In [2]:
```
from sympy import *
init_printing(use_unicode=true)
```

9.4

1

$$\begin{cases} \frac{dx}{dt} = x(1.5 - x - 0.5y) \\ \frac{dy}{dt} = y(2 - y - 0.75x) \end{cases}$$

In [4]:
```
x,y = symbols('x,y');
F = x*(3/2 -1*x -1*y/2);
G = y*(2 -1*y -3*x/4); F,G
```

Out[4]:
$$\left(x\left(-x - \frac{y}{2} + \frac{3}{2}\right),\ y\left(-\frac{3x}{4} - y + 2\right)\right)$$

In [5]:
```
solve(F)
```

Out[5]:
$$[\{x : 0\},\ \{y : 3 - 2x\}]$$

In [49]:
```
solve(G.subs(x,0))
```

Out[49]:
$$[0,\ 2]$$

In [50]:
```
solve(G.subs(y,3-2*x))
```

Out[50]:
$$\left[\frac{4}{5},\ \frac{3}{2}\right]$$

In [9]:
```
solve(F)[1][y].subs(x,3/2)
```

Out[9]:
$$0$$

In [8]:
```
solve(F)[1][y].subs(x,4/5)
```

Out[8]:
$$\frac{7}{5}$$

In [16]:
```
c1,c2,c3,c4 = Matrix([0,0]),Matrix([0,2]),Matrix([3/2,0]),Matrix([4/5
,7/5]); c1,c2,c3,c4
```

Out[16]:
$$\left(\begin{bmatrix}0\\0\end{bmatrix},\ \begin{bmatrix}0\\2\end{bmatrix},\ \begin{bmatrix}\frac{3}{2}\\0\end{bmatrix},\ \begin{bmatrix}\frac{4}{5}\\\frac{7}{5}\end{bmatrix}\right)$$

In [17]:
```
J = Matrix([[diff(F,x),diff(F,y)],[diff(G,x),diff(G,y)]]); J
```

Out[17]:
$$\begin{bmatrix} -2x - \frac{y}{2} + \frac{3}{2} & -\frac{x}{2} \\ -\frac{3y}{4} & -\frac{3x}{4} - 2y + 2 \end{bmatrix}$$

In [18]:
```
J1 = J.subs(x,0).subs(y,0); J1, J1.subs(x,0).subs(y,0).eigenvects()
```

Out[18]:
$$\left(\begin{bmatrix} \frac{3}{2} & 0 \\ 0 & 2 \end{bmatrix}, \left[\left(\frac{3}{2}, 1, \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right] \right), \left(2, 1, \left[\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right] \right) \right] \right)$$

In [19]:
```
J2 = J.subs(x,0).subs(y,2); J2, J2.subs(x,0).subs(y,2).eigenvects()
```

Out[19]:
$$\left(\begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & -2 \end{bmatrix}, \left[\left(-2, 1, \left[\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right] \right), \left(\frac{1}{2}, 1, \left[\begin{bmatrix} -\frac{5}{3} \\ 1 \end{bmatrix} \right] \right) \right] \right)$$

In [20]:
```
J3 = J.subs(x,3/2).subs(y,0); J3, J3.subs(x,3/2).subs(y,0).eigenvects
()
```

Out[20]:
$$\left(\begin{bmatrix} -\frac{3}{2} & -\frac{3}{4} \\ 0 & \frac{7}{8} \end{bmatrix}, \left[\left(-\frac{3}{2}, 1, \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right] \right), \left(\frac{7}{8}, 1, \left[\begin{bmatrix} -\frac{6}{19} \\ 1 \end{bmatrix} \right] \right) \right] \right)$$

In [21]:
```
J4 = J.subs(x,4/5).subs(y,7/5); J4,J4.eigenvects()
```

Out[21]:
$$\left(\begin{bmatrix} -\frac{4}{5} & -\frac{2}{5} \\ -\frac{21}{20} & -\frac{7}{5} \end{bmatrix}, \left[\left(-\frac{11}{10} - \frac{\sqrt{51}}{10}, 1, \left[\begin{bmatrix} \frac{2}{5\left(\frac{3}{10} + \frac{\sqrt{51}}{10}\right)} \\ 1 \end{bmatrix} \right] \right), \left(-\frac{11}{10} + \frac{\sqrt{51}}{10}, 1 \right.\right.$$

In [49]:
```
l1 = solve(10*x^2 + 22*x +7)[0]; l1
```

Out[49]:
$$-\frac{11}{10} - \frac{\sqrt{51}}{10}$$

In [50]:
```
l2 = solve(10*x^2 + 22*x +7)[1]; l2
```

Out[50]:
$$-\frac{11}{10} + \frac{\sqrt{51}}{10}$$

In [85]:
```
simplify((3-sqrt(51))*10*(J4-1*l1*eye(2))[0,0])/2 - 20*(J4-1*l1*eye(2
))[1,0]
```

Out[85]: 0

In [84]:
```
simplify((3-sqrt(51))*10*(J4-1*l1*eye(2))[0,1])/2 - 20*(J4-1*l1*eye(2
))[1,1]
```

Out[84]: 0

In [89]:
```
{l1: Matrix([-20*(J4-1*l1*eye(2))[1,1],20*(J4-1*l1*eye(2))[1,0]])}
```

Out[89]:

$$\left\{ -\frac{11}{10} - \frac{\sqrt{51}}{10} : \begin{bmatrix} 6 - 2\sqrt{51} \\ -21 \end{bmatrix} \right\}$$

