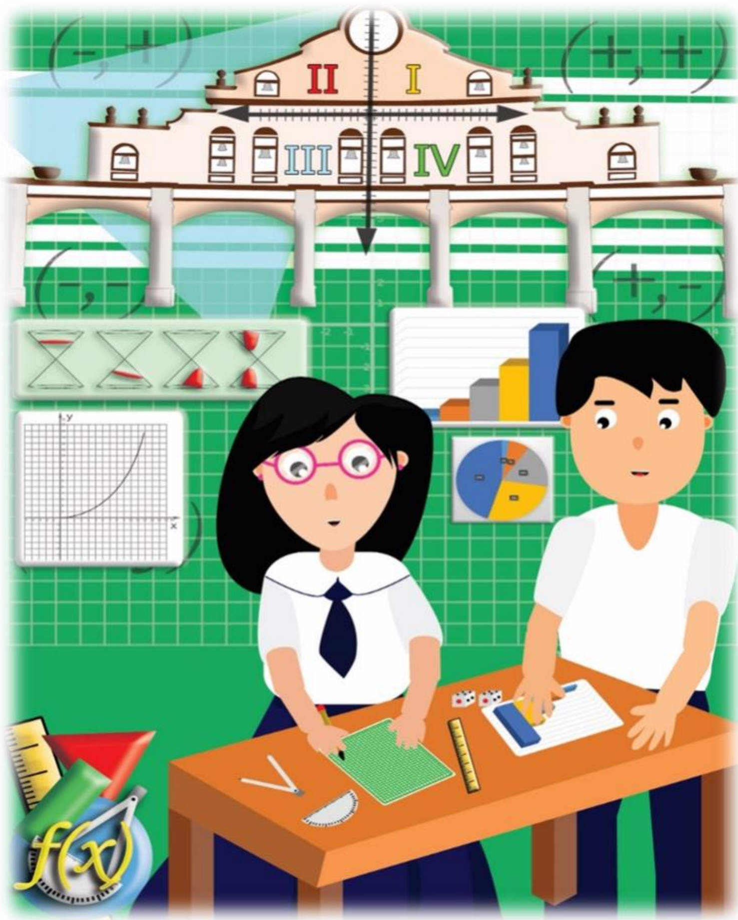
Senior High School

Department of Education
National Capital Region
SCHOOLS DIVISION OFFICE
MARIKINA CITY

STATISTICS AND PROBABILITY

Quarter 3: Module 2

Mean and Variance of Discrete Random Variable



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City of Good Character
DISCIPLINE • GOOD TASTE • EXCELLENCE

What I Need to Know

Grade 11 learners! In this module, you will learn how to:

Compute probabilities corresponding to a given random variable M11/12SP-llla-4, Illustrate the mean and variance of a discrete random variable M11/12SP-lllb-1 and Calculate the mean and the variance of a discrete random variable M11/12SP-lllb-2.

You can say that you have understood the lesson in this module if you can already:

1. Compute probabilities corresponding to a given random variable,
2. Can illustrate the mean and variance of a discrete random variable and
3. Calculate the mean and the variance of a discrete random variable.

LESSON 1: PROBABILITY OF A RANDOM VARIABLE

What I Know

Read each item carefully and encircle the letter that corresponds to the correct answer.

1. The process that generate a sets of data.
   A. Statistical Data — C. Statistical Probability
   B. Statistical Experiment — D. Statistical Probability Distribution
2. The possible outcomes of an experiment is called
   A. Sample — C. Space
   B. Sample Space — D. Statistics
3. A figure that uses branches to determine all the outcomes in an experiment.
   A. Experiment — C. Tree Diagram
   B. Listing — D. Trial Diagram
4. The measure of how likely a particular event will occur.
   A. Distribution — C. Sample
   B. Probability — D. Statistics
5. Determine the number of possible outcomes in tossing 2 dies.
   A. 20 — B. 25 — C. 36 — D. 45
6. A table, a formula or a graph showing the possible values of the random variable together with their corresponding probabilities.

