Math 146B Winter 2020 Name: Solutions

Quiz 3 Discussion Section: ______ NetID: ____________

1. Find the fundamental period of the function $\cos\left(\frac{n\pi x}{L}\right)$. (Circle only one)

   (a) 2π

   (b) $2L$

   (c) $\frac{2L}{n}$

   (d) $2n\pi$

$\cos\left(\frac{n\pi x}{L}+2\pi\right)=\cos\left(\frac{n\pi x}{L}\right)$.

$=\cos\left(\frac{n\pi}{L}\left(x+\frac{2\pi L}{n\pi}\right)\right)$.

$=\cos\left(\frac{n\pi}{L}\left(x+\frac{2L}{n}\right)\right)$.

2. Consider $y''+\lambda y=0$ with $y(0)=y(\pi)=0$, where $0<x<\pi$. We know only when $\lambda>0$, the equation has nontrivial solutions. Find those eigenvalues for λ. (Circle only one)

   (a) $\lambda=n^2$, for $n=1,2,3,\ldots$

   (b) $\lambda=\frac{(2n-1)^2}{4}$, for $n=1,2,3,\ldots$

   (c) $\lambda=n$, for $n=1,2,3,\ldots$

   (d) $\lambda=\frac{(2n-1)}{2}$, for $n=1,2,3,\ldots$

$\lambda>0$: $r^2+\lambda=0 \Rightarrow r=\pm\sqrt{\lambda}\,i$

$y=c_1\cos(\sqrt{\lambda}x)+c_2\sin(\sqrt{\lambda}x)$.

$y(0)=c_1=0$. $y(\pi)=c_2\sin(\sqrt{\lambda}\pi)=0$

$\Rightarrow \sin(\sqrt{\lambda}\pi)=0$. $\Rightarrow \sqrt{\lambda}=n$,

So $\lambda=n^2$ for $n=1,2,\cdots$

3. Use the method of separation of variables to reduce the partial differential equation $\frac{\partial u}{\partial t^2}=\frac{\partial u}{\partial x^2}$ into a pair of ordinary differential equations. Here $u=u(x,t)$. (Your answer may involve a constant λ.)

Let $u=X(x)\,T(t)$. $\neq 0$.

So $X\cdot T''=X''\cdot T$.

Then $\frac{T''}{T}=\frac{X''}{X}=-\lambda$ some constant.

So $T''+\lambda T=0$. and $X''+\lambda X=0$.