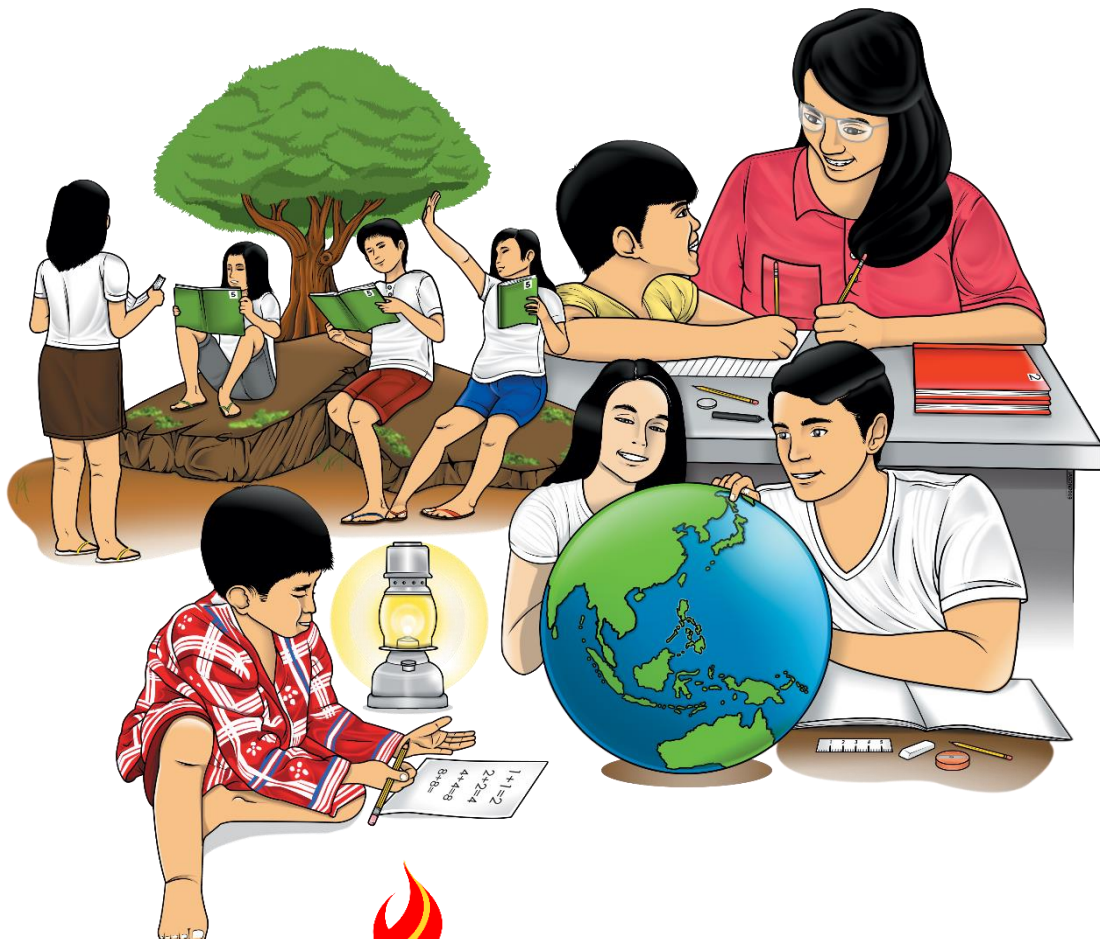


Mathematics

Quarter 4 – Module 6: Problems Involving Right Triangles



Mathematics – Grade 9
Alternative Delivery Mode
Quarter 4 – Module 6: Problems Involving Right Triangles
First Edition, 2020

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Mathematics

Quarter 4 – Module 6: Problems Involving Right Triangles

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

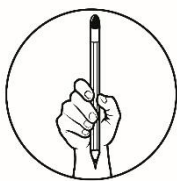


What I Need to Know

This module was designed and written with you in mind. It is here to help you master **Problems Involving Right Triangles**. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

