3}{8}$ C. $\frac{5}{8}$ D. $\frac{7}{8}$
 15. What is the probability of not getting true?
 A. $\frac{1}{8}$ B. $\frac{3}{8}$ C. $\frac{5}{8}$ D. $\frac{7}{8}$

LESSON 1: ILLUSTRATING A RANDOM VARIABLE (DISCRETE AND CONTINUOUS RANDOM VARIABLE)



What's In

Observe:

Random variable

X: {1, 2, 3, 4}

Y: {0, 1}

Z: {1, 2, 3}

Algebraic variable

$10 + x = 13$

$y = 3x + 2$

$x + y + z = 5$

1. What can you say about the variable in random variable?
2. What can you say about the variable in algebraic variable?
3. What is/are the difference/s of the variable in random variable and in algebraic variable?



What's New

A quantitative variable is a variable which can have a numerical value. It differs in amount or quantity. But is this variable can only be counted? Or is it possible to be measurable? To know more on the different types of quantitative variable do the activity 1.

Activity 1:

Write C if the given is countable and M if the given is measurable.

1. A dozen of eggs
2. Time to complete a task
3. The height of the horse
4. Number of questions in math test
5. Number of chairs inside the classroom

Questions:

1. How do you say that it is countable?



2. How do you say that it is measurable?
3. If in a dozen of eggs one was broken, do you consider the broken egg as a whole? Why? Why not?
4. A tape measure was used to measure the height of the horse. Do you think the height of the horse will be the same if a ruler is used? Why? Why not?



What is It

A **random variable** is a set whose elements are the numbers assigned to the outcomes of an experiment. It is usually denoted by uppercase letters such as X , whose elements are denoted by lower case letters x_1, x_2, x_3 , and so on.

Examples of Random Variables:

1. X : Tossing a coin
2. Y : Arrival of cars at a tollbooth during a minute
3. Z : Determining the number of people who arrive at a store in a ten-minute interval
4. A : Measuring the lengths of cars produced in factory
5. B : Sampling the volume of liquid nitrogen in a storage tank
6. C : Number of women among 10 newly hired teachers
7. D : Cost (rounded to the nearest Php) of a Statistics book
8. E : Number of eggs a hen lays
9. F : The amount of milk obtained from a cow
10. G : Amount of rainfall in the different cities in Metro Manila

The outcomes for random variables and their associated probabilities can be organized into distributions. The two types of distributions are discrete and continuous distribution.

Two Types of Random Variable

Discrete random variable has a finite number of possible values.

Examples:

1. The number of French fries a fast food restaurant serves everyday.
It is discrete because French fries in a fast food is serve as a whole.
2. The number of Filipinos infected by the COVID-19 everyday.
It is discrete because Filipinos infected by COVID 19 are counted as a whole. There is no $\frac{1}{2}$ people.
3. Randomly selecting 30 people who consume softdrinks everyday
It is discrete because people who consume softdrinks are counted as a whole. There is no $\frac{1}{2}$ people.
4. Counting the number of people who arrive at store during a one-hour period
It is discrete because people who arrive at store are counted as a whole. There is no $\frac{1}{2}$ people.
5. The number of eggs that a Tapsilog consume in a day
It is discrete because eggs consume in tapsilog business counted as a whole even it was cracked.



A random variable is **continuous random variable** if it assumes an infinite number of values. They are obtained by measuring. They often include fractions and decimals. Since continuous data must be measured, answer must be rounded because of the limits of the measuring device. Usually, answers are rounded to the nearest given unit.

Examples:

1. Measuring the length of newly design gown
It is continuous because length is obtained with the use of measuring tool.
2. The speed traveled by a vehicle
It is continuous because speed is obtained with the use of speedometer.
3. Volume of salt water in the container
It is continuous because volume is obtained with the use of measuring tool like cylinder.
4. Weight (in grams) of 5 randomly selected students of Grade 11
It is continuous because weight is obtained with the use of weighing scale.
5. The amount of milk obtained from a goat.
It is continuous because amount of milk is obtained with the use of measuring tool.



What's More

Identify whether the underlined words in the following statements are discrete or continuous. Write **D** if it is **discrete** and **C** if it is **continuous**.

