MATH 131–HOMEWORK 2

Due on Wednesday, Jan 23, before class.

Please read section §1.B. and §1.C.

Question 1 For each of the following subsets of $\mathbb{F}^3$, determine whether it is a subspace of $\mathbb{F}^3$:

(1) $S_1 = \{(x_1, x_2, x_3) \in \mathbb{F}^3 \mid x_1 + 2x_2 + 3x_3 = 0\}$;
(2) $S_2 = \{(x_1, x_2, x_3) \in \mathbb{F}^3 \mid x_1 + 2x_2 + 3x_3 = 4\}$;
(3) $S_3 = \{(x_1, x_2, x_3) \in \mathbb{F}^3 \mid x_1 x_2 x_3 = 0\}$;

If it is a subspace, prove it. If it is not a subspace, explain why.

Question 2 (10 points) Prove or give a counterexample:
The intersection of any two subspaces of V is a subspace.

Question 3 (10 points) Prove or give a counterexample: If U_1, U_2 and W are subspaces of V such that $U_1 + W = U_2 + W$, then $U_1 = U_2$.

Question 4 (10 points) Give an example of a nonempty subset U of $\mathbb{R}^2$ such that U is closed under addition, but U is not a subspace of $\mathbb{R}^2$.

Question 5 (10 points) Prove or disprove:

$$U = \{f \in \mathbb{R}[x] \mid f''(x) = 0\}$$

is a subspace of $\mathbb{R}[x]$.

Question 6 (10 points) For each of the following subset of $\mathtt{Mat}_{3\times 3}(\mathbb{F})$, determine whether it is a subspace of $\mathtt{Mat}_{3\times 3}(\mathbb{F})$.

(1) $S_1 = \{A \in \mathtt{Mat}_{3\times 3}(\mathbb{F}) \mid \mathtt{tr}\, A = 0\}$
(2) $S_2 = \{A \in \mathtt{Mat}_{3\times 3}(\mathbb{F}) \mid \det A = 0\}$

If it is a subspace, prove it. If it is not a subspace, explain why.

Here $\mathtt{tr}\, A$ stands for the trace of A, which is the sum of diagonal elements. More explicitly, let $M = (a_{ij})$ be a $n \times n$ matrix,

$$\mathtt{tr}\, M = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \cdots + a_{nn}.$$