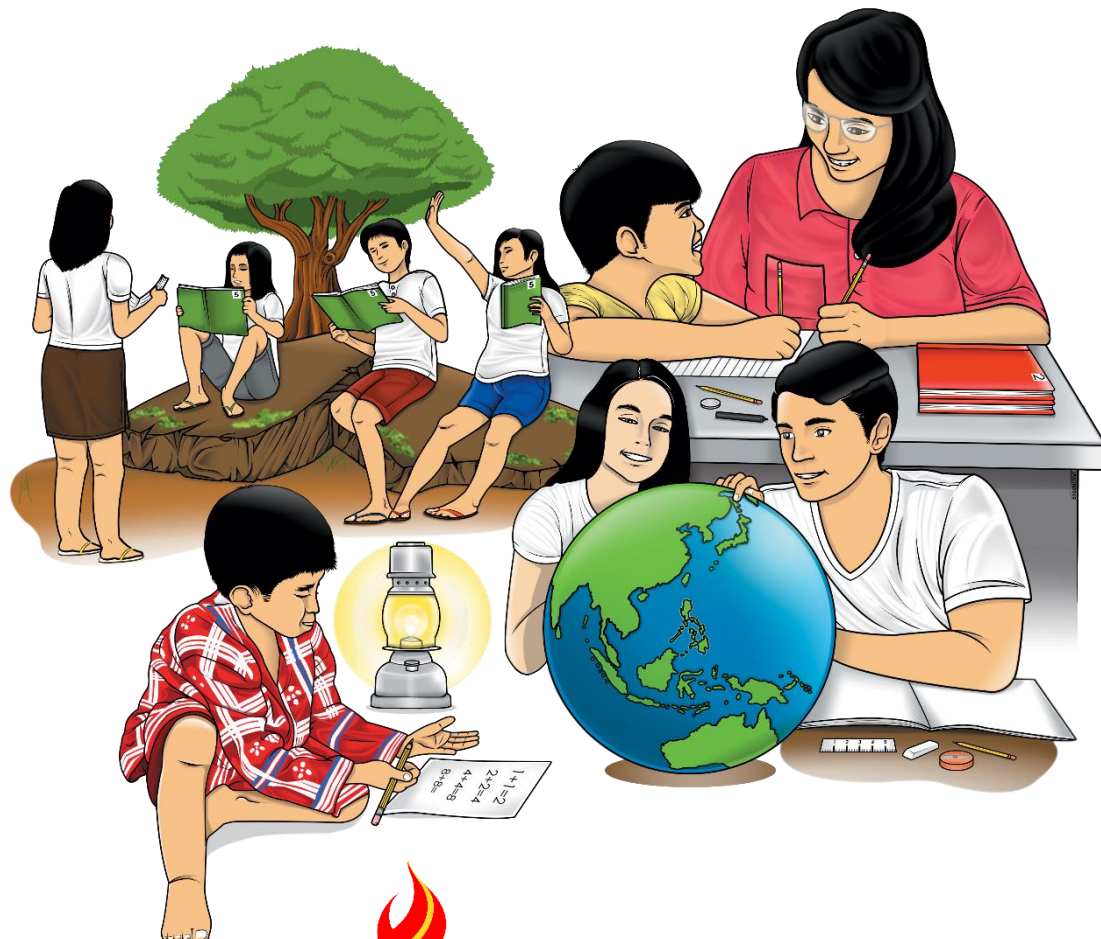


Mathematics

Quarter 4 – Module 2: Trigonometric Functions (csc, sec, and cot)



Mathematics – Grade 9
Alternative Delivery Mode
Quarter 4 – Module 2: Trigonometric Functions (csc, sec, and cot)
First Edition, 2020

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Mathematics

Quarter 4 – Module 2: Trigonometric Functions (csc, sec, and cot)

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

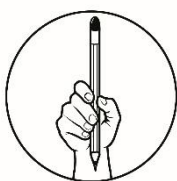
Thank you.