A. Probability Mass Data
B. Probability Mass Distribution
C. Probability Mass Function
D. Probability Mass Sample

7. Determine the number of possible outcomes in tossing 3 coins.
A. 8 B. 10 C. 12 D. 15

8. Determine the number of possible outcomes in tossing 2 coins and a die.
A. 25 B. 30 C. 32 D. 39

9. The number of possible outcomes in tossing 1 coin.
A. Invalid B. Determined C. Undetermined D. Valid

10. The probability mass function of COVID - 19 patient in one area in Kalumpang, Marikina.

X	1	2	3	4
P(X=x)	2/10	1/10	3/10	5/10

A. Invalid B. Determined C. Undetermined D. Valid

What's In

A random sample of 50 grade 11 students were evaluated for their absences. The data showed that in a certain 1-week period, 20 of them had no absences,18 had 1 absent, 6 had 2 absences, 4 had 3 absences and 2 had 4 absences. Let X be the number of absences.

a. What are the possible values of X?
b. What is the probability that a randomly chosen student had more than 1 absences?
c. What is the probability that a randomly chosen student had at most 2 absences?
d. What is the probability that a randomly chosen student that had at least 2 absences?

What's New

A **probability distribution** is an assignment of probabilities to the values of the random variable; it is a function that describes the possible outcomes that a random variable can attain. The abbreviation **pdf** is used for a probability distribution function.

For instance, a hospital researcher is interested in the number of times the average COVID - 19 patient call the nurse during a 12 hour shift. For a random sample of 50 patients, the following information was obtained:

Number of times the patients called the nurse during the 12 hr. shift	Number of patients who called the nurse
0	4
1	8
2	16
3	14
4	6
5	2

Let X be the number of times a patient calls the nurse during a 12-hour shift. For this case, the possible values of the random variable X are 0, 1, 2, 3, 4, and 5. Let P(x) the probability that X takes on value.

X	P(X)
0	4/50
1	8/50
2	16/50
3	14/50
4	6/50
5	2/50

There were 50 patients observed.

- 4 out of 0 patients did not ring the bell. So, the P(X) value for 0 will be 4/50. Thus, P(0) = 4/50.
- 8 out of 50 patients rung the bell only once. The P(X) value for 1 is 8/50, then P(1) = 8/50.

What will be the respective value of P(2)? P(3)?; P(4); and P(5)?

What is It

The **probability distribution** of a random variable distinguish probabilities of all the possible values of random variables. For a discrete random variable, the *probability distribution* is defined by a probability mass function which is denoted by P(X=x).

The **probability mass function** is a table, a formula, or a graph showing the possible values of the random variable together with the corresponding probabilities. The possible values of the random variables are called **mass points**.

Consider the following experiments of tossing a die. Let the random variable X be defined as the number of dots shown on the upturned face of the die. Recall that the possible values of X are 1, 2, 3, 4, 5, 6. Note that all of the probability of obtaining 1 is 1/6 and this probability is the same for all other possible values of X. The probability mass function of X if written using a formula is:

$$P(X) = \begin{cases} 1/6 & \text{where, the values of X are 1, 2, 3, 4, 5, 6} \\ 0 & \text{otherwise} \end{cases}$$