1. The number of cars sold by a car dealer yearly
2. The number of applicants who have applied for a job vacancy
3. Time required for a vehicle to cover 50 km
4. Number of airplanes in NAIA airport
5. Headcounts of students during the earthquake drill
6. Population of ants inside the cave
7. Scores of students in 30-item test
8. Number of patient arrivals per hour in medical clinic
9. Cost (rounded to nearest Php) of Statistics book
10. Average temperature in Baguio city



What I Have Learned

Fill in the blanks with the correct word/words.

1. A _____ is a set whose elements are the numbers assigned to the outcomes of an experiment.
2. It is usually denoted by _____ and elements are denoted by _____.
3. The two types of random variables are _____ and _____.



4. A discrete random variable has _____ outcome.
5. A _____ has uncountable number of outcome.
6. Suppose we flip a coin several times and count the number of heads. The number of heads could be any whole number. We could say that it is _____ random variable.
7. Suppose the manager mandates that all male employees must have a height between 64 and 74 inches. The height of the employees is _____ random variable.
8. Since continuous data must be measured, answer must be _____ because of the limits of the measuring device.



What I Can Do




Research Problem: The Relation between Height and Shoe Size in Adolescents
Task:

1. Collect some data about the research problem by asking your family members about their:
 - A. sex
 - B. age
 - C. shoe size
2. Use the formula to compute the height of your family members

Boys:
Height in centimeters = 5.3 times shoe size plus 133

Girls
Height in centimeters = 4.5 times shoe size plus 140
3. Based on the data you collected, are height and shoe size related? Why? Why not?
4. Using the data you have, identify which of these are discrete and continuous variables.

Rubrics:

	 3 pts	 2 pts	 1 pt.
1. Followed the directions and understood the concept			
2. Used unique strategy			
3. Displayed neat and tidy work			



4. Finished the task completely			
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Assessment

Choose the letter that best answers the question.

1. The correct example of discrete variable is _____.
 - A. Time taken to commute to work
 - B. The sum of the numbers rolled on a pair of dice
 - C. Sam ran 100 meters in 10.2 seconds
 - D. Weight of car travelling everyday
2. Which of the following variables can assume only a finite number of values and can be determined by simply counting?
 - A. Continuous Random Variable
 - B. Discrete Random Variable
 - C. Discrete Probability Distribution
 - D. Pay Off Value
3. Which of the following random variables is continuous?
 - A. The number of learning packets delivered in a day
 - B. The number of bottled water sold in a day
 - C. The number of people tested positive everyday
 - D. The amount of time it takes to conduct swab-test
4. Which of the following doesn't belong to the group?
 - A. Number of patients a doctor sees in a day
 - B. The Friday-night attendance in a cinema
 - C. The number of defective light bulbs
 - D. The distance between two persons
5. Which of the following random variables is NOT continuous?
 - A. Amount of gasoline in a car
 - B. Lifetime of an AAA battery
 - C. Number of goals scored by a football team
 - D. Time it takes to commute to work



Additional Activities

Read each item then choose the letter that corresponds to the correct answer.

1. This takes numerical values that describe outcomes of a chance process.
 - A. Probability distribution
 - B. Random variable
 - C. Standard deviation
 - D. Sample space
2. What is the term used for the set of all possible outcomes in an experiment?



- A. Random Variable
 - B. Statistics
 - C. Sample Space
 - D. Variable
3. Which of the following is a discrete random variable?
- A. The average amount of electricity consumed
 - B. The number of patients in a hospital
 - C. The amount of paint used in repainting a building
 - D. The average weight of female athletes
4. Which of the statements refers to discrete random variable?
- A. A discrete random variable has a number of possible values.
 - B. A discrete random variable has a countable number of possible values.
 - C. A discrete random variable has a measurable number of possible values.
 - D. A discrete random variable has an infinite number of possible values.
5. The correct examples of discrete variable.
- A. Time taken to commute to work
 - B. The sum of the numbers rolled on a pair of dice
 - C. Sam ran 100 meters in 10.2 seconds
 - D. Weight of car travelling everyday

LESSON 2: FINDING THE POSSIBLE VALUES OF A RANDOM VARIABLE



What's In

Write **T** if the sample space given is true and **F** if it is not. If your answer is F, write the correct sample space.

