

GENERAL MATHEMATICS

Quarter 1: Module 4.A Exponential Functions



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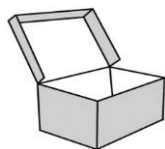
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What I Need to Know

Hello, Grade 11 learners! In this module, you will learn how to :

Represent real-life situations using exponential functions. **M11GM-Ie-3**

Distinguish between exponential function, exponential equation, and exponential inequality. **M11GM-Ie-4**

Solve exponential equations. **M11GM-Ie-f-1**

Solve exponential inequalities. **M11GM-Ie-f-1**

This module is divided into 4 lessons, namely:

Lesson 1: Representing Real-life Situations Using Exponential Functions.

Lesson 2: Exponential Functions, Equations, and Inequalities.

Lesson 3: Solving Exponential Equations

Lesson 4: Solving Exponential Inequalities

You can say that you have understood the lesson in this module if you can already:

1. define exponential functions;
2. illustrate exponential functions that represent-real life situations using exponential functions like population, half-life, and compound interest;
3. solve problems involving real-life situations using exponential functions;
4. distinguish exponential functions, equations, and inequalities;
5. use the properties in solving exponential equation and inequalities, and
6. solve exponential equations and inequalities.



What I Know

Choose the correct letter that corresponds to your answer.

1. The price of a car depreciates at 8% every year. a) How much will a car with an initial cost of P560 ,000 be after 3 years? b) How many years will a buyer wait for him to buy the car at half of its original price?

- A. Php 435 600.28 and 9 years C. Php 438 065.26 and 7 years
B. Php 436 065.28 and 8 years D. Php 434 065.29 and 8 years

2. The half- life of a radioactive substance is one hundred ninety-four days. How many days will it take for eighty percent of the substance to decay?

- A. 365 days B. 356 days C. 358 days D. 362 days

3. It is an expression of the form $(a)(b^x) + d$, where $b > 0$, $b \neq 1$.

- A. Exponential Equation C. Exponential Function
B. Exponential Expression D. Exponential Inequality

4. Which among the following is an exponential function?

- A. 2^{x+1} B. $y = 2^{x+1}$ C. $2^{x+1} > 2^2$ D. $2^{x+1} = 2^2$

5. Which of the following shows an exponential equation?

- A. 2^{x+1} B. $y = 2^{x+1}$ C. $2^{x+1} > 2^2$ D. $2^{x+1} = 2^2$

6. Given an exponential equation $3^{x+1} = 3^4$. What is the value of x?

- A. 1 B. 2 C. 3 D. 4

7. Solve $9^{x+1} = 27$.

- A. $-\frac{1}{2}$ B. $\frac{1}{2}$ C. -1 D. 1

8. Solve for $2^{5-x} = 16$.

- A. -9 B. -1 C. 1 D. 9

9. What is the base of the given exponential inequality $(\frac{1}{10})^x < (\frac{1}{10})^{x+4}$?

- A. $\frac{1}{100}$ B. 0 C. 10 D. 100

10. Solve for x: $10^x > 1000^{x+2}$.

- A. $x < -3$ B. $x > 3$ C. $x < -6$ D. $x > 6$





What's In

Given the following exponential expressions, complete the table of values and answer the questions that follow.

Exponential Expressions	$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$
2^x					
3^{-x}					
$(\frac{1}{2})^x$					

1. How do you evaluate the given exponential expression 2^x , 3^{-x} , and $(\frac{1}{2})^x$ if $x = 0, 1, 2, 3$, and 4 ?
2. What happens to the value of the expression if the exponent is equal to zero?
3. What happens to the values of the expression 2^x when the value of x increases?
4. What happens to the values of the expression $(\frac{1}{2})^x$ when the value of x increases?



What's New

Let us study this situation.

In a certain city with 5 barangays, the LGU (Local Government Unit) declared an outbreak status for COVID 19. The table below shows the status of COVID 19 for the first month.

First Month	No. of COVID19 Cases
Week 1	20
Week 2	40
Week 3	80
Week 4	160

Follow up Questions:

1. How many COVID19 cases were recorded during the first week?
2. What observations can you depict from the table? Explain your answer.
3. Write the formula to determine the number of Covid19 cases from week1 to week 4.



What is It

Exponential functions are used to model populations, carbon date artifacts, help coroners determine time of death, compute investments, and many other applications.

An **exponential function** with base b is a function of the form $f(x) = b^x$ or $y = b^x$ ($b > 0$, $b \neq 1$).

Example 1. Complete the table of values for the exponential functions $y = \left(\frac{1}{4}\right)^x$, 6^x and $y = (0.5)^x$.

