Use the hint, exists $\phi$ on $[a, b]$ with $\phi' = \gamma'/\gamma$ and $\phi(a) = 0$. That $\phi$ must be the one on $FTC$ 1, i.e. $\phi(x) = \int_a^x \frac{\gamma'(t)}{\gamma(t)} dt$

Since $\gamma$ exp$(-\phi)$ is constant, it follows exp$(\phi(b)) = 1 \implies \phi(b) = -2\pi i n$. So Ind$(\gamma)$ is some integer. Now compute,

$$\frac{1}{2\pi i} \int_0^{2\pi} \frac{ine^{int}}{e^{int}} dt = \frac{n}{2\pi} \int_0^{2\pi} dt = n$$

So, if $\gamma = e^{int}$ then Ind($\gamma$) $= n$.