- | | |
|--|-------------------------------|
| 1. Tossing a coin | $S = \{H, T\}$ |
| 2. Rolling a die | $S = \{1, 2, 3, 4, 5, 6\}$ |
| 3. A card is pick from 4 cards, which spell the word MATH | $S = \{m, a, t, h\}$ |
| 4. Picking 2 marbles, one at a time, from a bag that contains 2 blue and 2 red marbles | $S = \{(B,B), (R,R), (R,B)\}$ |
| 5. Tossing 2 coins | $S = \{HH, HT, TT\}$ |



What's New



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To find out if you are ready to learn this new topic, do the following activity:

Activity 1:

List the sample space of the following experiments.

Experiment	Sample space
1. Tossing a two coins	
2. Answer 3 questions with true/false	
3. Gender of the children if a family has two children	
4. Randomly getting three fruits in a jar with replacement consisting of 3 apples and 2 oranges	
5. Randomly choosing two names out of 8 people without replacement: John, Jack, Mary, Jhen, Cloth, Reny, Judy, Jess	

Questions:

1. Supposed you are interested in the number of head that came out in experiment 1, what are the possible outcomes?
2. Supposed you are interested only on true answer, what are the possible outcomes?
3. Supposed you are interested only on a child that has a gender of a boy, what is the possible outcome?
4. What are the possible outcome in experiment 3 if you are interested on getting only apples?
5. Supposed you are only interested on two names that start with J, what are the possible outcomes?



What is It

If two coins are tossed, what numbers can be assigned for the outcome of heads that will occur? If a dice is rolled, what number can be assigned for the outcome that gives a sum of 12? The answers to these questions require an understanding of random variables.

For instance, if two coin is tossed, the set of possible outcomes (S) of the experiment is:

$$S = \{TT, TH, HT, HH\}$$



Since we are interested in the number of heads that came out in the experiment, then we can assign:

- 0 no head came out
- 1 1 head came out
- 2 2 heads came out

Thus, we can write:

Sample Space	Number of Heads
TT	0
TH	1
HT	1
HH	2

From the table above, instead of writing **Number of Heads**, we can assign it as set X whose elements (x_1, x_2, x_3) are 0, 1, 2. In symbol,

$$X = \{0, 1, 2\}$$

Then X is called a **random variable**.

Example 1: Supposed a three questions was answered by true or false. Let X be the random variable consisting of True answer. Find the values of the random variable X .

Sample Space	Number of True
TTT	3
TTF	2
TFT	2
TFF	1
FTT	2
FTF	1
FFT	1
FFF	0

$$X = \{0, 1, 2, 3\}$$

Example 2: In a family with 2 children, let Y be the random variable consisting of girls. Find the values of the random variable Y .

Sample Space	Number of Girls
GG	2
GB	1
BG	1
BB	0

$$Y = \{0, 1, 2\}$$



Example 3: A jar contains 3 apples and 2 oranges. Three (3) fruits are randomly chosen with replacement. Let Z be the random variable representing an apple. Find the values of the random variable Y.

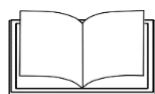
Sample Space	Number of Apple
AAA	3
AAO	2
AOA	2
OAA	2
AOO	1
OAO	1
OOA	1
OOO	0

Z = {0, 1, 2, 3, 4, 5}

Example 4: Randomly choosing two names out of 8 people without replacement: John, Jack, Mary, Jhen, Cloth, Reny, Judy, and Jess
Let X be the random variable representing a name that starts with J. Find the values of random variable X.

Sample Space	Name that starts with J
John, Jack	2
Jen, Mary	1
Cloth, Reny	0
Judy, Jess	2

Y = {0, 1, 2}



What's More

Illustrate the random variable by completing the table.

- Three coins are tossed. Let X be the random variable representing the number of tails occur. Find the values of the random variable X.

Possible outcome	Number of Tail



X = _____

2. A shipment of 4 cellphones contains 1 defective. If a store receives three of these cellphones at random, list the elements of the sample space S using the letters D for defective and N for non-defective respectively. Let X be the variable representing the defective, find the values of the random variable X.

Possible Outcome	Number of defective

X = _____





What I Have Learned

Suppose four coins are tossed simultaneously and the number of heads are recorded. Let x be the number of head appeared. If your task is to find the number of head appeared, what is the first step that you will do?

Suppose two balls are drawn in succession without replacement from an urn containing 5 red balls and 6 blue balls. What is a possible value of the random variable for the red balls? How did you get it?



