Senior High School

Department of Education
National Capital Region
SCHOOLS DIVISION OFFICE
MARIKINA CITY

MATHEMATICS

Quarter 1: Module 2A
RATIONAL FUNCTIONS

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What I Need to Know

This module was designed and written with you in mind. It is here to help you master the **Rational Functions**. The scope of this module is to represent real-life situations using rational functions,distinguish rational function, rational equation, and rational inequality, solve rational equations, and rational inequalities.

This module on rational functions will help you to learn and understand the content and how it is applicable to our daily life.

The module is divided into 4 lessons, namely:

Lesson 1: Real-life Situations Using Rational Functions
Lesson 2: Rational Function, Rational Equation, and Rational Inequality
Lesson 3: Rational Equations
Lesson 4: Rational Inequalities

After going through this module, you are expected to:

1. represent real-life situations using rational functions,
2. distinguish rational function, rational equation, and rational inequality,
3. solve rational equations and
4. solve rational inequalities.

What I Know

Choose the letter that corresponds to the exact answer.

1. The rational equation $y = \frac{2x^2}{x+1}$ can be written as________________.

   A. $f(x)=\frac{2x^2}{x+1}$ B. $f(y)=\frac{2x^2}{x+1}$ C. $f(x)=\frac{x+1}{2x^2}$ D. $f(y)=\frac{x+1}{2x^2}$

2. A function of the form $f(x)=\frac{p(x)}{q(x)}$; where p(x) and q(x) are polynomial functions and $q(x)\neq 0$ is a ____________________________.

   A. Polynomial Function B. Rational Function
   C. Radical Function D. Quotient Function

3. What makes the function $f(x)=\frac{1}{2^x+1}$ not a rational function?

   A. The numerator is not a polynomial.
   B. The denominator is not a polynomial.
   C. There is no exponent in the numerator.
   D. The numerator has no variable x.

4. If a runner can run around the 400-meter oval in t minutes. The speed of the runner can be expressed as a rational function f(t)=_________________.
   A. 400t B. $\frac{t}{400}$ C. $\frac{400}{t}$ D. $400t^2$
5. A 100 million pesos lotto jackpot price is to be divided equally among the lucky ticket holders of the six-number winning combination. Express the situation as a function f(n) if there are n number of ticket holders who got the six-number winning combination in the lotto draw.
   A. $f(n)=\frac{100\ 000\ 000}{n}$ C. $f(n)=\frac{100\ 000\ 000}{6n}$
   B. $f(n)=\frac{n}{100\ 000\ 000}$ D. $f(n)=\frac{6n}{100\ 000\ 000}$
6. Which of the following is an inequality involving rational expressions?
   A. Rational Expression C. Rational Function
   B. Rational Equation D. Rational Inequality
7. What is the least common denominator of the equation $\frac{1}{x-2}+\frac{1}{x^2-7x+10}=\frac{6}{x-2}$?
   A. (x-2)(x+5) B. (x-7)(x-3) C. (x-2)(x-5) D. $x^2-3x-10$
8. Solve for x: $\frac{7}{3x}+\frac{4}{6x^2}=\frac{5}{8x}-\frac{2}{3x^2}$
   A. $-\frac{32}{41}$ B. $-\frac{41}{32}$ C. $\frac{32}{41}$ D. $\frac{41}{32}$
9. Determine where the numerator and denominator are zeros for the rational inequality $\frac{x+3}{x-9}\leq 0$.
   A. 3 and 9 B. 3 and -9 C. -3 and -9 D. -3 and 9
10. Solve the rational inequality $\frac{1}{x}+10>0$. Write the solution set in interval notation.
   A. $(-\infty, -\frac{1}{10}) \cup (0, +\infty)$ C. $(-\infty, \frac{1}{10}) \cup (0, +\infty)$
   B. $(-\infty, 10)$ D. $(10, \infty)$

Lesson 1: Represents Real-Life Situations Using Rational Functions

What's In

A **polynomial function** p of degree n is a function that can be written in the form $f(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\ldots+a_1x+a_0$; where $a_0, a_1, \ldots, a_n \in R$, $a_0 \neq 0$ and n is a positive integer.

1. What makes $f(x)=2x^3+3x-\frac{1}{\sqrt{x}}$ not a polynomial function?
2. What makes $p(x)=2e^x+3x-15$ not a polynomial function?
3. What makes $r(m)=\pi m^{-1}+3em-1$ not a polynomial function?

What's New

All real numbers have a corresponding reciprocal/multiplicative inverse.

1. Do you agree in the given statement? Why?
2. If you disagree, can you give a real number that has no reciprocal/multiplicative inverse?
3. If x represents a real number, how can you express its reciprocal?
4. If x represents a real number, how can you express the statement as a function f(x)?

What is It

Dividing a polynomial by another polynomial forms a new kind of function which is called a rational function.

The statement "All real numbers have a corresponding reciprocal/multiplicative inverse" can be represented by the function $f(x) = \frac{1}{x}$, which is an example of a rational function.

Definition: A **rational function** is a function of the form $f(x)=\frac{p(x)}{q(x)}$; where p(x) and q(x) are polynomial functions and $q(x) \neq 0$. Thus, the function $f(x) = \frac{1}{x}$, $x \neq 0$, is undefined when the denominator is zero, which means that not all real numbers have a corresponding reciprocal and that is 0.