What I Need to Know

The learners will be able to:

Illustrate the six trigonometric functions: sine, cosine, tangent, **cosecant**, **secant**, and **cotangent**. M9GE-IVa-43.2



What I Know

Find out how much you already know about the module. Write the letter that you think is the best answer to each question on a sheet of paper. Answer all items. After taking and checking your answers in this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

- It is a trigonometric function the value of which is the quotient of the length of the hypotenuse and the length of the side opposite the given angle.
a. cosecant b. cotangent c. secant d. tangent
- It is a trigonometric function the value of which is the quotient of the length of the hypotenuse and the length of the side adjacent to the given angle.
a. cosecant b. cotangent c. secant d. tangent
- It is a trigonometric function the value of which is the quotient of the length of the side adjacent to and the length of the side opposite the given angle.
a. cosecant b. cotangent c. secant d. tangent

For items 4 – 9, refer to the figure at the right. Given is $\triangle XYZ$, a right triangle, with side lengths 8 cm, 15 cm, and 17 cm.

4. What is the value of $\csc X$?

a. $\frac{15}{17}$

b. $\frac{17}{15}$

c. $\frac{8}{15}$

d. $\frac{17}{8}$

5. What is the value of $\sec X$.

a. $\frac{15}{17}$

b. $\frac{17}{15}$

c. $\frac{8}{15}$

d. $\frac{17}{8}$

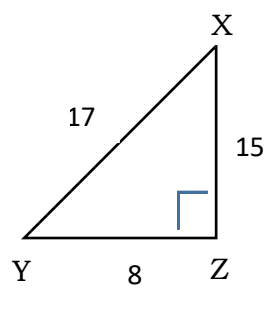
6. What is the value of $\cot X$?

a. $\frac{15}{17}$

b. $\frac{15}{8}$

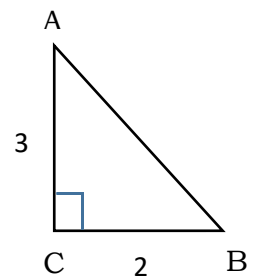
c. $\frac{8}{15}$

d. $\frac{17}{8}$



7. What is the value of $\cos X$?
 a. $\frac{15}{17}$ b. $\frac{17}{8}$ c. $\frac{8}{17}$ d. $\frac{17}{15}$
8. What is the value of $\sin X$?
 a. $\frac{15}{17}$ b. $\frac{15}{8}$ c. $\frac{8}{17}$ d. $\frac{17}{8}$
9. What is the value of $\tan X$?
 a. $\frac{15}{17}$ b. $\frac{15}{8}$ c. $\frac{8}{15}$ d. $\frac{17}{8}$
10. If $f(\theta) = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$, then $f(\theta)$ is _____ of θ .
 a. sine b. cosine c. tangent d. secant
11. With respect to the given angle, what is the ratio of the hypotenuse to the opposite side?
 a. secant b. cosine c. tangent d. cosecant

For items 12-13, refer to the figure at the right. Given is $\triangle ACB$, a right triangle as shown.



12. Which of the following ratio has the same value as $\csc A$?
 a. $\sec A$ b. $\csc B$ c. $\sec B$ d. $\sec C$
13. Which of the following is equal to $\frac{3}{2}$?
 a. $\tan A$ b. $\cot A$ c. $\sec A$ d. $\csc A$
14. In a right triangle PQR, $|PQ| = 12$ cm and $|QR| = 5$ cm. What is $\sec R$?
 a. $\frac{12}{13}$ b. $\frac{5}{13}$ c. $\frac{5}{12}$ d. $\frac{13}{5}$
15. Let $\triangle ABC$ be a right triangle such that $\angle A$ and $\angle B$ are acute angles. What is the relationship between $\tan A$ and $\cot B$?
 a. equal b. cofunction c. complementary d. reciprocal

Lesson

1

Trigonometric Functions

In the previous module, you have learned how to illustrate the three basic trigonometric functions namely sine, cosine, and tangent. In this module, you will learn how to define, illustrate and how to apply the reciprocals of the three basic functions namely secant, cosecant, and cotangent.

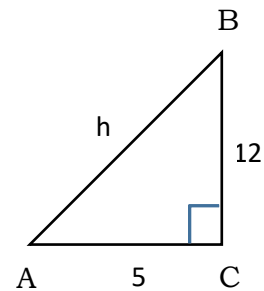


What's In

LET'S RECALL

Use the figure at the right to answer the following:

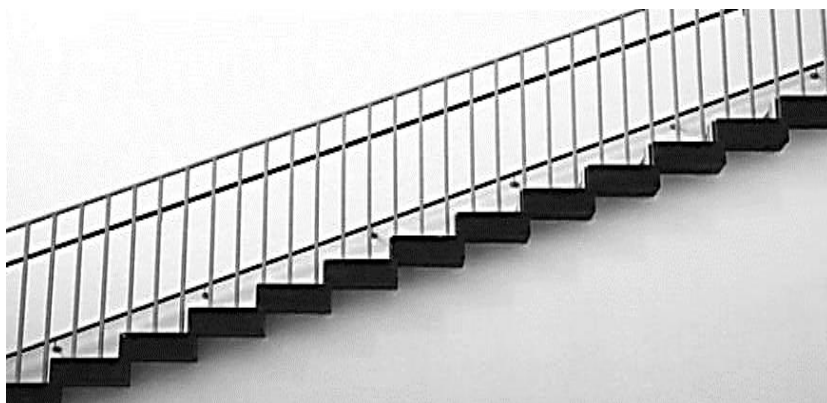
1. What mathematical formula will you use to find h ?
2. What is the value of h ?
3. What is the value of $\sin A$?
4. What is the value of $\cos A$?
5. What is the value of $\tan A$?



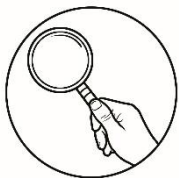
What's New

Here you'll learn the definitions of secant, cosecant, and cotangent functions and how to apply them.

While working to paint your grandfather's staircase, you are looking at the triangular shape on the wall that support the stairs. The staircase looks like this:

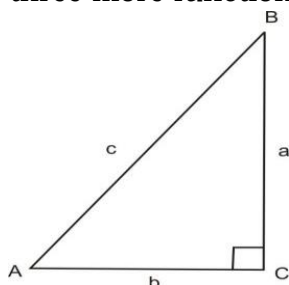


You are thinking about all the possible ratios between two sides. You already know the three common ratios using the functions **sine, cosine, and tangent**. How many other ratios are there? By the end of this module, you will know the other important ratios between two sides of a right triangle.



What is It

You already learned the three basic trigonometric functions and there are three more functions to think about:



$$\sin A = \frac{a}{c}$$

❖ Instead of $\frac{a}{c}$, we can consider $\frac{c}{a}$

$$\cos A = \frac{b}{c}$$

❖ Instead of $\frac{b}{c}$, we can consider $\frac{c}{b}$

❖ Instead of $\frac{a}{b}$, we can consider $\frac{b}{a}$

These new ratios define the **reciprocals of the first three trigonometric functions**.

The cosecant function (csc) is the reciprocal of the **sine function**. It is the ratio of the length of the hypotenuse to the length of the side opposite a given angle in a right triangle.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\csc A = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{c}{a}$$

The secant function (sec) is the reciprocal of the **cosine function**. It is the ratio of the length of the hypotenuse to the length of the side adjacent a given angle in a right triangle.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{c}{b}$$

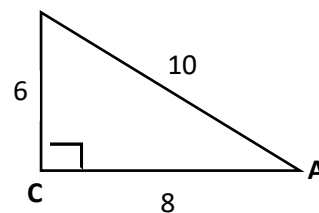
The cotangent function (cot) is the reciprocal of the **tangent function**. It is the ratio of the length of the adjacent side to the length of the opposite side of a given angle in a right triangle.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

$$\cot A = \frac{\text{adjacent}}{\text{opposite}} = \frac{b}{a}$$

Finding the values of the other trigonometric functions B

Let's study the following examples.



1. Use the figure at the right to find the values of $\csc A$, $\sec A$, and $\cot A$.

Solution:

Finding the value of the **cosecant function**. We know that cosecant function is the reciprocal of the sine function. Since sine function is the ratio of the length of the side opposite an angle to the length of the hypotenuse, then

cosecant function is the ratio of the length of the hypotenuse to the length of the side opposite an angle.

$$\csc A = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{10}{6} = \frac{5}{3}$$

Finding the value of the **secant function**. We know that the secant function is the reciprocal of the cosine function. Since cosine function is the ratio of the length of the side adjacent to an angle to the length of the hypotenuse, then secant function is the ratio of the length of the hypotenuse to the length of the side adjacent to an angle.

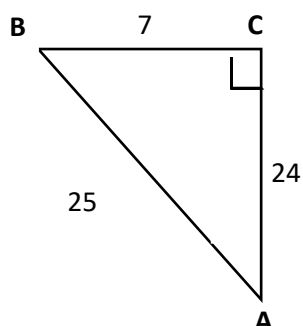
$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{10}{8} = \frac{5}{4}$$

Finding the **cotangent function**. We know that the cotangent function is the reciprocal of the tangent function. Since tangent function is the ratio of the length of the side opposite an angle to the length of the side adjacent to an angle, then cotangent function is the ratio of the length of the side adjacent to an angle to the length of the side opposite an angle.

$$\cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{8}{6} = \frac{4}{3}$$

2. Consider the triangle BCA below. Find the values of sine, cosine, and tangent of angle A, and then use those to get the values of secant, cosecant, and cotangent of the same angle.