What I Can Do

Luke has an ice cream stand. He made four flavors of ice cream: chocolate, mango, ube and oreo. Luke wishes to analyze if chocolate ice cream was bought by how many customers so that he can add more numbers of chocolate ice cream in a stand. Luke has collected data from the last week and check what kind of ice creams were bought by the first 4 person in each transaction. The data is shown in the table.

C – Chocolate

M – Mango

U – Ube

O – Oreo

Weekly Sales of the first 4 Person




Week	Ice cream bought
1	CCCU
2	CCUU
3	CMUO
4	OOUC
5	UMMM
6	CCCC
7	UOCC

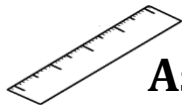
Task:



1. Let X be the random variable representing the number of chocolate occur. Find the values of the random variable X .
2. Based on the result, should Luke spend money to make more chocolate ice cream? Why? Why not?

Rubrics:

	 3 pts	 2 pts	 1 pt.
1. Followed the directions and understood the concept			
2. Used unique strategy			
3. Displayed neat and tidy work			
4. Finished the task completely			



Assessment

Read each item carefully then choose the letter that corresponds to the correct answer.

1. Two coins are tossed simultaneously and the number of tails are recorded. Let x be the number of tail appeared. What is x ?
 A. random variable C. Variable
 B. qualitative variable d. Probability variable
2. If two coins are tossed, which is not a possible value of the random variable for the number of tails?
 A. 0 B. 1 C. 2 D. 3
3. Which of these is not the possible values of the random variable for the number of girls of a family having 3 children?
 A. 1 B. 2 C. 3 D. 4
4. What is the sample space if two coins are tossed?
 A. {H, T}
 B. {HH, TT}
 C. {HH, HT, TH, TT}
 D. {HHH, TTT, HHT, HTH, HTT, THH, THT, TTH}
5. Given: Number of defects in a batch of 20 units. Let X be the random variable of the number of defects. What are the possible values of the random variable?
 A. $X \leq 20$ C. $X = 0, 1, 2, 3, \dots 20$
 B. $X \geq 20$ D. $X = 1, 2, 3, 4, 5, \dots 20$
6. Narra tree is 82 to 115 ft. high. Let x be the height of the Narra tree. What are the possible values of the random variable?



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LESSON 3: PROBABILITY DISTRIBUTION



What's In

In your Math 10 class in the junior high school, you learned how to find the probability of a particular event. Let us see if you can recall.

Find the probability of the following events.

1. Getting an odd number in a single roll of a die
2. Getting a sum of 11 when two dice are rolled
3. The probability that all children are girls if a couple has three children
4. Getting an ace card when a card is drawn from a deck of cards
5. Getting a blue ball from a box containing 3 red and 6 blue balls



What's New

Many variables in business, education, engineering, and other areas can be analyzed by using probability distribution.

Activity 1:

A basketball team has to play 3 games during the elimination round of a tournament. What is the probability that they will win:

- a. 0 games?
- b. 1 game?
- c. 2 games?



What is It

A probability distribution shows the relative probability that each outcome of an experiment will happen.

Example 1:



A basketball team has to play 3 games during the elimination round of a tournament. Let X be the random variable representing the winning games. Find the probability of each of the values of the random variable X .

Solution:

To find the probability of each values of the random variable X , the following steps may be followed:

1. Determine the sample space.

Sample Space:

$x = \{WWW, WWL, WLW, LWW, WLL, LWL, LLW, LLL\}$

2. Count the number of wins in each outcome in the sample space and assign this number to this outcome.

Possible Outcomes	Value of the Random Variable X (number of wins)
WWW	3
WWL	2
WLW	2
LWW	2
WLL	1
LWL	1
LLW	1
LLL	0

3. There are four possible values of the random variable X representing the number of wins. These are 0, 1, 2, and 3.

Number of wins	Probability $P(X)$
0	$\frac{1}{8}$
1	$\frac{3}{8}$
2	$\frac{3}{8}$
3	$\frac{1}{8}$

There are 8 possible outcomes and no win occur once, so the probability that we shall assign to the value of the random variable 0 is $\frac{1}{8}$.

There are 8 possible outcomes and 1 win occur three times, so the probability that we shall assign to the value of the random variable 1 is $\frac{3}{8}$.

There are 8 possible outcomes and 2 wins occur three times, so the probability that we shall assign to the value of the random variable 2 is $\frac{3}{8}$.