There are real-life situations that can be modeled by rational functions.

1. Use rational function to express your speed walking (t minutes) in going to school which is 300 meters away from your house.

From the formula $s = \frac{d}{t}$

where:

d=distance (300 meters)

t=time (t minutes)

$s = \frac{d}{t}$

$s = \frac{300}{t}$ replace d by 300

$f(t) = \frac{300}{t}$ rewrite s as a function in terms of t.

2. A rectangular lot with an area of 150 square meters needs to enclose by a fence. Let w be the width of the rectangular lot and l be the length of the rectangular lot. Find a function P(w) representing the perimeter of the fencing material required.

$A = lw$	Formula for the area of a rectangle
$150 = lw$	Replace A by 150 since the area of the rectangular lot is 150 square meters.
$\frac{150}{w} = l$	Solve for l in terms of w by dividing both sides by w since we need to find a function in P(w).
$P = 2l + 2w$	Formula in getting the perimeter of a rectangle
$P = 2\left(\frac{150}{w}\right) + 2w$	Replace l by $\frac{150}{w}$ and simplify.
$P = \frac{300}{w} + 2w$	Rewrite P to P(w) since the right hand side of the equation is expressed in terms of w.
$P(w) = \frac{300 + 2w^2}{w}$	Simplify.
$P(w) = \frac{2w^2 + 300}{w}$	Rearrange the terms in the numerator in descending order.

3. The local government of Marikina allotted P500,000 to be divided equally among the students of Malanday National High School for the reproduction of modules for the school year 2020-2021 since face to face classes are not possible to prevent the spread of covid 19 virus. Let x be the number of students in Malanday National High School. The express amount allotted for each student as a function g(x) is

$$g(x) = \frac{500\ 000}{x}$$

4. In order to qualify for the Championship in an inter-barangay basketball league, barangay Malanday needs to increase their winning percentage to get either in the first rank or in the second rank. If barangay Malanday has won 6 out of 25 games with a winning percentage of 24%. The express winning percentage of barangay Malanday as a function p(x), if x is the number of wins that the team needs to win to increase their winning percentage is

$$p(x) = \frac{6 + x}{25 + x}$$

What's More

Express the following situations as a function.

1. A private company would like to give ₱50,000.00 incentives to its employees who performed well in their job. Construct a function f(x) representing the amount that each employee receives if the incentive is to be divided equally among the x well-performing employees.

2. If a swimmer can swim one lap (50 meters) of an Olympic sized swimming pool in t minutes. Express the swimmer's speed as a function f(t).

3. A 5-liter alcohol solution contains 2 liters of ethyl alcohol concentrate which makes it a 40% alcohol solution. You want to increase the alcohol concentration of a solution to effectively fight the COVID 19 virus. Express the alcohol concentration of the solution as a function f(a), if a is the amount of ethyl alcohol concentrate in liters that must be added in the solution to increase its alcohol concentration percentage.

4. A $300m^2$-rectangular vegetable farm is to be enclosed by a fence. If l and w represent the length and width of the rectangle, respectively, find the function P(l) representing the perimeter of the required fencing material.

5. Express your speed as a function S(m) riding a bicycle in m minutes going to school which is 500 meters away from your house.

What I Have Learned

Fill in the blank. Write the word/s or statement/s that will complete the sentence.

1. A rational function is a ratio of two ________________.
2. A rational function is written in the form ______________.
3. A rational function is defined if the polynomial in the denominator of a rational function must not be equal to ______________________.

What I Can Do

Problem: A 500 square meter rectangular flower garden needs to be enclosed by a fence. If l and w represent the length and width of the rectangular flower garden, respectively, construct a function P(w) representing the perimeter of the required fencing material with the given conditions:

a. The longer side of the flower garden faces the river and does not require any fencing material.

b. The shorter side of the flower garden faces the river and does not require any fencing material.
c. The flower garden must be divided into two equal parts by putting a fence at the middle parallel to the shorter sides of the rectangular flower garden.
d. The flower garden must be divided into two equal parts by putting a fence at the middle parallel to the longer sides of the rectangular flower garden.

Rubrics for Scoring

Score	Description
15 points	Complete solutions with correct answers
10 points	75% correct solutions with incorrect answer.
5 points	50% correct solution with incorrect answer.
No point earned	No output at all

Assessment

Write the letter that corresponds to the correct answer on a clean sheet of paper.

1. Your father is driving a car in going to work which is 54 km away from your house. Express the speed of your father's car as function C(n) if it takes n minutes of travel from your house to his office.

   A. $C(n)=54n$ C. $C(n)=\frac{54}{n}$
   B. $C(n)=\frac{n}{54}$ D. $C(n)=54-n$

2. Is the function $f(x)=\frac{1}{x^{\sqrt{2}}}$ a rational function?

   A. False C. Sometimes true
   B. True D. Cannot be determined

3. If a man can jog around the 400-meter oval in m minutes, the speed of the jogger can be expressed as a rational function f(m)=____________.

   A. 400m B. $\frac{m}{400}$ C. $\frac{400}{m}$ D. $400m^2$

4. Which of the following is not a rational function?

   A. $f(x) = \frac{2x}{x-5}$ C. $f(x)=\frac{\sqrt{3x+2}}{2x}$
   B. $f(x) = \frac{x-2}{x+5}$ D. $f(x) = \frac{3x+8}{x^2+2x+5}$

5. Use rational function to express your speed in walking (n minutes) while going to school which is 400 meters away from your house.

