MATH 131–HOMEWORK 1

Due on Monday, Jan 14, before class.

Please read section §1.A.

Question 1 (10 points) Sketch to the following sets of points on the xy–plane:

(a) $\{(x, y) \in \mathbb{R}^2 \mid y = x^2\}$
(b) $\{(x, y) \in \mathbb{R}^2 \mid xy \geq 0\}$
(c) $\{(x, y) \in \mathbb{R}^2 \mid x + 2y = 0\}$
(d) $\{(x, y) \in \mathbb{R}^2 \mid x + 2y = 1\}$

Question 2 (10 points)

(1) compute $(1 + 4i)(1 - 4i)$
(2) compute the multiplicative inverse of $(1 + 4i)$ in $\mathbb{C}$ in the form $a + bi$.
(3) compute $(2 + i)/(1 + 4i)$ in $\mathbb{C}$ in the form $a + bi$.

Question 3 (10 points)

(1) Suppose a and b are real numbers, not both 0. Find real numbers c and d such that
$$1/(a + bi) = c + di.$$
Hint: using the product $(a + bi)$ times its complex conjugate $a - bi$.
(2) Show that for every $\alpha \neq 0 \in \mathbb{C}$, there exists a unique $\beta \in \mathbb{C}$ such that $\alpha\beta = 1$.

Question 4 (10 points) Let $x = (x_1, \ldots, x_n), y = (y_1, \ldots, y_n), z = (z_1, \ldots, z_n) \in \mathbb{F}^n$ and let $a, b \in \mathbb{F}$ be scalars. Prove that

(i)
$$(a + b)x = ax + bx.$$

(ii)
$$x + (y + z) = (x + y) + z.$$