There are 8 possible outcomes and 3 wins occur once, so the probability that we shall assign to the value of the random variable 3 is $1/8$.

Thus, the Probability Distribution and the corresponding probability of each the values of the random variable X are shown below.

The Probability Distribution of the Probability Mass Function of Discrete Variable X

Number of winning	0	1	2	3
Probability P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Example 2: In a family with 4 children, let Y be the random variable representing the number of children who are boys. Find the probability of each of the values of the random variable Y.

Solution:

Following the steps:

1. Determine the sample space.

Sample Space:

$x = \{BBBB, BBBG, BBGB, BBGG, BGBB, BGBG, BGGB, BGGG, GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG\}$

2. Count the number of wins in each outcome in the sample space and assign this number to this outcome.

Possible Outcomes	Value of the Random Variable Y (number of boys)
BBBB	4
BBBG	3
BBGB	3
BGBB	3
GBBB	3
BBGG	2
BGBG	2
BGGB	2
GBBG	2
GBGB	2
GGBB	2
BGGG	1



GBGG	1
GGBG	1
GGGB	1
GGGG	0

3. There are five possible values of the random variable Y representing the number of wins. These are 0, 1, 2, 3, and 4.

Number of boys	Probability P(Y)
0	$\frac{1}{16}$
1	$\frac{4}{16} = \frac{1}{4}$
2	$\frac{6}{16} = \frac{3}{8}$
3	$\frac{4}{16} = \frac{1}{4}$
4	$\frac{1}{16}$

There are 16 possible outcomes and no boys in the family will occur once, so the probability that we shall assign to the value of the random variable 0 is $1/16$.

There are 16 possible outcomes and 4 outcomes for having one boy in the family will occur, so the probability that we shall assign to the value of the random variable 1 is $\frac{4}{16}$ or in simplest form we have $1/4$.

There are 16 possible outcomes and 6 outcomes for having two boys in the family will occur, so the probability that we shall assign to the value of the random variable 2 is $\frac{6}{16}$ or in simplest form we have $3/8$.

There are 16 possible outcomes and 1 outcomes for having four boys in the family will occur, so the probability that we shall assign to the value of the random variable 3 is $\frac{4}{16}$ or in simplest form we have $1/4$.

Thus, the Probability Distribution and the corresponding probability of each the values of the random variable X are shown below.

The Probability Distribution of the Probability Mass Function of Discrete Variable Y

Number of boys	0	1	2	3	4
Probability P(Y)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Example 3: Construct a discrete probability of an experiment where the sum of the number shown on a pair of dice in a single throw is considered. Find the probability of getting a sum of 5, 6, 7, 8 and 9.



Solution:

Die A \ Die B	1	2	3	4	5	6
1	[1,1] = 2	[1,2] = 3	[1,3] = 4	[1,4] = 5	[1,5] = 6	[1,6] = 7
2	[2,1] = 3	[2,2] = 4	[2,3] = 5	[2,4] = 6	[2,5] = 7	[2,6] = 8
3	[3,1] = 4	[3,2] = 5	[3,3] = 6	[3,4] = 7	[3,5] = 8	[3,6] = 9
4	[4,1] = 5	[4,2] = 6	[4,3] = 7	[4,4] = 8	[4,5] = 9	[4,6] = 10
5	[5,1] = 6	[5,2] = 7	[5,3] = 8	[5,4] = 9	[5,5] = 10	[5,6] = 11
6	[6,1] = 7	[6,2] = 8	[6,3] = 9	[6,4] = 10	[6,5] = 11	[6,6] = 12

Sum of 5	[4,1], [3,2], [2,3], [1,4]	4
Sum of 6	[5,1], [4,2], [3,3], [2,4], [1,5]	5
Sum of 7	[6,1], [5,2], [4,3], [3,4], [2,5], [1,6]	6
Sum of 8	[6,2], [5,3], [4,4], [3,5], [2,6]	5
Sum of 9	[6,3], [5,4], [4,5], [3,6]	4

The discrete probability distribution is shown below:

Sum of x	5	6	7	8	9
P(x = x)	$\frac{4}{36} = \frac{1}{9}$	$\frac{5}{36}$	$\frac{6}{36} = \frac{1}{6}$	$\frac{5}{36}$	$\frac{4}{36} = \frac{1}{9}$

There are 36 possible outcomes and rolling the dice that shows a sum of 5 has 4 outcomes, so the probability that we shall assign to the value of the random variable 5 is $\frac{4}{36}$ or in simplest form we have $1/9$.