   A. f(n) = 400 – n C. $f(n)=\frac{400}{n}$

   B. $f(n) = \sqrt{400\text{-}n}$ D. $f(n)=\frac{n}{400}$

Additional Activities

Write the letter that corresponds to the correct answer on a clean sheet of paper.

1. Your mother is riding a jeepney going to Marikina public market in h hours which is 10 km away from your house. Express the speed of the jeepney as a function f(h)=________________.

   A. 10h B. $\frac{h}{10}$ C. $\frac{10}{h}$ D. $10h^2$

2. What makes the function $f(x)=\frac{e}{2^x+3x-5}$ not a rational function?
   A. The numerator is not a polynomial.
   B. The denominator is not a polynomial.
   C. There is no exponent in the numerator.
   D. The numerator has no variable x.

3. Use rational function f(w) to express the speed of the rolling ball (w minutes) from the first base to the second base which is 26.8 meters away from each other.

   A. f(w) = 26.8 – w C. $f(w)=\frac{26.8}{w}$

   B. $f(w) = \sqrt{26.8\text{-}w}$ D. $f(t)=\frac{w}{26.8}$

4. Your brother is driving a motorcycle in going to work which is 60 km away from your house. Express the speed of your brother's motorcycle as function C(n) if it takes n minutes of travel from your house to his office.

   A. C(n) = 60n C. $C(n) = \frac{60}{n}$

   B. $C(n) = \frac{n}{60}$ D. C(n) = 60-n

5. Is $2\pi x^{\sqrt{x}}y - ey = (x+1)(x-2)$ a rational function?

   A. True C. Sometimes true

   B. False D. Cannot be determined

Lesson 2: Rational Function, Rational Equation and Rational Inequality

What's In

Identify whether the following given expressions are rational or not and then explain your answer.

1. $\frac{1}{x^{2}-2x+3}$
2. $\frac{2x^2+1}{3}$
3. $\frac{2^{x+1}}{2x+1}$
4. $\frac{\pi x^3+3\pi}{\sqrt{x+1}}$
5. $\frac{1}{\frac{x^2-1}{x+3}}$

Answer the following questions briefly.

1. Why is the expression $\frac{\frac{3}{x-3}}{\frac{2x+5}{x+1}}$ a rational expression?
2. Which term in the expression $\frac{e^x}{x+3}$ makes it not a rational expression?
3. Why is the expression $\frac{3^x+2x-1}{x-1}$ not a rational expression?

What's New

Classify the following expressions into three groups based on the common characteristic that you observed.

$x^2+4x+3 \geq \frac{1}{x}$	$x^2-1 \leq \frac{3}{x}$	$\frac{x}{x+1}=12+y$	$\frac{x^2}{2+x}=\frac{2}{x}$
$2x^2+8=\frac{1}{x+1}$	$\frac{x+1}{x^2-1}>1-x$		
$f(x)=\frac{x}{x-1}$	$\frac{1}{3x^2}-7=-\frac{1}{x}$	$9x-y=\frac{10}{x}$	$\frac{4}{3}+x<\frac{1}{x}$

1. What is the common characteristic that you observed as your basis in classifying the expressions in the first group?
2. What is the common characteristic that you observed as your basis in classifying the expressions in the second group?
3. What is the common characteristic that you observed as your basis in classifying the expressions in the third group?

What is It

What you will learn here is about rational function, equation and inequality.

Let's have the following definitions:

A **rational function** is a function of the form $f(x) = \frac{p(x)}{q(x)}$; where p(x) and q(x) are polynomial functions, and q(x) ≠ 0. The domain of f(x) is all values of x where q(x) ≠ 0.

Example:

$$F(x) = \frac{x^2+2x+3}{x+1} \text{ or } y = \frac{x^2+2x+3}{x+1}$$

A **rational equation** is an equation involving rational expressions.

Example:

$$\frac{2}{x} - \frac{3}{2x} = \frac{1}{5}$$

A **rational inequality** is an inequality involving rational expressions.

Example:

$$\frac{5}{x-3} \leq \frac{2}{x}$$

Other Examples:

Rational Equation	a. $\frac{2+x}{x+2} = 5$ b. $\frac{x}{3} = \frac{5}{x-5}$
Rational Inequality	a. $3x > \frac{x}{x-2}$ b. $2x - 5 \geq \frac{2}{x-3}$
Rational Function	a. $f(x) = 2x^3 - 3x^2 + 3x - 2$ b. $g(x) = \frac{4x+6}{x+2}$

What's More

Determine whether the given is a rational function, a rational equation, a rational inequality, or none of these.

1. $y = \frac{x^2+3x+2}{x+4}$
2. $\frac{2}{x+4} = 8$
3. $\frac{3x+2}{x+4} = x\sqrt{x+1}$
4. $\frac{\sqrt{x+1}}{x-2} = 2x - 3$
5. $\frac{x^2+3x+2}{\frac{x+2}{x+3}} = \frac{x}{x-5}$
6. $\frac{3^x+2x-1}{x-1} \geq 2x-3$
7. $\frac{x}{x+4} < x+5$
8. $\frac{e^x}{x+3} = x - 5$
9. $\frac{\frac{3}{x-3}}{\frac{2x+5}{x+1}} = 2^{x+3}$
10. $\frac{x^2 + 3x}{4} = \frac{x+1}{x}$

What I Have Learned

Fill in the blank. Write the term/s or expression/s that will complete the sentence.