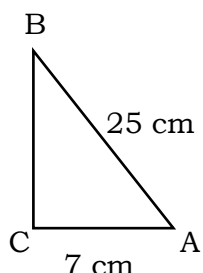
- uses trigonometric functions to solve real-life problems involving right triangles (**M9GE-IVf-g-46.1**)



What I Know

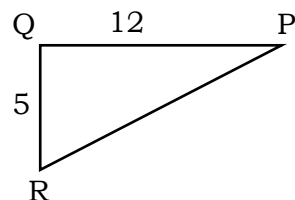
Let us find out how much you already know about right triangle. Take note of the items that you will not be able to answer correctly. Try to find the correct answer as you go through this module. Use Calculator if needed. Write your answer in a separate sheet of paper

Consider $\triangle BCA$ at the right for items 1 – 5.

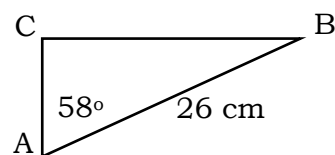


- 1) Which side of the triangle is the hypotenuse?
 a. \overline{BC} b. \overline{AB} c. \overline{AC} d. \overline{CB}
- 2) Which side is opposite of $\angle B$.
 a. \overline{BC} b. \overline{AB} c. \overline{AC} d. \overline{BA}
- 3) What is the length of \overline{BC} ?
 a. 25 cm b. 24 cm c. 14 cm d. 7 cm
- 4) What is the value of sine A?
 a. $\frac{7}{25}$ b. $\frac{24}{25}$ c. $\frac{7}{24}$ d. $\frac{24}{7}$
- 5) What is the value of tan B?
 a. $\frac{7}{25}$ b. $\frac{24}{25}$ c. $\frac{7}{24}$ d. $\frac{24}{7}$
- 6) Which of the following is the value of $\sin 10^\circ$?
 a. 0.1736 b. 0.1763 c. 0.1896 d. 0.2020

- 7) In the right $\triangle PQR$ at the right, $|PQ| = 12$ cm and $|QR| = 5$ cm. What is the value of $\cos R$?
 a. $\frac{12}{13}$ c. $\frac{5}{12}$
 b. $\frac{5}{13}$ d. $\frac{12}{13}$

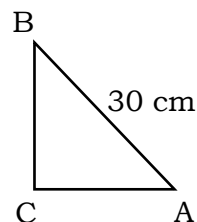


- 8) Given at the right is right $\triangle ACB$. Find $|AC|$.
 a. 15 cm c. 13 cm
 b. 13.78 cm d. 12 cm



- 9) What is the value of A to the nearest whole number if $\sin A = 0.529$?
- a. 30° b. 31° c. 32° d. 33°

- 10) Using $\triangle BCA$ at the right, if $m\angle A = 45^\circ$ and $|BA| = 30$ cm, what is $|BC|$ to the nearest whole number?
- a. 21 cm c. 23 cm
b. 22 cm d. 24 cm



- 11) If the legs of a right triangle are congruent, what is the measure of each acute angle in the triangle?
- a. 30° b. 45° c. 50° d. 60°
- 12) An airplane is flying at an altitude of 30,000 feet. If the angle of depression of the airport is 10° , how far must a radio signal travel from the airport to the plane in feet?
- a. 29,544 b. 30,463 c. 170,138 d. 172,763
- 13) A 50-foot ramp is used to load cargo onto an airplane. If the ramp makes a 23-degree angle with the ground, how far away is the foot of the ramp from the plane?
- a. 19.54 ft b. 21.22 ft c. 45 ft d. 46 ft
- 14) A 6-foot person casts a 5-foot shadow. What is the angle of elevation of the sun?
- a. 33.56° b. 39.81° c. 50.19° d. 56.44°
- 15) The angle of elevation of a 50-meter-high balloon from observer A is 45° and from observer B is 30° . What is the distance between the two observers if they are both located east of the balloon?
- a. 36.6 m b. 50 m c. 86.6 m d. 136.6 m

Lesson

1

Problems Involving Right Triangle

In Solving Right Triangles, you can use your knowledge of the Pythagorean Theorem and the six trigonometric functions. Solving right triangle means computing for the missing measures of sides and measures of angles. How you solve depends on how much information are given.

