Department of Education
National Capital Region
**SCHOOLS DIVISION OFFICE
MARIKINA CITY**

GENERAL MATHEMATICS

Quarter 1: Module 7
LOGARITHMIC FUNCTIONS

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What I Need to Know

Hello senior high school learners! In this module, you will learn how to

Represent a logarithmic function through its: (a) table of values, (b) graph, and (c) equation M11GM-li-2,

Find the domain and range of a logarithmic function M11GM-li-3,

Determine the intercepts, zeroes, and asymptotes of logarithmic functions M11GM-li-4 and

Solve problems involving logarithmic functions, equations and inequalities M11GM-lj-2.

You can say that you have understood the lesson in this module if you can already:

1. represent a logarithmic function through its: (a) table of values, graph, and equation,
2. sketch the graph of logarithmic function and its transformation,
3. find the domain and range of a logarithmic function,
4. determine the intercepts, zeroes and asymptotes of logarithmic functions and
4. solve problems involving logarithmic functions, equations and inequalities.

What I Know

Choose the correct letter that corresponds to the exact answer. Write your answer on another sheet of paper.

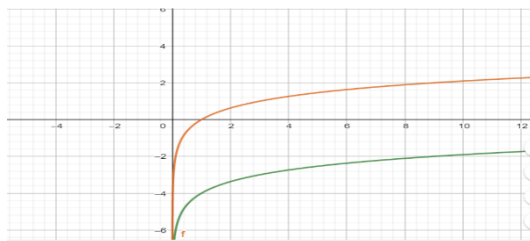1. Determine the missing value in this table of values for the function $y = 2^x$.

x	$y = 2^x$
-1	0.5
0	
1	2

   A. -1   B. 0   C. 1   D. 2

2. Which exponential function is decreasing?

   A. $y = \left(\frac{1}{3}\right)^x$ B. $y = 1.383^x$ C. $y = 7.7^x$ D. $y = \left(\frac{5}{2}\right)^x$

3. The graphs of $y = \log_3 x$ and its transformation image $y = g(x)$ are shown. Identify the transformations.

   A. A translation of 2 units right and 4 units down

   B. A horizontal compression by a factor of $\frac{1}{2}$

   C. A reflection in the x-axis
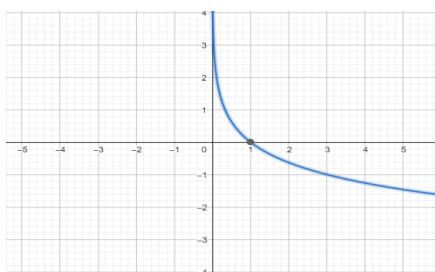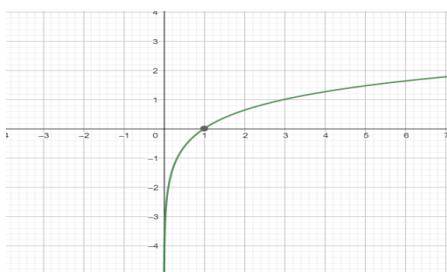
   D. A vertical stretch by a factor of 2

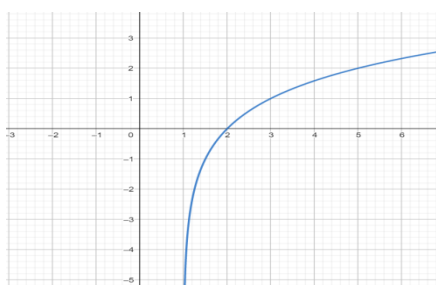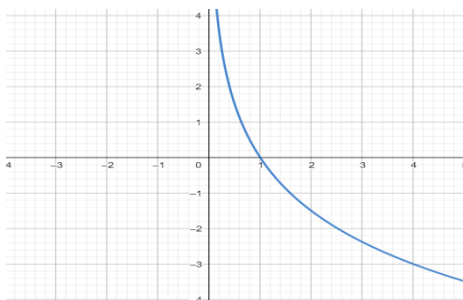4. Which exponential equation matches the graph shown?

A. $y = \left(\frac{1}{8}\right)^x$ B. $y = -\left(\frac{1}{8}\right)^x$ C. $y = 8^x$ D. $y = -8^x$

5. Given the function $f(x) = \log_3 x$. Graph the function and find the domain and range.

A. domain: $(0,\infty)$; range: $(-\infty,\infty)$ C. domain: $(0,\infty)$; range: $(-\infty-\infty)$

B. domain: $(1,\infty)$: range: $(-\infty,\infty)$ D. domain: $(1,\infty)$: range: $(0,\infty)$

6. Determine the domain and range of the function. F(x) =-2$^{(x + 3)}$ +4
   A. Domain: (-∞,4), Range:(-∞, ∞) C. Domain:(-∞, ∞), Range:(-∞,4)
   B. Domain: (-4, ∞), Range:(-∞, ∞) D. Domain:(-∞, ∞), Range:(-∞,∞)

7. Determine the domain and range of the function. F(x) = 3$^{(x + 3)}$ +1
   A. Domain: (-∞,1), Range:(-∞, ∞) C. Domain:(-∞, ∞), Range:(-∞,1)
   B. Domain: (-1, ∞), Range:(-∞, ∞) D. Domain:(-∞, -1), Range:(1,∞)
8. Which of the following is not a property of logarithmic functions?
   A. The domain is the set of all positive numbers.
   B. The range is the set of all positive real numbers.
   C. The x-intercept is 0.
   D. the vertical asymptote is the line x = 0.
9. Find the x-intercept of $f(x) = log_3(x - 1) - 2$
      A) x=10   B) x=9   C) x=8   D) x=7
   10. The vertical asymptote of $f(x) = log_3(x - 1)$ is at x=_____?
      A) -1   B) 0   C) 1   D) 2