There are 36 possible outcomes and rolling the dice that shows a sum of 6 has 5 outcomes, so the probability that we shall assign to the value of the random variable 6 is $5/36$.

There are 36 possible outcomes and rolling the dice that shows a sum of 7 has 6 outcomes, so the probability that we shall assign to the value of the random variable 7 is $\frac{6}{36}$ or in simplest form we have $1/6$.

There are 36 possible outcomes and rolling a dice that shows a sum of 8 has 5 outcomes, so the probability that we shall assign to the random value of the variable 8 is $5/36$.

There are 36 possible outcomes and rolling a dice that shows a sum of 9 has 4 outcomes, so the probability that we shall assign to the value of the random variable 9 is $\frac{4}{36}$ or in simplest form we have $1/9$.

Properties of a Probability Distribution:

From the preceding examples, notice that the probability of each value of the random variable must be between or equal to 0 and 1. In symbol, $0 \leq P(X) \leq 1$. The sum of the probabilities of all values of the random variable must be equal to 1. In symbol, $\Sigma P(X) = 1$.



What's More

Check Your Progress

Two fair 4-sided spinners, each bearing the numbers 1-4 are spun. The number of points awarded, W , is given by the difference between the two scores. Construct a probability distribution of a score with a difference of 2 or more.



What I Have Learned

Fill in the blank.

A table of values that shows the probability of any of the outcomes of an experiment is called _____.

In constructing probability distribution, you need to find the _____ of every outcome.

The probability of each value of the random variable must be between or equal to _____ and _____. In symbol, _____.

The sum of the probabilities of all values of the random variable must be equal to _____. In symbol, _____.



What I Can Do

Random events such as tossing coins are used in almost all books on probability. It is also a simple and popular technique to reach a decision that is free from all biases and judgments. This technique also doesn't involve any machines to






deliver result. It is the most trusted technique to resolve problems. Back to ancient time, the result of tossing coin was considered to be an expression of divine will. But is flipping a coin really fair?

Conduct an experiment to identify if really tossing a coin gives a fair result.

1. Toss a coin 50 times. Then record the outcome.
2. Construct a probability distribution on getting a head
3. Construct a probability distribution of getting a tail?
4. What are your observations during flipping a coin?
5. What is your conclusion based on the results you got?
6. Do you still think tossing a coin to decide who'll pay for lunch is a good idea?

Rubrics:

	 3 pts	 2 pts	 1 pt.
1. Followed the directions and understood the concept			
2. Used unique strategy			
3. Displayed neat and tidy work			
4. Finished the task completely			



Assessment

Carefully read each item then write the letter that corresponds to the correct answer.

1. Fair dice are rolled and the numbers that appear are added. What is the probability that this sum is at least 5?
 A. $\frac{5}{36}$ B. $\frac{1}{6}$ C. $\frac{5}{12}$ D. $\frac{5}{6}$



2. A fair coin is flipped three times. What is the probability of seeing heads exactly once?
- A. $1/8$ B. $1/6$ C. $3/8$ D. $1/2$
3. A fair coin is flipped three times. What is the probability of seeing heads at least once?
- A. $1/8$ B. $1/6$ C. $6/7$ D. $7/8$
4. The Sum Rule states that the sum of the probabilities of all instances of a random variables equals ____.
- A. -1 B. 0 C. 1 D. 2
5. Two fair dice are rolled and the numbers that appear are added. What is the probability that this sum is exactly 3?
- A. $1/18$ B. $1/12$ C. $1/4$ D. $1/6$
6. Let set $A = \{1, 2, 3, 4\}$ and set $B = \{2, 3, 5, 7\}$. One number is chosen at random from each set and their sum is calculated. What is the probability that this sum is exactly 6?
- A. $1/16$ B. $3/16$ C. $1/4$ D. $3/8$
7. Let set $A = \{1, 2, 3, 4\}$ and set $B = \{2, 3, 5, 7\}$. One number is chosen at random from each set and their sum is calculated. What is the probability that this sum is less than 6?
- A. $5/16$ B. $1/3$ C. $3/8$ D. $5/8$
8. If a fair coin is tossed twice, what is the probability of seeing heads exactly once?
- A. $1/3$ B. $1/2$ C. $2/3$ D. $3/4$
9. In a valid probability distribution, the probabilities must:
- A. Be between 0 and 1 and the sum of the probabilities is 1
- B. Be between 0 and 1 and the sum of the probabilities is less than 1
- C. Be between 0 and 1 and the sum of the probabilities is a positive number



D. Be between 0 and 1 and the sum of the probabilities is greater than 1

10. Find the missing value for $P(X=5)$ in the probability distribution:

Number of Toys | Probability

0		0.03
1		0.16
2		0.30
3		0.23
4		0.17
5		?

