MATH 151B HOMEWORK

Problem 1

Suppose f and g are complex differentiable function on $(0,1)$, $f(x) \to 0$, $g(x) \to 0$, $f'(x) \to A$, $g'(x) \to B$ as $x \to 0$ where A and B are complex numbers and $B \neq 0$. Prove that

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{A}{B}$$

Hint:

$$\frac{f(x)}{g(x)} = \left(\frac{f(x)}{x} - A\right) \cdot \frac{x}{g(x)} + A\frac{x}{g(x)}$$

Apply the L'Hôpital's rule rule to the real and imaginary part of $\frac{f(x)}{x}$ and $\frac{g(x)}{x}$.

Problem 2

Suppose α is increasing on $[a,b]$ and is continuous at $x_0 \in [a,b]$. Let f be the function $f(x_0) = 1$ and $f = 0$ otherwise. Prove that $f \in \mathfrak{R}(\alpha)$ and

$$\int_a^b f d\alpha = 0$$

Problem 3

Suppose $f \geq 0$ is continuous on $[a,b]$ and $\int_a^b f(x)dx = 0$. Prove that $f(x) = 0$ for all $x \in [a,b]$.

Problem 4

Suppose $f(x) = 0$ for all irrational x and $f(x) = 1$ for all rational x, prove that f is not Riemann integrable on any $[a,b]$ for any $a < b$.