LESSON 1: Graphing Logarithmic Functions

What's In

Let us recall what we have learned from the previous module by answering the following:

A.   Define logarithmic function, logarithmic equation and logarithmic inequality.

B.   Identify whether the given item is a logarithmic function, logarithmic equation and logarithmic inequality.

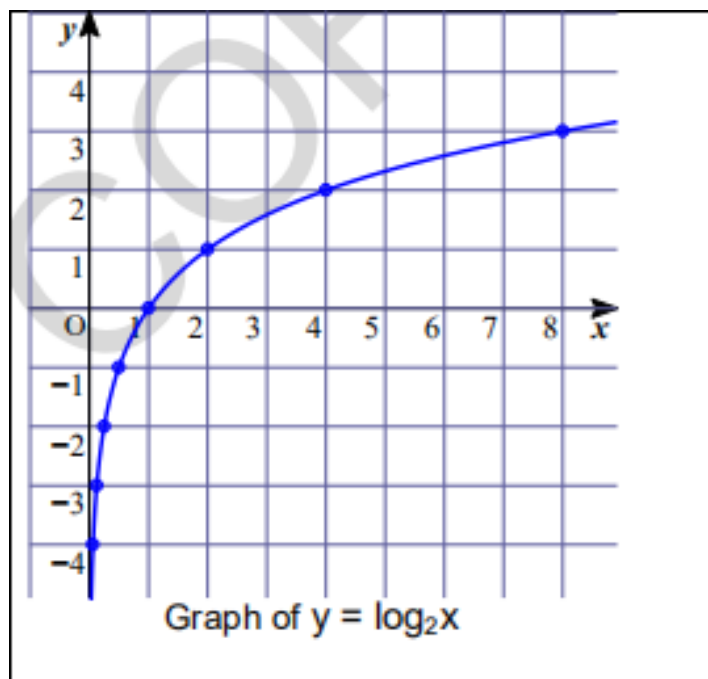C. Find the value/s of x in the following equations/inequalities.

1. log 5x = log 25

2. log (3x + 1) = log 2

3. log $_2$ (x +2) < log $_2$ 2x

? What's New

Below is the graph of the logarithmic function f(x) = log $_2$ x where b > 1. Consider it then, answer the questions that follow.

Graph of y = log₂x

1.   What are the possible values for x?
2.   What are the possible values for y?
3.   Is the function increasing? Why?
4.   What is the point of the graph that intersects the x axis?
5.   At what point y = 0?
6.   What is the point of the graph that intersects the y axis?
7.   At what point x = 0?

8.   What line is the graph approaches but will not intersect it?

Let us consider the graph of the logarithmic function $f(x) = \log_{1/2} x$ where $b < 1$. Consider it then, answer the questions that follow.
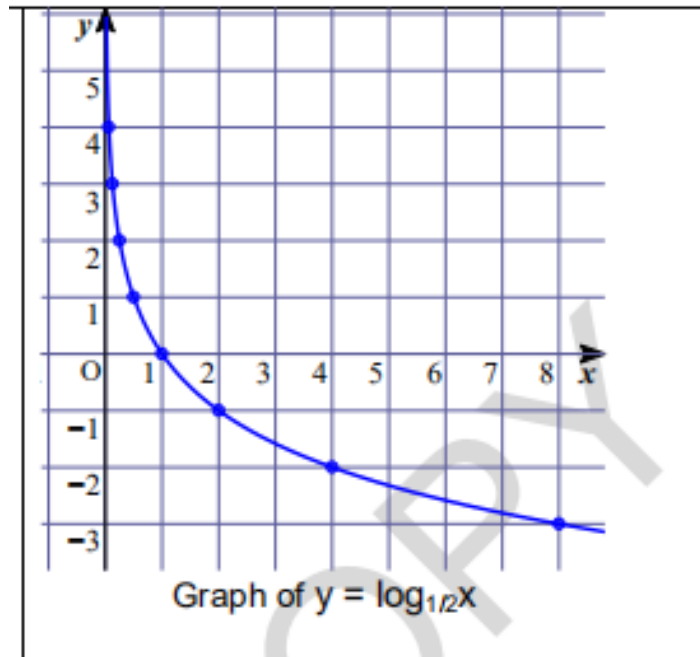

Graph of $y = \log_{1/2} x$

1.   What are the possible values for x?
2.   What are the possible values for y?
3.   Is the function increasing? Why?
4.   What is the point of the graph that intersects the x axis?
5.   At what point y = 0?
6.   What is the point of the graph that intersects the y axis?
7.   At what point x = 0?
8.   What line is the graph approaches but will not intersect it?

What is It

Notice that whenever we draw the graph of a function given its equation, we commonly construct a table, assign values for x to obtain its corresponding values. Then, we plot the points on the coordinate plane and connect them. In the case of a logarithmic function, we connect the points using a smooth curve.

Observe that in f(x) = $\log_2 x$ where b > 1, the function is increasing because as the value of x increases the value of y also increases. All possible values of x or the domain is x ≥ 0 while all possible values of y or the range is the set of real numbers. The point of the graph that intersects the x - axis is the point (1,0) which is also known as the x - intercept and its zero is 1. The graph does not intersect the y - axis so it has no y - intercept. The line that the graph approaches but will not intersect is the line x = 0. This line is known as the vertical asymptote of the function.

Similarly, Observe that in f(x) = $\log_{1/2} x$ where b < 1, the function is decreasing because as the value of x decreases the value of y also decreases. All possible values of x or the domain is x ≥ 0 while all possible values of y or the range is the set of real numbers. The point of the graph that intersects the x - axis is the point (1,0) which is also known as the x - intercept and its zero is 1. The graph does not intersect the y - axis so it has no y - intercept. The line that the graph approaches but will not intersect is the line x = 0. This line is known as the vertical asymptote of the function.

