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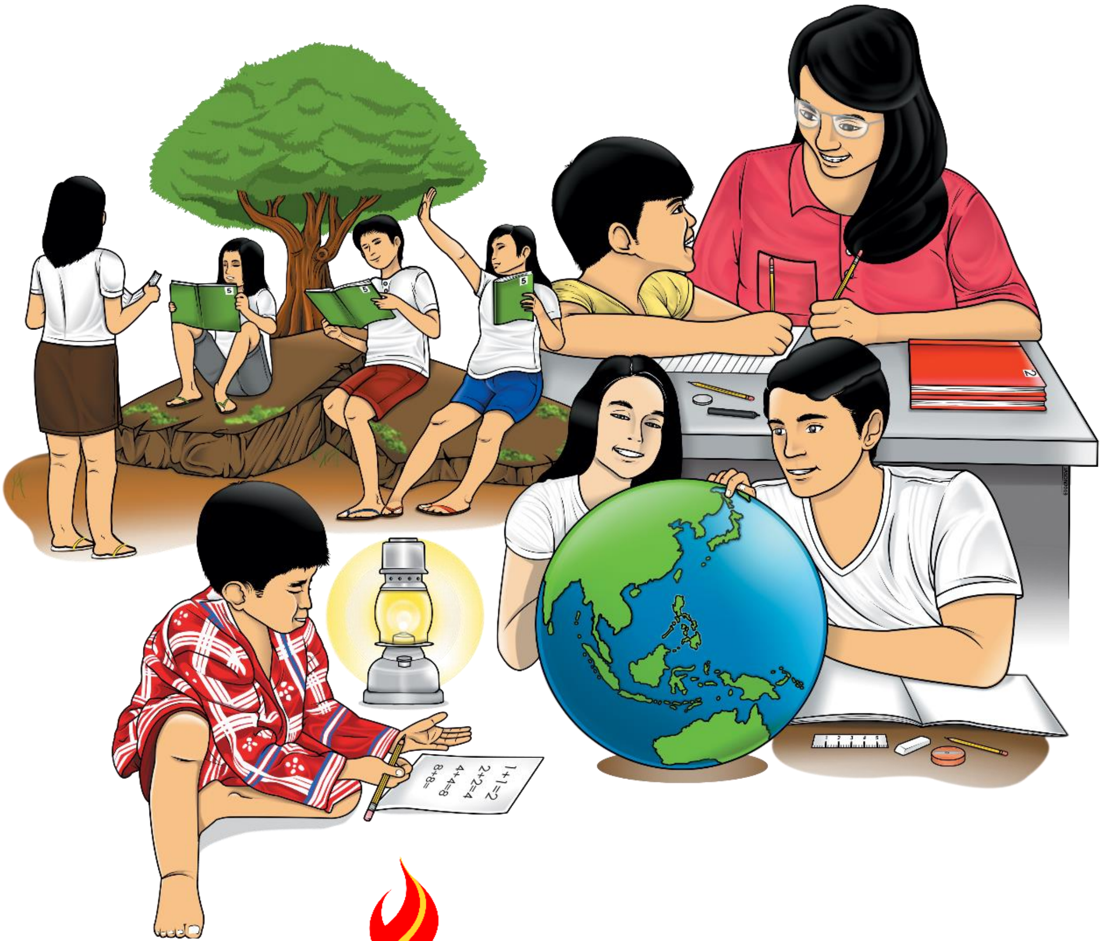
Mathematics

Quarter 2 – Module 8:
Operations on Radical Expressions



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Mathematics

Quarter 2 – Module 8:
Operations on Radical Expressions

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Lesson 1	Operations on Radical Expressions

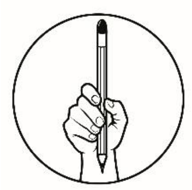
Lesson 1: Operations on Radical Expressions

Radical expressions are common in the formulas in businesses to calculate the unknown variables about depreciation, inflation and interests. Engineers also use radicals for measurements and calculations. Auto mechanics used radicals to calculate the car engine's efficiency. Rather than use a calculator to approximate the values of these radicals, they may express the answer as an exact value in radical form. In order for them to obtain the accurate and exact value in the calculations, they must have the knowledge and skills of the different operations on radical expressions.

What I Need to Know

The learners will be able to:

- Perform operations involving radical expressions.
- Solve simple problems involving operations on radicals.



What I Know

Find out how much you already know about the contents of the module. Write the letter of the correct answer in each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. Which of the following is a square root of 196?
   a. 98 b. 14 c. 392 d. 16

2. Between what two consecutive whole numbers does $\sqrt{31}$ lie?
   a. 4 and 5 b. 6 and 7 c. 5 and 6 d. 7 and 8

3. Subtract. $15\sqrt{5} - 17\sqrt{5}$
   a. $32\sqrt{10}$ b. $32\sqrt{5}$ c. $-2\sqrt{5}$ d. -2

4. Find $\sqrt{64}$.
a. –8 b. 32 c. 8 d. 4096

5. Evaluate and perform the operation of $-3\sqrt{20} - \sqrt{5}$
a. $-\sqrt{5}$ b. $-7\sqrt{5}$ c. $-3\sqrt{15}$ d. already simplified

6. Find the perimeter of a triangle whose side lengths are 15 cm, $8\sqrt{7}$ cm, and $\sqrt{112}$ cm. Give the answer as a radical expression in simplest form.
a. $(15 + 8\sqrt{7} + \sqrt{112}\,)$cm
b. $(15 + 12\sqrt{7}\,)\ cm$
c. $(15 + 24\sqrt{7}\,)\ cm$
d. $27\sqrt{7}$ cm

7. Simplify the expression $\sqrt{8y} + 5\sqrt{50y} - 2\sqrt{18y}$.
a. $13\sqrt{2y}$
b. $21\sqrt{2y}$
c. $(\sqrt{8} + 5\sqrt{50} - 2\sqrt{18})\sqrt{y}$
d. $882y$

8. Simplify the expression: $\frac{2}{-4x+4\sqrt{5x}}$
a. $-2x + 2\sqrt{5x}$
b. 1
c. $\frac{-x-\sqrt{5x}}{2x^2-10x}$
d. $\frac{1-\sqrt{5}}{12}$