Solution:

x	-2	-1	0	1	2
$y = \left(\frac{1}{4}\right)^x$	$y = \left(\frac{1}{4}\right)^x$ Given $= \left(\frac{1}{4}\right)^{-2}$ Substitute the value of x. $= (4^{-1})^{-2}$ Apply the Law of Negative exponent $= (4)^2$ Multiply the exponent $(-1)(-2)$ $= 16$ Evaluate 4^2	4	1	$\frac{1}{4}$	$\frac{1}{16}$
$y = 6^x$	$y = 6^x$ Given $= 6^{-2}$ Substitute the value of x. $= \frac{1}{6^2}$ Apply the Law of Negative exponent $a^{-n} = \frac{1}{a^n}$ $= \frac{1}{36}$ Evaluate 6^2 in the denominator	$\frac{1}{6}$	1	6	36



$y = (0.5)^x$	$y = (0.5)^x$				
	$= \left(\frac{1}{2}\right)^x$	Write 0.5 into a fraction form			
	$= \left(\frac{1}{2}\right)^{-2}$	Substitute the value of x			
	$= \left(\frac{1^{-2}}{2^{-2}}\right)$	Distribute the exponent -2 from the base $\frac{1}{2}$			
	$= \frac{1}{\frac{1}{4}}$	Apply the law negative exponent	2	1	0.5
	$= 1 \bullet \frac{4}{1}$	Multiply 1 from the reciprocal of $\frac{1}{4}$			0.25
	$= 4$	Cancel the common factor 1 and 1.			

Example 2. If $f(x) = 4^x$, evaluate $f(2)$, $f(-2)$, $f(0.4)$.

Solution: $f(x) = 4^x$

$f(2) = 4^2 = 16$
$f(-2) = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
$f(0.4) = 4^{0.4} = \sqrt[5]{4^2} = \sqrt[5]{16}$

Definition: Let b be a positive number not equal to 1. A transformation of exponential functions with base b is a function of the form $g(x) = a \cdot b^{x-c} + d$, where a , c and d are real numbers.

Applications of Exponential Functions

Compound interest, population growth, and exponential decay are the most common applications of exponential functions and their transformations in real-life.

1. Compound Interest: A starting amount of money called **principal** earns interest at the end of a given period of time such as one year. If the interest rate is compounded, the interest earned at the end of the period is added to the principal and this new amount will earn interest in the next period. The same process is repeated for each succeeding period. Interest previously earned will also earn interest in the next period.

Compound Interest

If a principal P is invested at an annual rate of r , compounded annually, then the amount after t years is given by $A = P(1+r)^t$.



Example: Mang Cardo deposits Php 12, 000.00 in the Veterans Bank that pays 3% compound interest annually. Define an exponential model for this situation. How much money will he have after 5 years?

Solution: Let t = time in years

Initially,

at, $t = 0$	Investment in 0 year	= 12, 000
at, $t = 1$	Investment in 1 year	= 12, 000 + 12, 000(0.03) = 12, 360.00
at, $t = 2$	Investment in 2 years	= 12, 360 + 12, 360(0.03) = 12, 730.80
at, $t = 3$	Investment in 3 years	= 12, 730.8 + 12, 730.8 (0.03) = 13, 112.72
at, $t = 4$	Investment in 4 years	= 13, 112.72 + 13, 112.72(0.03) = 13, 506.11
at, $t = 5$	Investment in 5 years	= 13, 506.11 + 13, 506.11(0.03) = 13, 911.29

It shows from the table, that the principal amount with the interest earned can be computed as follows:

Let t = time in years

$t = 0$	Php 12, 360
$t = 1$	Php 12, 000(1+0.03) = Php 12,000(1.03) = 12, 360.00
$t = 2$	Php 12, 000(1+0.03) ² = Php 12,000(1.03) ² = 12, 730.80
$t = 3$	Php 12, 000(1+0.03) ³ = Php 12,000(1.03) ³ = 13,112.72
$t = 4$	Php 12, 000(1+0.03) ⁴ = Php 12,000(1.03) ⁴ = 13, 506.11
$t = 5$	Php 12, 000(1+0.03) ⁴ = Php 12,000(1.03) ⁵ = 13, 911.29

Thus, the exponential model for this situation is $A = 12, 000(1.03)^t$ and the amount of money after 5 years is Php 13, 911.29.

2. Population Growth. At $t = 0$, initially there were 20 bacteria. Suppose that the bacteria doubles every 100 hours. Give an exponential model for the bacteria as a function of t .

Solution: Let y = number of bacteria

t = number of hours

Initially,

at $t = 0$	Number of bacteria = 20	or $y = 20$
at $t = 100$	Number of bacteria = 20(2)	or $y = 20(2)$
at $t = 200$	Number of bacteria = 20(2) ²	or $y = 20(2)^2$
at $t = 300$	Number of bacteria = 20(2) ³	or $y = 20(2)^3$
at $t = 400$	Number of bacteria = 20(2) ⁴	or $y = 20(2)^4$

An exponential model for this situation is $y = 20(2)^{\frac{t}{100}}$



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Exponential Models and Population Growth

Suppose a quantity y doubles every T units of time. If y_0 is the initial amount, then the quantity y after t units of time is given by $y = y_0(2)^{t/T}$.