1. A __________________ is an equation involving rational expressions.
2. A rational inequality is an inequality involving ____________________.
3. A rational function is a function of the form _________________________.
4. $\frac{x}{x-2} = \frac{x-1}{x^2-4} + \frac{1}{x+2}$ is a _____________.
5. $P(x) = \frac{x+2}{x-3}$ is a ________________________.
6. $\frac{(x+1)(x-1)}{2x} \geq \frac{x}{x^2-4}$ is a __________________________.

What I Can Do

Determine whether the given is a rational function, a rational equation, a rational inequality, or none of these. If the answer is none of these write the term/expression which makes it not a rational function, a rational equation, and a rational inequality.

1. $y = 5x^2 - 2x + 1$
2. $\frac{8}{x} - 5 = \frac{x}{2x-1}$
3. $\sqrt{x-2} = 4$
4. $\frac{x-1}{x+1} = x^2$
5. $y = \frac{7x^2 + 4\sqrt{x} + 1}{x^2 + 3}$
6. $6x - \frac{5}{x+3} \geq 0$
7. $f(x) = \frac{2x+1}{e^x - 1}$
8. $3^{2x-1} = 9^{x+1}$
9. $x(x+3) \leq \sqrt{x+3}$
10. $\sqrt{2x-5} \geq \sqrt{x+1}$

Assessment

Write the letter that corresponds to the correct answer on a clean sheet of paper.

1. $f(x) = \frac{x-4}{x^2 - 5x + 4}$ is a __________________________.
   A. Rational Expression C. Rational Function
   B. Rational Equation D. Rational Inequality
2. $\frac{x}{2x-5} + 4 = 5 - \frac{x}{x-1}$ is a ______________________.
   A. Rational Expression C. Rational Function
   B. Rational Equation D. Rational Inequality
3. $\frac{2x-5}{x^2-9} > \frac{5}{x+3} - 1$ is a _______________.
   A. Rational Expression C. Rational Function
   B. Rational Equation D. Rational Inequality
4. A function of the form $f(x) = \frac{p(x)}{q(x)}$, where $q(x) \neq 0$.
   A. Rational Expression C. Rational Function
   B. Rational Equation D. Rational Inequality
5. Which of the following shows rational function?
   A. $2x + 3 < 10$ B. $2x \geq x - 1$ C. $\frac{x+1}{2x} = 5$ D. $y = \frac{2x}{x^2 - 9}$

Additional Activities

Determine the term/expression that makes the following not a rational function, a rational equation, and a rational inequality.

1. $y = 5x^2 - 2x^{\sqrt{2}} + \sqrt{\frac{1}{5}}$

   A. $2x^{\sqrt{2}}$ B) x^2 C) $\sqrt{\frac{1}{5}}$ D) None of these

2. $\frac{\sqrt{8}}{x} - 5\sqrt{x} = \frac{x}{2x-1}$

   A. $\frac{\sqrt{8}}{x}$ B. $5\sqrt{x}$ C. x D. 2x-1

3. $(x-3)(x+3) \leq \sqrt{x+3}$

   A. x-3 B. x+3 C. $\sqrt{x+3}$ D. none of these

4. $y = \frac{7x^2 + 4\sqrt{x} + 1}{x^2 + 3}$

   A. $7x^2$ B. $4\sqrt{x}$ C. $x^2 + 3$ D) none of these

5. $3(2x-5) = 9^{x+1}$

   A. 9^{x+1} B. 2 C. 2x-5 D. none of these

Lesson 3: Rational Equations

What's In

A. Solve the following Equations.

1. $2x + 4 = 0$
2. $2(a - 6) = 0$
3. $x^2 - 9 = 0$
4. $y^2 = 25$
5. $(2x + 1)(x - 1) = 0$

B. Find the LCM of the following

1. 10 and 5
2. 15 and 6
3. $2(x + 3)$ and $4(x - 5)$
4. $2x(x + 4)(x - 3)$ and $5x^2(x + 4)(x - 4)$
5. $2(x - 3)$ and $3x^2(x - 4)$

Answer the following questions briefly:

1. How did you solve the following equations above? What properties of equations are involved?

2. Discuss how to find the LCD or LCM of a given number or expressions.

3. Why do we have to find LCD in solving rational equation?

What's New

In finding the LCD of two or more fractions, we get the least common multiple (LCM) of their denominators by listing them.

Example: $\frac{1}{3}$ and $\frac{1}{6}$

Their denominators are 3 and 6.

List the multiples of 3: 3, 6, 9, 12, 15, 18, 21, ...

List the multiples of 6: 6, 12, 18, 24, ...

The common multiples are 6, 12, 18. The least among these common multiples is 6. So, 6 is the Least Common Denominator of $\frac{1}{3}$ and $\frac{1}{6}$.

What is It

What you will learn here is all about solving rational equations. Here are some illustrative examples for your guide.

	Rational Equation
Definition	An equation involving rational expressions.
Examples	i. $\frac{5x}{6} - \frac{1}{2} = \frac{1}{3}$ ii. $\frac{1}{x} - \frac{2}{3x} = \frac{1}{5}$ iii. $\frac{x}{x+2} - \frac{1}{x-2} = \frac{8}{x^2-4}$ iv. $\frac{4x-6}{2x-3} = \frac{7}{x+1}$

A rational equation can be solved for all x values that satisfy the equation.