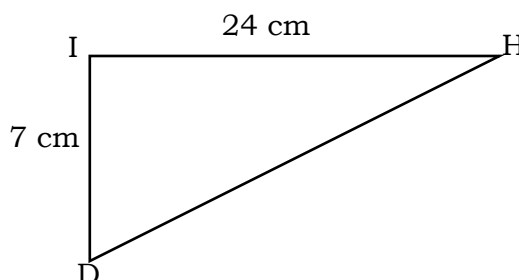
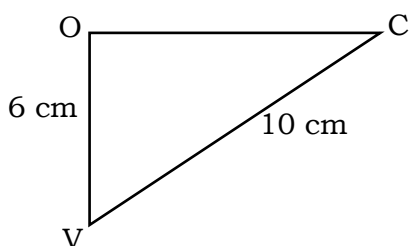


What's In

The concepts of trigonometric functions are essential in solving word problems involving right triangles. Many real-life problems involving right triangle can be solved using those functions. Let us recall these concepts by answering activities 1, 2, and 3.

Activity 1. Let's Match

Use the right $\triangle COV$ and right $\triangle HID$ in answering the matching exercises that follow. Match each item in column A to its corresponding value in column B. Write the letter of the correct answer on your answer sheet.



A. Trigonometric function values of an angle

Column A	Column B	
1) $\sin C$	a) $\frac{4}{5}$	f) $\frac{5}{3}$
2) $\tan C$	b) $\frac{24}{25}$	g) $\frac{5}{4}$
3) $\cos C$	c) $\frac{7}{24}$	h) $\frac{25}{24}$
4) $\sin V$	d) $\frac{3}{4}$	i) $\frac{7}{25}$
5) $\cos V$	e) $\frac{24}{7}$	j) $\frac{3}{5}$
6) $\sin D$		
7) $\cos D$		
8) $\tan D$		
9) $\cos H$		
10) $\tan H$		

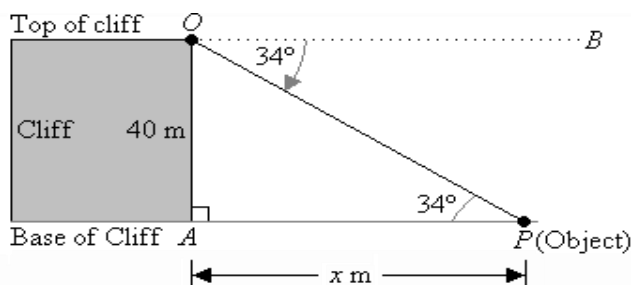
B. Relations of angles to sides of a right triangle.

Column A	Column B		
1) hypotenuse of $\triangle COV$	a) \overline{OC}	b) \overline{IH}	c) \overline{VC}
2) opposite side of $\angle C$			
3) adjacent side of $\angle C$			
4) opposite side of $\angle V$			
5) adjacent side of $\angle V$			
6) opposite side of $\angle H$	d) \overline{ID}	e) \overline{OV}	f) \overline{HD}
7) adjacent side of $\angle D$			
8) hypotenuse of $\triangle HID$			
9) opposite side of $\angle D$			
10) adjacent side of $\angle H$			

Activity 2. Use calculator to find the value of the following: (4 decimal place)

- | | | | |
|--------------------|-------|--------------------|-------|
| 1) $\sin 40^\circ$ | _____ | 4) $\tan 76^\circ$ | _____ |
| 2) $\cos 40^\circ$ | _____ | 5) $\sin 58^\circ$ | _____ |
| 3) $\tan 40^\circ$ | _____ | | |

Activity 3. Name the angle of elevation and angle of depression in the figure.



What's New

“A good way to integrate problem solving is to spend some time building positive attitudes and disposition about problem solving”.

