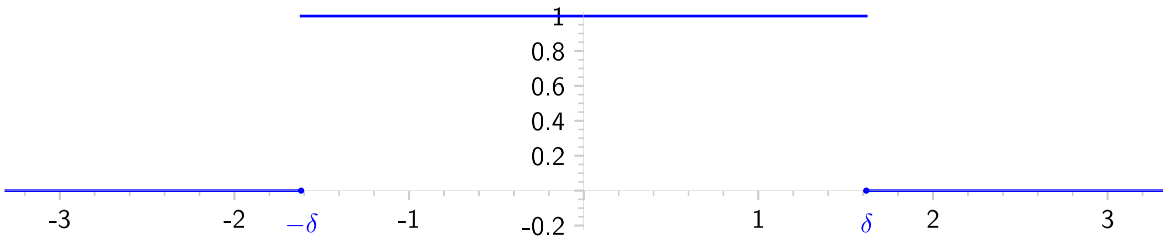
The following is the subset of the plot of f in $[-\pi, \pi]$.



(a)

Following formula (62)

$$c_m = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-imx}dx$$

Since f is 1 for $|x| < \delta$ and 0 for $\delta < |x| \leq \pi$, we can see that,

$$\begin{aligned}
m \neq 0 \implies c_m &= \frac{1}{2\pi}\int_{-\delta}^{\delta} e^{-imx}dx \\
&= \frac{1}{2\pi} - \frac{e^{-imx}}{im}|_{-\delta}^{\delta} \\
&= \frac{1}{2\pi} - \left(\frac{e^{-im\delta}}{im} - \frac{e^{im\delta}}{im}\right) \\
&= \frac{1}{2\pi}\left(\frac{e^{im\delta} - e^{-im\delta}}{im}\right) \\
&= \frac{1}{m\pi}\left(\frac{e^{im\delta} - e^{-im\delta}}{2i}\right) \\
&= \frac{1}{m\pi}\sin(m\delta) \\
m = 0 \implies c_0 &= \frac{1}{2\pi}\int_{-\delta}^{\delta} e^{0}dx \\
&= \frac{\delta - (-\delta)}{2\pi} = \frac{\delta}{\pi}
\end{aligned}$$

$$c_{-m} = \frac{\sin(-m\delta)}{-m\pi} = \frac{-\sin(m\delta)}{-m\pi} = \frac{\sin(m\delta)}{m\pi} = \overline{c_m}$$

So, f is real by the remark in page 186. Notice, $\cos(x) = \frac{e^{ix}+e^{-ix}}{2} \implies 2\cos(inx) = e^{inx} + e^{-inx}$

Then we can see that if we change m to n,

$$f(x) = \sum_{-\infty}^{\infty} c_n e^{inx} = c_0 + \sum_{n=1}^{\infty} c_n e^{inx} + \sum_{n=1}^{\infty} c_{-n}e^{i-nx} = c_0 + \sum_{n=1}^{\infty} c_n(e^{inx} + e^{i-nx}) = c_0 + \sum_{n=1}^{\infty} c_n 2\cos(nx)$$

So $b_n = 0$ for $n \in \mathbb{N}$ and $a_n = 2c_n$, for $(n = 0, 1, 2, \ldots)$.[1] That is $a_0 = \frac{2\delta}{\pi}$, and $a_n = \frac{2\sin(n\delta)}{n\pi}$ for $n \in \mathbb{N}$. Finally

$$f(x) = \frac{\delta}{\pi} + \sum_{n=1}^{\infty} \frac{2\sin(n\delta)}{n\pi}\cos(nx)$$

[1] This is why Boyce and DiPrima wrote $a_0/2$ in their definition of Fourier series.

(b)

$$\forall t \in [-\delta, \delta] \quad |f(0+t) - f(0)| = |1-1| = 0 \leq |t|$$

Thus by 8.14 $\lim_{n\to\infty} S_n(f;0) = f(0)$, which means

$$\begin{aligned}
\frac{\delta}{\pi} + \sum_{n=0}^{\infty} \frac{2\sin(n\delta)}{n\pi}\cos(0) = f(0) \implies & \frac{\delta}{\pi} + \sum_{n=0}^{\infty} \frac{2\sin(n\delta)}{n\pi} = 1 \\
\implies & \frac{2}{\pi}\sum_{n=0}^{\infty} \frac{\sin(n\delta)}{n} = 1 - \frac{\delta}{\pi} \\
\implies & \sum_{n=0}^{\infty} \frac{\sin(n\delta)}{n} = \frac{\pi}{2} - \frac{\delta}{2} \\
\implies & \sum_{n=0}^{\infty} \frac{\sin(n\delta)}{n} = \frac{\pi - \delta}{2}
\end{aligned}$$

(c) Formula (84) gives us what we need,

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |f(x)|^2 dx = \sum_{-\infty}^{\infty} |c_n|^2$$

$$\begin{aligned}
\frac{1}{2\pi}\int_{-\pi}^{\pi} |f(x)|^2 dx &= \frac{1}{2\pi}\int_{-\delta}^{\delta} |1|^2 dx \\
&= \frac{\delta - (-\delta)}{2\pi} \\
&= \frac{\delta}{\pi} \\
&= \frac{\delta^2}{\pi^2} + 2\sum_{n=1}^{\infty} \frac{\sin^2(n\delta)}{n^2\pi^2} \\
&= c_0^2 + 2\sum_{n=1}^{\infty} |c_n|^2 \\
&= c_0^2 + \sum_{n=1}^{\infty} |c_n|^2 + \sum_{n=1}^{\infty} |c_n|^2 \\
&= c_0^2 + \sum_{n=1}^{\infty} |c_{-n}|^2 + \sum_{n=1}^{\infty} |c_n|^2 \\
&= \sum_{-\infty}^{\infty} |c_n|^2
\end{aligned}$$

$$\begin{aligned}
\frac{\delta}{\pi} = \frac{\delta^2}{\pi^2} + 2\sum_{n=1}^{\infty} \frac{\sin^2(n\delta)}{n^2\pi^2} \implies & \delta\pi = \delta^2 + 2\sum_{n=1}^{\infty} \frac{\sin^2(n\delta)}{n^2} \\
\implies & \pi = \delta + 2\sum_{n=1}^{\infty} \frac{\sin^2(n\delta)}{n^2\delta} \\
\implies & \sum_{n=1}^{\infty} \frac{\sin^2(n\delta)}{n^2\delta} = \frac{\pi - \delta}{2}
\end{aligned}$$