1) $y'' - 2y' - 3y = 0 \Rightarrow r^2 - 2r - 3 = 0 \Rightarrow (r-3)(r+1) = 0$ so $y_h = C_1 e^{3t} + C_2 e^{-t}$

so $\boxed{y_p = A\sin(2t) + B\cos(2t)}$

2) $r^2 - 2r + 2 = 0$

$r = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm \frac{1}{2}\sqrt{-4} = 1 \pm i$

$y = C_1 e^t \cos(t) + C_2 e^t \sin(t)$

$y' = C_1 e^t \cos(t) - C_1 e^t \sin(t) + C_2 e^t \sin t + C_2 e^t \cos t$

$y(0) = 2 = C_1$

$y'(0) = 2 = C_1 + C_2 \quad \rightarrow C_2 = 0$

$\Rightarrow \boxed{C_1 = 2,\ C_2 = 0}$

3) $\begin{vmatrix} t+1 & 2t-1 & 3t^2+2 \\ 1 & 2 & 6t \\ 0 & 0 & 6 \end{vmatrix} = 6\big(2(t+1) + 2t - 1\big) = 6(2t+2+2t-1)$

$= 24t + 6 \neq 0$

so L. I.

$K_1 f_1 + K_2 f_2 + K_3 f_3 = t^2(3K_3) + t(K_1 + 2K_2) + (K_1 - K_2 + 2K_3) = 0$

$K_3 = 0$

$K_1 + 2K_2 = 0$

$K_1 - K_2 = 0$

$3K_2 = 0 \quad \Rightarrow K_1 = 0 \quad \Rightarrow K_2 = 0$

$\Rightarrow \boxed{K_1 = K_2 = K_3 = 0}$ so Linearly Independent