Procedure in Solving Rational Equations

To solve rational equations:

1. Determine the domain by excluding values that will make any of the denominators equal to zero.
2. Multiply each term of the equation by the LCD that will eliminate the denominators.
3. Check for extraneous solution.

Example 1: Solve for x: $\frac{5x}{6} - \frac{1}{2} = \frac{1}{3}$

Solution:

Since the denominators are constants, there will be no values that will make the denominators equal to zero that must be excluded in the domain. Therefore, the domain of the equation is the set of all real numbers or $\{x/x \in \mathbb{R}\}$. Having assumed that the solutions should be elements of this domain, we can multiply both sides of the equation by the LCD which is 6.

$\frac{5x}{6} - \frac{1}{2} = \frac{1}{3}$	Given
$(6)\left(\frac{5x}{6}\right) - (6)\left(\frac{1}{2}\right) = (6)\left(\frac{1}{3}\right)$	Multiply each term by the LCD which is 6 and simplify.
$5x - 3 = 2$	
$5x - 3 + 3 = 2 + 3$	Add 3 to both sides of the equation and simplify.
$5x = 5$	
$\frac{5x}{5} = \frac{5}{5}$	Divide both sides by 5 and simplify.
$x = 1$	

Check if x = 1 is a solution of $\frac{5x}{6} - \frac{1}{2} = \frac{1}{3}$.

$\frac{5x}{6} - \frac{1}{2} = \frac{1}{3}$	
$\frac{5(1)}{6} - \frac{1}{2} = \frac{1}{3}$	Replace x by 1 and simplify.
$\frac{5}{6} - \frac{1}{2} = \frac{1}{3}$	
$\frac{5}{6} - \frac{3}{6} = \frac{2}{6} = \frac{1}{3}$	Simplify the left-hand side of the equation by subtracting the fraction using the LCD which is 6.
$\frac{1}{3} = \frac{1}{3}$(True statement)	After simplifying the left-hand side of the equation, it is equal to the right-hand side of the equation.

Hence, x = 1 is a solution of $\frac{5x}{6} - \frac{1}{2} = \frac{1}{3}$.

Example 2: Solve for x: $\frac{1}{x} - \frac{2}{3x} = \frac{1}{5}$

Solution:
Equate each denominator to zero to determine the values that must be excluded in the domain.

$x = 0$ and $3x = 0$

$\frac{3x}{3} = \frac{0}{3}$ Divide both sides by 3 and simplify

$x = 0$

Since both denominators will be zero at x = 0, this value must be excluded in the domain.

Therefore, the domain of the equation is $\{x/x \neq 0\}$.

Having assumed that the solutions should be elements of this domain, we can multiply both sides of the equation by the LCD which is 15x.

$(15x)\left(\frac{1}{x}\right) - (15x)\left(\frac{2}{3x}\right) = (15x)\left(\frac{1}{5}\right)$	Multiply each term by the LCD which is 15x and simplify.
$15 - 10 = 3x$	Subtract the left-hand side of the equation.
$3x = 5$	
$\frac{3x}{3} = \frac{5}{3}$	Divide both sides of the equation by 3 and simplify.
$x = \frac{5}{3}$	

Check if $x = \frac{5}{3}$ is a solution of $\frac{1}{x} - \frac{2}{3x} = \frac{1}{5}$.

$\frac{1}{x} - \frac{2}{3x} = \frac{1}{5}$	
$\frac{1}{\frac{5}{3}} - \frac{2}{3\left(\frac{5}{3}\right)} = \frac{1}{5}$	Replace all x's by $\frac{5}{3}$ and simplify.
$\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$	
$\frac{1}{5} = \frac{1}{5}$ (True statement)	After simplifying the left-hand side of the equation, it is equal to the right-hand side of the equation.

Hence, $x = \frac{5}{3}$ is a solution of $\frac{1}{x} - \frac{2}{3x} = \frac{1}{5}$.

Example 3: Solve for x: $\frac{x}{x+2} - \frac{1}{x-2} = \frac{8}{x^2-4}$

Solution.
Factor each denominator in the fraction.: $\frac{x}{x+2} - \frac{1}{x-2} = \frac{8}{(x-2)(x+2)}$

Equate each denominator to zero and solve for x to determine values that must excluded in the domain.

$$x + 2 = 0 \qquad x - 2 = 0 \qquad \text{and} \qquad (x + 2)(x - 2) = 0$$
$$x + 2 - 2 = 0 - 2 \qquad x - 2 + 2 = 0 + 2 \qquad x + 2 = 0 \text{ or } x - 2 = 0$$
$$x = -2 \qquad x = 2 \qquad x = -2 \text{ or } x = 2$$

The values that will make the denominators equal to zero are -2 and 2. These must be excluded in the domain.

Therefore the domain of the equation is $\{ x \,/\, x \neq -2, 2\}$

Having assumed that the solutions should be elements of this domain, we can multiply both sides of the equation by the LCD which is (x + 2)(x - 2).

$$(x+2)(x-2)\left(\frac{x}{x+2}\right) - (x+2)(x-2)\left(\frac{1}{x-2}\right) = (x+2)(x-2)\left(\frac{8}{(x-2)(x+2)}\right)$$

Multiply each term by the LCD which is (x+2)(x-2) and simplify.

$$(x - 2)(x) - (x + 2)(1) = 8$$

Simplify the right-hand side of the equation by applying the distributive property of multiplication.

$$x^2 - 2x - x - 2 = 8$$

Simplify by combining similar terms.

$$x^2 - 3x - 2 = 8$$
$$x^2 - 3x - 2 + (-8) = 8 + (-8)$$

Add negative -8 to both sides of the equation and simplify.

$$x^2 - 3x - 10 = 0$$

Upon reaching this step, we can solve quadratic equation by factoring.

$$(x + 2)(x - 5) = 0$$

Factor the left-hand side of the equation.

$$x + 2 = 0 \quad \text{or} \quad x - 5 = 0$$

Apply the zero-product property by setting each factor to zero and solve for the values of x.

$$x = -2 \quad \text{or} \quad x = 5$$

However, only x = 5 belongs to the domain specified earlier. We call x = -2 as an extraneous solution (without going through the checking part when x = -2.)

Check if x = 5 is solution of $\frac{x}{x+2} - \frac{1}{x-2} = \frac{8}{x^2-4}$.

$$\frac{x}{x+2} - \frac{1}{x-2} = \frac{8}{x^2-4}$$

$\frac{5}{5+2} - \frac{1}{5-2} = \frac{8}{(5)^2-4}$

Replace all x's by 5 and simplify.

$\frac{5}{7} - \frac{1}{3} = \frac{8}{21}$

$\frac{15}{21} - \frac{7}{21} = \frac{8}{21}$

Subtract the left-hand side of the equation using the LCD which is 21.

$\frac{8}{21} = \frac{8}{21}$ (True statement)

After simplifying the left-hand side of the equation, it is equal to the right-hand side of the equation.

Hence, x = 5 is a solution of $\frac{x}{x+2} - \frac{1}{x-2} = \frac{8}{x^2-4}$.

Example 4: Solve for x: $\frac{4x-6}{2x-3} = \frac{7}{x+1}$

Solution:
Equate each denominator to zero to determine the values that must be excluded in the domain.

$2x - 3 = 0$ and $x + 1 = 0$

$2x = 3$ $x = -1$

$x = \frac{3}{2}$

The values that will make the denominators equal to zero are -1 and $\frac{3}{2}$. These must be excluded in the domain.

Therefore the domain of the equation is $\{ x / x \neq -1, \frac{3}{2}\}$.

Having assumed that the solutions should be elements of this domain, we can multiply both sides of the equation by the LCD which is (2x – 3) (x + 1) or, since the given equation is in proportion form. Hence, the product of the extremes 4x – 6 and x + 1 is equal to the product of the means 2x – 3 and 7.

$\frac{4x-6}{2x-3} = \frac{7}{x+1}$

$(4x - 6)(x + 1) = (7)(2x - 3)$

Either by multiplying both sides by the LCD which is (2x-3)(x+1) or by getting the product of the extremes equal to the product of the means, you will arrive to this equation after simplifying.

$4x^2 - 6x + 4x - 6 = 14x - 21$

Simplify the left-hand side of the equation by combining similar terms -6x and 4x which is equal to -2x.

$4x^2 - 2x - 6 + (-14x) + 21 = 14x + (-14x) - 21 + 21$

Set the equation to zero by adding -14x and 21 to both sides of the equation and simplify by combining similar terms (-2x and -14x which is equal to -16x) and (-6 and 21 which is equal to 15).

$4x^2 - 16x + 15 = 0$

Upon reaching this step, we can solve quadratic equation by factoring.

$(2x - 5)(2x - 3) = 0$

Factor the left-hand side of the equation.

$2x - 5 = 0 \quad \text{or} \quad 2x - 3 = 0$

Apply the zero-product property by equating each factor to zero and solve for the values of x.

$x = \frac{5}{2} \quad \text{or} \quad x = \frac{3}{2}$

However, only $x = \frac{5}{2}$ belongs to the domain specified earlier. We call $x = \frac{3}{2}$ as an extraneous solution (without going through the checking part when $x = \frac{3}{2}$).

Check if $x = \frac{5}{2}$ is solution of $\frac{4x-6}{2x-3} = \frac{7}{x+1}$.

$$\frac{4x-6}{2x-3} = \frac{7}{x+1}$$

$$\frac{4\left(\frac{5}{2}\right)-6}{2\left(\frac{5}{2}\right)-3} = \frac{7}{\left(\frac{5}{2}\right)+1}$$

Replace all x's by $\frac{5}{2}$ and simplify.

$$\frac{10-6}{5-3} = \frac{7}{\frac{5+2}{2}}$$

$2 = 2$ (True statement)

After simplifying both sides of the equation, the left-hand side is equal to the right-hand side of the equation.

Hence, $x = \frac{5}{2}$ is the only root of the given equation $\frac{4x-6}{2x-3} = \frac{7}{x+1}$.

What's More

Find the solution(s) to the following rational equations.

1. $\frac{1}{x} = \frac{2}{3x-1}$

2. $\frac{x-2}{2x+4} = \frac{1}{5}$

6. $\frac{18}{x^2-3x} = \frac{2x}{x-3} - \frac{6}{x}$

7. $\frac{3x}{x+2} + \frac{6}{x} + 4 = \frac{12}{x^2+2x}$

3. $\frac{1}{x} + \frac{1}{2x} = \frac{2}{3}$

4. $\frac{x+3}{2} - \frac{x}{4} = \frac{4}{3}$

5. $\frac{3}{x+1} - \frac{2}{x-1} = \frac{1}{x^2-1}$

8. $\frac{1}{x-2} + \frac{4}{x+9} = \frac{3}{x^2+7x-18}$

9. $\frac{3}{5x^2} - \frac{1}{5} = \frac{2}{25x^2}$

10. $\frac{9}{x+1} = \frac{2}{x^2-1}$

What I Have Learned

Fill in the blank. Write the statement/s or expression/s that will complete sentence.

1. In finding the LCD of any fractions, we get the ____________of the denominators.

2. The LCD of 2 and 5 is _________________.

3. A ____________________ is an equation involving rational expressions.

4. The LCM of the give rational equation $\frac{2}{2x} = \frac{4}{x^2}$ is ___________________.

5. If $\frac{3}{x-1} = \frac{4}{x+2}$, the value of x = ___________.

What I Can Do

Solve the following rational equation and show your complete solution. Write your answers on a clean sheet of paper.

1. $\frac{2w+5}{5} = \frac{8}{15}$

2. $\frac{3}{x+2} + \frac{4}{x-2} = \frac{2}{x^2-4}$

3. $\frac{5}{6x^2} = \frac{7}{2x} + \frac{1}{3x^2}$

Rubrics for Scoring

Score	Description
15 points	Complete solutions with correct answer.
10 points	75% correct solutions with incorrect answer.
5 points	50% correct solutions with incorrect answer.
No point earned	No output at all

Assessment

Direction: Choose the letter that corresponds to the exact answer.

1. Find the value of x in the equation $\frac{3}{x-2} - \frac{2}{x} = \frac{17}{x-2}$.

   A. $-\frac{1}{2}$ B. -2 C. $\frac{1}{4}$ D. 4

2. Solve for x: $\frac{7}{3x} + \frac{4}{6x^2} = \frac{5}{8x} - \frac{2}{3x^2}$

   A. $-\frac{32}{41}$ B. $-\frac{41}{32}$ C. $\frac{32}{41}$ D. $\frac{41}{32}$

3. Solve for x: $\frac{5}{x-2} = 7 - \frac{10}{x+2}$

   A. $\frac{6}{7}$ and 3
   B. $-\frac{6}{7}$ and 3
   C. $-\frac{6}{7}$ and -3
   D. $\frac{6}{7}$ and -3

4. What is the least common denominator of the given equation

   $$\frac{3}{x+2} + \frac{4}{x^2+4x+4} = \frac{6}{x+2}\ ?$$

   A. (x+2)(x+4)
   B. (x+2)(x+2)
   C. $x^2 + 4$
   D. (x+2)(x+2)(x+2)

5. What is the least common denominator of the given equation

   $$\frac{1}{x-2} + \frac{1}{x^2-7x+10} = \frac{6}{x-2}\ ?$$

   A. (x-2)(x+5)
   B. (x-7)(x-3)
   C. (x-2)(x-5)
   D. $x^2 - 7x + 10$

Additional Activities

Solve for x of the following.

1. $\frac{3x+1}{x-2} = 5$ 2. $\frac{x^2}{x-4} = \frac{x-3}{2x-1}$ 3. $\frac{10x}{5} - \frac{5}{10x} = 0$

Lesson 4: Rational Inequality

What's In

Find the solution set of the following quadratic inequalities.

1. $x^2 - 4x > -3$
2. $x^2 - x > 12$
3. $2x^2 < 9x + 5$
4. $-x^2 + 4 < 0$

Answer the following questions briefly.

1. What is quadratic inequality?
2. How do we solve quadratic inequalities?
3. Do we have same process in solving rational inequality? Explain your answer.

What's New

Consider the table below. Let's remember that sometimes we need to solve inequalities like the given examples on the table.

Symbol	**Words**	**Example**
$>$	greater than	$2x^2 + x > 2$
$<$	less than	$2x^2 < 8$
$\geq$	greater than or equal to	$3 \geq x^2$
$\leq$	less than or equal to	$3x^2 + 1 \leq 5x$

What is It

What you will learn here is all about solving rational inequality. Here are some illustrative examples for your guide.

Rational Inequality

Definition: An inequality involving rational expressions.

Examples:

i. $\frac{1}{x} - \frac{2}{3x} > \frac{1}{5}$

ii. $\frac{3x}{x+2} + \frac{6}{x} + 4 \leq \frac{12}{x^2+2x}$

iii. $\frac{1}{x-2} + \frac{4}{x+9} \geq \frac{3}{x^2+7x-18}$

A rational inequality can be solved for all x values that satisfy the inequality.

Procedure in Solving Rational Inequalities

To solve rational inequalities:

1. Rewrite the inequality as a single fraction on one side of the inequality symbol and zero on the other side.
   A. This step helps us apply the rule on the division of signs (for any real number).