Solution:



$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{7}{25}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{24}{25}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{7}{24}$$

Since we know that cosecant is the reciprocal of sine, secant is the reciprocal of cosine, and cotangent is the reciprocal of tangent, then their values are shown below.

$$\sec A = \frac{1}{\cos A} = \frac{25}{24} \qquad \csc A = \frac{1}{\sin A} = \frac{25}{7} \qquad \cot A = \frac{1}{\tan A} = \frac{24}{7}$$

3. Using the triangle BCA in example 2, find the trigonometric function values of angle B.

Solution:

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{24}{25}$$

$$\csc B = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{25}{24}$$

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{7}{25}$$

$$\sec B = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{25}{7}$$

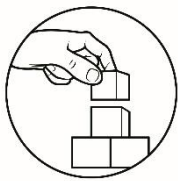
$$\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{24}{7}$$

$$\cot B = \frac{\text{adjacent}}{\text{opposite}} = \frac{7}{24}$$

From the trigonometric function values of angle A in example 2 and the trigonometric function values of angle B in example 3, the following relationships are established:

$\sin A = \cos B = \cos (90^\circ - A)$ $\cos A = \sin B = \sin (90^\circ - A)$
$\sec A = \csc B = \csc (90^\circ - A)$ $\csc A = \sec B = \sec (90^\circ - A)$
$\tan A = \cot B = \tan (90^\circ - A)$ $\cot A = \tan B = \cot (90^\circ - A)$

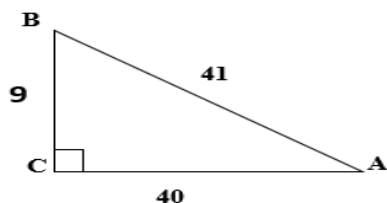
From these relationships, sine and cosine are cofunctions of complementary angles. Tangent and cotangent are also cofunctions of complementary angles, and similarly with secant and cosecant.



What's More

Activity 1: Matching Test

Directions: Use the $\triangle BCA$ below. Match each trigonometric function of an angle in column A to its corresponding value in column B and write the letter of the correct answer on the space provides



COLUMN A

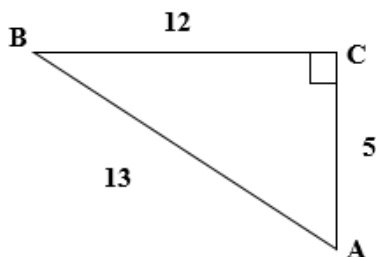
1. $\cot A$ _____
2. $\sec B$ _____
3. $\csc A$ _____
4. $\cot B$ _____
5. $\sec A$ _____

COLUMN B

- A. $\frac{41}{40}$
- B. $\frac{41}{9}$
- C. $\frac{9}{40}$
- D. $\frac{40}{9}$
- E. $\frac{40}{41}$

Activity 2: Alternative-Response Test

Directions: Refer to the $\triangle BCA$ below. Write True if the statement is correct and False if it is wrong. Write your answers on the line.

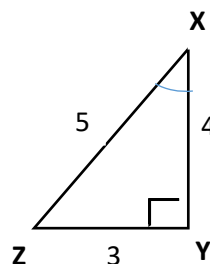


1. $\sec A = \frac{13}{12}$. _____
2. $\cot B = \frac{5}{13}$. _____
3. $\csc A = \frac{12}{13}$. _____
4. $\cot A = \frac{5}{12}$. _____
5. $\csc B = \frac{13}{5}$. _____

Activity 3: Supply Test

Directions: Refer to the $\triangle XYZ$ below, find the trigonometric functions values of $\angle X$.