9. Simplify the expression: $(\sqrt{18x} + 4\sqrt{8x} - \sqrt{50x})$.
a. $13\sqrt{2x}$
b. $6\sqrt{2x}$
c. $72x$
d. $(\sqrt{18x} + 4\sqrt{8} - \sqrt{50x})\sqrt{x}$

10. Simplify the expression: $(\sqrt{45b} + 4\sqrt{20b} - 2\sqrt{125b})$.
a. $\sqrt{5b}$
b. $12\sqrt{5b}$
c. $5b$
d. $(\sqrt{45b} + 4\sqrt{20} - 2\sqrt{125})\sqrt{b}$

11. Get the product in simplest form $\sqrt{2}\,(\sqrt{6} + \sqrt{7}\,)$.
a. $\sqrt{12} + \sqrt{14}$
b. $2\sqrt{6} + 2\sqrt{7}$
c. $\sqrt{26}$
d. $2\sqrt{3} + \sqrt{14}$

12. Write the product in simplest form. $\sqrt{9}\,(\sqrt{3} + 2\sqrt{2}\,)$
a. $9\sqrt{3} + 18\sqrt{2}$
b. $3\sqrt{11}$
c. $\sqrt{27} + \sqrt{72}$
d. $3\sqrt{3} + 6\sqrt{2}$

13. The area of a square garden is 85 square meters. Estimate the side length of the garden.
a. 9 m b. 7 m c. 11 m d. 12 m

14. Simplify $\sqrt{\frac{200}{49}}$.
a. $\frac{20}{7}$ b. $\frac{2}{7}$ c. $\frac{10\sqrt{2}}{7}$ d. $\frac{2\sqrt{10}}{7}$

15. A square stepping stone in Atlanta's Centennial Olympic Park measure $4\sqrt{2}$ feet on a side. Which of the following is TRUE of the area of the square stone?
a. The area is a indeterminate
b. The area is a rational number
c. The area is an irrational number
d. The area cannot be determined

What's In

Before we proceed, let us review simplifying radicals.

A radical expression is in its simplest form if:
- (i) there are ***no*** perfect nth powers as factors of the radicand,
- (ii) there are ***no*** fractions inside the radical, and
- (iii) there are ***no*** radicals in the denominator. The following laws of radicals can be useful to simplify radical expressions.

For any positive integers ***m*** and ***n*** and real numbers ***a*** and ***b***.

1. $(\sqrt[n]{a})^n = a$ ex. $\left(\sqrt[3]{5}\right)^3 = 5$
2. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ex. $\sqrt[5]{2x} = \sqrt[5]{2}\sqrt[5]{x}$ Product Rule for Radicals
3. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ ex. $\sqrt[7]{\frac{11}{y}} = \frac{\sqrt[7]{11}}{\sqrt[7]{y}}$ Quotient Rule for Radicals
4. $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$ex. $\sqrt[3]{7^{15}} = \left(\sqrt[8]{7}\right)^{15}$
5. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ ex. $\sqrt[3]{\sqrt[4]{3}} = \sqrt[(3)(4)]{3} = \sqrt[12]{3}$

Let us simplify the following radical expressions.

1. $\sqrt{49}$
2. $\sqrt[4]{81}$
3. $\sqrt[3]{-64n^3}$
4. $\sqrt[3]{250x^9}$
5. $\sqrt{\sqrt[3]{16}}$
6. $\sqrt[10]{625x^8}$
7. $\sqrt{\frac{3}{16}}$
8. $\sqrt[5]{\frac{-32y^5}{x^{20}}}$
9. $\frac{9}{\sqrt{3}}$
10. $\sqrt[3]{\frac{3}{5a^4}}$

What's New

Operations Involving Radicals

Recall that the radicals $x\sqrt{5}$ and $3\sqrt{5}$ are similar. The radicals $\sqrt[7]{2x}$ *and* $5y\sqrt[7]{2x}$ are also similar since they have the same index and same radicand. Radical expressions can be added, subtracted, multiplied or divided. For example, $\sqrt{3} + 5\sqrt{3} - 2\sqrt{3} = 4\sqrt{3}$ because $x + 5x - 2x = 4x$ is true for any value of x. Because x + 5x - 2x are expressions with like terms while $\sqrt{3} + 5\sqrt{3} - 2\sqrt{3}$radicals with same radicand. However, in $\sqrt{2} + 5\sqrt{3} - 2\sqrt{3}$, $\sqrt{2}$ and $\sqrt{3}$ cannot be combined because they are not similar. Hence, their sum can only be indicated as$\sqrt{2} + 3\sqrt{3}$. The next examples further illustrate the basic operations with radicals.

Addition and Subtraction of Radicals

Rule:
Simplify first the radicand, then perform addition and subtraction on the coefficient of the terms with similar radicand.

Examples:

1. $2\sqrt{3} + 4\sqrt{3}$
   Solution: Add the coefficients and annex their common radicand factor
   $$2\sqrt{3} + 4\sqrt{3} = (2+4)\sqrt{3}$$
   $$= 6\sqrt{3}$$
2. $5\sqrt{2} - \sqrt{2}$
   Solution: Subtract the coefficients and annex their common radicand factor
   $$5\sqrt{2} - \sqrt{2} = (5-1)\sqrt{2}$$
   $$= 4\sqrt{2}$$
3. $6\sqrt{3} + \sqrt{2} - 9\sqrt{2} + \sqrt{3}$

   Solution:
   $6\sqrt{3} + \sqrt{2} - 9\sqrt{2} + \sqrt{3} = \left(6\sqrt{3} + \sqrt{3}\right) + (\sqrt{2} - 9\sqrt{2})$ Group similar radicals
   $= 7\sqrt{3} - 8\sqrt{2}$ Add the coefficients of similar radicals

4. $\sqrt{m} - 8\sqrt[3]{m} - 7\sqrt{m} - \sqrt[3]{m}$
   Solution:
   $\sqrt{m} - 8\sqrt[3]{m} - 7\sqrt{m} - \sqrt[3]{m} = \left(\sqrt{m} - 7\sqrt{m}\right) + (-8\sqrt[3]{m} - \sqrt[3]{m})$ Group similar radicals
   $= -6\sqrt{m} - 9\sqrt[3]{m}$ Add the coefficients of similar radicals

