# Quiz 2 Solutions

1) $y'' + 9y = 4\sin(3t)$

- homog: $r^2 + 9 = 0 \Rightarrow r = \pm 3i \Rightarrow y_h = C_1\cos(3t) + C_2\sin(3t)$
- particular: Already have $y_h$ with $\sin(3t)$ and $\cos(3t)$ so need to multiply by $t$: $\boxed{y_p = t\left(A\cos(3t) + B\sin(3t)\right)}$

2) $y'' + 9y = 4\sin(3t)$

- from (1), $y_h = C_1\underbrace{\cos(3t)}_{y_1} + C_2\underbrace{\sin(3t)}_{y_2}$
- $y_p = u_1(t)y_1(t) + u_2(t)y_2(t)$

$$u_1 = \int \frac{g(t)W_1(t)}{W(t)}\,dt, \quad u_2 = \int \frac{g(t)W_2(t)}{W(t)}\,dt, \quad g(t) = 4\sin(3t)$$

$$W(t) = \begin{vmatrix} \cos(3t) & \sin(3t) \\ -3\sin(3t) & 3\cos(3t) \end{vmatrix} = 3\cos^2(3t) + 3\sin^2(3t) = 3$$

$$W_1(t) = \begin{vmatrix} 0 & \sin(3t) \\ 1 & 3\cos(3t) \end{vmatrix} = -\sin(3t), \quad W_2(t) = \begin{vmatrix} \cos(3t) & 0 \\ -3\sin(3t) & 1 \end{vmatrix} = \cos(3t)$$

$$\Rightarrow \boxed{y_p = -\frac{4}{3}\cos(3t)\int \sin^2(3t)\,dt + \frac{4}{3}\sin(3t)\int \sin(3t)\cos(3t)\,dt}$$

3) $\sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} 2a_n x^n$

Shift indices: $\sum_{n=0}^{\infty} (n+1)a_{n+1}x^n = \sum_{n=0}^{\infty} 2a_n x^n$

$\Rightarrow (n+1)a_{n+1} = 2a_n \quad \Rightarrow a_{n+1} = \frac{2}{n+1}a_n \quad \Rightarrow a_n = \frac{2}{n}a_{n-1}$

So,

$$a_n = \frac{2}{n}a_{n-1} = \frac{2}{n}\cdot\frac{2}{n-1}a_{n-2} = \frac{2}{n}\cdot\frac{2}{n-1}\cdot\frac{2}{n-2}a_{n-3} = \cdots = \frac{2^n}{n!}a_0$$

$$\Rightarrow \boxed{a_n = \frac{2^n}{n!}a_0, \quad n = 1, 2, \ldots}$$