Example 1: Sketch the graph of y = $2\log_2 x$ and determine its domain, range, x - intercept, zero and vetrical asymptote.

Solution:
The graph of y = $2\log_2 x$ is obtained from the graph of y = $\log_2 x$ by multiplying each y - coordinate by 2.

x	1/16	1/8	¼	½	1	2	4	8
$\log_2 x$	–4	–3	–2	–1	0	1	2	3
y =$2\log_2 x$	–8	–6	–4	–2	0	2	4	6

The graph is shown below.

The domain is x ≥ 0. The range is the set of real numbers. The x - intercept and zero is 1. The vertical asymptote is x = 0.
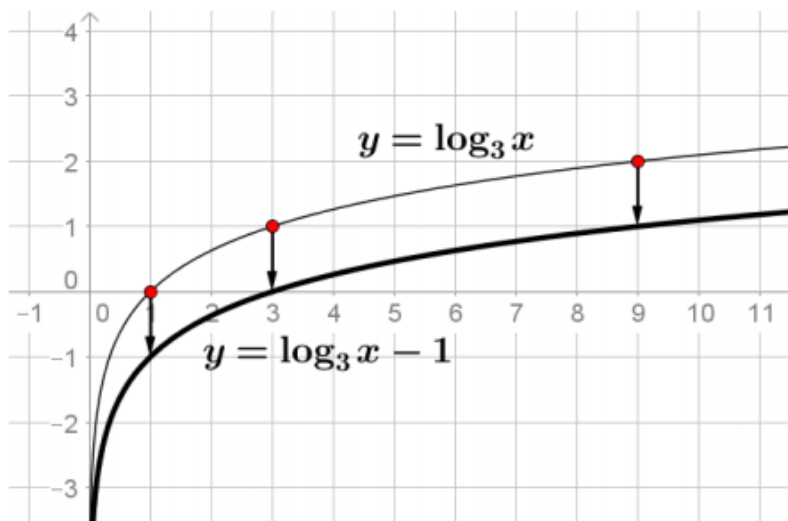
Example 2: Sketch the graph of y = log₃ x - 1 and determine its domain, range, x - intercept, zero and vetrical asymptote.

Solution:

The graph of y = log₃ x - 1 is obtained from the graph of y = log₃ x. The "-1" means vertical shift downward by 1 unit.

Some points on the graph of y = log₃ x are (1,0), (3,1) and (9,2). Shifting these points 1 unit down, we obtain (1,-1), (3,0) and (9,1). Plotting these points the graph is shown below.

The domain is x ≥ 0. The range is the set of real numbers. The x - intercept and zero is 3. The vertical asymptote is x = 0. The x - intercept can also be solved algebraically by setting y = 0 in y = log₃ x - 1.

$0 = \log_3 x - 1$

$\log_3 x = 1$

$X = 3^1 = 1$

Example 3:Sketch the graph of y = log $_{0.25}$ (x + 2) and determine its domain, range, x - intercept, zero and vetrical asymptote.

Solution:

Note that 0 < b < 1. Rewrite the equation y = log $_{0.25}$ (x + 2) into y = log $_{0.25}$ [x - (- 2). The "-2" means a horizontal shift of 2 units to the left.

The graph of y = log $_{0.25}$ (x + 2) is obtained from the graph of y = log $_{0.25}$ x. Some points on the graph of y = log $_{0.25}$ x are (1,0), (4,-1) and (0.25,1). Shifting these points 2 units to the left, we obtain (-1,0), (2,-1) and (-1.75,1). Plotting these points, the graph is shown below.
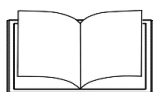

The domain is x $\geq$ -2. The range is the set of real numbers. The x - intercept and zero is -1. The vertical asymptote is x = -2.

The examples above can be generalized to form the following guidelines for graphing transformations of logarithmic functions:

Graph of f(x) = a·log$_b$(x – c) + d

- The value of b (either b > 1 or 0 < b < 1) determines whether the graph is increasing or decreasing.
- The value of a determines the stretch or shrinking of the graph. Further, if a is negative, there is a reflection of the graph about the x-axis.
- Based on y = a·log$_b$x, the vertical shift is d units up (if d > 0) or d units down (if d < 0), and the horizontal shift is c units to the right (if c > 0) or c units to the left (if c < 0).

What's More

Sketch the graph of each logarithmic function and determine its domain, range, x - intercept, zero and vertical asymptote.

1. $y = \log_6 x$

2. $y = \log_2 x + 5$

3. $f(x) = -\log_4(x - 2)$

What I Have Learned

To summarize what we have learned in this lesson, answer the following:

A.   What are the steps in drawing the graph of a logarithmic function?

_____
_____
_____
_____
_____

B.   In $y = \log_b x$ where $b > 1$, the function is _____ because as the value of x increases the value of y also increases. The domain is _____ while the range is the set of _____ numbers. The x - intercept and its zero is _____. The vertical asymptote of the function is the line _____.

In $y = \log_b x$ where $0 < b < 1$, the function is _____ because as the value of x increases the value of y also increases. The domain is _____ while the range is the set of _____ numbers. The x - intercept and its zero is _____. The vertical asymptote of the function is the line _____.

