MATH 131–HOMEWORK 6

Due on Wednesday, Feb 20, before class.

Please read section 3.B.

Question 1 (10 points) Prove that there does not exist a linear map $T : \mathbb{R}^5 \to \mathbb{R}^5$ such that

$$\texttt{range}\, T = \texttt{null}\, T.$$

Question 2 (10 points) Prove or give a counterexample:

If $T : V \to W$ is a linear map and $v_1, \dots, v_n$ spans V, then $T(v_1), \dots, T(v_n)$ spans $\texttt{range}(T)$.

Question 3 (10 points) Prove or give a counterexample: Let $T : V \to W$ and $S : V \to W$ are two linear maps. If $\texttt{null}\, T = \texttt{null}\, S$, then $T = S$.

Question 4 (10 points) (10 points) Suppose V and W are both finite-dimensional. Prove that there exists an injective linear map from V to W if and only if $\dim V \leq \dim W$.

Question 5 (10 points) Prove that there does not exist a linear map from $\mathbb{F}^5$ to $\mathbb{F}^2$ whose null space equals

$$\{(x_1, x_2, x_3, x_4, x_5) \mid x_1 = 3x_2;\ x_3 = x_4 = x_5\}.$$

Question 6 (10 points) Suppose $T \in \mathcal{L}(V, W)$ is injective and $\{v_1, \dots, v_n\}$ is linearly independent in V. Prove or disprove that $\{T(v_1), T(v_2), \dots, T(v_n)\}$ is linearly independent in W.