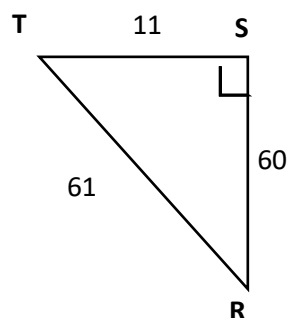
1. $\sin X =$	4. $\csc X =$
2. $\cos X =$	5. $\sec X =$
3. $\tan X =$	6. $\cot X =$



Activity 4: Supply Test

Directions: Refer to the $\triangle TSR$ below, then find the values of the following:

1. $\csc T =$	4. $\csc R =$
2. $\sec T =$	5. $\sec R =$
3. $\cot T =$	6. $\cot R =$



What I Have Learned

The cosecant function (\csc) is the reciprocal of the sine function. It is the ratio of the length of the hypotenuse to the length of the side opposite a given angle in a right triangle.

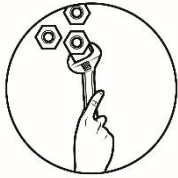
$$\csc A = \frac{\text{hypotenuse}}{\text{opposite}}$$

The secant function (\sec) is the reciprocal of the cosine function. It is the ratio of the length of the hypotenuse to the length of the side adjacent a given angle in a right triangle.

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent}}$$

The cotangent function (\cot) is the reciprocal of the tangent function. It is the ratio of the length of the adjacent side to the length of the opposite side of a given angle in a right triangle.

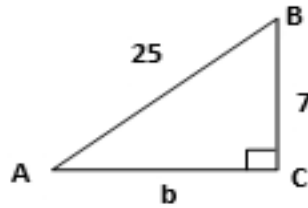
$$\cot A = \frac{\text{adjacent}}{\text{opposite}}$$



What I Can Do

Directions: Use the $\triangle BCA$ below and find the value of each of the following. Write your answers below.

1. b
2. $\csc A$
3. $\sec A$
4. $\cot A$
5. $\csc B$
6. $\sec B$
7. $\cot B$

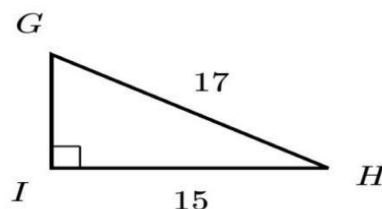


Assessment

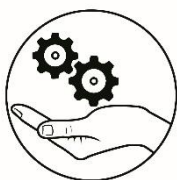
DIRECTIONS: Answer each of the following items accurately. Encircle the letter of the correct answer.

1. It is the ratio of the length of the hypotenuse to the length of the side opposite the given angle in a right triangle.
a. Cosecant b. Cosine c. Cotangent d. Secant
2. It is the ratio of the length of the hypotenuse to the length of the side adjacent to the given angle in a right triangle.
a. Cosecant b. Cosine c. Cotangent d. Secant
3. It is the ratio of the length of the adjacent side to the length of the side opposite of the given angle in a right triangle.
a. Cosecant b. Cosine c. Cotangent d. Secant

For items 4 - 10, use $\triangle GIH$ at the right



4. What is the length of GI?
a. 8 b. 9 c. 10 d. 11
5. What is the value of $\csc G$?
a. $\frac{15}{8}$ b. $\frac{8}{15}$ c. $\frac{17}{8}$ d. $\frac{17}{15}$
6. What is the value of $\sec G$?
a. $\frac{15}{8}$ b. $\frac{8}{15}$ c. $\frac{17}{8}$ d. $\frac{17}{15}$
7. What is the value of $\cot G$?
a. $\frac{15}{8}$ b. $\frac{8}{15}$ c. $\frac{17}{8}$ d. $\frac{17}{15}$
8. What is the value of $\csc H$?
a. $\frac{15}{8}$ b. $\frac{8}{15}$ c. $\frac{17}{8}$ d. $\frac{17}{15}$
9. What is the value of $\sec H$?
a. $\frac{15}{8}$ b. $\frac{8}{15}$ c. $\frac{17}{8}$ d. $\frac{17}{15}$
10. What is the value of $\cot H$?
a. $\frac{15}{8}$ b. $\frac{8}{15}$ c. $\frac{17}{8}$ d. $\frac{17}{15}$



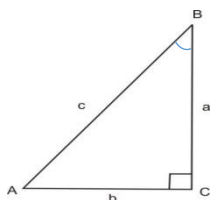
Additional Activities

As the measure of an acute angle in a right triangle increases, will the value of the cotangent of the angle increase or decrease? Explain.