C.   What are the guidelines for graphing transformations of logarithmic functions $y = a \bullet \log_b (x - c) + d$?
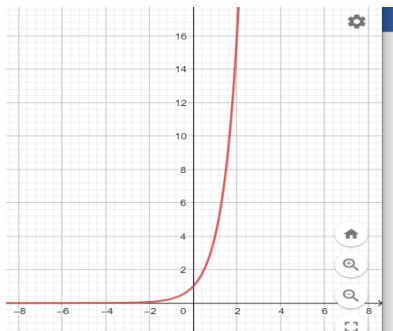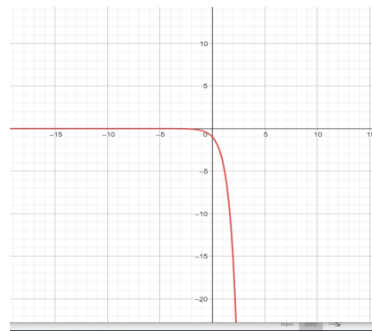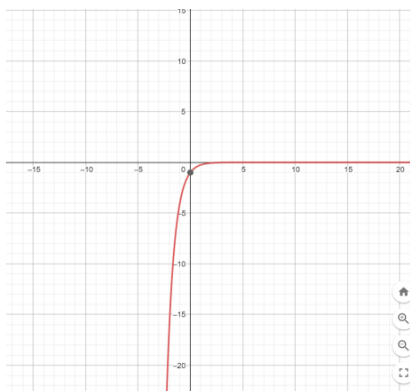
Sketch the graph and determine its domain, range, x - intercept, zero and vertical asymptote.

1. $y = \log_4 x$
2. $y = \log_{1/4} x$
3. $y = \log_2 (x+2)$

Rubrics

Score	Description
15 points	90-100% Correct answers with concrete explanation and there's corresponding neatness on the output.
10 points	60-89% Correct answers with an explanation and there's corresponding neatness on the output.
5 points	Incomplete answer with 50% incorrect answers and there's no corresponding neatness on the output.
No point earned	No output at all

Assessment

Choose the correct letter that corresponds to the exact answer.

1. The graph of $y = \log_2 x$ is compressed horizontally by a factor of 1 7, and then translated 8 units down. Identify the equation of the image graph.

A. $y-8 = -\log_2 (-7x)$ B. $y = 7\log_2 (x+8)$ C. $y = -8\log_2 (x-7)$ D. $y+8 = \log_2 (7x)$

2. Which graph represents the function $y = (\frac{1}{4})^x$.
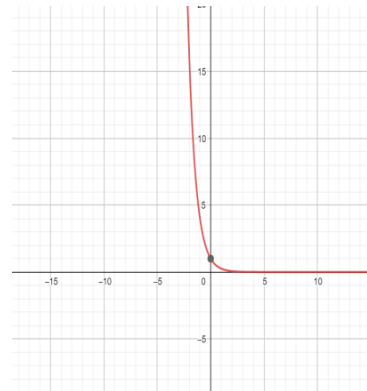
   A.   C.
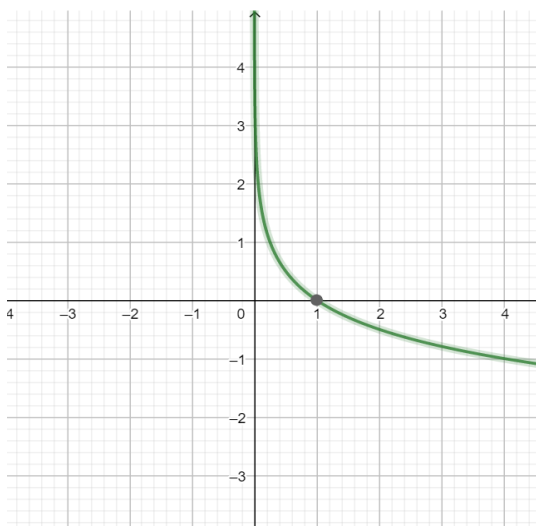
B.

D.