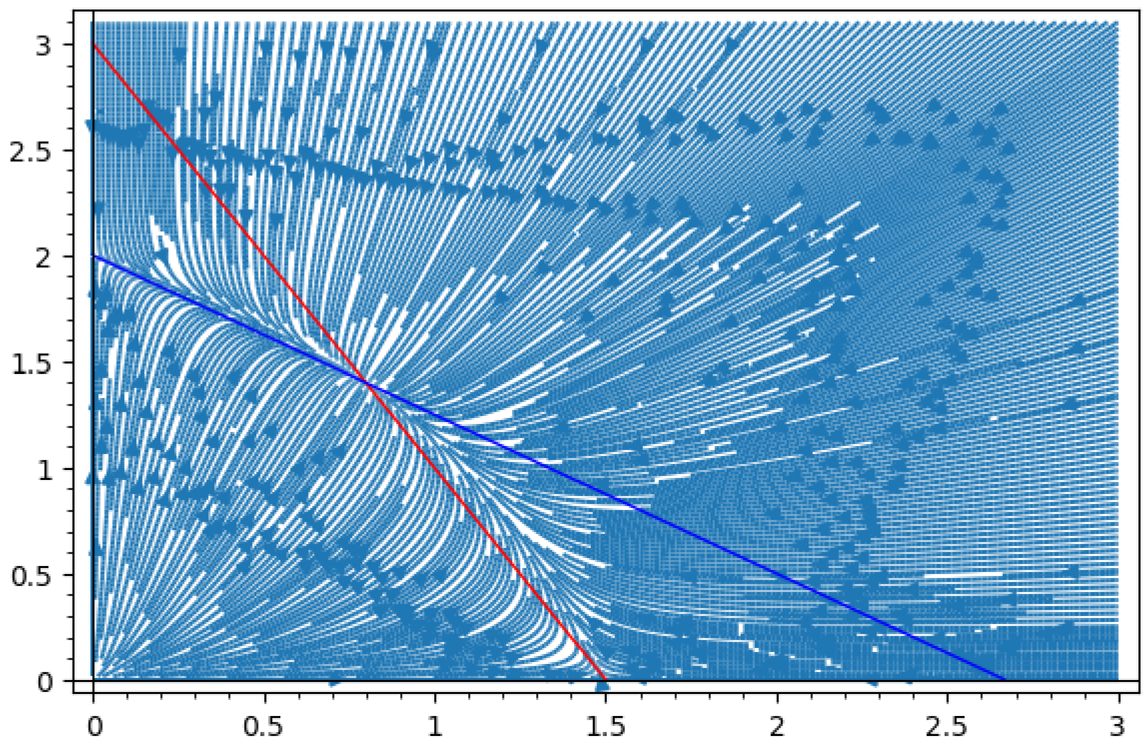
In [90]:
```
{l2: Matrix([-20*(J4-1*l2*eye(2))[1,1],20*(J4-1*l2*eye(2))[1,0]])}
```

Out[90]:

$$\left\{ -\frac{11}{10} + \frac{\sqrt{51}}{10} : \begin{bmatrix} 6 + 2\sqrt{51} \\ -21 \end{bmatrix} \right\}$$

In [61]:
```
x1,x2 = var('x1 x2');
f = x1*(3/2 -1*x1 -1*x2/2);
g = x2*(2 -1*x2 -3*x1/4);
SP = streamline_plot((f,g), (x1,0,3), (x2,0,3.1),density = 7)
Z1 = sage.plot.line.line([[0,3],[3/2,0]],color='red')
Z2 = sage.plot.line.line([[0,2],[8/3,0]],color='blue')
SP+Z1+Z2
```

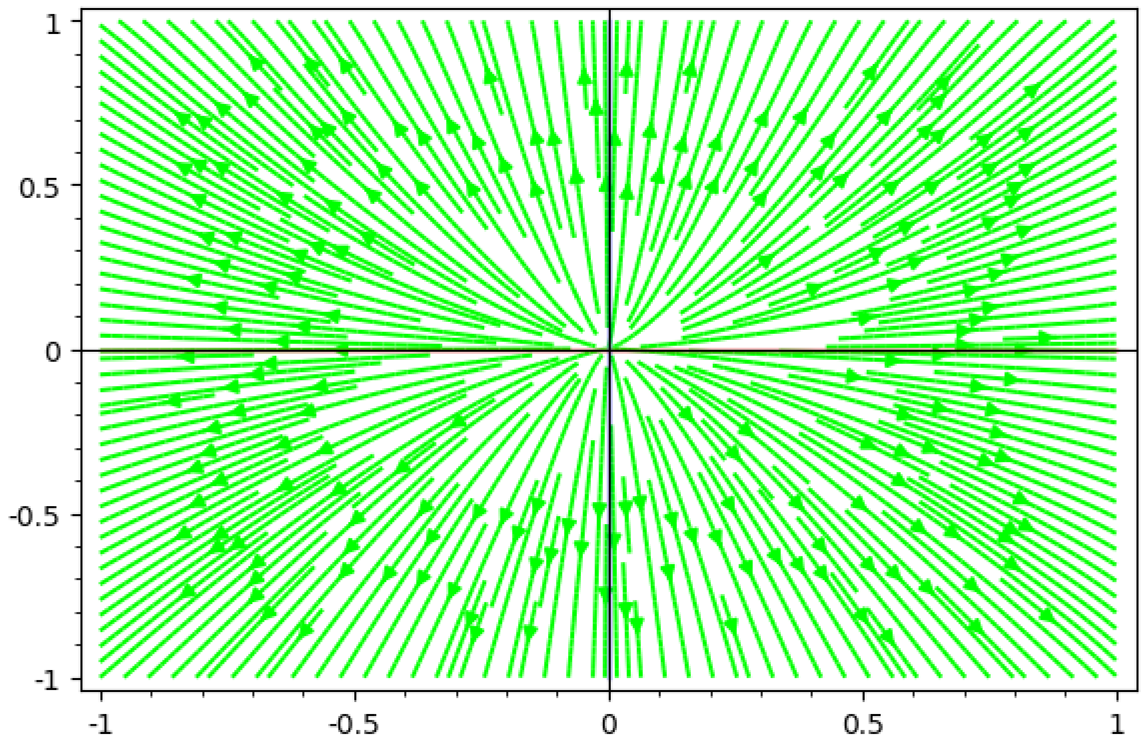
Out[61]:



In [150]:

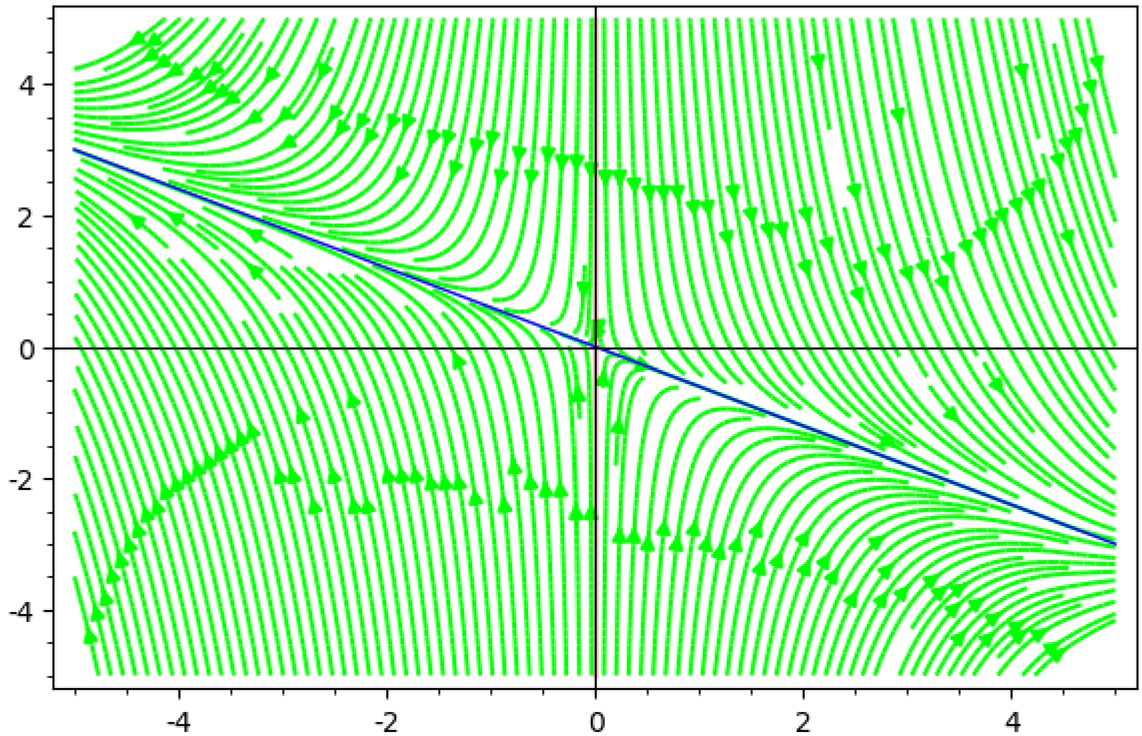
```
x1,x2 = var('x1 x2');
f = 3*x1/2;
g = 2*x2;
SP = streamline_plot((f,g), (x1,-1,1), (x2,-1,1),density = 3,color='lime')
Z1 = sage.plot.line.line([[1,0],[-1,0]],color='red')
Z2 = sage.plot.line.line([[0,1],[0,-1]],color='blue')
SP+Z1+Z2
```

Out[150]:



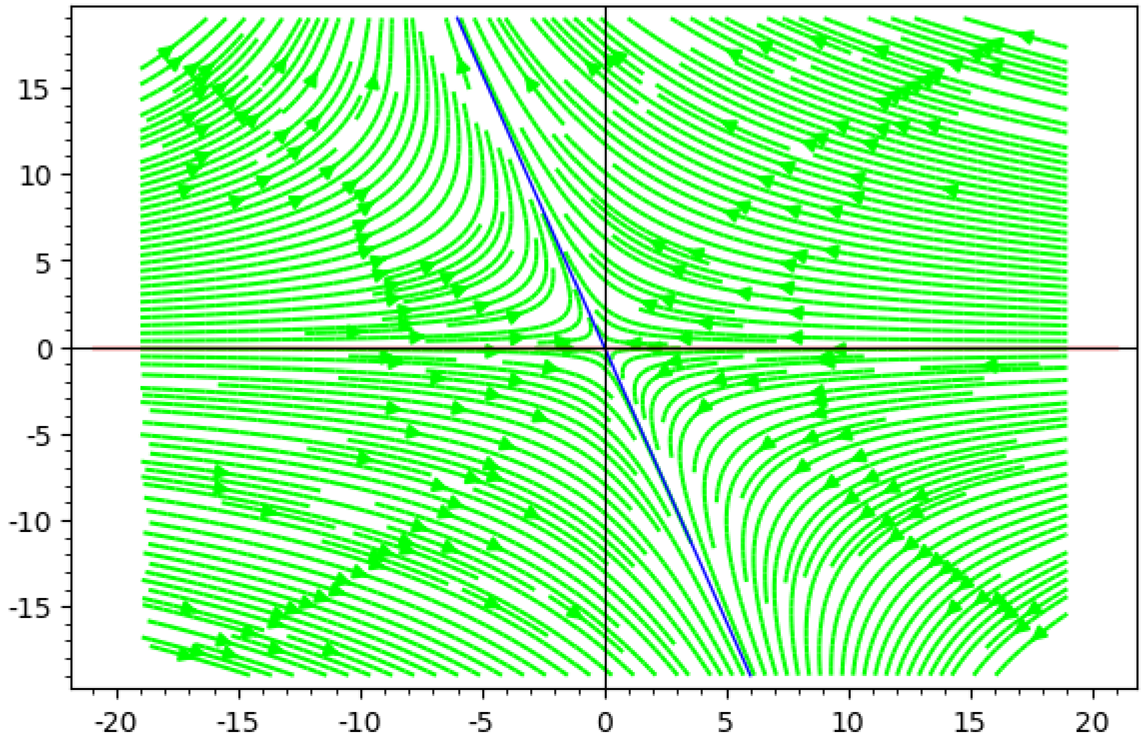
In [149]:
```
x1,x2 = var('x1 x2');
f = x1/2;
g = -3*x1/2-2*x2;
SP = streamline_plot((f,g), (x1,-5,5), (x2,-5,5),density = 3,color='l
ime')
Z1 = sage.plot.line.line([[0,-5],[0,5]],color='red')
Z2 = sage.plot.line.line([[-5,3],[5,-3]],color='blue')
SP+Z1+Z2
```