5. $\sqrt[3]{54x} + 2\sqrt[3]{16x}$
   Solution: The radicals are dissimilar but may be similar after simplifying the radicand of each radical.
   $\sqrt[3]{54x} + 2\sqrt[3]{16x} = \sqrt[3]{27 \cdot 2x} + 2\sqrt[3]{8 \cdot 2x}$ Factor each radicand
   $= 3\sqrt[3]{2x} + 2(2\sqrt[3]{2x})$ Simplify
   $= 3\sqrt[3]{2x} + 4\sqrt[3]{2x}$ Multiply
   $= 7\sqrt[3]{2x}$ Add the coefficients

6. $2\sqrt{12} - \sqrt{48} + 3\sqrt{27}$

   Solution:
   $2\sqrt{12} - \sqrt{48} + 3\sqrt{27} = 2\sqrt{4 \cdot 3} - \sqrt{16 \cdot 3} + 3\sqrt{9 \cdot 3}$ Simplify the radicand if possible
   $= 2\left(2\sqrt{3}\right) - (4\sqrt{3}) + 3(3\sqrt{3})$ Simplify
   $= 4\sqrt{3} - 4\sqrt{3} + 9\sqrt{3}$ Multiply
   $= 9\sqrt{3}$ Combine the coefficients

7. $\frac{\sqrt{20}}{2} + \sqrt{\frac{5}{4}}$

Solution:

$\frac{\sqrt{20}}{2} + \sqrt{\frac{5}{4}} = \frac{\sqrt{20}}{2} + \frac{\sqrt{5}}{\sqrt{4}}$ Distribute the radical into numerator and denominator

$= \frac{\sqrt{4 \cdot 5}}{2} + \frac{\sqrt{5}}{\sqrt{4}}$ Simplify the radicand

$= \frac{2\sqrt{5}}{2} + \frac{\sqrt{5}}{2}$ Simplify

$= \frac{2\sqrt{5}+\sqrt{5}}{2}$

$= \frac{3\sqrt{5}}{2}$ or $\frac{3}{2}\sqrt{5}$ Add the fractions

8. $10\sqrt{\frac{49}{2}} - \sqrt{50}$

Solution:

$10\sqrt{\frac{49}{2}} - \sqrt{50} = 10\frac{\sqrt{49}}{\sqrt{2}} - \sqrt{25 \cdot 2}$ Distribute the radical into numerator and denominator

$= 10\frac{7}{\sqrt{2}} - 5\sqrt{2}$ Simplify

$= 10\left(\frac{7}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}\right) - 5\sqrt{2}$ Rationalize since there is a radical in the denominator

$= 10\left(\frac{7\sqrt{2}}{2}\right) - 5\sqrt{2}$ Get the LCD. LCD is 2

$= \frac{70\sqrt{2}}{2} - \frac{10\sqrt{2}}{2}$ Write as similar fractions

$= \frac{60\sqrt{2}}{2}$ Subtract

$= 30\sqrt{2}$ Simplify

9. $\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}}$

Solution:

$\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}} = \frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}}$ Distribute the radical into numerator and denominator

$= \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ Rationalize since there is a radical in the denominator

$= \frac{\sqrt{6}}{3} + \frac{\sqrt{6}}{2}$ Get the LCD. LCD is 6

$= \frac{2\sqrt{6}}{6} + \frac{3\sqrt{6}}{6}$ Write as similar fractions

$= \frac{5\sqrt{6}}{6}$ Add

10. Find the perimeter of a triangle with sides measure $6\sqrt{10}$ cm, $5\sqrt{10}$ cm and $11\sqrt{10}$cm.

Solution: To find the perimeter of a triangle, add the measures of the three sides.

$P = a + b + c$

$P = 6\sqrt{10} + 5\sqrt{10} + 11\sqrt{10}$

$P = 22\sqrt{10}$ cm

The perimeter of the triangle is $22\sqrt{10}$ cm.

Multiplication of Radicals

Rule:
In multiplying radical expressions, we consider the following:

- ***The indices are the same***.

Apply the product rule for radicals, $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$, and if necessary, simplify the resulting radicand.

Examples:

1. $\sqrt{3} \cdot \sqrt{2}$
   Solution:

$\sqrt{3} \cdot \sqrt{2} = \sqrt{3 \cdot 2} = \sqrt{6}$ Product Rule for Radicals

2. $\sqrt{7} \cdot \sqrt{14}$
   Solution 1:
   $\sqrt{7} \cdot \sqrt{14} = \sqrt{7 \cdot 14}$ Product Rule for Radicals
   $= \sqrt{98}$ Factor out the greatest square power of 98
   $= \sqrt{49 \cdot 2}$
   $= 7\sqrt{2}$ Simplify

   Solution 2:
   $\sqrt{7} \cdot \sqrt{14} = \sqrt{7} \cdot \sqrt{7 \cdot 2}$ Factor the radicand

   $= \sqrt{7^2 \cdot 2}$ Product Rule for Radicals
   $= 7\sqrt{2}$ Simplify

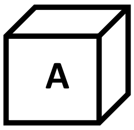
3. $\sqrt[4]{4y^3} \cdot \sqrt[4]{12y^2}$
   Solution:
   $\sqrt[4]{4y^3} \cdot \sqrt[4]{12y^2} = \sqrt[4]{48y^5}$ Product Rule for Radicals

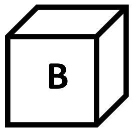
   $= \sqrt[4]{16y^4 \cdot 3y}$ Factor out the greatest fourth power
   $= 2y\sqrt[4]{3y}$ Simplify

4. $4\sqrt[3]{3}(2\sqrt[3]{6} - 7\sqrt[3]{5})$
   Solution:
   $4\sqrt[3]{3}(2\sqrt[3]{6} - 7\sqrt[3]{5}) = 4\sqrt[3]{3} \cdot 2\sqrt[3]{6} - 4\sqrt[3]{3} \cdot 7\sqrt[3]{5}$ Distributive Property
   $= 8\sqrt[3]{18} - 28\sqrt[3]{15}$ Product Rule for Radicals

5. $(\sqrt{5} - 2\sqrt{15})(\sqrt{5} + \sqrt{15})$
   Solution:
   $(\sqrt{5} - 2\sqrt{15})(\sqrt{5} + \sqrt{15}) = \sqrt{5}(\sqrt{5} + \sqrt{15}) - 2\sqrt{15}(\sqrt{5} + \sqrt{15})$ Distributive Property
   $= \sqrt{25} + \sqrt{75} - 2\sqrt{75} - 2\sqrt{225}$ Product Rule for Radicals
   $= 5 + 5\sqrt{3} - 10\sqrt{3} - 30$ Simplify
   $= -25 - 5\sqrt{3}$ Combine like terms

6. Compare the volume of the cubes.

$\sqrt{x}$ 



$\sqrt[6]{x^3}$

Solution: The formula to find the volume of a cube is $V = s^3$.