PROBLEM – BASED LEARNING WORKSHEET

Cofunctions

Based on the definitions of trigonometric functions and the figure below, we get the following.



$$\sin B = \frac{b}{c}$$

$$\csc B = \frac{c}{b}$$

$$\cos B = \frac{a}{c}$$

$$\sec B = \frac{c}{a}$$

$$\tan B = \frac{b}{a}$$

$$\cot B = \frac{a}{b}$$

Since $\angle A$ and $\angle B$ are complementary angles, $\angle A$ is the complement of $\angle B$ and $\angle B$ is the complement of $\angle A$. ($m\angle A + m\angle B = 90^\circ$), we have

$$\sin B = \sin(90^\circ - A) = \cos A$$

$$\csc B = \csc(90^\circ - A) = \sec A$$

$$\cos B = \cos(90^\circ - A) = \sin A$$

$$\sec B = \sec(90^\circ - A) = \csc A$$

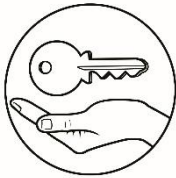
$$\tan B = \tan(90^\circ - A) = \cot A$$

$$\cot B = \cot(90^\circ - A) = \tan A$$

The relationships above explain how the cosecant, cosine, and cotangent functions derived their names. Also, we can say that the function of one acute angle is equal to the cofunction of its complement. **Cofunctions** are functions of complementary angles. It is convenient to arrange the functions in pairs as follows: sine and cosine, secant and cosecant, tangent and cotangent. In any pair, either function is the cofunction of the other.

Let's Analyze

1. What does it mean when two angles are complementary?
2. How many pairs of cofunctions in the trigonometric functions?
3. The values of the cofunctions of complementary angles are _____.
4. If A and B are complementary angles, what is equal to $\csc B$?
5. What is equal to $\cot 32^\circ$?



Answer Key

Grade 9 _ Quarter 4 _ Module 2			
What I Know:			
1. A	6. B	11. D	1. A
2. C	7. A	12. C	2. C
3. B	8. C	13. B	3. B
4. D	9. C	14. D	4. D
5. B	10. B	15. A	5. B
What's In:			
1. Pythagorean Formula	2. 13	3. 12/13	4. 5/13
	5. 5/12		
What's More:			
Activity 1			
1. D	1. FALSE	1. FALSE	1. 61/60
2. B	2. FALSE	2. 4/5	2. 61/11
3. B	3. FALSE	3. 3/4	3. 11/60
4. C	4. TRUE	4. 5/3	4. 61/11
5. A	5. TRUE	5. 5/4	5. 61/60
Activity 2			
1. FALSE	1. 3/5	1. 61/60	1. 61/60
2. FALSE	2. 4/5	2. 61/11	2. 61/11
3. FALSE	3. 3/4	3. 11/60	3. 11/60
4. TRUE	4. 5/3	4. 61/11	4. 61/11
5. TRUE	5. 5/4	5. 61/60	5. 61/60
6. 4/3	6. 4/3	6. 60/11	6. 60/11
Activity 3			
Activity 4			
Problem-Based			
1. The sum of the measures of the two angles is 90°	1. A	1. 24	1. 24
2. 3 pairs	2. D	2) 25/7	2) 25/7
3. Equal	3. C	3) 25/24	3) 25/24
4. Sec A	4. A	4) 24/7	4) 24/7
5. Tan 58°	5. D	5) 25/24	5) 25/24
	6. C	6) 25/7	6) 25/7
	7. B	7) 7/24	7) 7/24
	8. C		
	9. D		
	10. A		

References

To further explore the concept learned today and if it possible to connect the internet, you may visit the following links:

<https://www.ck12.org/flx/render/embeddedobject/1365>

<https://www.shmoop.com/trig-functions/trig-ratios.html>

https://media1.shmoop.com/images/algebra-ii/alg2_trig_graphik_3.png

<https://www.math-aids.com/Geometry/Trigonometry/>

<https://cdn.kutasoftware.com/Worksheets/Geo/9-Trigonometric%20Ratios.pdf>

<https://4.files.edl.io/87f1/03/10/20/175447-0b12f4cc-a7cd-45cb-83e8-bbf009f8b01b.pdf>

Orlando A. Oronce, Marilyn O. Mendoza (2010) Work text in Mathematics e-math Advanced Algebra and Trigonometry Rex Book Store 856 Nicanor Reyes, Sr. Street 1977 C.M. Recto Avenue, Manila, Philippines.

Soledad Jose Dilao, Ed D. Fernando B. Orines, Julieta G. Bernabe (2013)Advances Algebra Trigonometry and Statistics JTW Corporation Araneta Ave. Quezon City Philippines

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Department of Education - Bureau of Learning Resources (DepEd-BLR)

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