Problem 2. (12 points)

(2.1)

Find all solutions to

$$u_t - 7u_x = 0. \tag{1}$$

(2.2)

Find all solutions to (1) with $u(0, x) = x^2 + 1$.

(2.3)

Let V be the vector field given for every (t, x) in $\mathbb{R}^2$ by

$$V(t, x) = \langle 1, 2x \rangle.$$

What are the integral curves of V? (Just so you don't make a silly error, note that these curves are not lines.) Use this to find all solutions of the equation

$$u_t + 2xu_x = 0.$$

(2.4)

Suppose $u(t, x)$ is a differentiable function on $\mathbb{R}^2$ and for each nonnegative real number C, $u(t, x)$ is constant on the curves given by

$$\frac{t^2}{4} + x^2 = C.$$

Find a first order homogeneous differential equation for u so that the initial condition

$$u(0, x) = \cos\bigl(x^2\bigr)$$

uniquely determines u, and then solve the initial value problem. Note that you should know right away what a general solution looks like and then be able to determine without even calculating what the solution to the initial value problem is. The main part of the problem is to come up with and write down the differential equation.