$s_A = \sqrt{x}$ $\qquad$ $s_B = \sqrt[6]{x^3}$

$V_A = (\sqrt{x})^3$ $\qquad$ $V_B = (\sqrt[6]{x^3})^3$ (substitute the value of s to the formula $V = s^3$)

$V_A = \sqrt{x^3}$ $\qquad$ $V_B = \sqrt[6]{x^9}$

$V_A = x\sqrt{x}$ $\qquad$ $V_B = \sqrt{x^3}$

$V_B = x\sqrt{x}$

The two cubes have the same volume.

When the indices are different,

Do the following:

1. Write the radicals as expressions with rational or fractional exponents.
2. Change the fractional exponents into similar fractions.
3. Rewrite the product as a single radical.
4. Simplify the resulting radicand if necessary.

Examples:

1. $\sqrt[3]{3} \cdot \sqrt{3}$

Solution:

$\sqrt[3]{3} \cdot \sqrt{3} = 3^{\frac{1}{3}} \cdot 3^{\frac{1}{2}}$ Write using rational exponents

$= 3^{\frac{2}{6}} \cdot 3^{\frac{3}{6}}$ Write the exponents in similar fractions

$= \sqrt[6]{3^2 \cdot 3^3}$ Write as a single radical

$= \sqrt[6]{3^5}$ Product of Powers Rule

$= \sqrt[6]{243}$ Simplify

2. $\sqrt{2} \cdot \sqrt[3]{2} \cdot \sqrt[4]{2}$

Solution:

$\sqrt{2} \cdot \sqrt[3]{2} \cdot \sqrt[4]{2} = 2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{1}{4}}$ Write using rational exponents

$= 2^{\frac{6}{12}} \cdot 2^{\frac{4}{12}} \cdot 2^{\frac{3}{12}}$ Write the exponents as similar fractions

$= \sqrt[12]{2^6 \cdot 2^4 \cdot 2^3}$ Write as a single radical

$= \sqrt[12]{2^{13}}$ Product of powers rule

$= \sqrt[12]{2^{12} \cdot 2}$ Factor out the greatest 12^{th} power

$= 2\sqrt[12]{2}$ Simplify

3. $\sqrt{2} \cdot \sqrt[3]{4}$

Solution:

$\sqrt{2} \cdot \sqrt[3]{4} \;=\; 2^{\frac{1}{2}} \cdot 4^{\frac{1}{3}}$ Write using rational exponents

$= 2^{\frac{3}{6}} \cdot 4^{\frac{2}{6}}$ Write the exponents in similar fractions

$= \sqrt[6]{2^3 \cdot 4^2}$ Write as a single radical

$= \sqrt[6]{2^3 \cdot (2^2)^2}$ 4 can be expressed in base 2, $4 = 2^2$

$= \sqrt[6]{2^3 \cdot 2^4}$ Power of a Power Rule of Exponents

$= \sqrt[6]{2^7}$ Add the exponents

$= \sqrt[6]{2^6 \cdot 2}$ Extract the radicand with power of 6

$= 2\sqrt[6]{2}$ Simplify

4. $5\sqrt[3]{a^2} \cdot 7\sqrt[5]{a^6}$

Solution:

$5\sqrt[3]{a^2} \cdot 7\sqrt[5]{a^6} \;=\; 5(a)^{\frac{2}{3}} \cdot 7(a)^{\frac{6}{5}}$ Write using rational exponents

$= 5(a)^{\frac{10}{15}} \cdot 7(a)^{\frac{18}{15}}$ Write the exponents as similar fractions

$= 5 \cdot 7 \cdot \sqrt[15]{a^{10} \cdot a^{18}}$ Write as a single radical

$= 35\sqrt[15]{a^{28}}$ Add the exponents

$= 35\sqrt[15]{a^{15} \cdot a^{13}}$ Factor out the greatest 15th power

$= 35a\sqrt[15]{a^{13}}$ Simplify

Division of Radicals

Rule:

To divide a radical by another radical, rewrite the expression into fractional form and then rationalize the result like what we do in simplifying radicals involving fractions. In some cases, it is much easier to apply the Quotient Rule for Radicals, that is $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, before rationalizing the denominator. To rationalize a denominator that has two terms where one or all of them having a square root or radical, we multiply the denominator by its conjugate. Conjugates are the sum and difference of the same two terms. The product of two conjugates results in a difference of two squares and has no radicals.

Examples:

1. $\sqrt{72} \div \sqrt{2}$

Solution 1:

$\sqrt{72} \div \sqrt{2} \;=\; \frac{\sqrt{72}}{\sqrt{2}}$ Rewrite in fraction form

$= \frac{\sqrt{72}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ Rationalize the denominator by multiplying $\frac{\sqrt{2}}{\sqrt{2}}$ since $\sqrt{2} \cdot \sqrt{2} = 2$

$= \frac{\sqrt{144}}{\sqrt{4}}$ Multiply

$= \frac{12}{2}$ Simplify the radicals

$= 6$

Solution 2:

$\sqrt{72} \div \sqrt{2} = \sqrt{\frac{72}{2}}$ Quotient Rule for Radicals

$= \sqrt{36}$ Simplify

$= 6$

2. $\sqrt{6x} \div \sqrt{3x}$

Solution:

$\sqrt{6x} \div \sqrt{3x} = \sqrt{\frac{6x}{3x}}$ Quotient Rule for Radicals

$= \sqrt{2}$ Simplify

3. $\sqrt[3]{18y^2} \div \sqrt[3]{12y}$

Solution:

$\sqrt[3]{18y^2} \div \sqrt[3]{12y} = \sqrt[3]{\frac{18y^2}{12y}}$ Quotient Rule for Radicals

$= \sqrt[3]{\frac{3y}{2}}$

$= \frac{\sqrt[3]{3y}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$ Rationalize the denominatorby multiplying $\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$ since $\sqrt[3]{2} \cdot \sqrt[3]{2^2} = 2$

$= \frac{\sqrt[3]{12y}}{2}$ Simplify

4. $\sqrt{2} \div (2 + \sqrt{3})$

Solution:

$\sqrt{2} \div \left(2 + \sqrt{3}\right) = \frac{\sqrt{2}}{2+\sqrt{3}}$ Rewrite in fraction form

$= \frac{\sqrt{2}}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$ Rationalize the denominator by multiplying to its conjugates

Take note that conjugates are the sum and difference of the same two terms. The product of two conjugates results in a difference of two squares and has no radicals.