Out[149]:



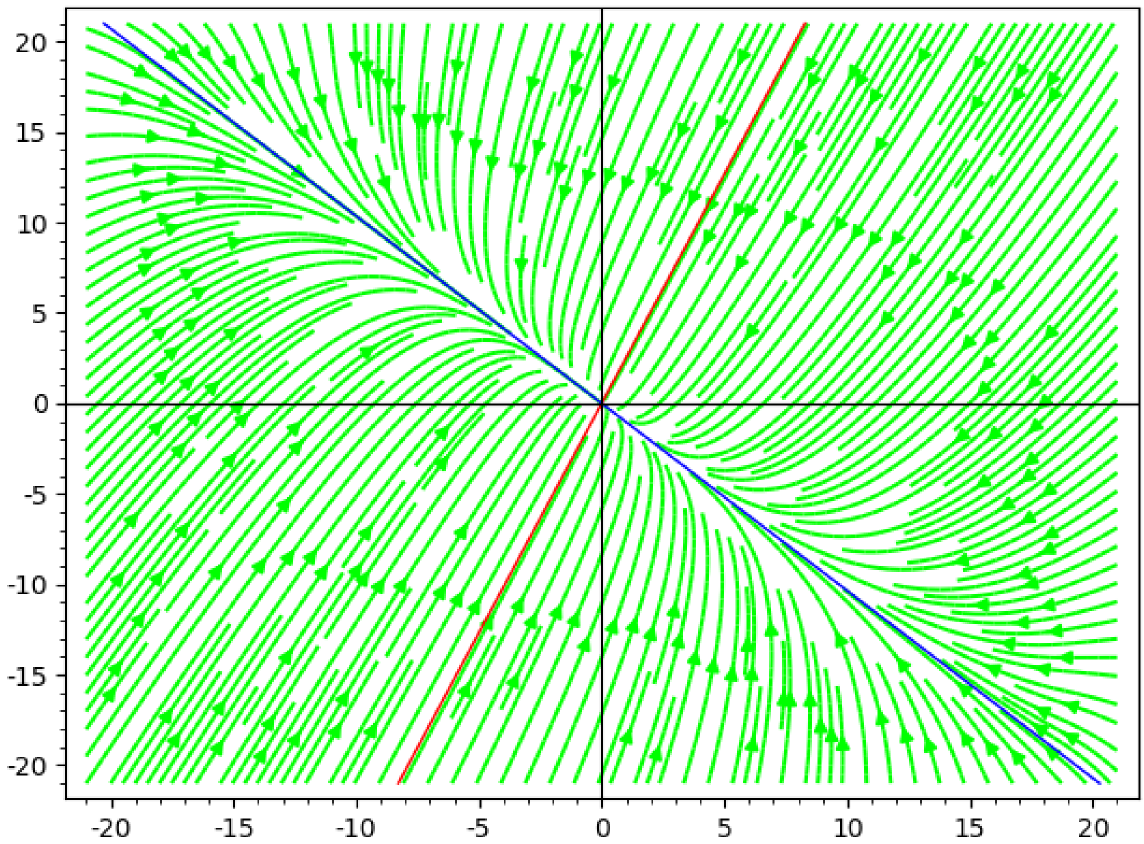
In [148]:
```
x1,x2 = var('x1 x2');
f = -3*x1/2 -3*x2/4;
g = 7*x2/8;
SP = streamline_plot((f,g), (x1,-19,19), (x2,-19,19),density = 3,colo
r='lime')
Z1 = sage.plot.line.line([[21,0],[-21,0]],color='red')
Z2 = sage.plot.line.line([[-6,19],[6,-19]],color='blue')
SP+Z1+Z2
```

Out[148]:



In [1]:
```
x1,x2 = var('x1 x2');
f = -4*x1/5 -2*x2/5;
g = -21*x1/20 -7*x2/5;
SP = streamline_plot((f,g), (x1,-21,21), (x2,-21,21),density = 3,colo
r='lime')
Z1 = sage.plot.line.line([[-6+2*sqrt(51),21],[6-2*sqrt(51),-21]],colo
r='red')
Z2 = sage.plot.line.line([[-6-2*sqrt(51),21],[6+2*sqrt(51),-21]],colo
r='blue')
SP+Z1+Z2
```

Out[1]:



3

$$\begin{cases} \frac{dx}{dt} = x(1.5 - 0.5x - y) \\ \frac{dy}{dt} = y(2 - y - 1.125x) \end{cases}$$

In [13]:
```
x,y = symbols('x,y');
F = x*(3/2 -1*x/2 -1*y);
G = y*(2 -1*y -9*x/8); F,G
```

Out[13]:

$$\left(x\left(-\frac{x}{2} - y + \frac{3}{2}\right),\ y\left(-\frac{9x}{8} - y + 2\right)\right)$$

In [14]:
```
solve(F)
```

Out[14]:

$$\left[\left\{ x : 0\right\}, \ \left\{ y : \frac{3}{2} - \frac{x}{2}\right\}\right]$$

In [15]:
```
solve(G.subs(x,0))
```

Out[15]:

$$[0, \ 2]$$

In [16]:
```
solve(G.subs(y,3/2-1*x/2))
```

Out[16]:

$$\left[\frac{4}{5}, \ 3\right]$$

In [17]:
```
solve(F)[1][y].subs(x,3)
```

Out[17]:

$$0$$

In [18]:
```
solve(F)[1][y].subs(x,4/5)
```

Out[18]:

$$\frac{11}{10}$$

In [19]:
```
c1,c2,c3,c4 = Matrix([0,0]),Matrix([0,2]),Matrix([3,0]),Matrix([4/5,1
1/10]); c1,c2,c3,c4
```

