Problem 1. (10 points)

(a) Suppose that $f: \mathbb{R} \to \mathbb{R}$, that f is differentiable, and that x_0 is a real number. Use the mean value theorem to show that there is a unique solution to the initial value problem

$$\begin{cases} f'(x) = 0 \\ f(x_0) = 5. \end{cases}$$

(b) Verify that if v and w are solutions to the inhomogeneous problem

$$3u_t + t^2 u_x + u = 2x + t^3,$$

then $v - w$ is a solution to the homogeneous problem

$$3u_t + t^2 u_x + u = 0.$$

(c) Verify that $u(t, x) = \sin(x^2 t)$ is a solution to the equation

$$t u_{tx} - u_x + 2x^3 t^2 u = 0.$$