$= \frac{2\sqrt{2}-\sqrt{6}}{4-3}$ Distributive Property

$= \frac{2\sqrt{2}-\sqrt{6}}{1}$ Simplify

$= 2\sqrt{2} - \sqrt{6}$ Write in lowest term

5. $\sqrt{xy} \div (\sqrt{x} - \sqrt{y})$

Solution:

$\sqrt{xy} \div \left(\sqrt{x} - \sqrt{y}\right) = \frac{\sqrt{xy}}{\sqrt{x}-\sqrt{y}}$ Rewrite in fraction form

$= \frac{\sqrt{xy}}{\sqrt{x}-\sqrt{y}} \cdot \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ Rationalize the denominator by multiplying to its conjugates

$= \frac{\sqrt{x^2y}+\sqrt{xy^2}}{x-y}$ Distributive Property

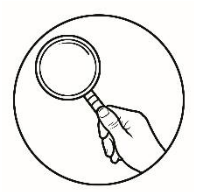
$= \frac{x\sqrt{y}+y\sqrt{x}}{x-y}$ Simplify

6. The area A of a parallelogram is $7\sqrt{12}$square units while its base b is $7\sqrt{3}$ units. Find the height of the parallelogram.
   Solution: The area of a parallelogram is given by the formula $A = bh$. To find the height of the parallelogram, we need to divide its area by the measure of the base.

$$h = A \div b$$

$$h = 7\sqrt{12} \div 7\sqrt{3} = \frac{7\sqrt{12}}{7\sqrt{3}} = \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2 \text{ units}$$

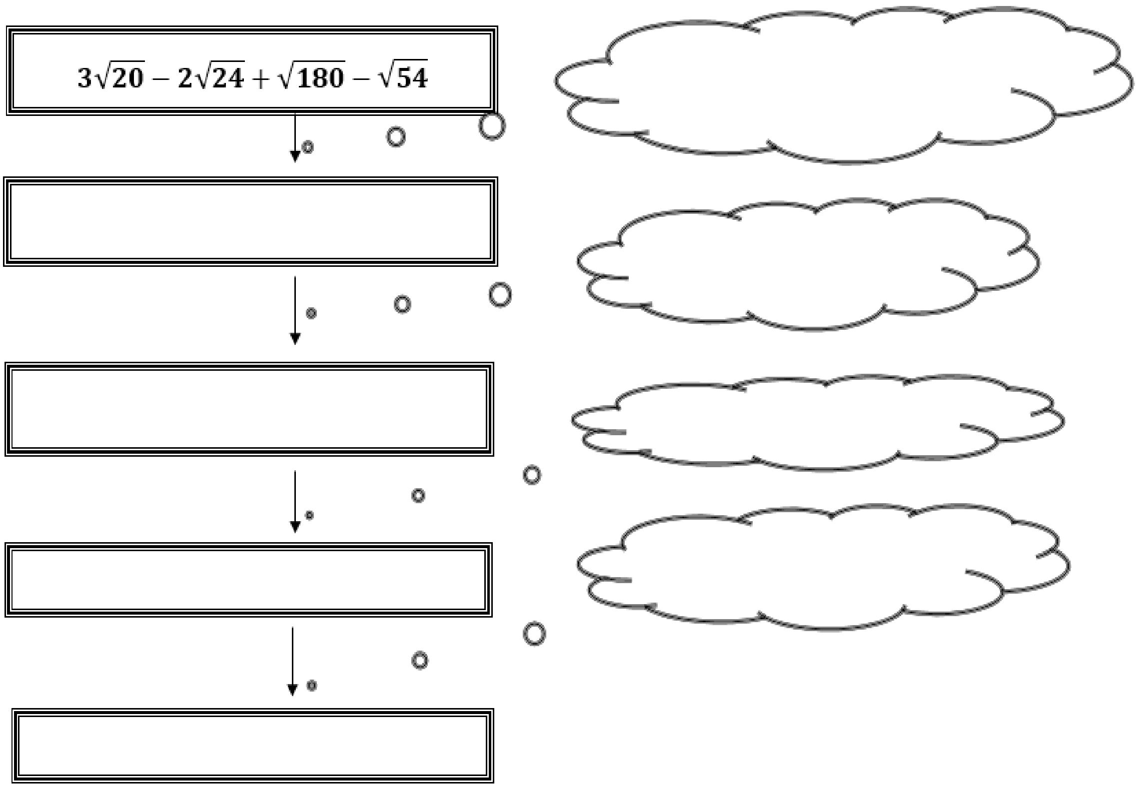
The height of the parallelogram is 2 units.



