MATH 148 HOMEWORK 8

1. Consider the planar system

$$\begin{cases} x' = x - xy \\ y' = -y + xy, \end{cases}$$

where $x \geq 0$ and $y \geq 0$.

(1) Find a non-trivial first integral of the system.
(2) Then using this first integral to show that all trajectories in the first qudrant $\{(x, y) : x > 0, y > 0\}$ are cycles. Here we count equilibria as trivial cycles.
(3) Sketch the phase portraits.

2. Consider the planar system

$$\begin{cases} x' = 2x - x^2 - xy \\ y' = 3y - y^2 - 2xy, \end{cases}$$

where $x \geq 0$ and $y \geq 0$.

(1) Show that to consider the future of the all trajectories in the dynamical system, one may only need to consider trajectories in a rectangle

$$\mathcal{R} = \{(x, y) : 0 \leq x \leq k,\ 0 \leq y \leq k\}$$

for some large k. Explain that the rectangle $\mathcal{R}$ is positively invariant.
(2) Find all the equilibiria and classify them via the linearization theorem.
(3) Use the all the informations above to determine the $\omega(\vec{x})$ for all $\vec{x} = (x, y)$ with $x \geq 0$ and $y \geq 0$.

3. For the two systems considered in problems 1 and 2, which one is a model of competitive species? Which is a model of predator-prey? Why?