Math 146B Winter 2020 Name: Solutions

Quiz 2 Discussion Section: ____ NetID: ____

1. By using the undetermined coefficients method, to find a particular solution of the linear nonhomogeneous equation $y'' + 9y = 4\sin(3t)$, which of the following y_p we should try? (Circle only one)

   (a) $y_p = 4\sin(3t)$

   (b) $y_p = A\sin(3t)$

   (c) $y_p = A\sin(3t) + B\cos(3t)$

   (d) $y_p = t(A\sin(3t) + B\cos(3t))$

   modify by multiplying t, since $\begin{cases} y_1 = \cos(3t) \\ y_2 = \sin(3t). \end{cases}$

2. By using the variation of parameters method, to find a particular solution of the linear nonhomogeneous equation $y''+9y = 4\sin(3t)$, which of the following y_p we should use? (Circle only one)

   (a) $y_p = -\frac{4}{3}\cos(3t)\int \sin^2(3t)\,dt$

   (b) $y_p = \frac{4}{3}\sin(3t)\int \sin(3t)\cos(3t)\,dt$

   (c) $y_p = -\frac{4}{3}\cos(3t)\int \sin^2(3t)\,dt + \frac{4}{3}\sin(3t)\int \sin(3t)\cos(3t)\,dt$

   (d) none of above

   let $\begin{cases} y_1 = \cos(3t) \\ y_2 = \sin(3t). \end{cases}$ $W(y_1,y_2) = 3.$

   $\Rightarrow \begin{cases} u_1 = -\int \frac{4\sin(3t)\sin(3t)dt}{3} \\ u_2 = \int \frac{4\sin(3t)\cos(3t)dt}{3} \end{cases}$

3. Assume $\sum_{n=1}^{\infty} na_n x^{n-1} = \sum_{n=0}^{\infty} 2a_n x^n$. Determine a_n. (Circle only one)

   (a) $a_n = \frac{a_0}{n}$ for $n = 1, 2, \ldots$, where a_0 is arbitrary.

   (b) $a_n = \frac{a_0}{n!}$ for $n = 1, 2, \ldots$, where a_0 is arbitrary.

   (c) $a_n = \frac{2a_0}{n!}$ for $n = 1, 2, \ldots$, where a_0 is arbitrary.

   (d) $a_n = \frac{2^n a_0}{n!}$ for $n = 1, 2, \ldots$, where a_0 is arbitrary.

   $\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n = \sum_{n=0}^{\infty} 2a_n x^n$

   $\Rightarrow (n+1)a_{n+1} = 2a_n$ for $n=0,1,\cdots$

   So $a_1 = 2a_0.$

   $2a_2 = 2a_1$

   $3a_3 = 2a_2$

   $\vdots$

   $n a_n = 2a_{n-1}.$

   (multiply all)

   $\Rightarrow n!\,\cancel{a_1}\cancel{a_2}\cdots a_n = 2^n a_0 \cancel{a_1}\cdots\cancel{a_{n-1}}$

   $\Rightarrow n!\,a_n = 2^n a_0$

   So $a_n = \frac{2^n}{n!}a_0.$