# MATH 131–HOMEWORK 4

Due on Monday, Feb 4, before class.

Please read section §2.A, §2.B.

**Question 1** (10 points)
Suppose $v_1, \ldots, v_m$ is linearly independent in $V$ and $w \in V$. Prove that if

$$v_1 + w, \ldots, v_m + w$$

is linearly dependent, then $w \in \mathtt{Span}(v_1, \ldots, v_m)$.

**Question 2** (10 points) Let $V = \{A \in \mathtt{Mat}_{2\times 2}(\mathbb{F}) \mid \mathtt{tr}\, A = 0\}$. Find a basis of $V$.

**Question 3** (10 points) Let $U$ be the subspace of $\mathbb{C}^5$ defined by:

$$U = \{(z_1, z_2, z_3, z_4, z_5) \in \mathbb{C}^5 : 6z_1 = z_2 \text{ and } z_3 + 2z_4 + 3z_5 = 0\}.$$

(a) Find a basis of $U$.
(b) Extend the basis in part (a) to a basis of $\mathbb{C}^5$.

**Question 4** (10 points) Prove or disprove: there exists a basis $p_0, p_1, p_2, p_3$ of $P_3(\mathbb{F})$ such that none of the polynomial $p_0, p_1, p_2, p_3$ has degree 2.

**Question 5** (10 points) Let $p_0 = 1+x, p_1 = 1+3x+x^2, p_2 = 2x+x^2, p_3 = 1+x+x^2 \in \mathbb{R}[x]$.
(a) Show that $p_0, p_1, p_2, p_3$ spans the vector space $P_2(\mathbb{R})$.
(b) Reduce the list $p_0, p_1, p_2, p_3$ to a basis of $P_2(\mathbb{R})$.

**Question 6** (10 points) Suppose $v_1, v_2, v_3, v_4$ is a basis of $V$. Prove that

$$v_1 + v_2, v_2 + v_3, v_3 + v_4, v_4$$

is also a basis of $V$.