-Randy Charles

One way to systematically approach problem solving is to use step-by-step process, sometimes referred to as Problem Solving Guidelines.

1. Understand the Problem What is the situation all about? What are you trying to find out? What are the key data/ conditions? What are the assumptions?	2. Develop a plan Have you ever worked a similar problem before? Can you estimate or calculate? What strategies can you use?
3. Implement the Plan What is the solution? Did you interpret correctly? Did you calculate correctly? Did you answer the questions?	4. Look Back Could you work the problem the other way? Is there another solution? Is the answer reasonable?

Many practical problems in trigonometry can be solved using right triangle in the situations. A sketch or drawing is helpful in setting up an appropriate triangle that involves a right triangle. Identify the right angle and the other angles in the right triangle. Label the sides and angles of the right triangle using the given information. Let the missing measure of a side or missing measure of an angle be represented by a variable. Hope you can become a successful problem solver.

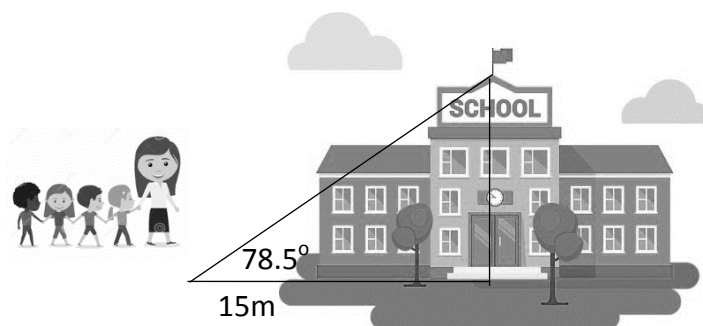


What is It

Problems Involving Right Triangles

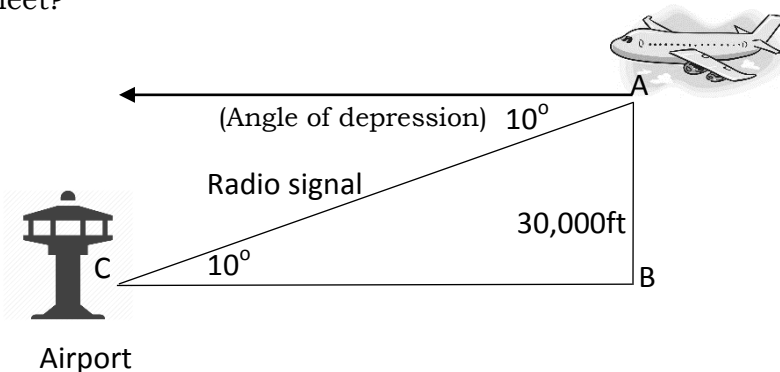
Everybody is Looking Up

Mrs. Penaflor and her class went out into the street to find the height of the school building. They made a simple sextant using a board protractor and a tripod. Each of the 18 students in the class measured the angle of elevation of the top of the building from a point 15 meters from the foot of the building and got an average angle measurement of 78.50° . Find the height of the school building to the nearest whole number. Do you think the school is in the countryside or in the city? Explain.



WHAT DO YOU THINK OF THIS!

- 1) S.O.S. An airplane is flying at an altitude of 30,000 feet. If the angle of depression of the airport is 10° , how far must a radio signal travel from the airport to the plane in feet? What is the horizontal distance of the plane from the airport in feet?



From the given problem 1, a right triangle can be formed by three points namely 1) the location of the airplane (A) at the time of observation, 2) a point on the ground that directly below the plane (B), and 3) the location of the airport (C). So, the right triangle is $\triangle ABC$ with right angle at B.

