(2) $S_2 = \{A \in \text{Mat}_{3\times 3}(\mathbb{F}) |\ \det A = 0\}$

$\forall A, B \in S_2$:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \text{ and } B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$\Rightarrow A + B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \\ a_{31}+b_{31} & a_{32}+b_{32} & a_{33}+b_{33} \end{pmatrix}$$

$$\begin{aligned}
&\Rightarrow \det A + B = \\
&+ (a_{11}+b_{11})([a_{22}+b_{22}][a_{33}+b_{33}] - [a_{32}+b_{32}][a_{23}+b_{23}]) \\
&- (a_{12}+b_{12})([a_{21}+b_{21}][a_{33}+b_{33}] - [a_{31}+b_{31}][a_{23}+b_{23}]) \\
&+ (a_{13}+b_{13})([a_{21}+b_{21}][a_{32}+b_{32}] - [a_{31}+b_{31}][a_{22}+b_{22}]) \\
&= \\
&(a_{11}+b_{11})(a_{22}a_{33}+b_{22}a_{33}+a_{22}b_{33}+b_{22}b_{33}-a_{32}a_{23}-b_{32}a_{23}-a_{32}b_{23}-b_{32}b_{23}) \\
&-(a_{12}+b_{12})(a_{21}a_{33}+b_{21}a_{33}+a_{21}b_{33}+b_{21}b_{33}-a_{31}a_{23}-b_{31}a_{23}-a_{31}b_{23}-b_{31}b_{23}) \\
&(a_{13}+b_{13})(a_{21}a_{32}+b_{21}a_{32}+a_{21}b_{32}+b_{21}b_{32}-a_{31}a_{22}-b_{31}a_{22}-a_{31}b_{22}-b_{31}b_{22}) \\
&= \\
&a_{11}a_{22}a_{33} + a_{11}b_{22}a_{33} + a_{11}a_{22}b_{33} + a_{11}b_{22}b_{33} \\
&- a_{11}a_{32}a_{23} - a_{11}b_{32}a_{23} - a_{11}a_{32}b_{23} - a_{11}b_{32}b_{23} \\
&+ b_{11}a_{22}a_{33} + b_{11}b_{22}a_{33} + b_{11}a_{22}b_{33} + b_{11}b_{22}b_{33} \\
&- b_{11}a_{32}a_{23} - b_{11}b_{32}a_{23} - b_{11}a_{32}b_{23} - b_{11}b_{32}b_{23} \\
&\\
&-a_{12}a_{21}a_{33} - a_{12}b_{21}a_{33} - a_{12}a_{21}b_{33} - a_{12}b_{21}b_{33} \\
&+ a_{12}a_{31}a_{23} + a_{12}b_{31}a_{23} + a_{12}a_{31}b_{23} + a_{12}b_{31}b_{23} \\
&- b_{12}a_{21}a_{33} - b_{12}b_{21}a_{33} - b_{12}a_{21}b_{33} - b_{12}b_{21}b_{33} \\
&+ b_{12}a_{31}a_{23} + b_{12}b_{31}a_{23} + b_{12}a_{31}b_{23} + b_{12}b_{31}b_{23} \\
&\\
&a_{13}a_{21}a_{32} + a_{13}b_{21}a_{32} + a_{13}a_{21}b_{32} + a_{13}b_{21}b_{32} \\
&- a_{13}a_{31}a_{22} - a_{13}b_{31}a_{22} - a_{13}a_{31}b_{22} - a_{13}b_{31}b_{22} \\
&b_{13}a_{21}a_{32} + b_{13}b_{21}a_{32} + b_{13}a_{21}b_{32} + b_{13}b_{21}b_{32} \\
&- b_{13}a_{31}a_{22} - b_{13}b_{31}a_{22} - b_{13}a_{31}b_{22} - b_{13}b_{31}b_{22} \\
&\\
&\Rightarrow \det A + B = \\
&= \\
&\det A \\
&+a_{11}b_{22}a_{33} + a_{11}a_{22}b_{33} + a_{11}b_{22}b_{33} \\
&- a_{11}b_{32}a_{23} - a_{11}a_{32}b_{23} - a_{11}b_{32}b_{23} \\
&+ b_{11}a_{22}a_{33} + b_{11}b_{22}a_{33} + b_{11}a_{22}b_{33} \\
&- b_{11}a_{32}a_{23} - b_{11}b_{32}a_{23} - b_{11}a_{32}b_{23} \\
&\\
&-a_{12}b_{21}a_{33} - a_{12}a_{21}b_{33} - a_{12}b_{21}b_{33} \\
&+ a_{12}b_{31}a_{23} + a_{12}a_{31}b_{23} + a_{12}b_{31}b_{23} \\
&- b_{12}a_{21}a_{33} - b_{12}b_{21}a_{33} - b_{12}a_{21}b_{33} \\
&+ b_{12}a_{31}a_{23} + b_{12}b_{31}a_{23} + b_{12}a_{31}b_{23}
\end{aligned}$$

$$
\begin{aligned}
&+a_{13}b_{21}a_{32} + a_{13}a_{21}b_{32} + a_{13}b_{21}b_{32} \\
&- a_{13}b_{31}a_{22} - a_{13}a_{31}b_{22} - a_{13}b_{31}b_{22} \\
&+ b_{13}a_{21}a_{32} + b_{13}b_{21}a_{32} + b_{13}a_{21}b_{32} \\
&- b_{13}a_{31}a_{22} - b_{13}b_{31}a_{22} - b_{13}a_{31}b_{22} \\
&+ \det B \\
&=
\end{aligned}
$$

$$
\begin{aligned}
&0 \\
&+ a_{11}b_{22}a_{33} + a_{11}a_{22}b_{33} + a_{11}b_{22}b_{33} \\
&- a_{11}b_{32}a_{23} - a_{11}a_{32}b_{23} - a_{11}b_{32}b_{23} \\
&+ b_{11}a_{22}a_{33} + b_{11}b_{22}a_{33} + b_{11}a_{22}b_{33} \\
&- b_{11}a_{32}a_{23} - b_{11}b_{32}a_{23} - b_{11}a_{32}b_{23} \\
&\\
&-a_{12}b_{21}a_{33} - a_{12}a_{21}b_{33} - a_{12}b_{21}b_{33} \\
&+ a_{12}b_{31}a_{23} + a_{12}a_{31}b_{23} + a_{12}b_{31}b_{23} \\
&- b_{12}a_{21}a_{33} - b_{12}b_{21}a_{33} - b_{12}a_{21}b_{33} \\
&+ b_{12}a_{31}a_{23} + b_{12}b_{31}a_{23} + b_{12}a_{31}b_{23} \\
&\\
&+a_{13}b_{21}a_{32} + a_{13}a_{21}b_{32} + a_{13}b_{21}b_{32} \\
&- a_{13}b_{31}a_{22} - a_{13}a_{31}b_{22} - a_{13}b_{31}b_{22} \\
&+ b_{13}a_{21}a_{32} + b_{13}b_{21}a_{32} + b_{13}a_{21}b_{32} \\
&- b_{13}a_{31}a_{22} - b_{13}b_{31}a_{22} - b_{13}a_{31}b_{22} \\
&+0 \\
&=
\end{aligned}
$$

$$
\begin{aligned}
&+a_{11}b_{22}a_{33} + a_{11}a_{22}b_{33} + a_{11}b_{22}b_{33} \\
&- a_{11}b_{32}a_{23} - a_{11}a_{32}b_{23} - a_{11}b_{32}b_{23} \\
&+ b_{11}a_{22}a_{33} + b_{11}b_{22}a_{33} + b_{11}a_{22}b_{33} \\
&- b_{11}a_{32}a_{23} - b_{11}b_{32}a_{23} - b_{11}a_{32}b_{23} \\
&\\
&-a_{12}b_{21}a_{33} - a_{12}a_{21}b_{33} - a_{12}b_{21}b_{33} \\
&+ a_{12}b_{31}a_{23} + a_{12}a_{31}b_{23} + a_{12}b_{31}b_{23} \\
&- b_{12}a_{21}a_{33} - b_{12}b_{21}a_{33} - b_{12}a_{21}b_{33} \\
&+ b_{12}a_{31}a_{23} + b_{12}b_{31}a_{23} + b_{12}a_{31}b_{23} \\
&\\
&+a_{13}b_{21}a_{32} + a_{13}a_{21}b_{32} + a_{13}b_{21}b_{32} \\
&- a_{13}b_{31}a_{22} - a_{13}a_{31}b_{22} - a_{13}b_{31}b_{22} \\
&+ b_{13}a_{21}a_{32} + b_{13}b_{21}a_{32} + b_{13}a_{21}b_{32} \\
&- b_{13}a_{31}a_{22} - b_{13}b_{31}a_{22} - b_{13}a_{31}b_{22}
\end{aligned}
$$

$$\Rightarrow A + B \notin S_1$$

$$
\begin{aligned}
&\Rightarrow \det A + B = 0 \text{ if} \\
&+a_{11}b_{22}a_{33} + a_{11}a_{22}b_{33} + a_{11}b_{22}b_{33} \\
&- a_{11}b_{32}a_{23} - a_{11}a_{32}b_{23} - a_{11}b_{32}b_{23} \\
&+ b_{11}a_{22}a_{33} + b_{11}b_{22}a_{33} + b_{11}a_{22}b_{33}
\end{aligned}
$$

$$
\begin{aligned}
&- b_{11}a_{32}a_{23} - b_{11}b_{32}a_{23} - b_{11}a_{32}b_{23} \\
&\\
&-a_{12}b_{21}a_{33} - a_{12}a_{21}b_{33} - a_{12}b_{21}b_{33} \\
&+ a_{12}b_{31}a_{23} + a_{12}a_{31}b_{23} + a_{12}b_{31}b_{23} \\
&- b_{12}a_{21}a_{33} - b_{12}b_{21}a_{33} - b_{12}a_{21}b_{33} \\
&+ b_{12}a_{31}a_{23} + b_{12}b_{31}a_{23} + b_{12}a_{31}b_{23} \\
&\\
&+a_{13}b_{21}a_{32} + a_{13}a_{21}b_{32} + a_{13}b_{21}b_{32} \\
&- a_{13}b_{31}a_{22} - a_{13}a_{31}b_{22} - a_{13}b_{31}b_{22} \\
&+ b_{13}a_{21}a_{32} + b_{13}b_{21}a_{32} + b_{13}a_{21}b_{32} \\
&- b_{13}a_{31}a_{22} - b_{13}b_{31}a_{22} - b_{13}a_{31}b_{22} \\
&= 0
\end{aligned}
$$