MATH 148 FINAL EXAM, SPRING 2019

Please return the exam into my office (Skye Hall 256) at 11am on Jun 12. You may only consult the resources that we posted at iLearn or your notes. **You must work independently.** In particular, you may not work with others. You may not email your instructor or TA for math quesitons during the exam time.

Unreasonably highly similar exam copies will be reported as cheating case to the Student Conduct & Academic Integrity Programs.

1. (8 pts) Construct a planar system with the following properties. Note you have to write down the explicit ODEs.

(1) There are 3 limit cycles and one equilibrium point at the origin.
(2) Two cycles repel nearby trajectories and one attracts nearby trajectories.
(3) The attracting limit cycle sits in-between the two repelling cycles.
(4) What's type of the equilibrium point?

2. (10 pts) Consider the following family of planar linear systems

$$\begin{cases} x' = bx + ay \\ y' = -x + by, \end{cases}$$

where $a, b \in \mathbb{R}$ are parameters.

(1) Determine for which choices of a and b, it's a Hamiltonian system.
(2) For the choice of a, b from part (1), you may note that it may be considered as a one-parameter family of planar linear systems in one of the parameter. Find all values of that parameter at which a bifurcation occurs.
(3) Describe the bifurcation by sketching the phase portraits for certain choices of the parameters before and after the bifurcation point.

3. (10 pts) Consider the following planar system

$$\begin{cases} x' = 4y - 4y^3 \\ y' = 2x - 2, \end{cases}$$

(1) Show that there is a homoclinic trajectory. Precisely, it's an equilibrium point attached by two homoclinic orbits.
(2) Show that all other trajectories than the homoclinic trajectory are cycles via the Poincaré-Bendixon theorem. Here we count equiibria as trivial cycles.
(3) Sketch the phase portraits.

4. (12 pts) Consider a planar system

$$\begin{cases} x' = 3x - x^2 - xy \\ y' = 2y - y^2 - \frac{1}{2}xy, \end{cases}$$

where $x \geq 0, y \geq 0$. Suppose x and y are the populations of two species.

(1) Is this a predator-prey model or a competitive species model? Explain your reasoning by tranlating the math relation given by the ODEs back to the real life relation between the two species.
(2) Can two competitive species co-exist in this model?
(3) Combining the vector fields, certain positively invariant set, Poincaré-Bendixon theorem, and linearization theorem to determine the ω-limit set $\omega(\vec{x})$ for each $\vec{x} = (x, y)$ in question.
(4) Can this system have a non-trivial first integral? Explain your reasoning.

5. (5 pts) Which part of the course materials is your favorite one? Note it can be a theorem, an example, a notion, or one of the chapters. Explain why it's your favorite one. (*This is bascially a free 5 pts. Just state some of your understanding or opinon for your favorite part of the course materials*)