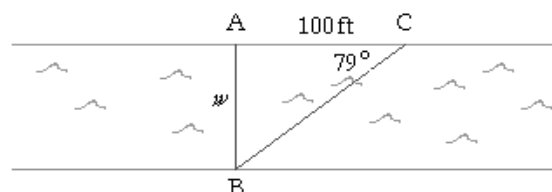
Mat Thinks...	Tanya Thinks...
<p>The involved sides of the right triangle are the hypotenuse and the leg opposite to $\angle ACB$. I'll use the sine function.</p> <p>Let h be the length of the hypotenuse.</p> $\sin C = \frac{\text{Opposite}}{\text{hypotenuse}}$ $\sin 10^\circ = \frac{30,000}{h}$ $h = \frac{30,000}{\sin 10^\circ}$ $h = \frac{30,000}{0.173648}$ $h = 172,763 \text{ feet}$	<p>The missing measure is the length of the hypotenuse and given measure of $\angle ACB$ is 10°, then the measure of $\angle CAB$ is 80°.</p> <p>Let h be the length of the hypotenuse.</p> $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos 80^\circ = \frac{30,000}{h}$ $h = \frac{30,000}{\cos 80^\circ}$ $h = \frac{30,000}{0.173648}$ $h = 172,763 \text{ feet}$

Can you solve for the horizontal distance of the plane to the airport? Compare your solution to your classmates. Note: You should give importance of the accurate drawing or illustration when applying trigonometric functions to word problems.

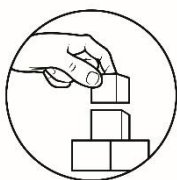
- 2) To measure the width of a river, two trees at points A and B, on the opposite banks of a river are identified. Segment AC along one bank is perpendicular to segment BA and is measured to be 100 feet. Angle ACB is measured to be 79° . How far apart are the two trees? or What is the width w of the river?

Since the side of the right triangle whose measure is given, is adjacent to the given angle, and the side opposite the given angle is asked, then the tangent function is to be used.

$$\begin{aligned}\tan 79^\circ &= \frac{w}{100} \\ w &= 100 (\tan 79^\circ) \\ w &= 100 (5.1446) \\ w &= 514.46 \text{ ft}\end{aligned}$$



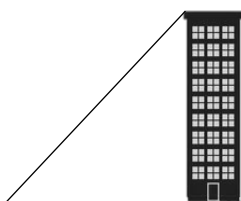
The width of the river is 514.46 ft which is also the distance between the two trees A and B.



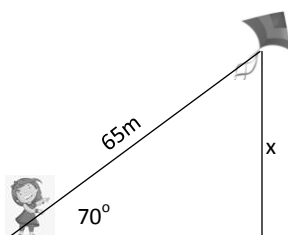
What's More

Solve the following. Label the figure.

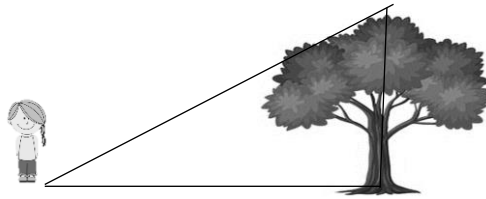
- 1) A building casts a shadow of 110 feet. If the angle of elevation of the top of the building from the tip of the shadow is 29° , find the height of the building.



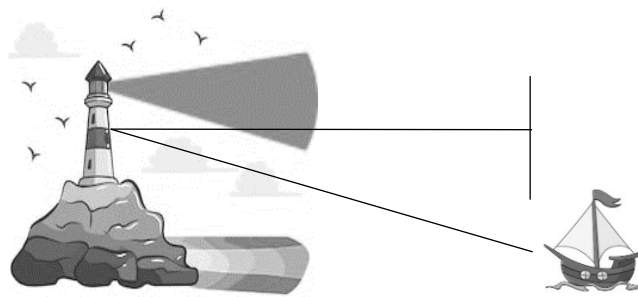
- 2) Julia is flying a kite whose string makes a 70° angle with the ground. The kite string is 65 meters long. How far is the kite from the ground?