The **half-life** of a radioactive substance is the time it takes for half of the substance to decay.

3. **Exponential Decay.** Suppose that the half-life of a certain radioactive substance is 10 days and there are 10g initially, determine the amount of substance remaining after 30 days and give an exponential model for the amount of remaining substance.

Solution: Let t = time in days
Initially,

at, $t = 0$	Amount of Substance = 10 g
at $t = 10$ days	Amount of Substance = $10 \text{ g } (\frac{1}{2}) = 5\text{g}$
at, $t = 20$ days	Amount of Substance = $10\text{g } (\frac{1}{2})^2 = 2.5\text{g}$
at, $t = 30$ days	Amount of Substance = $10\text{g } (\frac{1}{2})^3 = 1.25\text{g}$

An exponential model for this situations is $y = 50(\frac{1}{2})^{\frac{t}{10}}$

Exponential Functions and Half-life

If the half-life of a substance is T units, and y_0 is the amount of the substance corresponding to $t = 0$, then the amount y of substance remaining after t units of time is given by $y = y_0(\frac{1}{2})^{\frac{t}{T}}$

4. Natural Exponential Function.

While an exponential function may have various bases, a frequently used base is irrational number e , whose value is approximately 2.71828.

Definition

The **natural exponential function** is the function $f(x) = e^x$.



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Example: The predicted population of a certain city is given by $P = 250,000e^{(0.03y)}$, where y is the number of years after 2022. Predict the population for the year 2028.

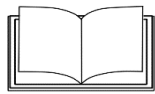
Solution: Given: The number of years from 2022 to 2028 is 6 so the value of $y = 6$.

$$P = 250,000e^{(0.03y)}$$

$$P = 250,000(2.71828)^{(0.03)(6)}$$

$$P = 299,304.30$$

Thus, the predicted population from year 2022 to 2028 is 299,304.



What's More

Solve the following:

1. A sky diver jumps from a reasonable height above the ground. The air resistance that she experiences is proportional to her velocity, and the constant of proportionality is 0.2. It can be shown that the download velocity of the sky diver at time t is given by:

$$v(t) = 80 (1 - e^{0.2t})$$

where t is measured in seconds and $v(t)$ is measured in feet per second (ft/s).

- What is the initial velocity of the sky diver?
- What is the velocity after 5 seconds? After 10 seconds?

2. The amount in grams of a radioactive substance present at a time t is

$$y = 250 \left(\frac{1}{2} \right)^{0.030t} \text{ where } t \text{ is time in days.}$$

- What is the amount present initially?
- What is the amount after 100 days?

2. The population of a city is $P = 1\,000\,000 (2^{0.02t})$ where t is measured in months.

- What is the initial population of the city?
- What is the population of the city after 50 months?



What I Have Learned

Fill in each blank with an appropriate word to complete the sentence.

1. An _____ with base b is a function of the form $f(x) = b^x$ or $y = b^x$ ($b > 0$, $b \neq 1$).
2. A transformation of exponential functions with base b is a function of the form _____.
3. If a principal P is invested at an annual rate of r , compound annually, then the amount after t years is given by _____.
4. The _____ of a radioactive substance is the time it takes for half of the substance to decay.
5. The _____ is the function $f(x) = e^x$



What I Can Do

Solve the following problems.

1. A radioactive substance decays in such a way that the amount of mass remaining after t days is given by the function $m(t) = 13e^{-0.015t}$ where $m(t)$ is measured in kilograms.
 - a. What is the mass at time $t = 0$
 - b. How much of the mass remains after 45 days?
2. A bacterial culture is growing according to the formula $y = 10\,000e^{0.6x}$ Where x is the time in days.
 - a. What is the number of bacteria after 1 week?
 - b. How many bacteria are there after 12 hours?

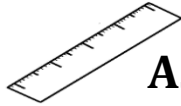
Rubrics for problem solving

Score	Description
15 points	Complete solutions and correct answer
10 points	Incomplete solutions and correct answer
5 points	Incomplete solutions and incorrect answer
No point earned	No output at all



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Assessment

Choose the correct letter that corresponds to the exact answer.

1. The cost of a new car is \$32,000. It depreciates at a rate of 15% per year. This means it loses 15% of each value each year. Find the value of the car when it is four years old.

- A. \$16 255 B. \$16 704 C. \$15 456 D. \$ 5 754

2. The population of a town is estimated to increase by 15% per year. The population today is 20 thousand. Find out what the population will be ten years from now.

