MATH 151B HOMEWORK

Problem 1

Consider the $f(x)$ on $[0,1]$ such that $f(x)=0$ if $x=\frac{1}{2^n}$ for some positive integer n and $f(x)=1$ otherwise. Prove that $f(x)$ is integrable and compute its integral.

Problem 2

Suppose $c_n \geq 0$ such that $\sum C_n$ converges and s_n is a sequence of distinct points in (a,b). Let

$$\alpha(x)=\sum_{n=1}^{\infty} c_n I(x-s_n)$$

Let f be bounded on (a,b) and continuous at s_n for each n. Is is true that $f \in \mathfrak{R}(\alpha)$?

Problem 3

Let γ_1 be a curve defined on $[a,b]$ and ϕ be a continuous 1-1 mapping of $[c,d]$ onto $[a,b]$ such that $\phi(c)=a$. Define $\gamma_2(s)=\gamma_1(\phi(s))$. Prove that γ_2 is rectifiable if and only if γ_1 is rectifiable. In that case, prove that they have the same length.

Problem 4

Suppose α increasing monotonically on $[a,b]$, g is continuous and $g(x)=G'(x)$. Prove that

$$\int_a^b G d\alpha = G(b)\alpha(b)-G(a)\alpha(a)-\int_a^b \alpha(x)g(x)dx.$$

Hint For any partition P, choose $t_i \in (x_{i-1},x_i)$ so that

$$g(t_i)\Delta x_i = G(x_i)-G(x_{i-1})$$

Show that

$$\sum_{i=1}^{n} \alpha(x_i)g(t_i)\Delta x_i = G(b)\alpha(b)-G(a)\alpha(a)-\sum_{i=1}^{n} G(x_{i-1})\Delta\alpha_i$$