MATH 151B HOMEWORK 3

Problem 1

Let f be a continuous real function on $\mathbb{R}$. If $f'(x)$ exists for all $x \neq 0$ and $f'(x) \to 3$ as $x \to 0$, does it follows that $f'(0)$ exists?

Problem 2

Let f be a twice-differentiable real function on $\mathbb{R}$ and M_0, M_1 and M_2 be the least upper bounds of $|f(x)|$, $|f'(x)|$ and $|f''(x)|$. Prove that

$$M_1^2 \leq 4M_0M_2.$$

Hint: Use the Taylor's theorem with $n = 2$.

Problem 3

Suppose f is defined in a neighborhood of x and suppose $f''(x)$ exists. Show that

$$\lim_{h \to 0} \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x).$$

Problem 4

Recall that a continuous function f on (a, b) is convex if and only if for all $x, y \in (a, b)$

$$f(\frac{x+y}{2}) \leq \frac{1}{2}(f(x) + f(y)).$$

Let f be a differentiable real function on (a, b). Prove that f is convex if f' is monotonically increasing.