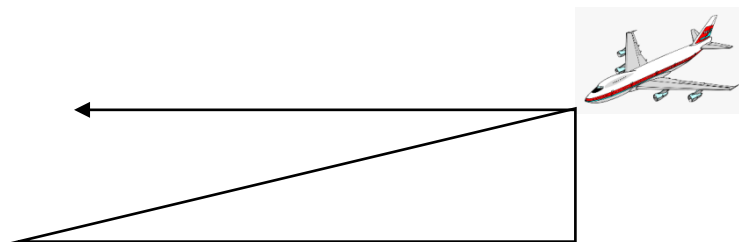
- 3) Daisy wants to calculate the height of a tree in her garden. She stands 20 meters away from the foot of the tree. Using a clinometer, she measures the angle of elevation of the top of the tree to be 52° . How tall is the tree?



- 4) Bon Voyage. From a lighthouse 250 feet above sea level, a ship is spotted with an angle of depression of 8 degrees. Approximately how far out at sea is the ship?



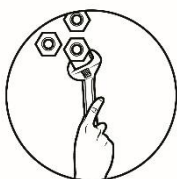
- 5) An airplane pilot finds the angle of depression of the airport to be 10° . If the altitude of the airplane at that moment is 15,000 feet, what is the horizontal distance of the plane from the airport in feet?





What I Have Learned

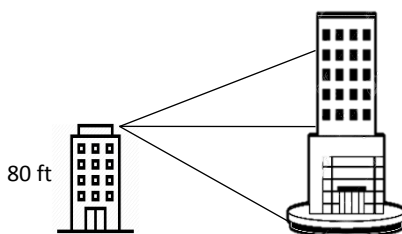
- The angle of depression equals the angle of elevation because they are alternate interior angles if the parallel lines are cut by a transversal which is usually the line of sight.
- To solve problems involving angle of elevation or angle of depression; first, carefully draw the figure (look for the right triangle); second, label the illustration using the given measures; third, determine the known side if it is opposite or adjacent to the given angle or the hypotenuse of the right triangle. Next determine the trigonometric function to be used then set up the equation and solve.



What I Can Do

With given examples above, are you ready now to solve more problems? Let's try to answer first the following.

- 1) After reading the problem what will you do next?
- 2) If the given measures are length of the hypotenuse and a measure of an acute angle in a right triangle, and the unknown is a length of a leg, what trigonometric function are you going to use?
- 3) If the given are the measures of two legs and you are asked to find the measures of the acute angles, what trigonometric functions will you use?
- 4) Label the figure. Create a problem using the figure.



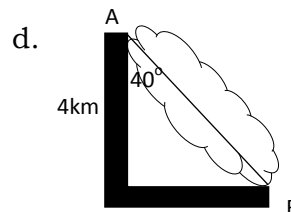
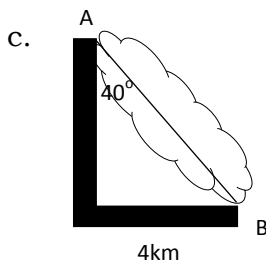
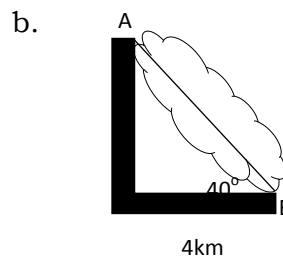
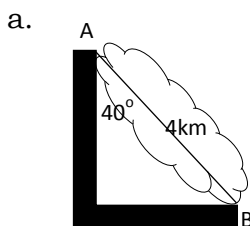


Assessment

Read and answer each of the following problems carefully. Label the figure and solve. Use a separate sheet of paper (5pts each)

- A. A north-south road and an east-west road end at the shore of a lake at points A and B, respectively. From point A, the line of sight to point B makes a 40° angle with the north-south road. Point B is known to be 4 km from the intersection of the roads.