What is It

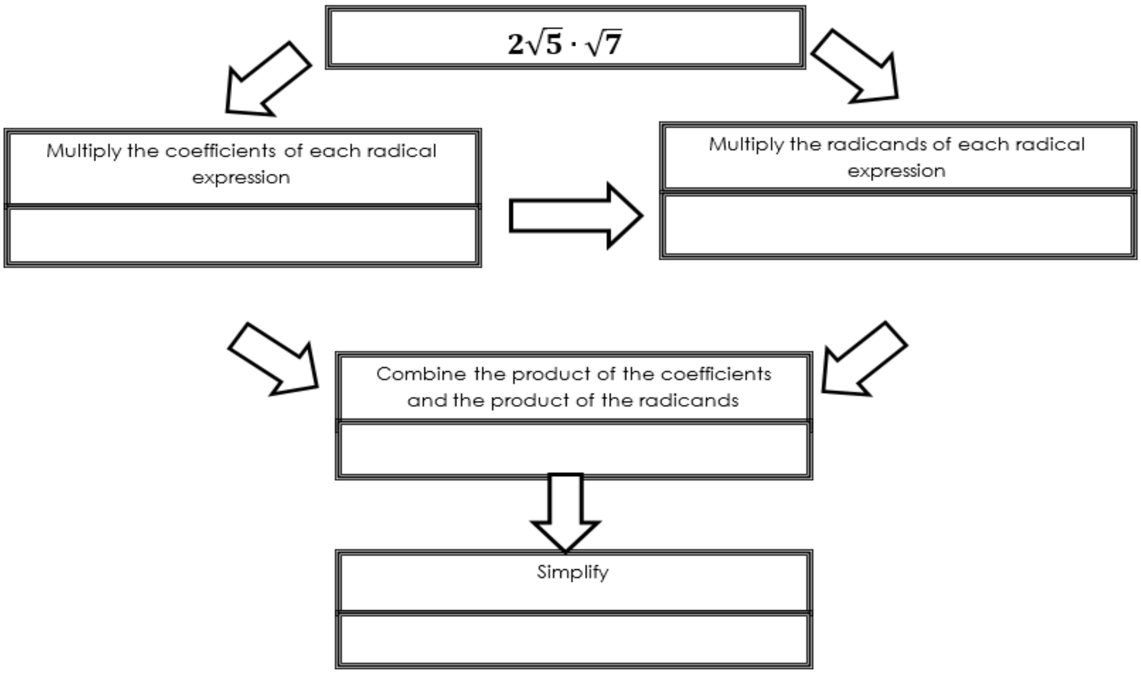
Perform the operation on radical expressions with the problem solving maps as guide.

ACTIVITY 1
ADDITION AND SUBTRACTION OF RADICAL EXPRESSIONS
STUDENT'S WORKSHEET ON MULTI-RULE MAP (MRM)



What's More

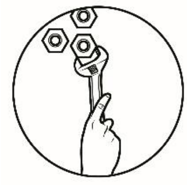
ACTIVITY 2
MULTIPLYING RADICAL EXPRESSIONS
STUDENT'S WORKSHEET ON MATH-BREAKER MAP (MBM-STRUCTURED)



What I Have Learned

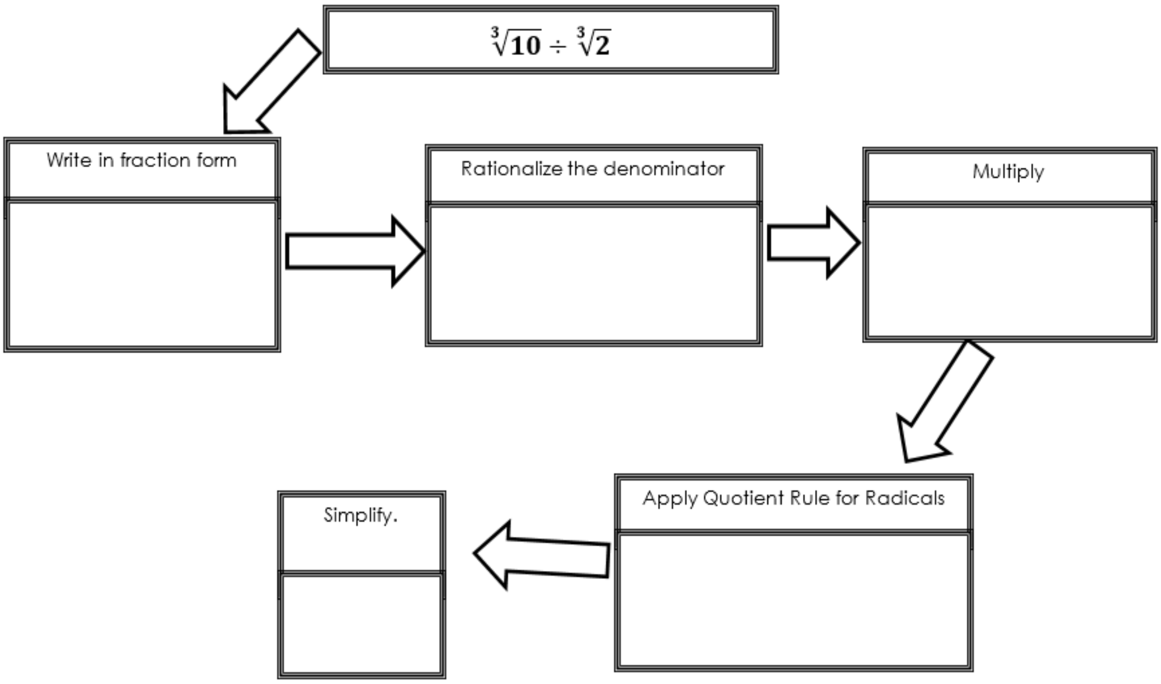
Operations with radicals include:

1. Addition and Subtraction: Combine like radicals by adding or subtracting their coefficients and copy their common radicand. The sum or difference of unlike radicals, unless written to equivalent or like radicals, cannot be expressed as a single term.
2. Multiplication: Multiply the coefficients and multiply the radicands then simplify. The product rule for radicals is applied for multiplying radicals.
3. Division: For all radicals whose denominators are not equal to zero, divide the coefficients and divide the radicands then simplify. The quotient rule for radicals is applied when dividing radicals.
4. To rationalize a denominator that has two terms where one or all of them having a square root or radical, we multiply the denominator by its conjugate. Conjugates are the sum and difference of the same two terms. The product of two conjugates results in a difference of two squares and has no radicals.



What I Can Do

ACTIVITY 3
DIVIDING RADICALS
STUDENT'S WORKSHEET ON MATH-BREAKER MAP (MBM- STRUCTURED)



Assessment

Read the questions carefully. Write the letter of the correct answer.