Out[19]:

$$\left(\left[\begin{matrix}0\\0\end{matrix}\right], \ \left[\begin{matrix}0\\2\end{matrix}\right], \ \left[\begin{matrix}3\\0\end{matrix}\right], \ \left[\begin{matrix}\frac{4}{5}\\\frac{11}{10}\end{matrix}\right]\right)$$

In [20]:
```
J = Matrix([[diff(F,x),diff(F,y)],[diff(G,x),diff(G,y)]]); J
```

Out[20]:

$$\left[\begin{matrix}- x - y + \frac{3}{2} & - x\\- \frac{9 y}{8} & - \frac{9 x}{8} - 2 y + 2\end{matrix}\right]$$

In [21]:
```
J1 = J.subs(x,0).subs(y,0); J1, J1.subs(x,0).subs(y,0).eigenvects()
```

Out[21]:

$$\left(\left[\begin{matrix}\frac{3}{2} & 0\\0 & 2\end{matrix}\right], \ \left[\left(\frac{3}{2}, \ 1, \ \left[\left[\begin{matrix}1\\0\end{matrix}\right]\right]\right), \ \left(2, \ 1, \ \left[\left[\begin{matrix}0\\1\end{matrix}\right]\right]\right)\right]\right)$$

In [22]:
```
J2 = J.subs(x,0).subs(y,2); J2, J2.subs(x,0).subs(y,2).eigenvects()
```

Out[22]:

$$\left(\left[\begin{matrix}- \frac{1}{2} & 0\\- \frac{9}{4} & -2\end{matrix}\right], \ \left[\left(-2, \ 1, \ \left[\left[\begin{matrix}0\\1\end{matrix}\right]\right]\right), \ \left(- \frac{1}{2}, \ 1, \ \left[\left[\begin{matrix}- \frac{2}{3}\\1\end{matrix}\right]\right]\right)\right]\right)$$

In [23]:
```
J3 = J.subs(x,3).subs(y,0); J3, J3.subs(x,3/2).subs(y,0).eigenvects()
```

Out[23]:

$$\left(\begin{bmatrix}-\frac{3}{2} & -3\\ 0 & -\frac{11}{8}\end{bmatrix},\ \left[\left(-\frac{3}{2},\ 1,\ \left[\begin{bmatrix}1\\0\end{bmatrix}\right]\right),\ \left(-\frac{11}{8},\ 1,\ \left[\begin{bmatrix}-24\\1\end{bmatrix}\right]\right)\right]\right)$$

In [26]:
```
J4 = J.subs(x,4/5).subs(y,11/10); J4,J4.eigenvects()
```

Out[26]:

$$\left(\begin{bmatrix}-\frac{2}{5} & -\frac{4}{5}\\ -\frac{99}{80} & -\frac{11}{10}\end{bmatrix},\ \left[\left(-\frac{3}{4}+\frac{\sqrt{445}}{20},\ 1,\ \left[\begin{bmatrix}\frac{4}{5\left(\frac{7}{20}-\frac{\sqrt{445}}{20}\right)}\\1\end{bmatrix}\right]\right),\ \left(-\frac{\sqrt{445}}{20}-\frac{3}{4},\right.\right.$$

In [24]:
```
l1 = solve(x^2 + 3*x/2 -11/20)[0]; l1
```

Out[24]:

$$-\frac{3}{4}+\frac{\sqrt{445}}{20}$$

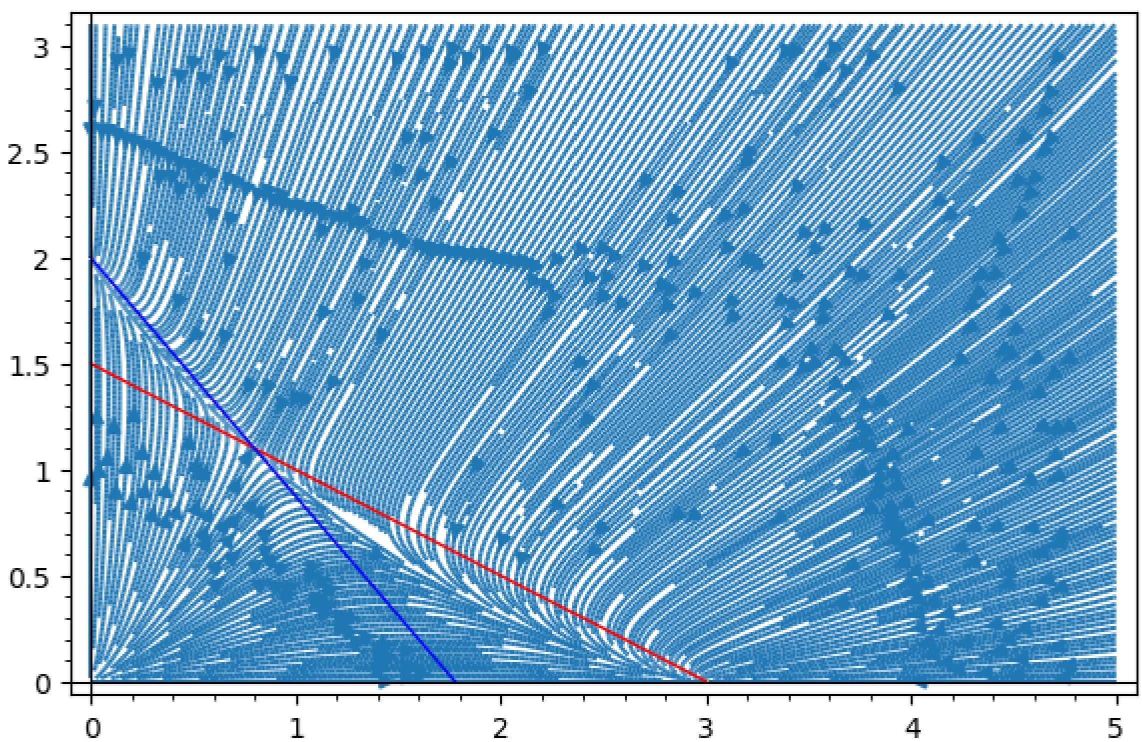
In [25]:
```
l1 = solve(x^2 + 3*x/2 -11/20)[0]; l1
```