A. 0.01

B. 0.11

C. 0.21

D. 0.31



Additional Activities

Construct a probability distribution for rolling a single die.

SUMMATIVE TEST

Read each item carefully. Then, choose the letter of the correct answer.

1. Which one of the following is discrete data?

- A. Ana is 150 cm tall.
- B. Ana ran 100 meters in 15 seconds.
- C. Ana weighs 50 kg.
- D. Ana has two brothers and one sister.

2. Which of these is continuous variable?

- A. The number of mangoes produced by my mangoes tree each year
- B. Number of texts you send each day



- C. The distance that a cyclist rides each year
 - D. The number of people that it would take to paint a house
3. Which one of the following is discrete data?
- A. speed of a car
 - B. time needed to finish the test
 - C. number of death per day attributed to COVID 19
 - D. amount of sugar in a cup

4-6: Consider the following discrete probability distribution:

X	0	1	2	3	4
P(X)	0.3	0.2	0.1	0.15	0.25

4. What is the sample space of the experiment?
- A. {0.1, 0.15, 0.2, 0.25, 0.3}
 - C. {0, 1, 2, 3, 4}
 - B. {0.1, 0.15, 0.2, 0.25}
 - D. {0, 1, 2, 3}
5. What are the sample points associated to $0.1 \leq P(X) \leq 0.3$?
- A. {1, 2, 3, 4}
 - C. {0, 1, 2, 3}
 - B. {0, 1, 2, 3, 4}
 - D. {1, 3}
6. What are the sample points associated to $P(X) \leq 0.2$?
- A. {1, 2}
 - B. {1, 2, 3}
 - C. {1}
 - D. {2, 3}

For questions 7-9.

7. Suppose two balls are drawn in succession without replacement from an urn containing 5 red balls and 6 blue balls. Which is a possible value of the random variable for the red balls?
- A. 2
 - B. 3
 - C. 4
 - D. 5
8. From question number 7, what is the sample space?
- A. $S = \{RR, RB, BR, BB\}$
 - B. $S = \{RR, RB, BR\}$
 - C. $S = \{RRR, RBR, BRR, BBB, RRB\}$
 - D. $S = \{RR, RB, BR, BBR\}$
9. If X be the random variable representing the blue balls, what is the value of Z?
- A. $X = \{1, 2, 3, 4\}$
 - C. $X = \{0, 1, 2\}$
 - B. $X = \{0, 1, 2, 3, 4\}$
 - D. $X = \{1, 2, 3\}$
10. Suppose four coins are tossed simultaneously and the number of heads are recorded. Let X be the number of head appeared. If your task is to find the number of head appeared, what is the first step you will do?



- A. Count the number of heads in each outcome in the sample space.
 - B. Assign a number for this outcome
 - C. Determine the sample space
 - D. All of these
11. What is the term used for the set of all possible outcomes in an experiment?
- A. Random Variable
 - B. Statistics
 - C. Sample Space
 - D. Variable
12. Which of the statements refers to discrete random variable?
- A. A discrete random variable has a number of possible values.
 - B. A discrete random variable has a countable number of possible values.
 - C. A discrete random variable has a measurable number of possible values.
 - D. A discrete random variable has an infinite number of possible values.
13. Which of the following doesn't belong to the group?
- A. Number of patients a doctor sees in a day
 - B. The Friday-night attendance in a cinema
 - C. The number of defective light bulbs
 - D. The distance between two persons
14. Which of the following variables can assume only a finite number of values and can be determined by simply counting?
- A. Continuous Random Variable
 - B. Discrete Random Variable
 - C. Discrete Probability Distribution
 - D. Pay off Value
15. You have a bag containing 3 red chips and 6 white chips and you draw 4 chips. Let random variable Y be the number of red chips drawn from the bag out of 4 draws without replacement. Find the values of the random variable Y .
- A. $Y = \{1, 2, 3, 4\}$
 - B. $Y = \{0, 1, 2, 3, 4\}$
 - C. $Y = \{0, 1, 2, 3\}$
 - D. $Y = \{1, 2, 3, 4, 5\}$
16. What is the sample space suppose two coins are tossed?
- A. $\{H, T\}$
 - B. $\{HH, TT\}$
 - C. $\{HH, HT, TH, TT\}$
 - D. $\{HHH, TTT, HHT, HTH, HTT, THH, THT, TTH\}$
17. A couple has four children. Let "b" = the number of boys they have. What are the possible instances of "b"?



