Math 151B Final Exam

NAME (please print):

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6	Total
3 pts.	3 pts.	3 pts.	3 pts.	3 pts.	3 pts.	18 pts

Please write the pledge below in your own handwriting, fill in the required information, and sign. “On my honor, I (your name) have neither given nor received any unauthorized aid on this examination. I have never used unauthorized tools or look up unauthorized materials. I finished my exam within 3 hours”

Pledge:

Start Time and Date:

End Time and Date:

Signature:

Problem 1 (3 pts)

If f is continuous on $[0,1]$ and for all n,

$$\int_0^1 f(x)x^n dx = 0.$$

Prove that $f(x) = 0$.

Problem 2 (3 pts)

Suppose $c_n \geq 0$ such that $\sum C_n$ converges and x_n is a sequence of distinct points in (a, b). Let

$$\alpha(x) = \sum_{n=1}^{\infty} c_n I(x - x_n)$$

Let f be bounded on (a, b) and continuous at x_n for each n. Prove that $f \in \mathfrak{R}(\alpha)$.

Problem 3 (3 pts)

Let γ_1 be a curve defined on $[a, b]$ and ϕ be a continuous 1-1 mapping of $[c, d]$ onto $[a, b]$ such that $\phi(c) = a$. Define $\gamma_2(s) = \gamma_1(\phi(s))$. Prove that γ_2 is rectifiable if and only if γ_1 is rectifiable. In that case, prove that they have the same length.

Problem 4 (3 pts)

If

$$C_0 + \frac{C_1}{2} + \cdots + \frac{C_{n-1}}{n} + \frac{C_n}{n+1} = 0$$

for real constants $C_0, \ldots, C_n$, prove that the equation

$$C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n = 0$$

has at least one real root between 0 and 1.

Problem 5 (3 pts)

Suppose $f(x) = 0$ for all irrational x and $f(x) = 1$ for all rational x, prove that f is not Riemann integrable on any $[a, b]$ for any $a < b$.

Problem 6 (3 pts)

If $\sum |c_n| < \infty$ and x_n is a sequence of distinct points in (a, b), show that

$$f(x) = \sum_{k=1}^{\infty} c_k I(x - x_k)$$

converge uniformly and that f is continuous for every $x \neq x_n$.