MATH 148 STUDY GUIDE

Chap 1. Introduction to dynamical systems and 1D (one dimensional) systems

- Statement of the Picard's theorem and counter examples
- Blow-up solutions and global existence of ODEs
- What are dynamical systems and what do we care about in dynamical systems
- First order, 1 variable autonomous ODE and 1D dynamical systems.
- Equilibria and their types
- Dynamicas of 1D systems via equilibria
- Bifurcation in a 1-parameter family of 1D systems

Chap 2. Planar linear systems

- Definiton and different forms of planar linear systems $\vec{x}' = A\vec{x}$
- Theories and computing solutions for planar linear systems
- Classification of the equlibrium point $\vec{0}$ via eigenvalues of the matrix A
- Dynamics and phase portraits of planar linear systems
- Bifurcation in a 1-parameter family of planar linear systems

Chap 3. Nonlinear planar systems I–Linearization Theorem

- Limit cycles and systems in polar coordinates
- Definiton of dynamical systems and their relations with ODE systems
- Definition of topoloical conjugacy and why its an equivalent relation
- Topolocial conjugacy preserves equilibria and cycles
- Dynamical description of the orgin via its stable and unstable manifolds
- Topolocial conjugacy preserves the type of the orgin of linear systems
- Hyperbolic linear systems
- Statement of the linearization theorem
- Definition and classificaiton of hyperbolic equilibria of nonlinear systems
- Topological conjugacy preserves the types of hyperbolic equilibria
- Application of the linearization theorem

Chap 4. Nonlinear planar systems II–Hamiltonian systems

- Definiton of planar Hamiltonian systems
- Definition of first integrals
- Hamiltonian function as a first integral
- Level sets of a first integral are invariant
- Dynamics and phase portrats of Hamiltonian systems via the level sets of its Hamiltonian function
- Examples of Hamiltonian systems, in particular the undamped pendulum
- Statement of the Liouville theorem and its consequences

Chap 5. Nonlinear planar systems III–Poincaré-Bendixon theorem

- positively and negatively invariant sets, invariant set
- ω-limit point and set, α-limit point and set, and their dynamical meaning
- ω or α-limit sets are invariant
- local section, flow box, times of first arrival and return to a local section
- monotone sequences along a trajectory or along a line segament
- Proposition 5.1: consider a sequence that is both on a local section and on a trajectory. If it's monotone on the trajectory, then it's monotone on the local section.
- Proposition 5.2: if $\vec{y}$ is a ω-limit point of some point $\vec{x}$, then the trajectory of $\vec{y}$ meet any local section at most at one point.
- **Poincaré-Bendixon theorem**: for any $\vec{x}$, if $\omega(\vec{x})$ ($\alpha(\vec{x})$, resp.) is bounded, nonempty, and contains no equlibirium point, then $\omega(\vec{x})$ ($\alpha(\vec{x})$, resp.) is a cycle. Moreover, either $x \in \omega(\vec{x})$ and the trajectory of $\vec{x}$ is the cycle $\omega(\vec{x})$, or $x \notin \omega(\vec{x})$ and $\omega(\vec{x})$ is a limit cycle that attracts (repels, resp.) some nearby trajectories.
- Corollaries and applications of the Poincaré-Bendixon theorem.

Chap 6. Applications to biological models

- Defintion and setup of the predator-prey model
- Computation of the first integral of predator-prey models
- Dynamics of the predator-prey model via first integral and Poincaré-Bendixon
- Defintion and setup of the competitive species model
- Positive invariance of large rectangles of the competitive species model
- Classification and dynamics of the the competitive species models via null-clines and linearization