- A. 70 911 people C 79 911 people B. 67 911 people D. 80 911 people

3. Mina bought a diamond ring for P60 000. If the value of the ring increases at a constant rate of 4% per year, how much will the ring be in 21 years?

- A. Php 136 726.08 C. Php 138 456.68
B. Php 135 720.9 D. Php 134 754.5

4. Suppose \$4000 is invested at 6% interest compounded annually. How much money will there be in the bank at the end of 5 years? At the end of 20 years?

- A. \$ 4 352.80 and \$ 13 828. 55 C. \$ 5 435.75 and \$ 13 456.6
B. \$ 5 352.90 and 12 828.54 D. \$ 5 345.65 and \$ 12 875.75

5. The local government projects that the town's population will grow at a constant rate of 32% per year. At this rate, how many years will it take to be five times of its current size?

- A. 5 years B. 7 years C. 4 years D. 6 years



Additional Activities

In the research study of a biologist, there are 30 bacteria which have been recorded at $t = 0$. Suppose that the bacteria double in every 100 hours, present an exponential model for the bacteria as a function of t . Write your complete solution in the box.





What's In

Answer the following:

1. If $f(x) = 5^x$, evaluate $f(2)$, $f(-2)$, $f(0)$, and $f(\frac{1}{2})$
2. In the previous lesson, what real-life situations are using exponential functions?
3. In the exponential function $f(x) = b^x$, x is the _____.
4. In the formula $A = P(1+r)^t$, What is P ?
5. What do you call the function $f(x) = e^x$?



What's New

Considering the items below, find for their similarities and differences. Then, answer the questions that follow.

a. $4^{x-2} = 64$

b. $y = 6^x$

c. $3^x \geq 27$

1. What are the similarities of a, b and c?
2. What makes a and c different from b?
3. What makes a and b different from c?



What is It

Here, we will distinguish exponential functions, exponential equations and exponential inequality.

Definition:

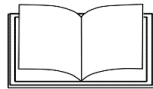
Exponential equation: It is an equation that involves the expression $b^{P(x)}$, where $b > 0$ and $P(x)$ is a polynomial.

Exponential Function: It is a function defined by $f(x) = a \cdot b^{P(x)} + h$ where a, b, h are real numbers; $a \neq 0$, $b > 0$ and $P(x)$ is a polynomial.

Exponential Inequality: An inequality of the $b^x < c$ or $b^x > c$.

Illustrative Examples:

- a. $f(x) = 4^{x-1}$ is an exponential function.
- b. $5^{2x} = 25$ and $3^x = 9^{x+1}$ are exponential equations.
- c. $2^{x+1} > 8$ is an exponential inequality.



What's More

Determine if the given item is an exponential function, an exponential equation, an exponential inequality or not any of them.

1. $g(x) = 7x^2$	Answer
2. $3 \geq (\frac{1}{3})^x$	Answer
3. $3^{2x} = y$	Answer
4. $2(10^{x+3}) = 300$	Answer
5. $6 < 12^{x+2}$	Answer



What I Have Learned

Fill in each blank with an appropriate word/s to complete the statement/s.

_____1_____ is an expression of the form _____2_____, where $(b>0, b \neq 1)$. We defined an exponential equation as an equation that involves the expression _____3_____ where $b>0$ and $P(x)$ is a _____4_____. _____5_____ is a function defined by $f(x) = a \cdot b^{P(x)} + d$. Where $a, b, h \in$ _____6_____, $a \neq 0$, _____7_____ and $P(x)$ is a polynomial. Exponential inequality An inequality of the _____8_____ or $b^x \geq c$.



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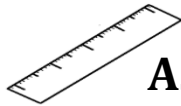


What I Can Do

Classify the given items below by writing them under the correct column.

$y = 5^x - 1$	$9^{x+1} = 3^x$	$3 \leq 3^x + 2$	$x^5 + 1 < 3x^3$
$100000 = 10^{2x}$	$f(x) = 3^x + 2$	$64^x + \frac{1}{2} = 0$	$\left(\frac{1}{3}\right)^{x+2} = \left(\frac{1}{9}\right)^x$
$3^x < (3^3)^{x-2}$	$8^x = \frac{1}{64}$	$4^x = 1024$	$y = 6^x - 1$

Exponential Function	Exponential Equation	Exponential Inequality



Assessment

Choose the letter that corresponds to your answer.