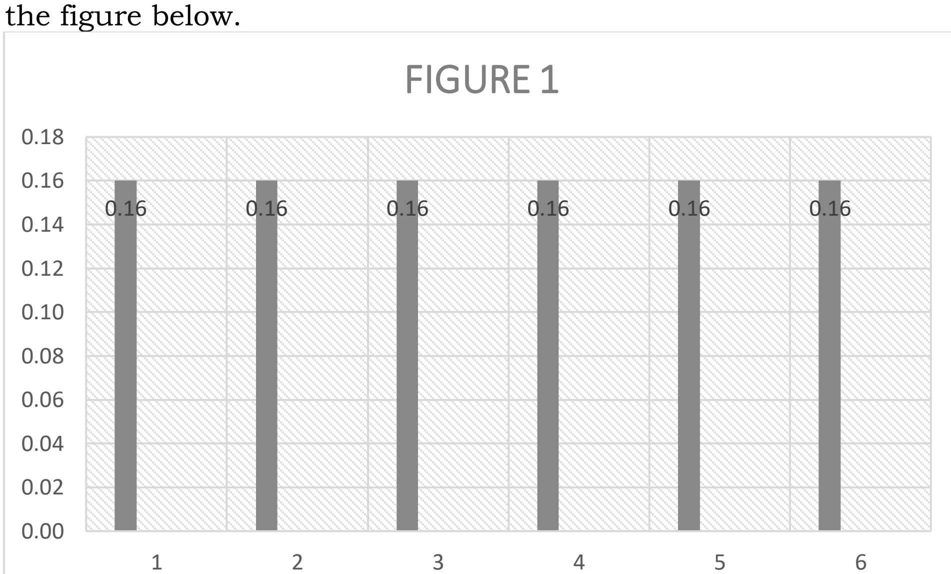
Note that the *probability mass function* may also be presented in tabular form. The first row of the table presents the possible values of x, while the second row shows the corresponding probability of each value. The tabular representation of P(X) in the previous example is given below.

Table 1: *Probability Mass Function* of X, where X is the number of dots on the upturned face of die

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

Another way of showing the probability mass function is by using a probability histogram, which is a figure or diagram that contains rectangles entered at each mass point. The height or length of the rectangle corresponds to the probability of that particular mass point. The x – axis represents the possible values of X, while the y-axis represents the probabilities which is the same idea presented in the tabular representations.

Refer to the figure below.



****Graphical representation of a probability mass function of X, where X represents the number of showing on the upturned face of a die.*

In general the properties of a probability mass function are as follows.

1) P(X) ≥ 0 (Read as, *"Probability of X is greater than or equal to 0"*)

2) $\sum_{i=1}^{k} P(Xi) = 1$
(Read as, "*The summation of P(X_i) where i is ranging from 1 to k.*")
where xi is a mass point

Let us have some examples on how to compute the *probability mass function.* Consider the two properties discuss above. Don't forget to validate the given examples below. Let's start now.

1. Two coins are tossed. Let X be the number of heads. Obtain the probability mass function of X.

SOLUTION: Recall that the sample space S for the experiment.

Possible Outcomes	X (no. of heads)
HH	2
TT	0
HT	1
TH	1

- Probability of getting no heads or (TT): **P (0) = 1/4**

  (On 4 possible outcomes presented on the table, there is only one possible outcome that may produce no head, that is why P(0) = ¼).

- Probability of getting 1 head or (HT, TH): **P (1) = 2/4 or 1/2**

  (There are 2 possible outcomes that may produce 1 head, HT and TH. Adding their respective P(X), ¼ + ¼ is equal to 2/4 or ½. Making, P(1) = ½.)

- Probability of getting 2 heads or (HH): **P (2) = 1/4**

  Why do you think P(2) is equal to ¼?

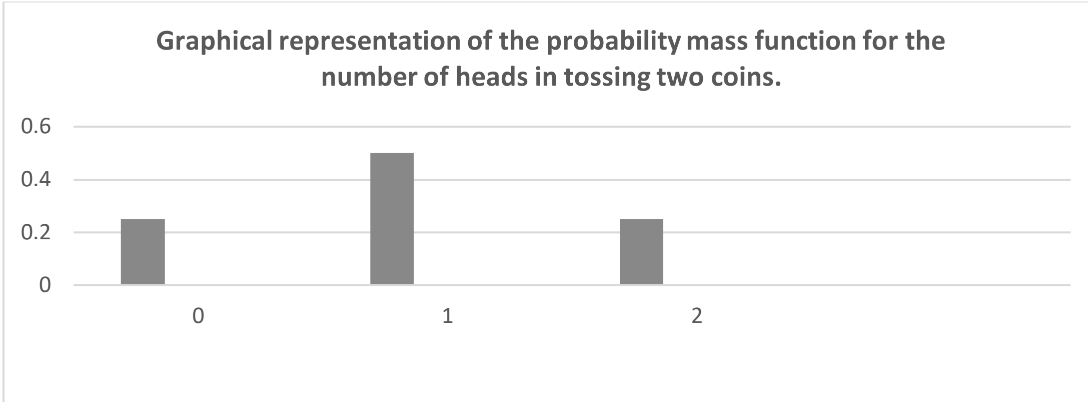
In tabular form, the *probability mass function* of X is shown below.