1. Which is the appropriate drawing of the situation?



2. How far is point A from the intersection?

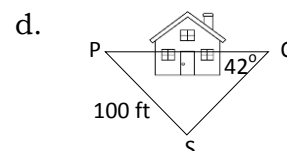
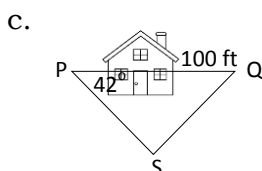
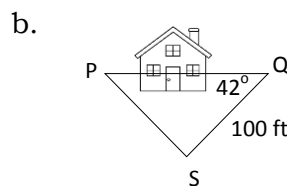
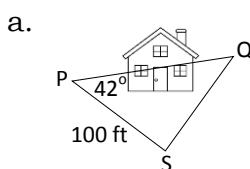
- a. 3.38 km b. 4.76 km c. 5.22 km d. 6.22 km

3. How far is it across the lake from A to B?

- a. 3.38 km b. 4.76 km c. 5.22 km d. 6.22 km

- B. Points P and Q are on the opposite sides of a house. Point S is 100 ft away from point P so that angle S measures 90° and angle P measures 42° .

4. Which is the appropriate illustration of the situation?



5. How far is point P from point Q?

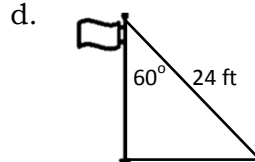
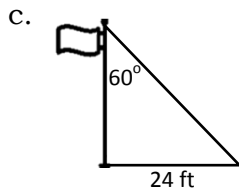
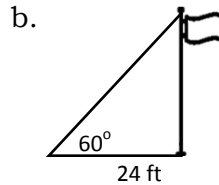
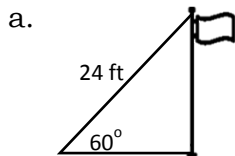
- a. 90.04 ft b. 111.06 ft c. 134.56 ft d. 149.45ft

6. How far is point S from point Q?

- a. 90.04 ft b. 111.06 ft c. 134.56 ft d. 149.45ft

C. A flagpole casts a 24-foot shadow when the angle of elevation of the sun is 60° .

7. Which is the appropriate illustration of the situation?

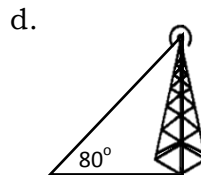
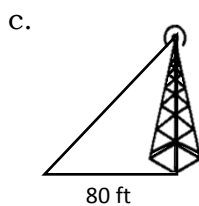
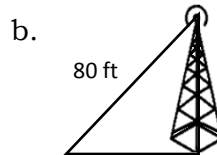


8. What is the height of the pole?

- a. 48 ft. b. 41.57 ft. c. 20.78 ft d. 13.86 ft

D. A supporting cable is attached to the top of an 80-foot tower.

9. Which is the appropriate illustration of the situation?



10. How long is the cable if it forms a 70° angle with the ground?

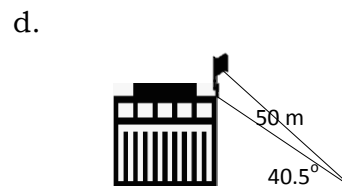
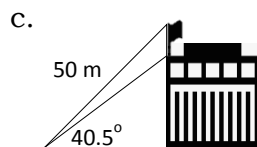
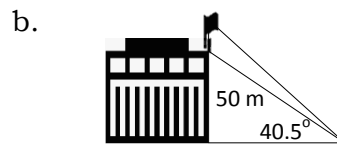
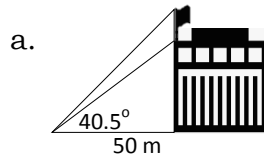
- a. 219.80 ft. b. 85.13 ft c. 29.11 ft d. 27.36 ft

11. If the cable forms a 70° angle with the ground, how far is the base of tower to the cable on the ground?

- a. 219.80 ft. b. 85.13 ft c. 29.11 ft d. 27.36 ft

- E. From point P on the ground, 50 m from a building, the angle of elevation of the base of a flagpole at the top of the building is 40.5° and the angle of elevation of the flag is 45.3° .