- Which of the following is an exponential function?
A. $3^{x+2} < 81$ B. $125 = 5^x$ C. $y = 7^{x-1}$ D. $64^x = 2^{2x+2}$
- In which of the following is $4^{x-1} = 16$ classified?
A. Exponential Equation C. Exponential Function
B. Exponential Inequality D. Exponential Model
- Which is true about an exponential function?
A. It involves radical expressions.
B. It contains the inequality symbol.
C. It is written in the form $f(x) = b^x$, where $b > 0$, $b \neq 0$
D. It has a numerical exponent only.
- Which of the following shows an exponential equation?
A. 2^{x+1} B. $y = 2^{x+1}$ C. $2^{x+1} > 2^2$ D. $2^{x+1} = 2^2$
- Which of the following is an exponential inequality?
A. $4^{x-2} > 64^{2x}$ B. $4x - 1 < 16x^2$ C. $y = 3^x$ D. $f(x) = 10^x$



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Additional Activities

Identify which of the following are exponential functions, exponential equations, and exponential inequalities.

1. $5 < 125^x$

2. $y = 100^x$

3. $7^{x+1} = 49$

4. $4^{x+2} = 64$

5. $\left(\frac{2}{3}\right)^x \geq \frac{21}{8}$

Lesson 3

Solving Exponential Equation.



What's In

A. Determine whether the following is an exponential function, exponential equation or exponential inequality.

1. $5^x = 125^{x+1}$

2. $81 < 3^x$

3. $f(x) = 4^{x-1}$

4. $10^{x+2} > 1000$

5. $y = 6^x$



What's New

Supply the missing parts of the given equation then, answer the questions that follow.

1. $2^3 = 8$

2. $4^3 = 64$



3. $4^4 = 256$

4. $3^4 = 81$

5. $5^3 = 125$

- A. How do you obtain 8 in item no.1?
 B. What is another way to represent 64 in exponential form?
 C. How many factors of 4 are there in 256?
 D. In item number 4, how will you find the value of the exponent if the base is 3?



What is It

Here, we will learn how to solve exponential equations with the same base.

Exponential equation is an equation involving exponential expressions, of the form $b^x = c$, where $b > 0$ and $b \neq 1$.

Example: a. $5^x = 25$ b. $2^x = 8^{x-4}$ c. $\left(\frac{1}{100}\right)^{x+3} = \left(\frac{1}{1000}\right)^x$

In solving exponential equations with the same base, we use the **One-to-one Property** of exponential functions.

If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$. It only means that, if the bases are the same or equal, then the exponents must be equal.

Example 1: Solve for $5^x = 25$.

Note: If you notice, the bases are not equal. We have to express 25 as 5^2 in order for both sides of the equation to have equal bases.

Solution:

$5^x = 25$	Given
$5^x = 5^2$	Since 25 can be written as $5 \bullet 5$, then rewrite 25 as base of 5, apply the product rule $b^m \bullet b^n = b^{m+n}$, so we have 5^2
$x = 2$	Apply the one-to-one property of exponent: if $b^x = b^y$, then $x_1 = x_2$



Checking: Substitute the value of $x = 2$ from the given exponential equation.

$$5^x = 25$$

$$5^2 = 5^2$$

$$25 = 25$$

Example 2: Solve for $2^x = 8^{x-4}$.

Note: If you notice, the bases are not equal but we can rewrite 8 in exponential form with base 2.

Solution:

$2^x = 8^{(x-4)}$	Given
$2^x = (2^3)^{(x-4)}$	Since 8 can be written as $2 \bullet 2 \bullet 2$, then express 8 as 2^3 by applying the product rule
$2^x = 2^{3(x-4)}$	Since the base are equal, we can apply the power to a power rule $(b^m)^n = b^{(m)(n)}$
$x = 3(x-4)$	Apply the one-to-one property of exponent: if $b^x = b^y$, then $x_1 = x_2$
$x - 3x = 3x - 12 - 3x$	Solve for x by applying Addition Property of Equality
$x - 3x = -12$	APE
$-2x = -12$	Combining like terms
$x = 6$	Apply Division Property of Equality

Checking: Substitute the value of $x = 6$ from the given exponential equation.

$$2^x = 8^{x-4}$$

$$2^6 = 8^{6-4}$$

$$2^6 = 8^2$$

$$64 = 64$$

Example 3: Solve for x $(\frac{1}{100})^{x+3} = (\frac{1}{1000})^x$.

Note: The bases of the given exponential equation are not equal. Then we can rewrite $\frac{1}{100}$ and $\frac{1}{1000}$ as same base of $\frac{1}{10}$. We know that 100 can be expressed as 10^2 and 1000 as 10^3 .



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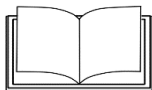
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Solution:

$(\frac{1}{100})^{x+3} = (\frac{1}{1000})^x$	Given
$(\frac{1}{10^2})^{x+3} = (\frac{1}{10^3})^x$	Since both denominators are not equal, express 100 and 1000 as a base of 10. 100 can be written as $10 \bullet 10$, and 1000 can be also written as $10 \bullet 10 \bullet 10$, then express 100 as 10^2 and 1000 as 10^3 by applying the product rule
$(\frac{1}{10})^{2(x+3)} = (\frac{1}{10})^{3(x)}$	Since the bases are equal ,apply the power to a power rule $(b^m)^n = b^{(m)(n)}$
$2x + 6 = 3x$	Apply the one-to-one property of exponent: if $b^x = b^y$, then $x_1 = x_2$
$-x = -6$	Solve for x by applying Addition Property of Equality
$x = 6$	Multiply both side by -1

Checking: Substitute the value of $x = 6$ from the given exponential equation.

$$\begin{aligned}
 (\frac{1}{100})^{x+3} &= (\frac{1}{1000})^x \\
 10^{-2(x+3)} &= 10^{-3(x)} \\
 10^{-2(6+3)} &= 10^{-3(6)} \\
 10^{-18} &= 10^{-18}
 \end{aligned}$$



What's More

Solve for the value of x in the following exponential equations.

- $2^x = 256$
- $10^x = 1000^{x-4}$
- $7^x = 49^{x+2}$
- $(\frac{1}{9})^x = (\frac{1}{27})^{x+2}$
- $4^{x+2} = 16^{x+1}$



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What I Have Learned

Given with the following exponential equation, supply the missing parts of the table by writing the correct solution and reason.

1. $6^x = 36$

$6^x = 36$	_____
_____	Since 36 can be written as $6 \bullet 6$, then rewrite 36 as base of 6, Apply the product rule $b^m \bullet b^n = b^{m+n}$, so we have 6^2
$x = 2$	_____

_____	Given
_____	Since 64 can be written as $4 \bullet 4 \bullet 4$, then express 64 as 4^3 by applying the product rule
$4^x = (4^3)^{(x-4)}$	_____
_____	Apply the one-to-one property of exponent: if $b^x = b^y$, then $x_1 = x_2$
$x = 3x - 12$	_____
$-2x = -12$	_____
_____	Multiply both side by $(-1/2)$



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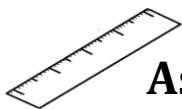
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What I Can Do

Complete the table below.

Exponential equation	Equal Base	Value of x
$5^x = \frac{1}{125}$		
$4^{3x+2} = 1024$		
$49^x = 16807^{x+4}$		
$6^{2x}, 6^{2-4x}$		



Assessment

Choose the letter that corresponds to the exact answer.

For item numbers 1 and 2, refer to the given equation $125^{x+2} = 25^{x-5}$

- What is the equal base of 125 and 25?
A. 5 B. 10 C. 25 D. 125
- What is the value of x in the given equation?
A. -16 B. -4 C. 4 D. 16
- What is the equal base for the given exponential equation $9^{x^2} = 27^{x-1}$?
A. 1 B. 3 C. 6 D. 9
- Solve for $3^x = 9^{x-2}$.
A. 2 B. 3 C. 4 D. 6
- Solve for $2^{5-x} = 16$.
A. -9 B. -1 C. 1 D. 9



Additional Activities

Solve for x.

- $3^{2x-3} = 3^{-x}$
- $8^{4x} = 8$
- $\frac{5^x}{5^x} = 5^{-2x}$
- $3^{-2x} \cdot 3^x = 27$
- $2^x \cdot \frac{1}{32} = 32$



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What's In

Solve the value of x:

1. $8^{2x} = 4$

4. $2^{x+4} = 2^3$

2. $4^{2x+1} = 32$

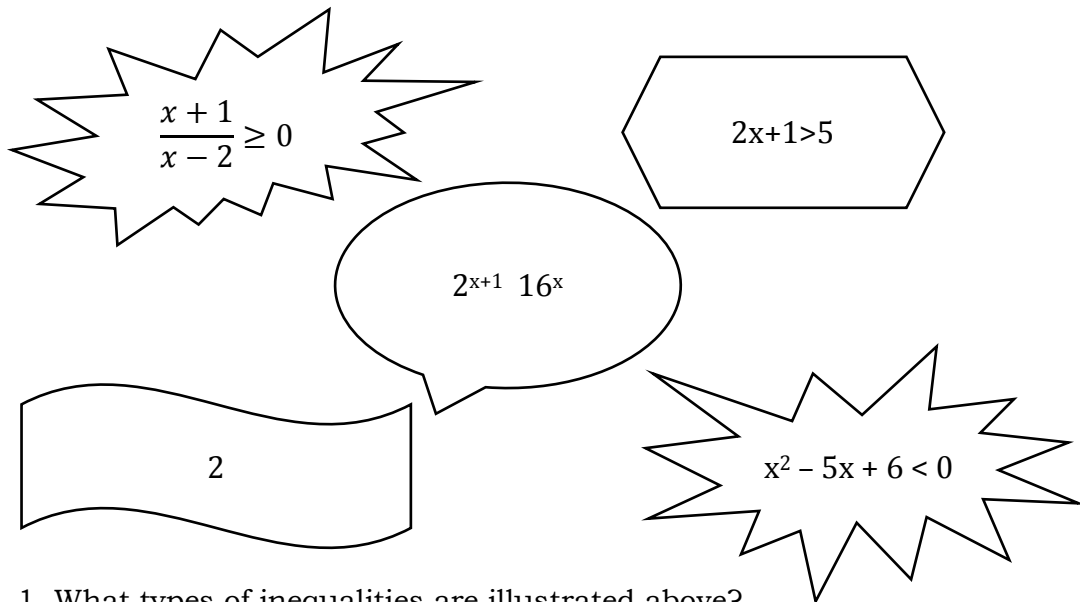
5. $16^x = 8$

3. $9^{x+4} = 27$



What's New

Consider the following inequalities and answer the questions that follow.



1. What types of inequalities are illustrated above?
2. How do we solve those inequalities?
3. Is there any rule in solving exponential inequalities?



What is It

Here, we will be studying the property of exponential inequalities that will help us in solving exponential inequalities.

Property of Exponential Inequalities

If $b > 1$, then the exponential function $y = b^x$ is increasing for all values of x . It means that $b^x < b^y$ if and only if $x < y$.

If $0 < b < 1$, then the exponential function $y = b^x$ is decreasing for all values of x . It means that $b^x > b^y$ if and only if $x < y$.

Example 1: Solve the inequality $2^x < 4^{x-2}$.

Solution: If you notice, the bases are not the same. However, 2 and 4 can be written by expressing the terms into the same base 2.

$2^x < 4^{x-2}$	Given
$2^x < 2^{2(x-2)}$	Express 4 as 2^2 for both sides of the inequality to have the same bases.
$x < 2x - 4$	Since $b > 1$, then the direction of the inequality is retained
$x+4 < 2x - 4 + 4$	To solve for x , Use APE.
$4 < 2x - x$	Combining like terms
$4 < x$	

Therefore, the solution set to the inequality is $\{x \in \mathbf{R} / x > 4\}$.

Example 2: Solve the inequality $(\frac{3}{2})^x \geq \frac{8}{27}$.

Solution: The bases are not the same but we can rewrite $\frac{8}{27}$ into $\frac{2^3}{3^3}$.

$(\frac{3}{2})^x \geq \frac{8}{27}$	Given
$(\frac{3}{2})^x \geq \frac{2^3}{3^3}$	Express 8 as 2^3 and 27 as 3^3
$(\frac{3}{2})^x \geq \{(\frac{3}{2})^{-1}\}^3$	Transform $\frac{2^3}{3^3}$ into $\{(\frac{3}{2})^{-1}\}^3$ by using the negative law of exponent to have the same bases.
$(\frac{3}{2})^x \geq (\frac{3}{2})^{-3}$	Multiply -1 and 3
$x \geq -3$	Since $b > 1$, then the direction of the inequality is retained



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Therefore the solution set to the inequality is $\{x \in \mathbf{R} / x \geq -3\}$.

Example 3: Solve the inequality $74 - 3x > 49^{-x-1}$.

Solution: The bases are not the same but ,7 and 49 can be written using 7 as the base.

$74 - 3x > 49^{-x-1}$	Given
$74 - 3x > 7^{2(-x-1)}$	Express 49 as 7^2 for both sides of the inequality to have the same bases.
$74 - 3x > 7^{-2(-x-1)}$	Since the inequality is already expressed in the same base , just drop the base , retain the exponent
$4 - 3x > -2(-x - 1)$	Multiply $2(-x - 1)$
$4 - 3x > -2x - 2$	Since $b > 1$, then the direction of the inequality is retained
$-3x + 2x > -2 - 4$	Use APE to solve for the value of x
$-x > -6$	Multiply both sides by negative 1
$x < 6$	Since we multiply both sides by negative, we have to reverse the inequality sign

Therefore, the solution set to the inequality is $\{x \in \mathbf{R} / x < 6\}$.



What's More

Solve for x:

1. $10^x > 100$

2. $(\frac{2}{3})^x \leq \frac{9}{4}$

3. $2^{2-x} < 64$

4. $8^{3-x} \geq 64^{-x-1}$

5. $(\frac{5}{7})^{x+1} \leq \frac{49}{25}$



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What I Have Learned

Fill in each blank with the correct answer.

1. The value of x in the inequality $(\frac{1}{5})^x \geq \frac{1}{25}$ is _____.
2. The equal base to the inequality $(\frac{1}{5})^x \geq \frac{1}{25}$ is _____.
3. If $b > 1$, then the exponential function $y = b^x$ is _____ for all.
4. If $0 < b < 1$, then the exponential function $y = b^x$ is _____ for all x .
5. If the base is greater than 1, then _____ of the inequality is retained.