Table 1: Probability Mass Function for the Number of Heads in Tossing Two Coins

X	0	1	2
P(X)	1/4	1/2	1/4

Note that the two properties of a probability mass function are satisfied. All the probability values are non-negative, and between o and 1; and the probabilities sum up to 1.

Figure 1: Graphical representation of the *probability mass function* for the number of heads in tossing two coins



2. Suppose that the *probability mass function* is defined by the following formula.

$$P\ (X = x) = \begin{cases} X/10, \ x = 1,2,3,4 \\ 0, \text{ otherwise} \end{cases}$$

Make a tabular representation of the probability mass function. Then verify that the properties of a probability mass function are satisfied.

SOLUTION: Substitute each given value of x to the formula to get the corresponding probability.

x	1	2	3	4
P (X = x)	1/10	2/10	3/10	4/10

Note that P (X=x) ≥ 0 for all values of x. Also
$\sum_{i=1}^{k} \boldsymbol{P(X = xi) = 1}$

1/10 + 2/10 + 3/10 + 4/10 = 10/10 or 1

What's More

Solve the problem.

The daily demand for copies of a movie magazine at a variety store has the *probability distribution* as follows.

Number of Copies (X)	Probability P(X)
0	0.06
1	0.14
2	0.16
3	0.14
4	0.12
5	0.10
6	0.08
7	0.07

8	0.06
9	0.04
10	0.03

A. What is the probability that three or more copies will be demanded in a particular day?
B. What is the probability that the demand will be at least two but not more than six?

What I Have Learned

Fill in the blank with the right word/s.

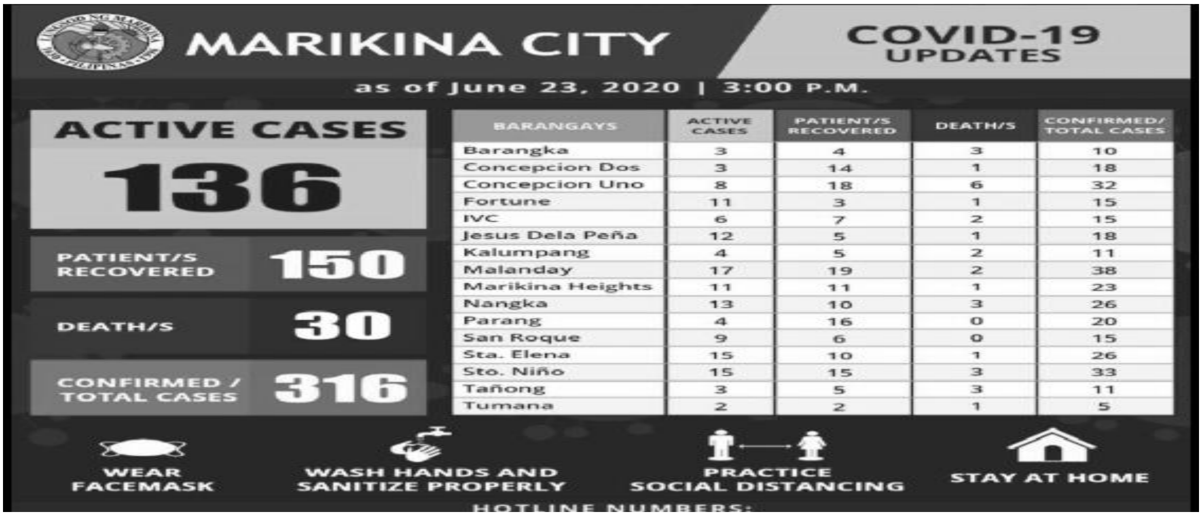
1. The probability of getting 2 head in tossing two coins is _____.
2. The _________________________ of a random variable distinguishes probabilities to the possible values of random variables. For a discrete random variable, the probability distribution is defined by a probability mass function denoted by P(X=x).
3. The _________________________ is a table, a formula, or a graph showing the possible values of the random variable together with the corresponding probabilities.
4. The possible values of the random variable are called ________________________.
5. What are the general properties of a probability mass function?