Out[25]:

$$-\frac{3}{4}+\frac{\sqrt{445}}{20}$$

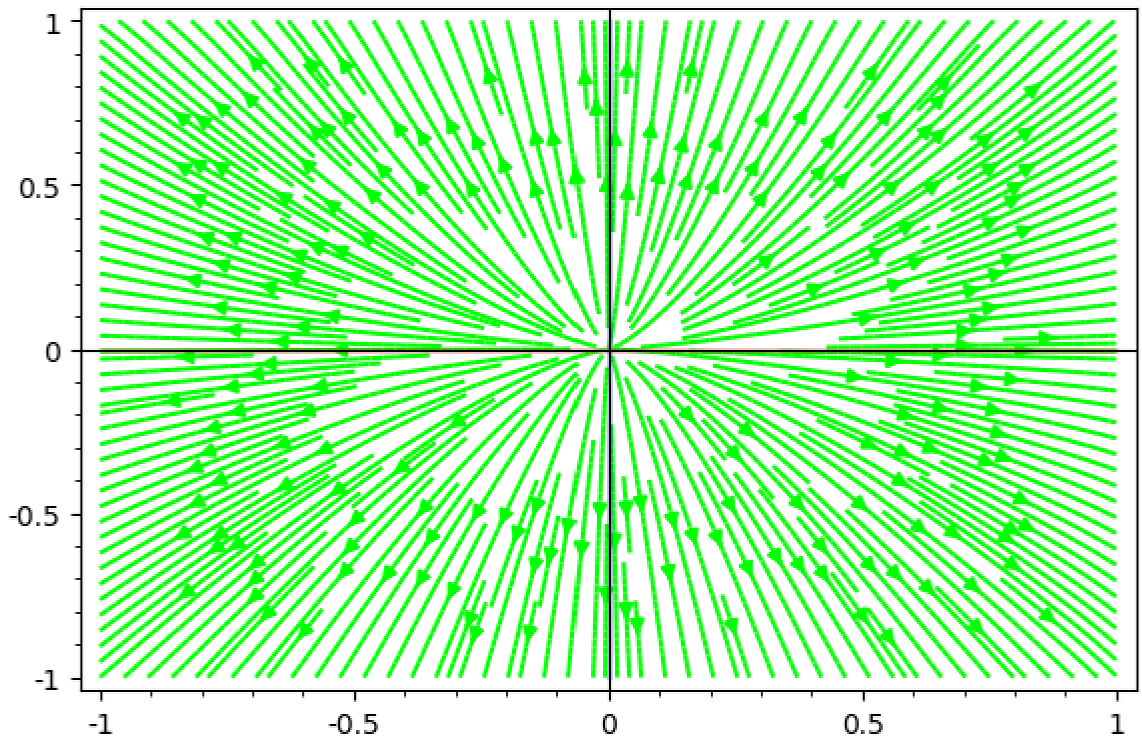
In [59]:
```
x1,x2 = var('x1 x2');
f = x1*(3/2 -1*x1/2 -1*x2);
g = x2*(2 -1*x2 -9*x1/8);
SP = streamline_plot((f,g), (x1,0,5), (x2,0,3.1),density = 7)
Z1 = sage.plot.line.line([[0,3/2],[3,0]],color='red')
Z2 = sage.plot.line.line([[0,2],[16/9,0]],color='blue')
SP + Z1 + Z2
```

Out[59]:



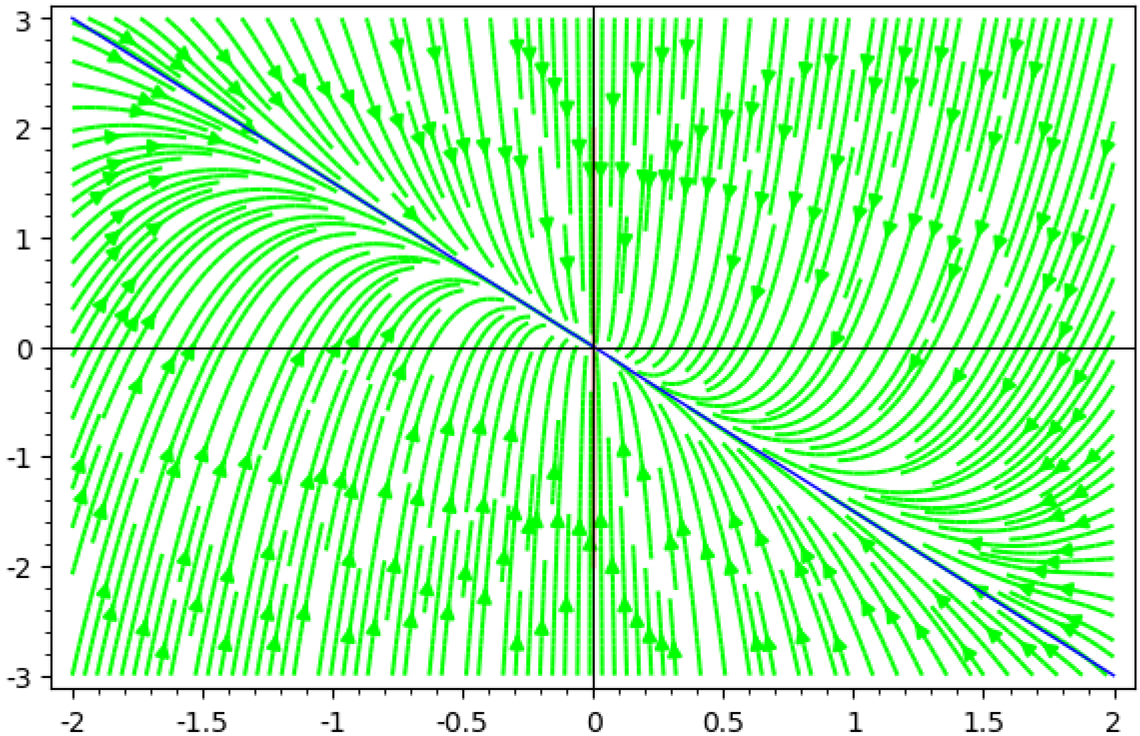
In [31]:
```
x1,x2 = var('x1 x2');
f = 3*x1/2;
g = 2*x2;
SP = streamline_plot((f,g), (x1,-1,1), (x2,-1,1),density = 3,color='lime')
Z1 = sage.plot.line.line([[1,0],[-1,0]],color='red')
Z2 = sage.plot.line.line([[0,1],[0,-1]],color='blue')
SP+Z1+Z2
```

