

## MATH 151B MIDTERM SAMPLE EXAM

### Problem 1

Every rational number  $x$  can be written in the form  $x = \frac{m}{n}$  where  $n > 0$  and  $m$  and  $n$  are integers without common divisors. When  $x = 0$ , we take  $n = 1$ . Consider the following function  $f$  on  $\mathbb{R}$  where  $f(x) = 0$  if  $x$  is irrational and  $f(x) = \frac{1}{n}$  if  $x = \frac{m}{n}$  is rational. Prove that  $f$  is continuous at every irrational point and  $f$  has a simple discontinuity at every rational point.

### Problem 2

Suppose  $f$  is real and three times differentiable on  $[-1, 1]$ , such that

$$f(-1) = 0, \quad f(0) = 0, \quad f(1) = 1, \quad f'(0) = 0.$$

Prove that  $f^{(3)}(x) \geq 3$  for some  $x \in (-1, 1)$ .

### Problem 3

Suppose  $\alpha$  is continuous and increasing on  $[a, b]$  and  $f$  is the function such that  $f(x_0) = 1$  and  $f(x) = 0$  if  $x \neq x_0$ . Prove that

$$\int f d\alpha = 0.$$

### Problem 4

Suppose  $f$  and  $g$  are complex differentiable function on  $(0, 1)$ ,  $f(x) \rightarrow 0$ ,  $g(x) \rightarrow 0$ ,  $f'(x) \rightarrow A$ ,  $g'(x) \rightarrow B$  as  $x \rightarrow 0$  where  $A$  and  $B$  are complex numbers and  $B \neq 0$ . Prove that

$$\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{A}{B}$$