1. When do you say that two radicals are alike?
   a. when their indices are the same
   b. when their radicands are the same
   c. when their integral exponents are the same
   d. when their indices and radicands are the same

2. The product of $(5\sqrt{2}+4)(5\sqrt{2}-4)$ is __________.
   a. -34   b. 34   c. 26   d. -26

3. Which of the following set of radicals can be added or subtracted?
   a. $\sqrt{xy},\ 3\sqrt{5x}$ c. $3m\sqrt{n},\ m^3\sqrt{n}$
   b. $2\sqrt{7x},\ 3\sqrt{63x}$ d. $\sqrt[3]{xy^2},\ 3\sqrt[3]{x^2y}$

4. Three of the radicals below can be expressed as similar radicals. Which of the following is NOT similar to the other given radicals?

a. $\sqrt{44}$ b. $\sqrt{99}$ c. $\sqrt{110}$ d. $\sqrt{176}$

5. The combined form of $64\sqrt{3} - 4\sqrt{48} - 5\sqrt{48}$is ____.

a. $3\sqrt{8}$ b. $28\sqrt{3}$ c. $8\sqrt{3}$ d. $3\sqrt{48}$

6. Divide: $\frac{\sqrt[3]{48x^3y^7}}{\sqrt[3]{8x^5y}}$

a. $\frac{x^3\sqrt{8}}{y}$ b. $\frac{y^2\sqrt[3]{6x}}{x}$ c. $\frac{x^2\sqrt[3]{6}}{y}$ d. $\frac{y^3\sqrt{6}}{x}$

7. What is the equivalent of$\sqrt[3]{4} + \sqrt[5]{2}$ using exponential notation?

a. $4^{\frac{1}{3}} + 2^{\frac{1}{5}}$ b. $4^3 + 2^5$ c. 6^8 d. $6^{\frac{1}{8}}$

8. What is the result after simplifying $2\sqrt{3} + 4\sqrt{3} - 5\sqrt{3}$?

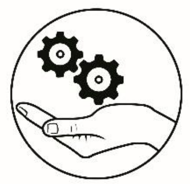
a. $-\sqrt{3}$ b. $\sqrt{3}$ c. $11\sqrt{3}$ d. $21\sqrt{3}$

9. Evaluate$(2^{-4})(2^{10})$.

a. 2^{14} b. 2^{-14} c. 64   d. $\frac{1}{2}$

10. The length of a rectangle is $2\sqrt{50}$ meters and its width is $3\sqrt{8}$ meters. Find its area.

a. $5\sqrt{58}$ sq. meters   c. $32\sqrt{2}$ sq. meters
b. 120 sq. meters   d. 124 sq. meters



Additional Activities

What's the Message?
Decode the message by performing the operations on radicals. Write the letter that corresponds to the answer in the box provided for.

$\sqrt{7} + \sqrt{7}$	$2\sqrt{6} - 4\sqrt{6}$	$5\sqrt[3]{3a} + 5\sqrt[3]{3a} - 9\sqrt[3]{3a}$	$6\sqrt{2} - 5\sqrt{8} + 2\sqrt{32}$
A	B	C	D
$5\sqrt{27} - 2\sqrt[3]{27} + \sqrt[3]{27}$	$\sqrt{12} - 5\sqrt[3]{8} - 7\sqrt{20}$	$\sqrt{4} \cdot \sqrt{6}$	$5\sqrt[3]{a} \cdot 4\sqrt{a^3}$
E	I	L	R
$\sqrt[4]{3}(\sqrt[4]{2} + \sqrt[4]{4})$	$\sqrt{75} \div \sqrt{3}$	$\sqrt[3]{4x^2} \div \sqrt[3]{x}$	$1 \div (\sqrt{x} - \sqrt{y})$
S	T	U	Y

Answer Box

$2\sqrt{7}$	$\sqrt[3]{3a}$	$\sqrt[3]{3a}$	$\sqrt[3]{4x}$	$20a\sqrt[6]{a^5}$	$2\sqrt{7}$	$\sqrt[3]{3a}$	$\frac{\sqrt{x}+\sqrt{y}}{x-y}$

$-2\sqrt{6}$	$\sqrt[3]{4x}$	$2\sqrt{3}-10-14\sqrt{5}$	$2\sqrt{6}$	$4\sqrt{2}$	$\sqrt[4]{6}+\sqrt[4]{12}$

$\sqrt[3]{3a}$	$20a\sqrt[6]{a^5}$	$15\sqrt{3}-3$	$4\sqrt{2}$	$2\sqrt{3}-10-14\sqrt{5}$	$-2\sqrt{6}$	$2\sqrt{3}-10-14\sqrt{5}$	$2\sqrt{6}$	$2\sqrt{3}-10-14\sqrt{5}$	5	$\frac{\sqrt{x}+\sqrt{y}}{x-y}$

E-Search

To further explore the concept learned today and if it's possible to connect the internet, you may visit the following links:
https://www.youtube.com/watch?v=4Gq3LPORQU
https://www.youtube.com/watch?v=X8zEiBT_zi0
https://www.youtube.com/watch?v=8zPqSDcA90I
https://www.youtube.com/watch?v=NxRDUc3p0QQ
Prentice Hall Algebra 1 Teaching Resource
https://1.cdn.edl.io/x1HxFjyVLo8pgAr2evQRFJb68knnbUOXdiUn0W67oRsODBSt.pdf
http://jeff560.tripod.com/mathword.html

PROBLEM – BASED WORKSHEET

TLE PROJECT

Norman and Benedict are Grade 9 students. Mr. Rivera, their TLE teacher, asked them to bring wood boards of any shapes and sizes to be used in making utility boxes. Norman brought a triangular board while Benedict brought a rectangular board whose dimensions are shown below.

$\sqrt{48}$ **dm** $\sqrt{27}$ **dm**

$\sqrt{125}$ **dm** $\sqrt{48}$ **dm**

LET'S ANALYZE

1. What grade level are Norman and Benedict in?
2. What is their project in TLE?
3. What shape did Norman use for the base of his box? What about Benedict?
4. If the boxes should be 5 dm in height, how much wood would be needed to make the sides of each box?
5. Whose utility box has the more space? Why did you say so?