What I Can Do

Find the *Probability Mass Function* of the given data below in three different ways: Definition, Graph and Formula

When X = 0 is an active case, 1 is recovered, 2 is death

a. Kalumpang
b. Sta. Elena



MARIKINA CITY COVID-19 UPDATES

as of June 23, 2020 | 3:00 P.M.

ACTIVE CASES	136
PATIENT/S RECOVERED	150
DEATH/S	30
CONFIRMED / TOTAL CASES	316

BARANGAYS	ACTIVE CASES	PATIENT/S RECOVERED	DEATH/S	CONFIRMED/ TOTAL CASES
Barangka	3	4	3	10
Concepcion Dos	3	14	1	18
Concepcion Uno	8	18	6	32
Fortune	11	3	1	15
IVC	6	7	2	15
Jesus Dela Peña	12	5	1	18
Kalumpang	4	5	2	11
Malanday	17	19	2	38
Marikina Heights	11	11	1	23
Nangka	13	10	3	26
Parang	4	16	0	20
San Roque	9	6	0	15
Sta. Elena	15	10	1	26
Sto. Niño	15	15	3	33
Tañong	3	5	3	11
Tumana	2	2	1	5

WEAR FACEMASK
WASH HANDS AND SANITIZE PROPERLY
PRACTICE SOCIAL DISTANCING
STAY AT HOME
HOTLINE NUMBERS:

RUBRICS

POINTS	4	3	2	1
Understands the Problem	Identifies special factors that influences the approach before starting the problem	Understands the problem	Understands enough to solve part of the problem or to get part of the solution	Doesn't understand enough to get started or make progress
Uses Information Appropriately	Explains why certain information is essential to the solution	Uses all appropriate information correctly	Uses some appropriate information correctly	Uses inappropriate information
Applies Appropriate Procedures	Explains why procedures are appropriate for the problem	Applies completely appropriate procedures	Applies some appropriate procedures	Applies inappropriate procedures
Uses Representations	Uses a representation that is unusual in its mathematical precision	Uses a representation that clearly depicts the problem	Uses a representation that gives some important information about the problem	Uses a representation that gives little or no significant information about the problem
Answers the Problem	Correct solution of problem and made a general rule about the solution or extended the solution to a more complicated solution	Correct solution	Copying error, computational error, partial answer for problem with multiple answers, no answer statement, answer labeled incorrectly	No answer or wrong answer based upon an inappropriate plan

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Assessment

Carefully read each item and choose the letter that corresponds to the correct answer.

1. The possible outcomes of an experiment is called___________.
   A. Sample B. Sample Space C. Space D. Variable
2. Determine the number of possible outcomes in tossing 2 dies.
   A. 20 B. 25 C. 36 D. 45
3. A figure that uses branches to determine all the outcome of an experiment.
   A. Experiment C. Tree Diagram
   B. Listing D. Probability Distribution
4. The measure of how likely a particular event will occur is called ________.
   A. Distribution B. Probability C. Sample D. Variable

5. It is the process that generate sets of data.
   A. Statistical Data C. Statistical Probability
   B. Statistical Experiment D. Statistics Variability
6. A table, a formula or a graph showing the possible values of the random variable together with their corresponding probabilities.
   A. Probability Mass Data C. Probability Mass Function
   B. Probability Mass Distribution D. Variable Mass Function
7. Determine the number of possible outcomes in tossing 2 dies.
   A. 12 B. 20 C. 36 D. 71
8. Determine the number of possible outcomes in tossing 5 coins and a die.
   A. 125 B. 192 C. 320 D. 450
9. How many are the sample space of tossing two coins?
   A. 1 B. 4 C. 8 D. 12
10. What are the possible values of tossing two coins?
   A. H,T B. H,H,T,T C. HH, HT, TH, TT D. T,H

Additional Activities

The following data shows the probabilities for the number of cars sold in a given day at a car dealer store.

