151A—7

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14 Let $I = [0,1]$ be the closed unit interval. Suppose f is a continuous mapping of I into I. Prove that $f(x) = x$ for at least one x in I.

pf.

Let $g : I \to I; x \mapsto (f-1)(x)$. Where $1 : I \to I; x \mapsto x$.

$f : I \to I \implies f(x) \geq 0$

$\implies f(0) \geq 0$

$\implies f(1) \leq 1 \implies f(1) - 1 \leq 1 - 1 = 0$

$\implies g(0) = f(0) - 0 \geq 0$

$\implies g(1) = f(1) - 1 \leq 0$

g is continuous as it is the difference of continuous functions.

$[0,1]$ is closed. So, the function g has a zero on I by the intermediate value theorem.

$\implies \exists x \in I : g(x) = 0$

$\implies f(x) - x = 0$

$\implies f(x) = x$