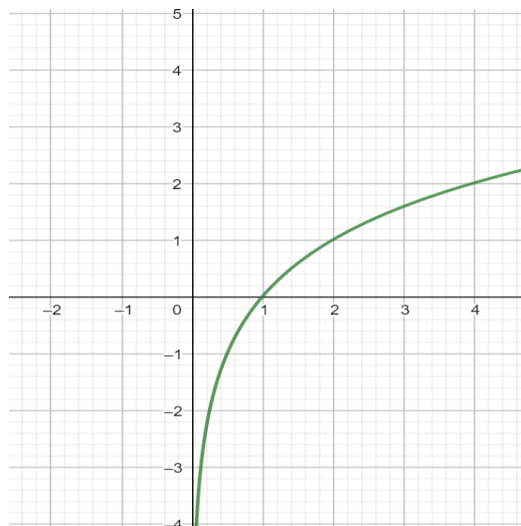

3. Which graph represents the function f(x) = $\log_{1/4} x$.
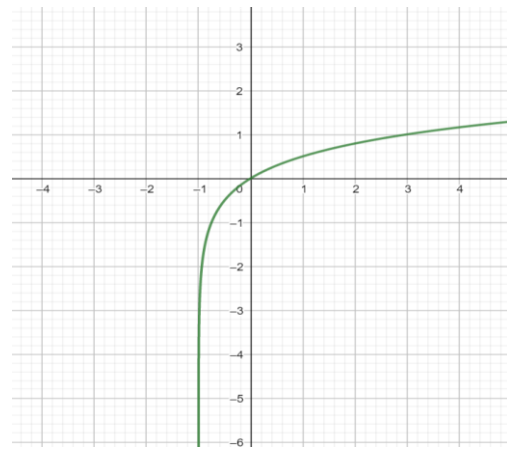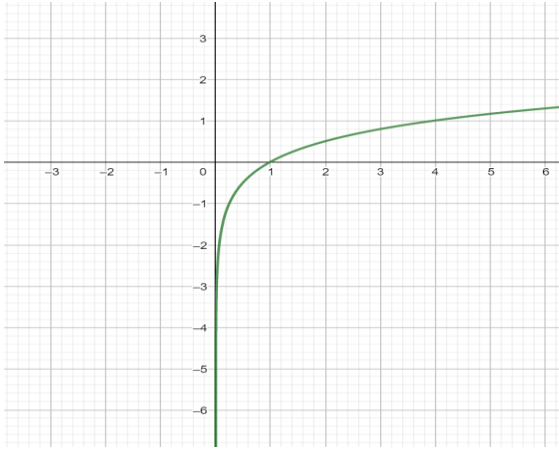
A.
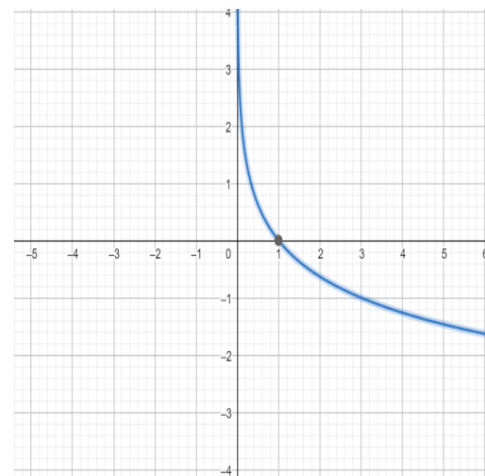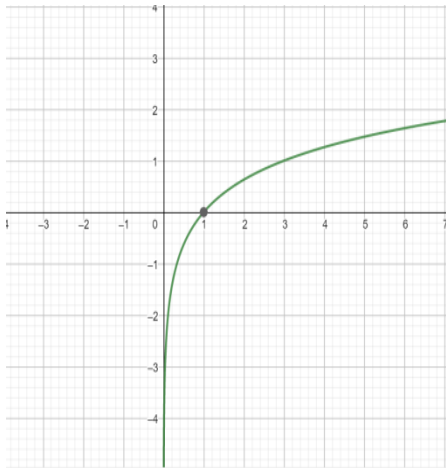
C.

B.   D.

4. Given the function f(x) = log $_3$ x. Graph the function and find the domain and range.

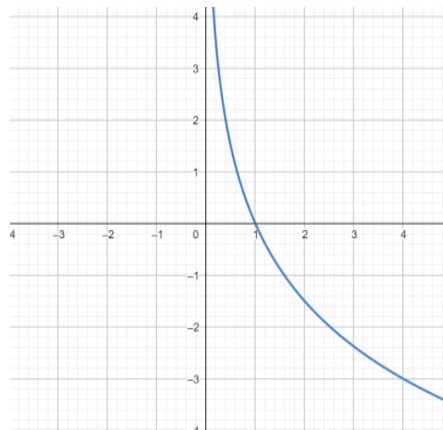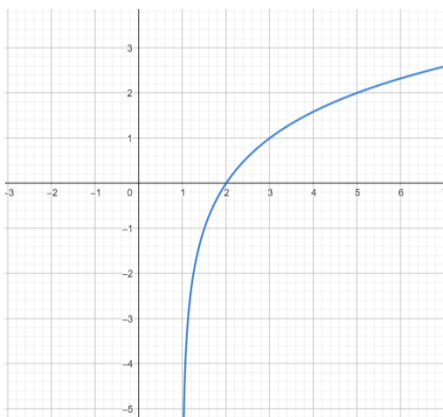   A. domain: (0,∞); range: (-∞,∞)   C. domain: (0,∞); range: (-∞-∞)

   B. domain: (1,∞): range: (-∞,∞)   D. domain: (1,∞): range: (0,∞)

5. Determine the domain and range of the function. F(x) =-2$^{(x + 3)}$ +4
   A. Domain: (-∞,4), Range:(-∞, ∞) C. Domain:(-∞, ∞), Range:(-∞, 4)
   B. Domain: (-4, ∞), Range:(-∞, ∞) D. Domain:(-∞, ∞), Range :(-∞,∞)

6. Which of the following is not a property of logarithmic functions?
   A. The domain is the set of all positive numbers.
   B. The range is the set of all positive real numbers.
   C. The x-intercept is 0.
   D. the vertical asymptote is the line x = 0.
7. Find the y-intercept of $y = log_2(3x - 5) + 3$
   A.2   B) 1   C) 0   D) none
8. The vertical asymptote of $f(x) = log_3(x + 1)$ is at x=_____?
   A.-1   B) 0   C) 1   D) 2
9. Find the x-intercept of $f(x) = log_3(x + 1) + 2$
   A) $-\frac{8}{9}$ B) $-\frac{9}{8}$ C) $\frac{8}{9}$ D) $\frac{9}{8}$
10. What is the behavior of the graph of the logarithmic function?
       $f(x) = log_3(1 - x)$ below the x-axis as the values of x gets closer to 1?
       A) Continuously increases without bounds
       B) Continuously decreases without bounds
       C) Cannot be determined
       D) None of the above

Note: In graphing logarithmic functions:
(a) x and y coordinates are interchanged
(b) graphs are reflections of each other along the line $y = x$
(c) domain of original function is the range of the inverse; range of the original function is the domain of the inverse
(d) horizontal asymptote of the function is the vertical asymptote of the inverse; vertical asymptote of the function is the horizontal asymptote of the inverse
(e) y-intercept of the function is x-intercept of the inverse; x-intercept of the function is y-intercept of the inverse

(f) behavior can be inspected after reflecting graphs along the line $y = x$

Additional Activities

A. In one Cartesian coordinate plane, sketch the graph of the following.
1. $y = \log_2 x$

2. $y = \log_3 x$

3. $y = \log_4 x$

4. $y = \log_5$

B. Determine the domain and range of the following logarithmic functions:
1. $y = 3 \log_3 x$

2. $y = \log_3 x$

3. $f(x) = \log_3 (x + 3) - 1$

4. $g(x) = \log_6 (x - 4) + 2$

5. $h(x) = \log (x - 2) + 5.$

C. Choose the letter that corresponds to the exact answer.
1) Express $y = 3^{2-x}$ logarithmic form.
    A) $log_y(2 - x) = 3$ C) $log_y 3 = 2 - x$
    B) $log_3(2 - x) = y$ D) $log_3 y = 2 - x$
2) 2) Find the x-intercept of $f(x) = log_3(x + 1) - 2$
    A) -9   B) -8   C) 8   D) 9
3) Evaluate $log_3 \frac{1}{729}$
    A) -6   B) -7   C) -8   D) -9
4) Express $6 = log_2 64$ in exponential form.
    A) $2^{64} = 6$ B) $2^6 = 64$ C) $64^2 = 6$ D) $6^2 = 64$
5) Find the y-intercept of $y = log_2(3x + 2) - 5$
    A) -4   B) -3   C) -2   D)

LESSON 2: Solving Problems Involving Logarithmic Function, Equation and Inequalities

What I Know

Choose the letter that corresponds to the exact answer.