Out[31]:



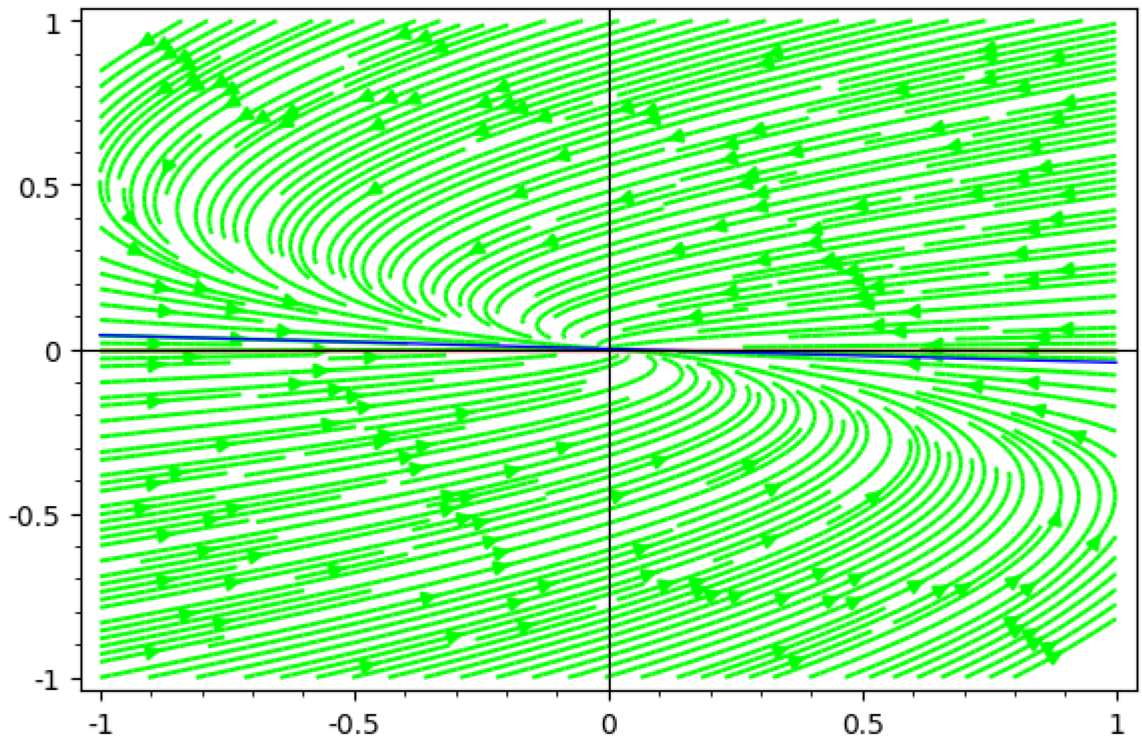
In [48]:
```
x1,x2 = var('x1 x2');
f = -0.5*x1;
g = -9*x1/4-2*x2;
SP = streamline_plot((f,g), (x1,-2,2), (x2,-3,3),density = 3,color='l
ime')
Z1 = sage.plot.line.line([[0,-2],[0,2]],color='red')
Z2 = sage.plot.line.line([[-2,3],[2,-3]],color='blue')
SP+Z1+Z2
```

Out[48]:



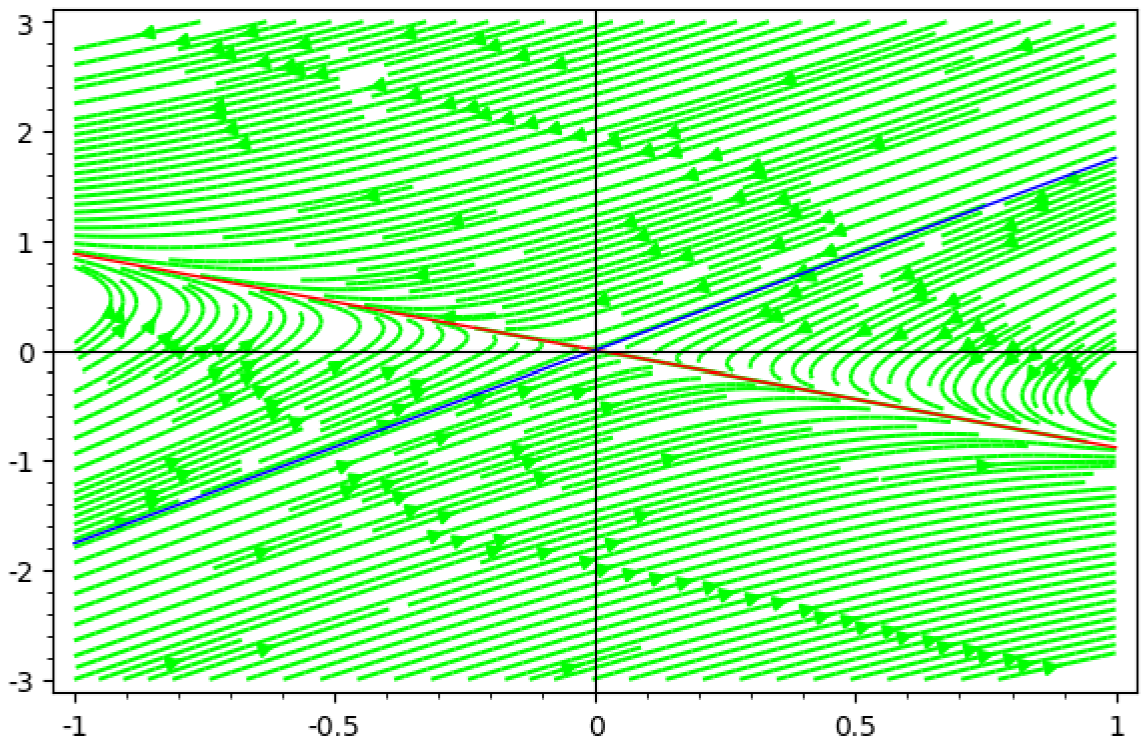
In [53]:
```
x1,x2 = var('x1 x2');
f = -1.5*x1 -3*x2;
g = -11*x2/8;
SP = streamline_plot((f,g), (x1,-1,1), (x2,-1,1),density = 3,color='l
ime')
Z1 = sage.plot.line.line([[1,0],[-1,0]],color='red')
Z2 = sage.plot.line.line([[-1,1/24],[1,-1/24]],color='blue')
SP+Z1+Z2
```