12. Which is the appropriate illustration of the situation?



13. What is the height of the building?

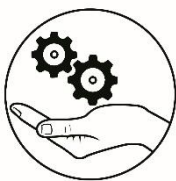
- a. 38.02 m b. 42.70 m c. 50.53 m d. 58.54 m

14. How far is the flag from the ground?

- a. 38.02 m b. 42.70 m c. 50.53 m d. 58.54 m

15. What is the height of the flagpole? (nearest tenths)

- a. 4.68 m b. 7.83 m c. 8.01 m d. 15.84 m

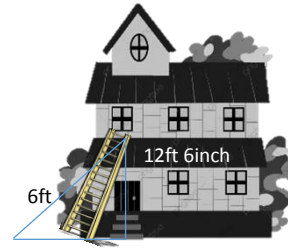


Additional Activities

If you are inside the elevator, how will you apply social distancing when the floor of the elevator is only 0.9m x 0.9m?

PROBLEMS INVOLVING RIGHT TRIANGLES

1. A 16-foot ladder is leaning against a house. It touches the bottom of a window that is 12 feet 6 inches above the ground. What is the measure of the angle that the ladder makes with the ground? Let θ be the measure of the angle that the ladder makes with the ground. A picture of the problem is drawn at the right.



We have the measure of the side opposite the angle in question as well as the hypotenuse of the right triangle. I can solve for the unknown in terms of the known using the appropriate function which is the sine function:

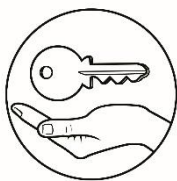
$$\sin \theta = \frac{12\text{ft } 6\text{in}}{16\text{ft}}$$

First, I need to convert all measures of lengths into inches and so.

$$\sin \theta = \frac{150\text{in}}{192\text{in}}$$

With the several examples that were solved, are you ready now to solve more problems? Let's try to answer first the following.

- 1) After reading the problem what will you do next?
- 2) If the given measures are length of the hypotenuse and a measure of an acute angle in a right triangle, and the unknown is a length of a leg, what trigonometric function are you going to use?
- 3) If the given are the measures of two legs and you are asked to find the measures of the acute angles, what trigonometric functions will you use?
- 4) Can we solve the problem number 1 using another trigonometric function? What other trigonometric function can we use?
- 5) Show another solution for problem 1.



Answer Key

PROBLEM - BASED LEARNING WORKSHEET

- 1) Illustrate/draw figure and label it.
- 2) Sine or cosine
- 3) Tangent or cotangent
- 4) Yes, we can use cosine, but first we will use Pythagorean theorem to find the other side.
- 5)

$$b^2 = 192^2 - 144^2 \quad b = 127$$

$$\cos x = \frac{127}{192}$$

$$\cos x = 0.66146$$

$$x = 48.6^\circ$$

ADDITIONAL ACTIVITIES

- Students will make their own problem about the given figure.
- Stand on the four corners of the elevator.

ASSESSMENT

1. C	6. A	11. C
2. B	7. B	12. A
3. D	8. B	13. B
4. A	9. A	14. C
5. C	10. B	15. B

WHAT I CAN DO

- 1) Illustrate the problem.
- 2) Cosine or Sine Function
- 3) Tangent Function
- 4) Answers may vary

WHAT'S MORE

- 1) $x = 60.97^\circ$
- 2) $x = 61.08^\circ$
- 3) $x = 25.60^\circ$
- 4) $d = 85069.2$ ft

WHAT'S IN

Activity 1

A	1) J
	2) D
	3) A
	4) A
	5) J
	6) B
	7) I
	8) E
	9) B
	10) C

Activity 2

1) 0.6428
2) 0.7660
3) 0.8391
4) 4.0108
5) 0.8480

Activity 3

Angle of Elevation of the top of the cliff from the object: $\angle APO$

Angle of Depression of the object from the top of the cliff: $\angle BOP$

WHAT I KNOW

1. B	6. A	11. B
2. C	7. B	12. D
3. B	8. B	13. D
4. B	9. C	14. C
5. C	10. A	15. A

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Problems and Illustrations

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<https://www.youtube.com/watch?v=ZPLJaXCmzfo>

<https://www.cut-the-knot.org/pythagoras/cosine2.shtml>

<https://www.mathsisfun.com/algebra/trig-sine-law.html>

<https://www.mathsisfun.com/algebra/trig-cosine-law.html>

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