1) The sound produces by a passing band has an intensity of 10^8 watts/m^3. What is its decibel level?
   A) 100   B) 200   C) 300   D) 400

2) If the pitch that the orchestras tune to has a frequency of 220 cycles per second. What is its corresponding nth key on the piano?
   A) 27th B) 37th C) 47th D) 57th

3) An earthquake released 10^{10} joules of energy. Find its magnitude.
   A) 7.73   B) 7.37   C) 3.73   D) 3.37

4) A 1-L solution contains 0.0001 moles of hydrogen ion. What is its pH level?
   A) 4   B) 5   C) 6   D) 7

5) If a gunshot from a rifle produces a sound intensity of 10^4 watts/m^3. Do you need to wear an ear protection at that level inside the firing range?
   A) Always   B) Sometimes   C) Never   D) Cannot be determined

6) How long will it take for a deposit of ₱250 000.00 to grow to ₱320 000 at 4% annual interest rate compounded quarterly?
   A) 2.6 years   B) 6.2 years   C) 26 years   D) 62 years

7) In how many years will a computer set depreciates from ₱49 990.00 to ₱29 500.00 if the value decreases by 12% annually?
   A) 4.13 years   B) 4.31 years   C) 41.3 years   D) 43.1 years

8) In how many years will a population grows from 100 000 to 315 000 if the population increases by 3% per year?
   A) 38.28 years   B) 38.82 years   C) 83.28 years   D) 83.82 years

9) A rock concert produces a sound with an intensity of 10^7 watts/m^3. What is classification according to its decibel level?
   A) Comfortable hearing level   C) Threshold of pain
   B) Intrusive   D) None of the above

10) How long will it take ₱25 000.00 to double if it is invested in a savings account that pays 1% annual interest rate compounded semi-annually?
    A) 49.69 years   B) 49.96 years   C) 69.49 years   D) 69.94 yea

What's In

Simplify the following by applying the properties of the logarithm.

1) $log\ log\ 10^{-5}\ =?$
2) $ln\ ln\ e =?$
3) $ln\ ln\ 1 =?$

Guide Questions:

1) What property of logarithm did you apply to get $log\ log\ 10^{-5}$ without using a calculator?
2) Why is $log\ log\ 1 = 0$?
3) Why is the $log\ log\ 10 = 1$?

❓ What's New

Answer the following questions.

1) Why do the scientists need to measure the magnitude of an earthquake?
2) How do the chemists compute for the pH level of a solution?
3) Why is it important to know the decibel level of the sound?

What is It

Getting the magnitude of an earthquake, the pH level of a solution, the decibel level of the sound and the time it takes for a certain amount to grow in an investment are some of the real-life situations using logarithm.

To solve problems involving logarithm, we will apply the five-step rule:

1) Identify what are given in the problem.
2) Determine what is asked in the problem.
3) Indicate the formula to be used.
4) Show your complete solution.
5) Give your answer in a complete sentence.

The magnitude of an earthquake is measured by Richter Scale which is given by the formula: $R = \frac{2}{3} log \frac{E}{10^{4.4}}$ where E is the amount of energy in joules released by an

earthquake and the following descriptions of various earthquake magnitudes is given in the table.

Magnitude	Description
1.0	Scarcely Perceptible-perceptible to people under favorable circumstances.
2.0	Slightly Felt-Hanging objects swing slightly.
3.0	Weak-Vibration is felt like one passing of a light truck. Hanging objects swing moderately.
4.0	Moderately Strong-Felt generally by people indoors and by some people outdoors.
5.0	Strong-Some shaking and rocking felt throughout building. Hanging objects swing violently.
6.0	Very Strong-Some people lose their balance. Heavy objects or furniture move or may be shifted.
7.0	Destructive-People find it difficult to stand in upper floors. Some cracks may appear. Limited liquefaction, lateral spreading and landslides are observed. Trees are shaken strongly.
8.0	Very Destructive-Many well-built buildings are considerably damaged. Liquefaction and lateral spreading cause man-made structure to sink, tilt or topple. Fissures and faults rapture may be observed.
9.0	Devastating-Most buildings are totally damaged. Bridges and elevated concrete structures are toppled or destroyed. Landslides and liquefaction with lateral spreading and sand boil are widespread.
10.0	Completely Devastating-Practically all man-made structures are destroyed. Massive landslides and liquefaction, large-scale subsidence and uplifting of landforms and many ground fissures are observed. Changes in river courses occur.

Example 1: The BIG ONE is forecasted to release a $10^{15.2}$ joules of energy due to the movement of the west valley fault. Find its corresponding magnitude and description.

Solution:

E=$10^{15.2}$ joules

R (magnitude)=?