Out[53]:



In [62]:
```
x1,x2 = var('x1 x2');
f = -2*x1/5 -4*x2/5;
g = -99*x1/80 -11*x2/10;
SP = streamline_plot((f,g), (x1,-1,1), (x2,-3,3),density = 3,color='l
ime')
Z1 = sage.plot.line.line([[1,(7-sqrt(445))/16],[-1,-1*(7-sqrt(445))/1
6]],color='red')
Z2 = sage.plot.line.line([[1,(7+sqrt(445))/16],[-1,-1*(7+sqrt(445))/1
6]],color='blue')
SP+Z1+Z2
```

Out[62]:



9.5

5

$$\begin{cases} \frac{dx}{dt} = x(-1 + \frac{5x}{2} - \frac{3y}{10} - x^2) \\ \frac{dy}{dt} = y(-\frac{3}{2} + x) \end{cases}$$

In [96]:
```
x,y = symbols('x,y');
F = x*(-1 +5*x/2 -3*y/10 -x^2);
G = y*(-3/2 +x); F,G
```

Out[96]:

$$\left(x\left(-x^2 + \frac{5x}{2} - \frac{3y}{10} - 1\right),\ y\left(x - \frac{3}{2}\right)\right)$$

In [68]:
```
solve(G)
```
Out[68]:
$$\left[\left\{ x : \frac{3}{2} \right\}, \ \{y : 0\} \right]$$

In [70]:
```
solve(F.subs(y,0))
```
Out[70]:
$$\left[0, \ \frac{1}{2}, \ 2\right]$$

In [72]:
```
solve(F.subs(x,3/2))
```
Out[72]:
$$\left[\frac{5}{3}\right]$$

In [81]:
```
c1,c2,c3,c4 = Matrix([0,0]),Matrix([1/2,0]),Matrix([2,0]),Matrix([3/2
,5/3]); c1,c2,c3,c4
```
Out[81]:
$$\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}, \ \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \ \begin{bmatrix} \frac{3}{2} \\ \frac{5}{3} \end{bmatrix} \right)$$

In [97]:
```
    J = Matrix([[diff(F,x),diff(F,y)],[diff(G,x),diff(G,y)]]); J
```
Out[97]:
$$\begin{bmatrix} -x^2 + x\left(\frac{5}{2} - 2x\right) + \frac{5x}{2} - \frac{3y}{10} - 1 & -\frac{3x}{10} \\ y & x - \frac{3}{2} \end{bmatrix}$$

In [99]:
```
J1 = J.subs(x,0).subs(y,0); J1, J1.subs(x,0).subs(y,0).eigenvects()
```
Out[99]:
$$\left(\begin{bmatrix} -1 & 0 \\ 0 & -\frac{3}{2} \end{bmatrix}, \ \left[\left(-\frac{3}{2}, \ 1, \ \left[\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right] \right), \ \left(-1, \ 1, \ \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right] \right) \right] \right)$$

In [88]:
```
J2 = J.subs(x,1/2).subs(y,0); J1, J1.subs(x,1/2).subs(y,0).eigenvects
()
```
Out[88]:
$$\left(\begin{bmatrix} -1 & 0 \\ 0 & -\frac{3}{2} \end{bmatrix}, \ \left[\left(-\frac{3}{2}, \ 1, \ \left[\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right] \right), \ \left(-1, \ 1, \ \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right] \right) \right] \right)$$

In [90]:
```
J3 = J.subs(x,2).subs(y,0); J1, J1.subs(x,2).subs(y,0).eigenvects()
```
Out[90]:
$$\left(\begin{bmatrix} -1 & 0 \\ 0 & -\frac{3}{2} \end{bmatrix}, \ \left[\left(-\frac{3}{2}, \ 1, \ \left[\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right] \right), \ \left(-1, \ 1, \ \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right] \right) \right] \right)$$

In [95]:
```
J4 = J.subs(x,3/2).subs(y,5/3); J1, J1.subs(x,3/2).subs(y,5/3).eigenv
ects()
```

Out[95]:

$$\left(\begin{bmatrix} -1 & 0 \\ 0 & -\frac{3}{2} \end{bmatrix}, \left[\left(-\frac{3}{2}, 1, \left[\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right]\right), \left(-1, 1, \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right]\right)\right]\right)$$

In [104]:
```
x1,y1 = var('x1,y1');
f = x1*(-1 +5*x1/2 -3*y1/10 -x1^2);
g = y1*(-3/2 +x1);
streamline_plot((f,g), (x1,0,3), (y1,0,3.1),density = 7)
```

Out[104]: