MATH 151B HOMEWORK 1

Problem 1

Let f be defined for all real x, and suppose that

$$|f(x) - f(y)| < (x - y)^2$$

for all real x and y. Prove that f is constant.

Problem 2

If

$$C_0 + \frac{C_1}{2} + \cdots + \frac{C_{n-1}}{n} + \frac{C_n}{n+1} = 0$$

for real constants $C_0, \ldots, C_n$, prove that the equation

$$C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n = 0$$

has at least one real root between 0 and 1.

Problem 3

Suppose f is continuous for $x \geq 0$, $f'(x)$ exists for $x > 0$, $f(0) = 0$, and f' is monotonically increasing. Define, for $x > 0$

$$g(x) = \frac{f(x)}{x}.$$

Prove that g is monotonically increasing.

Problem 4

Suppose f is differentiable in (a, b) and $f'(x) > 0$. Prove that f is strictly increasing in (a, b). Let g be its inverse function. Prove that g is differentiable and that

$$g'(f(x)) = \frac{1}{f'(x)}.$$