

## MATH 151B HOMEWORK

### Problem 1

If the sequences  $\{f_n\}$  and  $\{g_n\}$  converge uniformly on  $E$ , prove that  $\{f_n + g_n\}$  converge uniformly on  $E$ .

### Problem 2

If the sequences  $\{f_n\}$  and  $\{g_n\}$  defined on  $E$  such that

- (1)  $\sum f_n$  converge uniformly on  $E$
- (2)  $g_n$  converge to 0
- (3)  $g_1(x) \geq g_2(x) \geq g_3(x) \geq \cdots$  for every  $x \in E$

Prove that  $\sum f_n g_n$  converge uniformly on  $E$ .

### Problem 3

If  $\sum |c_n| < \infty$  and  $x_n$  is a sequence of distinct points in  $(a, b)$ , show that

$$f(x) = \sum_{k=1}^{\infty} c_k I(x - x_k)$$

converge uniformly and that  $f$  is continuous for every  $x \neq x_n$ .

### Problem 4

Suppose  $\{f_n\}$  is an equicontinuous sequence of functions on  $[a, b]$  and  $\{f_n\}$  converge point-wise on  $[a, b]$ . Prove that it converges uniformly.