What I Can Do

1. Give 3 examples of exponential inequalities and solve for the value of x . Write your brief explanation of how you solved for the value of x .
2. Interpret the Covid- 19 situation of Marikina City by using exponential inequality. (use the Marikina PIO report).

Scoring Rubrics					
Criteria	Beginning 1	Developing 2	Accomplished 3	Exemplary 4	Score
Neatness	The output was poorly created and difficult to read	The output was sloppy and difficult to read	The output is generally neat and readable	The output was done very neat and easy to read	
Accuracy	Majority of the data are incorrect and somehow irrelevant	The data are inaccurately represented, contain major errors	Data representation contains minor errors	All data are accurately represented	
Content (Application in real life)	Student lacks understanding of the concept	Students has limited understanding of the concept	Student's understanding of the concept is evident	Student's understanding of concept is clearly evident	

Time Management	The output was submitted late, 3-5 days after the deadline	The output was submitted late, 1-2 days after the deadline	The output was submitted on time	The output was submitted ahead of time.	
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Assessment

Choose the letter that corresponds to the exact answer.

- Which of the following is an exponential inequality?
A. $2^x = 4$ B. $2^x > 8$ C. $y = 2^{x+2} + 4$ D. $2^x + 4$
- Solve the inequality $2^x < 4^{x-2}$.
A. $x < -4$ B. $x > -4$ C. $x < 4$ D. $x > 4$
- What is the solution set to the inequality $2^x < 4^{x-2}$?
A. $\{x \in \mathbb{R} / x < -4\}$ B. $\{x \in \mathbb{R} / x > -4\}$ C. $\{x \in \mathbb{R} / x < 4\}$ D. $\{x \in \mathbb{R} / x > 4\}$
- Solve the inequality $4^{x+2} > 8$.
A. $x > -\frac{1}{2}$ B. $x > \frac{1}{2}$ C. $x < \frac{1}{2}$ D. $x < -\frac{1}{2}$
- What is the solution set in the given inequality $3^x > 9^{x+1}$?
A. $\{x \in \mathbb{R} / x < -2\}$ B. $\{x \in \mathbb{R} / x > -2\}$ C. $\{x \in \mathbb{R} / x < 2\}$ D. $\{x \in \mathbb{R} / x > 2\}$



Additional Activities

Solve for x:

- $9^x > 27$
- $\left(\frac{4}{5}\right)^x \leq \frac{125}{64}$
- $8^{5-x} < 64$
- $9^{3-2x} \geq 81^{-x-1}$
- $\left(\frac{1}{7}\right)^{x+1} \leq 49$



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Summative Test

1. The half-life of a radioactive substance is one hundred ninety-four days. How many days will it take for eighty percent of the substance to decay?
A. 365 days B. 356 days C. 358 days D. 362 days
2. A radioactive substance has a half-life of one week. In other words, at the end of every week the level of radioactivity is half of its value at the beginning of the week. The initial level of radioactivity is 20 counts per second. Find the radioactivity left after three weeks.
A. 2.5 B. 1.5 C. 3.5 D. 4.5
3. Which of the following shows an exponential equation?
A. 5^{x+1} B. $f(x) = 5^{x+1}$ C. $5^{x+1} > 5^2$ D. $5^{x+1} = 5^2$
4. Which among the following is an exponential function?
A. 5^{x+1} B. $f(x) = 5^{x+1}$ C. $5^{x+1} > 5^2$ D. $5^{x+1} = 5^2$
5. Which of the following is an exponential inequality?
A. 5^{x+1} B. $f(x) = 5^{x+1}$ C. $5^{x+1} > 5^2$ D. $5^{x+1} = 5^2$
6. Solve for x: $e^{x+3} = \frac{1}{e^2}$.
A. -5 B. -1 C. 1 D. 5
7. Solve $3^{x+1} = 27$.
A. $-\frac{1}{2}$ B. $\frac{1}{2}$ C. -2 D. 2
8. Solve the inequality $5^{x+2} > 25$.
A. $x > -\frac{1}{2}$ B. $x > \frac{1}{2}$ C. $x < 0$ D. $x > 0$
9. What is the solution set in the given inequality $2^x > 4^{x+1}$?
A. $\{x \in \mathbb{R} / x < -2\}$ B. $\{x \in \mathbb{R} / x > -2\}$ C. $\{x \in \mathbb{R} / x < 2\}$ D. $\{x \in \mathbb{R} / x > 2\}$
10. Solve for x: $36^x > 216^{x+2}$.
A. $x < -3$ B. $x > 3$ C. $x < -6$ D. $x > 6$





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