   Assuming that the divisor is non-zero:

   B. If the dividend and divisor are like in sign, the quotient is positive.
   C. If the dividend and divisor are unlike in sign, the quotient is negative.
2. Determine over what intervals the fraction takes on positive and negative values.
   A. Locate the x values for which the rational expression is zero or undefined (factoring the numerator and denominator is a useful strategy).
   B. Mark the numbers found in (2A) on a number line. Use a shaded circle to indicate that the value is included in the solution set, and a hollow circle if it is not.
3. By these markings, we divide the number line into intervals. Select a test point within the interior of each interval in (2B).
4. The sign of the rational expression at this test point is also the sign of the rational expression at each interior point in the interval.
5. Summarize the intervals containing the solutions.

> **Warning!**
>
> It is not valid to multiply both sides of an inequality by a variable or an expression. Recall that
> - Multiplying both sides of an inequality by a positive number retains the direction of the inequality, and
> - Multiplying both sides of an inequality by a negative number reverses the direction of the inequality.
>
> Since the sign of a variable or an expression (containing a variable) is unknown, then it is not valid to multiply both sides of an inequality by this expression.

Example 1: Solve the inequality $\frac{2x}{x+1} \geq 1$.

Solution**:** (a) Rewrite the inequality as a single fraction on one side, and 0 on the other side..

$$\frac{2x}{x+1} - 1 \geq 0$$

$$\frac{2x-(x+1)}{x+1} \geq 0$$

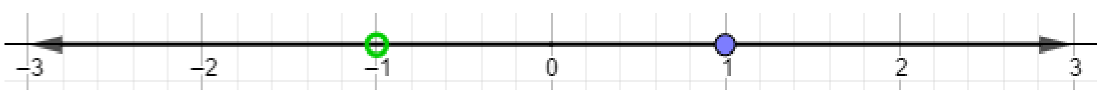
$$\frac{x-1}{x+1} \geq 0$$

(b) The value x = 1 is included in the solution since it makes the fraction equal to zero, while x = -1 makes the fraction undefined. Mark these on a number line. Use a shaded circle for x = 1 (a solution) and an unshaded circle for x = -1 (not a solution).

(c) Choose convenient test point in the interval determined by -1 and 1 to determine the sign of $\frac{x-1}{x+1}$ in these intervals. Construct a table of sign as shown below.

Interval	$x < -1$	$-1 < x < 1$	$X \geq 1$
Test point	x = -2	x = 0	x = 2
x - 1	-	-	+
x + 1	-	+	+
$\frac{x-1}{x+1}$	+	-	+

(d) Since we are looking for the intervals where the fraction is positive or zero, we determine the solution intervals to be $x < -1$ and $x \geq 1$. Plot these intervals on the number line.



The solution set is $\{x \in R \mid x < -1 \; or \; x \geq 1\}$.
It can also be written using interval notation: $(-\infty, -1) \cup [1, \infty)$.

What's More

Determine the solution set of the following inequalities.

1. $\frac{3}{x} > 4$
2. $\frac{x+3}{2x-1} < 0$
3. $\frac{3}{5(x-1)} \geq 0$
4. $\frac{2x+4}{x+1} \geq 0$
5. $\frac{x}{x+1} + 1 \leq -5$

What I Have Learned

Complete the sentence by filling up the appropriate expression/term/s in the blank.

1. An inequality involving rational expression is called a _______________.
2. Solving rational inequality means finding the values of the variable which will satisfy the __________________.

3. Assuming that the divisor is non-zero:
   A. If the dividend and divisor are like in sign, the quotient is __________.
   B. If the dividend and divisor are unlike in sign, the quotient is __________.

What I Can Do

Solve the following inequalities.

1. $\frac{8}{4x+10} \leq 0$
2. $\frac{b+12}{b-6} > 0$
3. $\frac{4c+1}{3c-2} + 3 \geq 4$
4. $\frac{2x-7}{x+1} \leq -7$
5. $y-5 < -\frac{9}{y+5}$

Rubrics for Scoring

Score	Description
15 points	Complete solutions with correct answer.
10 points	75% correct solutions with incorrect answer.
5 points	50% correct solutions with incorrect answer.
No point earned	No output at all

Assessment

Write the letter of the correct answer on a clean sheet of paper.

1. Find the solution of the rational inequality $\frac{x+3}{x-9} \leq 0$.
   A. $-3 \leq x < 9$
   B. $-9 \leq x < 3$
   C. (-3,9)
   D. (-3,9]
2. Solve the inequality $y - 5 < -\frac{9}{y+5}$.
   A. $(-\infty, -5) \cup (-4, 4)$
   B. $(5, \infty) \cup (-4, 4)$
   C. $(-\infty, -5) \cup (-4, \infty)$
   D. $(-\infty, -5) \cup (4, \infty)$
3. Determine where the numerator and denominator are zeros for the rational inequality $\frac{x+1}{x-5} \leq 0$.
   A. 1 and 5
   B. 1 and -5
   C. -1 and -5
   D. -1 and 5

4. Find the solution of the rational inequality $\frac{x}{x-7} > 0$.
   A. $-1 \leq x < 5$
   B. $-5 \leq x < 1$
   C. (-1,5)
   D. (-1,5]
5. Solve the rational inequality $\frac{1}{x} + 10 > 0$. Write the solution set in interval notation.
   A. $(-\infty, 10)$
   B. $(10, -\infty)$
   C. $(-10, \infty)$
   D. $(10, \infty)$

Additional Activities

Solve the following inequalities.

1. $\frac{1}{x+2} > 0$
2. $\frac{x-5}{2x-1} \geq 0$
3. $\frac{x^2-1}{x^2-x-2} > 0$

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