- 

A

B

C



What I Know

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. D | 2. D | 3. D | 4. A | 5. A |
| 6. C | 7. D | 8. D | 9. A | 10. C |
| 11. D | 12. C | 13. B | 14. A | 15. A |

LESSON 1

What' In

1. Random variable used uppercase letter
2. Algebraic variable used lower case letter
3. Random variable shows a possible value unlike algebraic variable is an unknown value.

What's New

- | | | | | |
|------|------|------|------|------|
| 1. C | 2. M | 3. M | 4. C | 5. C |
|------|------|------|------|------|

Questions:

1. It can be counted.
2. It is measurable because it needs measuring instrument.
3. No because there is no $1/2$ egg, etc. egg counted only by 1, 2,
4. No, Because in terms of measurement precision is used. It depends on the instrument and the person measuring it.

What's More

- | | | | | |
|---|------|------|------|-------|
| D | 2. D | 3. C | 4. D | 5. D |
| D | 7. D | 8. D | 9. D | 10. C |

What I have Learned

1. random variable
3. discrete and continuous
5. continuous
7. continuous
2. upper case and lowercase
4. countable
6. discrete
8. rounded off

What I Can Do

Different answer

Assessment:

- | | | | | |
|------|------|------|------|------|
| 1. D | 2. B | 3. D | 4. D | 5. C |
|------|------|------|------|------|

Additional Activity

- | | | | | |
|------|------|------|------|------|
| 1. B | 2. C | 3. B | 4. B | 5. B |
|------|------|------|------|------|



LESSON 3

What? In

- | | | | | |
|------------|--------|--------|---------|--------|
| 1/2 | 2. 1/8 | 3. 1/8 | 4. 1/13 | 5. 2/3 |
| What's New | 1/3 | b. 2/3 | c. 1 | |

What's More

Two fair 4-sided spinners, each bearing the numbers 1-4 are spun. The number of points awarded, W , is given by the difference between the two scores. Construct a probability distribution of a score with a difference of 2 or more.

The Probability Distribution of the Probability Mass Function of Discrete Variable W

What I Can Do

Different answer

What I have Learned

1. sample space 2. Probability 3. 0 and 1 4. 1 5. $\sum P(X)=1$

Assessment:

1. D 2. D 3. D 4. C 5. A 6. B 7. A 8. A 9. A 10. B

Additional Activity

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

Summative Test

1. D 2. C 3. C 4. C 5. B 6. D 7. A 8. A 9. C 10. C
11. C 12. B 13. D 14. B 15. B 16. C 17. C 18. C 19. D 20. A

Development Team of the Module

Writer: Jenifer B. Mesias (SEHS)

Editors: Freddie V. Viernes (PSDS), Imelda B. Olvida (MHHS – Principal IV),
Claudia Ayroso (KNHS), Joseph V. Crame (CIS), Marie Jeanne M. De Chavez (MHS)

Internal Reviewer: Dominador J. Villafria (EPS-Mathematics)

Cover Illustrator & Layout Artist: Joel J. Estudillo (SNNHS)

Management Team:

Sheryll T. Gayola

Assistant Schools Division Superintendent
OIC, Office of the Schools Division Superintendent

Elisa O. Cerveza

Chief, CID
OIC, Office of the Assistant Schools Division Superintendent

Dominador J. Villafria

Education Program Supervisor-Mathematics

Ivy Coney A. Gamatero

Education Program Supervisor– LRMS

For inquiries or feedback, please write or call:

Schools Division Office- Marikina City

Email Address: sdo.marikina@deped.gov.ph

191 Shoe Ave., Sta. Elena, Marikina City, 1800, Philippines

Telefax: (02) 682-2472 / 682-3989



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