Answer Key

WHAT I KNOW

1. B
2. C
3. C
4. C
5. B
6. B
7. B
8. C
9. B
10. A
11. D
12. D
13. A
14. C
15. B

WHAT'S IN

1. 7
2. 3
3. -4n
4. $5x^3\sqrt[3]{2}$
5. $\sqrt[3]{4}$
6. $\sqrt[5]{25x^4}$
7. $\frac{\sqrt{3}}{4}$
8. $\frac{-2y}{x^4}$
9. $3\sqrt{3}$
10 $\frac{\sqrt[3]{75a}}{5a^2}$

ADDITIONAL ACTIVITY

"Accuracy Builds Credibility"

ASSESSMENT

1. D
2. B
3. B
4. C
5. B
6. B
7. A
8. B.
9. C
10.B

PISA – BASED WORKSHEET

1. Grade 9
2. Making a utility box
3. Norman used a triangular base while Benedict's box has a rectangular base.
4. Norman's box needed $20\sqrt{3} + 25\sqrt{5} + 5\sqrt{175}$ sq. dm. of wood while Benedict's box needed $70\sqrt{3}$ sq. dm wood.
5. Norman's box has more space because it has a bigger volume.

WHAT IS IT

ACTIVITY 1
ADDITION AND SUBTRACTION OF RADICAL EXPRESSIONS
STUDENT'S WORKSHEET ON MULTI-RULE MAP (MRM)

$3\sqrt{20} - 2\sqrt{24} + \sqrt{180} - \sqrt{54}$

Factor the radicands.

$3\sqrt{4\cdot5} - 2\sqrt{4\cdot6} + \sqrt{36\cdot5} - \sqrt{9\cdot6}$

Get the roots of each factor.

$6\sqrt{5} - 4\sqrt{6} + 6\sqrt{5} - 3\sqrt{6}$

Group similar radicals.

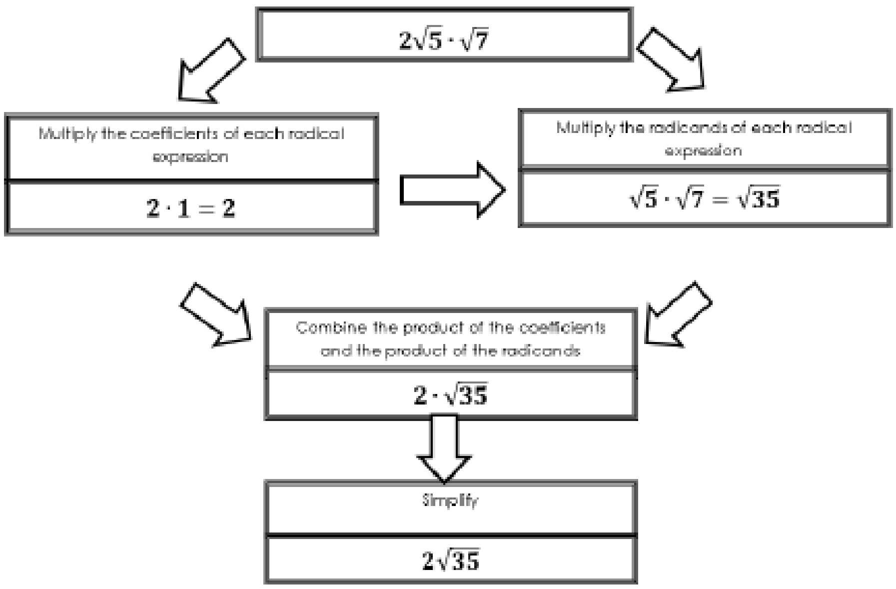
$(6\sqrt{5} + 6\sqrt{5}) - (4\sqrt{6} - 3\sqrt{6})$

Add the coefficients of similar radicals

$12\sqrt{5} - 7\sqrt{6}$

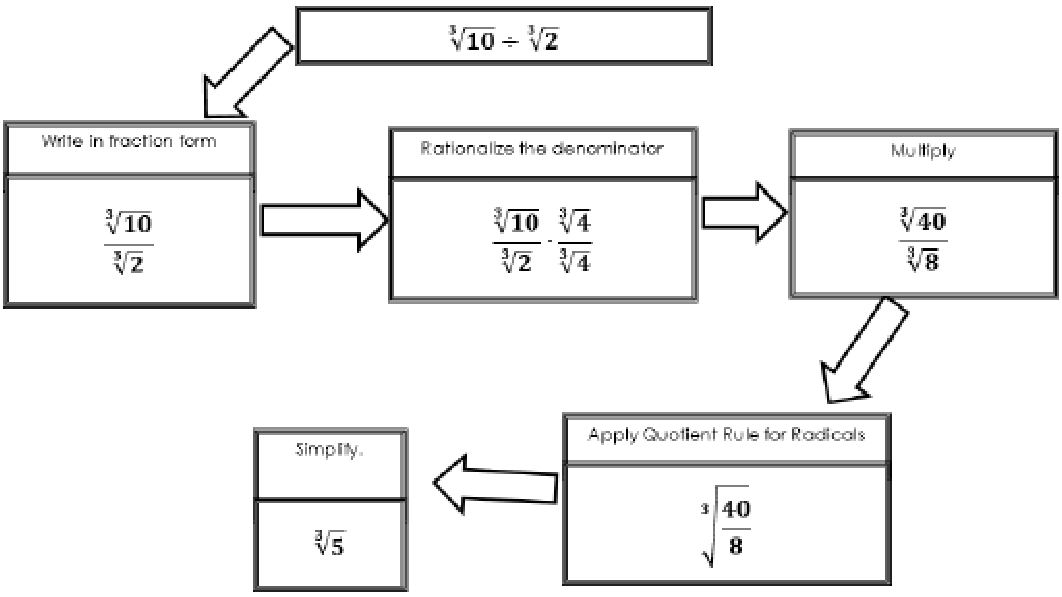
WHAT'S MORE

ACTIVITY 2
MULTIPLYING RADICAL EXPRESSIONS
STUDENT'S WORKSHEET ON MATH-BREAKER MAP (MBM-STRUCTURED)



WHAT I CAN DO

ACTIVITY 3
DIVIDING RADICALS
STUDENT'S WORKSHEET ON MATH-BREAKER MAP (MBM- STRUCTURED)



References

Nivera, G.C. &Lapinid, M. C.(2013). Grade 9 Mathematics Patterns and Practicalities. Salesiana Books by Don Bosco Press
Oronce, O. A. & Mendoza, M. O. C. (2003). Exploring Mathematics II Intermediate Algebra, Rex Book Store
DepEd Project Ease Mathematics Module 6 (downloaded from LRMDS)
Validated Problem Solving Maps Worksheet

For inquiries or feedback, please write or call:

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