Substitute the given value into the formula $R = \frac{2}{3} log \frac{E}{10^{4.4}}$ and simplify.

  a) From the formula: $R = \frac{2}{3} log \frac{E}{10^{4.4}}$ replace E by $10^{15.2}$ and simplify.

$$R = \frac{2}{3} log \frac{10^{15.2}}{10^{4.4}}$$
$$R = \frac{2}{3} log \, log \, 10^{10.8}$$
$$R = \frac{2}{3}(10.8)$$
$$R = 7.2 \text{ Slightly felt}$$

The BIG ONE will have a magnitude of 7.2 which is destructive where limited liquefaction, lateral spreading and landslides are observed.

Chemists classify the acidity or alkalinity of a substance according to its pH level which is given in the formula: pH=-log$[H^+]$ where H^+ is the number of moles of hydrogen ion in a solution.

where a pH level > 7 is classified as alkaline

pH level < 7 is classified as acidic

pH level = 7 is classified as neutral.

Example 2: Suppose a 1-liter solution contains 0.00000000001 moles of hydrogen ion. Find its pH level and classify whether it is acidic, alkaline or neutral.

Solution:

H^+=0.00000000001 or 10^{-11}

pH=?

Substitute the given value into the formula pH=-log$[H^+]$ and simplify.

From the formula:

pH=-log$[H^+]$ replace H^+ by 0.00000000001 or 10^{-11}

pH=-log$[10^{-11}]$=11 alkaline

The solution has a pH level of 11 and classified as alkaline.

In acoustics, the decibel level of the sound is given by the formula:

$D = 10log \frac{I}{10^{-12}}$ Where I is the sound intensity in watts/m^3. The classification and descriptions are given in the table below.

Level	Description
Under 60 decibels	Comfortable hearing levels
60-85 decibels	Intrusive and interferes with conversation.
Between 85-90 decibels	Hearing damage begins (unprotected exposure for 8 hours)
90-100 decibels	No more than 15 minutes of unprotected exposure recommended.
Over 100 decibels	Regular exposure of more than one-minute risks permanent damage.
Over 125 decibels	Threshold of pain begins.

and above	

Example 3: Suppose a stereo produces a sound intensity of 10^5. Find its decibel level, classification and give some precautionary measure.

$D = 10log\frac{I}{10^{-12}}$ replace I 10^5 and simplify

$D(s) = 10log\frac{10^5}{10^{-12}}$

$D(s) = 10\ log\ log\ 10^{17}$

$D(s) = 10(17)$

$D(s) = 170$ decibel

The stereo produces a sound with a 170 decibel level which is classified as threshold of pain and a regular exposure of more than minute will cause permanent hearing damage. It is not advisable to listen to the stereo at a high volume.

The nth note up the keyboard on a piano when played, is given by the formula:

$n = 1 + 12log_2\frac{f}{27.5}$, where f is the frequency of the pitch in cycles per second.

Example 4: Find the nth note up the keyboard on a piano that produces a pitch frequency of 440 cycles per second.

Solution:
f= 440 hertz (cycles per second)
n= nth note (key) on a piano

Substitute the given data into the formula $n = 1 + 12log_2\frac{f}{27.5}$ and simplify.

$n = 1 + 12log_2\frac{f}{27.5}$ replace f by 440 and simplify

$n = 1 + 12log_2\frac{440}{27.5}$

$n = 1 + 12log_2 16$

$n = 1 + 12(4)$

$n = 49^{th}$ key

The 49th key on the piano produces a pitch frequency of 440 hertz (cycles per second)

The time it takes for an investment to grow to a certain amount at a given interest rate is given by: $t = \frac{loglog\left(\frac{A}{P}\right)}{nloglog\left(1+\frac{r}{n}\right)}$

Where:
P: principal amount
A: accumulated amount
r: interest rate
n: number of times a year the interest is paid
t: number of time intervals that have passed

Example 5: How long will it take for a time deposit of ₱500 000.00 to double at 5% annual interest rate compounded monthly.

Solution:

P= 500 000

A=1 000 000

r= 5% or 0.05

n=12 (compounded monthly)

t=?

Substitute these given data in the problem into the formula and simplify.

$$t = \frac{\log\log\left(\frac{A}{P}\right)}{n\log\log\left(1+\frac{r}{n}\right)}$$

$$t = \frac{\log\log\left(\frac{1\,000\,000}{500\,000}\right)}{12\log\log\left(1+\frac{0.05}{12}\right)}$$

$$t = \frac{\log\log\left(2\right)}{12\log\log\left(1+\frac{0.05}{12}\right)}$$

$t = 13.9$ years

It will take 13.9 years for the P500 000.00 deposit to double at 5% compounded monthly

The time it takes for the value of a property/population to increase at a given rate of increase is given by:

$$t = \frac{\log\log\left(\frac{y}{a}\right)}{\log(1+r)}$$

Where:

y: value after the rate of increase is applied at a given number of time intervals that have passed.

a: initial value

r: rate of increase

t: number of time intervals that have passed

Example 6: How long will it take for a lot worth 1 000 000 to triple its value if the real property increases at 10% annually?

Solution:

y=3 000 000

a= 1 000 000

r= 10% or 0.1

t=?

Substitute these given data in the problem into the formula and simplify: $t = \frac{\log\log\left(\frac{3\,000\,000}{1\,000\,000}\right)}{\log(1+0.1)}$

$$t = \frac{\log\log\left(3\right)}{\log(1.1)}$$

$t = 11.53$ years

It will take 11.53 years for the value of a lot to triple at 10% annual rate. The time it takes for the value of a property/population to decrease at a given rate of decrease is given by:

$$t = \frac{\log \log \left(\frac{y}{a}\right)}{\log(1-r)}$$

Where:

y: value after the rate of increase is applied at a given number of time intervals that have passed.

a: initial value

r: rate of increase

t: number of time intervals that have passed

Example 7: The price of Samsung Galaxy note20 depreciates at 5% annually. How long will it take for its brand new price worth ₱72 000.00 to depreciate into half of its price?