Number of car/s (X)	Probability P(X)
0	0.100
1	0.150
2	0.250
3	0.140
4	0.090
5	0.080
6	0.060
7	0.050
8	0.040
9	0.025
10	0.015

a. Find $P(X \leq 2)$.

b. Find $P(X \geq 7)$.

c. Find $P(1 \leq X \leq 5)$.

LESSON 2: FINDING THE MEAN AND VARIANCE OF A DISCRETE

RANDOM VARIABLE

What's In

In this time of uncertainty where it is difficult to trust anyone, your family is your grip. Your parents are always there to give you a safe environment, provide food, clothes and good education. Do you have siblings? Your siblings are also there for you to provide guidance and assistance regardless of the situation you are facing. Parents and siblings are very important part of your lives.

Have you ever wondered about the average size of a Filipino family based in the Philippine Statistics Authority? In 2016 the average size of a Philippine household is 5.

It is so great to know some fact regarding average, especially of those variables that greatly affects our lives. The mean or average and the variance of a random variable will be the focus of this module.

Try this!

1. Find the average allowance of 5 students from Kalumpang National High School.
   Ana 30, Joseph 50, Lito 50, Ryan 100 and Jessica 80
2. Find the average height of 10 Grade 11 students.

Cris	5.3	Rose	4.9
Ariel	4.11	Jen	5.3
Tommy	5.4	Ayen	4.11
Jim	6.0	Bianca	5.0
Francis	5.8	Tam	5.0

What's New

Let us familiarize ourselves with the term Mean and Variance of a Discrete Random Variable. Let say the grade 11 students were asked to estimate the length (in inches) of a table, the error in the estimated values were recorded and tabulated as follows:

Error (X)	2	3	4	5	6
P(X)	0.25	0.1	0.3	0.15	0.20

Task:

1. Get the product of Error(x) and P(x) for each column.
2. Add all the product.

What is It

Definition of Terms:

- ❖ A **measure of variability or the spread of the values** for a random variable X is the variance. The variance measures the degree of spread of the different values of the random variable about its expected value or mean.
- ❖ **Discrete** are random variables obtained by counting.
- ❖ Event is the subset of a sample space (S).
- ❖ **Mass points** are the possible values of the random variable.
- ❖ **Null Set or Empty Set** is only possible to have an event that contains no sample point.
- ❖ **Probability Histogram** is a figure or diagram that contains rectangles centered at each mass point.
- ❖ **Probability** is the measure of how likely a particular event will occur.
- ❖ **Sample Point** is the element of a sample space.
- ❖ **Sample Space** is the set of all possible outcomes of an experiment.
- ❖ **Statistical Experiment** is the processes that generate a set of data.
- ❖ The **mean** of a random variable X is not prediction of the values of X but only an expectation.
- ❖ **Tree diagram** is a figure that uses branches to determine all the outcomes of an experiment.

Mean of a Discrete Random Variable

The mean of the discrete random variable X is also called the **expected value** of X. The expected value of X is denoted by E(X). To compute the mean of a discrete random variable, the formula below is used.

$E(X) = \mu_x = \Sigma\ [xi * P(x_i)]$

where x_i is the value of the random variable for outcome i, μ_x is the mean of random variable X, and $P(x_i)$ is the probability that the random variable will be outcome.

Variance of a Discrete Random Variable

Most of the time, the values of the random variable are not constant. Thus, it is important to say something about the variations in the distribution. A **measure of variability or the spread of the values** for a random variable X is the variance.

The variance measures the degree of spread of the different values of the random variable about its expected value or mean. The weighs used are the corresponding probabilities given in the *probability mass function*.

Suppose the *probability mass function* of a random variable X is given in the table below:

X	x_1	x_2	x_3	....	x_n
P(X=x)	$P(x_1)$	$P(x_2)$	$P(x_3)$		$P(x_n)$

Then, the variance of the random variable is

$$Var(X) = \sigma_x^2 = \sum_{i=1}^{k}[(xi - \mu)^2 * P(xi)]$$

Example 1

In a recent little league softball game, each player went to bat 4 times. The number of hits made by each player is described by the following probability distribution.

