MATH 131–HOMEWORK 9

Due on Wednesday, Mar 13, before class.

Please read section 3.E.

Question 1 (10 points) Suppose V is finite-dimensional, with $\dim V = n \geq 1$. Prove that there exist 1–dimensional subspaces $U_1, \ldots, U_n$ of V such that

$$V = U_1 \oplus \cdots \oplus U_n.$$

Question 2 (10 points) Suppose that U and V are subspaces of $\mathbb{R}^8$ such that $\dim U = 3$, $\dim W = 5$, and $U + W = \mathbb{R}^8$. Prove that $\mathbb{R}^8 = U \oplus W$.

Question 3 (10 points) Prove or give a counterexample:

if U_1, U_2, W are subspaces of V such that

$$U_1 \oplus W = U_2 \oplus W,$$

then $U_1 = U_2$.

Question 4 (10 points)

Suppose $U = \{(x, x, y, y) \in \mathbb{R}^4 \mid x, y \in \mathbb{R}\}$. Find a subspace W of $\mathbb{R}^4$ such that

$$\mathbb{R}^4 = U \oplus W.$$

Question 5 (10 points) Let $U = \{p \in P_4(\mathbb{R}) : p''(4) = 0\}$. (In homework 5, we have computed a basis of U and extended it to a basis of $P_4(\mathbb{R})$.)

Find a subspace W of $P_4(\mathbb{R})$ such that $P_4(\mathbb{R}) = W \oplus U$. Justify your answer.

Question 6 (10 points) Suppose $\phi \in \mathcal{L}(V, \mathbb{F})$. Suppose $u \in V$ is not in $\texttt{Null}\,\phi$. Let $U = \texttt{Span}(u)$. Prove that

$$V = \texttt{Null}\,\phi \oplus U.$$