Worksheet 1

Not due but *highly* recommended. It will earn you **extra credit** for one third of a homework.

Problem 1. Use the mean value theorem to show that there is a unique solution to the initial value problem

$$\begin{cases} x'(t) = 0 \\ x(t_0) = c. \end{cases}$$

Problem 2. Suppose that u is a function of (t, x) and that

$$\begin{cases} \frac{\partial u}{\partial t} = 0 \\ u(0, x) = f(x), \end{cases}$$

where f is a continuously differentiable function. Solve for u. Take $f(x)$ to be equal to x^2 and then once again solve for u.

Problem 3. Suppose that at every point in the t-x plane, the derivative of the function u in the direction v is zero. Take v to be equal to $\langle a, b \rangle$.

(a) Let V be the constant vector field

$$V(t, x) = v.$$

Find all curves in the plane that are integral curves of V.

(b) Find a system of coordinates (τ, y) so that

$$\frac{\partial f}{\partial y} = v \cdot \nabla f.$$

(c) What curves are the level sets of the y coordinate and how do these curves intersect the integral curves of V?

Problem 4. Suppose that a and b are constants. Solve the equation

$$au_t + bu_x = 0 \quad \text{with} \quad u(0, x) = f(x).$$

Refer to Problem 3 and follow the steps for that problem.

Problem 5. Suppose that u is a function on $\mathbb{R}$. Solve the equation

$$u' + 3u = 0.$$

Additionally, solve the inhomogeneous problem

$$u' + 3u = t + 1.$$

Problem 6. Solve the equation

$$u_t + 2u_x + 15u = 10x + 5t + 5.$$

Compare this carefully with Problem 5. What happens if we change the inhomogeneous part to be different. For example, solve the equation

$$u_t + 2u_x + 15u = 2x + t.$$

Problem 7. Suppose that u is a function of the pairs (t, x).

(a) Find the integral curves for the vector field V given by

$$V(t, x) = \langle 1, 2t \rangle.$$

(b) Find a solution to the homogeneous problem

$$u_t + 2tu_x = 0.$$

(c) Find a solution to the homogeneous problem

$$u_t + 2tu_x + 3u = 0.$$

(d) Find a solution to the inhomogeneous problem

$$u_t + 2tu_x + 3u = x - t^2.$$

Problem 8. Can you see a pattern developing in how we solve these kinds of problems. When will it be straightforward to solve such problems and what kind of difficulties arise in solving problems of this type? Given an example of a problem that should be solvable, but where it will by technically difficult if not impossible to get a nice solution in a closed form.