Number of hits, x	**Probability, P(x)**
0	0.10
1	0.20
2	0.30
3	0.25
4	0.15

What is the mean and variance of the probability distribution?

Solution

Solving for the mean of the probability distribution

$E(X) = \Sigma [x_i * P(x_i)]$

$E(X) = \{ (0 * .10)+(1 * 0.20)+ (2 * 0.30) + (3 * 0.25) + (4 * 0.15) \} = 2.15$

Solving for the variance of the probability distribution

$Var(X) = \sigma_x^2 = \sum_{i=1}^{k}[(xi - \mu)^2 * P(xi)]$

$Var(X) = [(0 – 2.15)^2 * .10] + [(1 – 2.15)^2 * .20] + [(2 – 2.15)^2 * .30] + [(3 – 2.15)^2 * .25] + [(4 – 2.15)^2 * .15]$

$= 1.4275$

$= 1.43$

Note: The symbol “*” is denoted by multiplication.

What’s More

Calculate the mean and the variance for a random variable, *X* defined the number of tails in four tosses of a coin. Make a table for probability distribution.

Remember: T represent a tail and H, a head. X denotes the number of tails in four tosses of a coin. X takes the values: 0, 1, 2, 3, 4.

What I Have Learned

The MEAN of the random variable is

____

____

The VARIANCE of the random variable is

____

____

The formula used in finding the mean is ____

The formula used in finding the variance is ____________________________________

What I Can Do

The number of items sold per day at a retail store, with its corresponding probabilities is shown in the table. Find the mean and the variance of the *probability distribution.*

Number of Items Sold (X)	Probability P(X)
19	0.20
20	0.20
21	0.30
22	0.20
23	0.10

POINTS	4	3	2	1
Understands the Problem	Identifies special factors that influences the approach before starting the problem	Understands the problem	Understands enough to solve part of the problem or to get part of the solution	Doesn't understand enough to get started or make progress
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Assessment

Read each item carefully and encircle the letter of the correct answer.

1. A figure or diagram that contains rectangles centered at each mass point.
   A. Histogram　　B. Mass Distribution
   C. Probability Histogram　　D. Tree Diagram
2. It is a figure that uses branches to determine all the outcomes of an experiment.
   A. Histogram　　C. Statistical Data
   B. Tree diagram　　D. Venn Diagram
3. It is a possible to have an event that contains no sample point.
   A. Mass Point　　C. Sample Point
   B. Null Set　　D. Sets
4. The element of a sample space.
   A. Experiment　　C. Sample Space
   B. Sample Point　　C. Statistical Data
5. The measure of how likely a particular event will occur.
   A. Experiment　　C. Sample Point
   B. Probability　　D. Sample Space
6. The possible values of the random variable are called _____.
   A. Event　　B. Mass Points
   C. Mass Space　　D. Probability
7. The processes that generate a set of data.
   A. Experiment　　C. Statistical data
   B. Statistics　　D. Statistical Experiment
8. The set of all possible outcomes of an experiment.
   A. Event　　B. Probability　　C. Sample space　　D. Variables
9. The subset of a sample space (S).
   A. Event　　B. Sample　　C. Space　　D. Variable
10. These are random variables obtained by counting.
    A. Continuous　　B. Discrete　　C. Ordinal　　D. Variable

Additional Activities

Additional Activity:

1. A life insurance policy gives a benefit of P2,000,000 for death caused by an accident. P 1,000,000 for death due to other causes, P 500,000 for permanent disability. In the event of death, a person who had already applied for permanent disability benefits would no longer be entitled to the death benefit. Based on the past experience of the insurance company, for a specific age interval, the *probability mass function* for the amount of benefit A for the events mentioned above is

a	P 2,000,000	P 1,000,000	P 500,000	0
P(a)	.0002	.0008	.001	.988

Find the expected amount of benefit A and the variance.

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https://openstax.org/books/introductory-statistics/pages/4-1-probability-distribution-function-pdf-for-a-discrete-random-variable

Answer Key

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