MATH 131–HOMEWORK 5

Due on Monday, Feb 11, before class.

Please read section §2.C and §3.A.

Question 1 (10 points) Suppose U and W are both five-dimensional subspaces of $\mathbb{R}^9$. Prove that $U \cap W \neq \{0\}$.

Question 2 (10 points) Let $U = \{p \in P_4(\mathbb{R}) : p''(4) = 0\}$.

(a) Show that U is a subspace of $P_4(\mathbb{R})$.
(b) Find a basis of U.
(c) Extend the basis in part (b) to a basis of $P_4(\mathbb{R})$.

Question 3 (10 points) $\mathbb{C}^n$ can be think as $\mathbb{C}$–vector space and $\mathbb{R}$–vector space. Find the bases of $\mathbb{C}^n$ over $\mathbb{C}$ and $\mathbb{R}$ respectively. Compute $\dim_{\mathbb{C}} \mathbb{C}^n$ and $\dim_{\mathbb{R}} \mathbb{C}^n$.

Question 4 (10 points) Suppose $b, c \in \mathbb{R}$. Define $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T(x, y, z) = (2x - 4y + 2z + b, 6x + cxyz).$$

Show that T is linear if and only if $b = c = 0$.

Question 5 (10 points) Suppose $T \in T(V, W)$ and $v_1, \ldots, v_m$ is a list of vectors in V such that $Tv_1, \ldots, Tv_m$ is a linearly independent list in W. Prove or give a counterexample: $v_1, \ldots, v_m$ is linearly independent.

Question 6 (10 points) Give an example of a function $\phi : \mathbb{R}^2 \to \mathbb{R}$ such that

$$\phi(av) = a\phi(v)$$

for all $a \in \mathbb{R}$ and all $v \in \mathbb{R}^2$ but ϕ is not linear.