Problem 4. (12 points)

Take k to be a positive real number. Suppose that u is the solution to the diffusion equation

$$u_t = ku_{xx}$$

on the closed interval $[0, \ell]$.

(4.1)

Solve the differential equation given that

$$u(t, 0) = u(t, \ell) = 0 \quad \text{and} \quad u(0, x) = \sin\left(\tfrac{2\pi x}{\ell}\right) + 3\sin\left(\tfrac{5\pi x}{\ell}\right).$$

(4.2)

Solve the differential equation given that

$$u_x(t, 0) = u_x(t, \ell) = 0 \quad \text{and} \quad u(0, x) = 1 + 3\cos\left(\tfrac{6\pi x}{\ell}\right) + 2\cos\left(\tfrac{7\pi x}{\ell}\right).$$