

**Problem 1.** (12 points)

(1.1)

Calculate the following:

$$\frac{\partial}{\partial x}(x^2 + y^2 - 3xy + \sin(x^2y + y^2)).$$

(1.2)

Verify that the function  $u$ , defined by

$$u(t, x) = t \sin(x - t^2),$$

is a solution to the equation

$$u_x^2 + tuu_t - 2t^2u_xu_{xx} = t^2.$$

(1.3)

Suppose that  $D$  is the ellipse

$$D = \left\{ (x, y) \in \mathbb{R}^2 : \frac{x^2}{4} + y^2 < 10 \right\}$$

and that  $f$  is a differentiable function defined on all of  $\mathbb{R}^2$ . Suppose that  $(x_0, y_0)$  is in  $\partial D$ , the boundary of  $D$ . Denote by  $\frac{\partial f}{\partial n}(x_0, y_0)$  the derivative of  $f$  in the direction of the outward pointing unit normal at the point  $(x_0, y_0)$ . Given that

$$\frac{\partial f}{\partial x}(2, 3) = 2 \quad \text{and} \quad \frac{\partial f}{\partial y}(2, 3) = 2,$$

calculate  $\frac{\partial f}{\partial n}(2, 3)$ .