MATH 131–**HOMEWORK** 8

Due on Wednesday, Mar 6, before class.

Please read section 3.D. The total score of this homework set is $12 + 9 + 1 + 3 = 25$.

Question 1 (12 points) Determine the following linear maps of vector spaces over $\mathbb{R}$ are isomorphism or not. If it is an isomorphism, find its inverse map. (Hint: inverse of matrices.) If it is not an isomorphism, briefly explain why.

(1) (Rotation by 60°)

$$L : \mathbb{R}^2 \to \mathbb{R}^2$$
$$(x, y) \mapsto (\frac{x}{2} - \frac{\sqrt{3}}{2}y, \frac{\sqrt{3}}{2}x + \frac{1}{2}y).$$

(2) (Reflection about x–axis)

$$L : \mathbb{R}^2 \to \mathbb{R}^2$$
$$(x, y) \mapsto (x, -y).$$

(3)

$$L : \mathbb{R}^3 \to \mathbb{R}^3$$
$$(x, y, z) \mapsto (x + 2y + 3z, 4x + 5y + 6z, 7x + 8y + 9z).$$

Question 2 (12 points) Determine the following spaces are isomorphic or not. If they are isomorphic, give one isomorphism explicitly.

(1) $\mathcal{L}(\mathbb{R}^2, \mathbb{R}^5)$ and $\mathbb{R}^7$.
(2) $\texttt{Span}((1, 1, 0), (2, 5, 6))$ and $\mathbb{R}^3$.
(3) $\{(x, y, z) \in \mathbb{R}^3 \mid 2x + 2y + z = 0\}$ and $\mathbb{R}^2$.

Question 3 (9 points) Suppose $T \in \mathcal{L}(U, V)$ and $S \in \mathcal{L}(V, W)$ are both invertible linear maps. Prove that $ST \in \mathcal{L}(U, W)$ is invertible and $(ST)^{-1} = T^{-1}S^{-1}$.

Question 4 (9 points) Suppose V is a finite-dimensional and $S, T \in \mathcal{V}$. Prove that ST is invertible if and only if both S and T are invertible.

Question 5 (9 points) Suppose V is finite-dimensional and $\dim V > 1$. Prove that the set of noninvertible operators on V is not a subspace of $\mathcal{L}(V)$. (Hint: you have seen this example in previous homeworks when $\dim V = 3$.)