Solution:

$$t = \frac{\log\log \left(\frac{y}{a}\right)}{\log(1-r)}$$
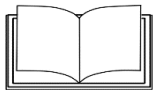
Where:

y: 36 000

a: 72 000

r: 5% or 0.05

t: ?

Substitute these given data in the problem into the formula and simplify:

$$t = \frac{\log \log \left(\frac{36\,000}{72\,000}\right)}{\log(1-0.05)}$$

$$t = \frac{\log \log (0.5)}{⍰⍰⍰ \log (0.95)}$$

$t = 13.51$ years

It will take 13.51 years for the price of Samsung Galaxy note20 to depreciate into half of it brand new price.

What's More

Solve the following problems following the five-step rule.

1) The earthquake in Davao last December 2019 produced a $10^{14.8}$ energy. Find its corresponding magnitude and classification.
2) A 1-liter substance contains 0.00001 moles of hydrogen ion. Find its corresponding pH level and classify whether the substance is acidic, alkaline or neutral.
3) Your earphones produced a sound intensity of 10^{-6} watts/m^3. Find its decibel level and classification. Is it advisable to listen you're your earphones on at that level for a long period of time? Why?
4) If the pitch that the orchestras tune to has a frequency of 880 hertz (cycles per second). Find the corresponding nth key on the piano.
5) A car is worth ₱1 345 000.00, after how many years will it cost ₱650 000.00 if it depreciates in value at 7% a yea

💡 What I Have Learned

Fill in the Blank: Write the term/expression that will complete the statement/s.
1. Real-life situations can be solved using logarithmic functions such as _____, _____and _____.
2. To get the magnitude of an earthquake, we use the formula ____.
3. To get the decibel level of a sound, we use the formula _____.

4. The time it takes for an investment to grow at given compound interest is given by _____.

5. The time it takes for the property to increase to a desired value at a given rate of increase is given by _____.

6. The time it takes for the property to decrease in value at a given rate of decrease is given by _____.

📋 What I Can Do

Express the following situations as a function f(a).

1) A motorcycle is worth ₱345 000.00, after how many years will it cost ₱150 000.00 if it depreciates in value at 3% a year.
2) Find the magnitude and the classification of an earthquake that produces 10^8 joules of energy.
3) If the pitch that the orchestras tune to has a frequency of 3520 cycles per second. Find the corresponding nth key on the piano.
4) A 1-liter substance contains 0.0000001 moles of hydrogen ion. Find its pH level and classify whether it is acidic, alkaline or neutral.
5) A car muffler produced a sound intensity of 10^{-3} watts/m^3. Find its decibel level. Is it safe for the passenger having a car muffler's sound at that decibel level?

Rubrics

Score	Description
15 points	90-100%Correct answers with concrete explanation and there's corresponding neatness on the output.
10 points	60-89% Correct answers with an explanation and there's corresponding neatness on the output.
5 points	Incomplete answer with 50% incorrect answers and there's no corresponding neatness on the output.
No point earned	No output at all

Assessment

Choose the letter that corresponds to the exact answer.
1) If a bell produces a sound intensity of 10^3 watts/m^3. What is its corresponding decibel level?
   A) 100   B) 150   C) 200   D) 250
2) A solution contains 0.000001 moles of hydrogen ion. What is its classification?
   A) acidic   C) neutral
   B) alkaline   D) cannot be determined
3) In how many years will a population grows from 100 000 to 615 000 if the population increases by 4% per year?
   A) 46.13 years   B) 46.31 years   C) 64.13 years   D) 64.31 years
4) How long will it take for a deposit of ₱25 000.00 to grow to ₱42 000 at an 12% annual interest rate compounded quarterly?
   A) 4.39 years   B) 4.93 years   C) 43.9 years   D) 49.3 years

5) How long will it take ₱25 000.00 to quadruple, if it is invested in a savings account that pays 2% annual interest rate compounded semi-annually?

A) 6.67 years   B) 6.76 years   C) 69.7 years   D) 96.7 years

Additional Activities

1) The sound produces by a passing band has an intensity of 10^6 watts/m^3. What is its classification according to its decibel level?

A) 8   B) 18   C) 180   D) 1 800

2) If the pitch that the orchestras tune to has a frequency of 1760 hertz (cycles per second). Find its nth key on the piano.

A) 43rd   B) 53rd   C) 63rd   D) 73rd

3) An earthquake released 10^9 joules of energy. What is its classification according to its magnitude?

A) Weak   B) Strong   C) Destructive   D) Devastating

4) A solution contains 0.000000001 moles of hydrogen ion. What is its classification?

A) alkaline   B) acidic   C) neutral   D) Cannot be determined

5) If a loudspeaker produces a sound intensity of 10^9 watts/m^3. What is its corresponding decibel level?

A) 120   B) 150   C) 180   D) 210

6) How long will it take for a deposit of ₱250 000.00 to grow to ₱420 000 at an 8% annual interest rate compounded quarterly?

A) 6.55 years   B) 55.6 years   C) 56.5 year   D) 65.5 years

7) How many years will it take for the price of an e-bike to depreciate in value from ₱179 990.00 to ₱139 500.00 if the value decreases by 5% annually?

A) 4.79 years   B) 4.97 years   C) 7.49 years   D) 7.94 years

8) In how many years will a population grows from 10 000 to 65 000 if the population increases by 5% per year? Express the time as a function f(r).

A) 36.38 years   B) 36.83 years   C) 38.36 years   D) 38.63 years

9) A dog's bark produces a sound with an intensity of 10^{-2} watts/m^3. What is its decibel level?

A) 100   B) 120   C) 150   D) 180

10) How long will it take ₱250 000.00 to quadruple, if it is invested in an account that pays 15% annual interest rate compounded semi-annually?

A) 9.58 years   B) 9.85 years   C) 19.8 years   D) 91.8 years

References

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