$$\zeta(2) = \frac{\pi^2}{6}$$

Consider $f(x) = x$ for x in $[-\pi, \pi]$,

$$C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\,dx = 0 \qquad \text{Since } x \text{ is an odd function}$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} x e^{inx}\,dx \qquad n \neq 0$$

$$= \frac{1}{2\pi}\left(x\frac{e^{inx}}{in}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{e^{inx}}{in}dx \right) \qquad u = x \quad dv = e^{inx}dx$$

$$du = dx \quad v = \frac{e^{inx}}{in}$$

$$= \frac{1}{2\pi}\left(\pi\frac{e^{in\pi}}{in} - (-\pi)\frac{e^{-in\pi}}{in} - \frac{e^{inx}}{(in)^2}\Big|_{-\pi}^{\pi} \right)$$

$$= \frac{1}{2\pi}\left(\pi\frac{e^{in\pi}}{in} - (-\pi)\frac{e^{-in\pi}}{in} - \left(\frac{e^{in\pi}}{(in)^2} - \frac{e^{-in\pi}}{(in)^2}\right) \right)$$

$$= \frac{1}{2\pi}\left(\pi\frac{e^{in\pi}}{in} + \pi\frac{e^{-in\pi}}{in} - \frac{e^{in\pi}}{(in)^2} + \frac{e^{-in\pi}}{(in)^2} \right) \qquad e^{\pm in\pi} = \cos(n\pi) \pm i\sin(n\pi) = \cos(n\pi) = (-1)^n$$

$$= \frac{1}{2\pi}\left(\pi\frac{(-1)^n}{in} + \pi\frac{(-1)^n}{in} - \frac{(-1)^n}{(in)^2} + \frac{(-1)^n}{(in)^2} \right) \qquad = \frac{1}{2\pi}\left(\pi\frac{(-1)^n}{in} + \pi\frac{(-1)^n}{in} - \cancel{\frac{(-1)^n}{(in)^2}} + \cancel{\frac{(-1)^n}{(in)^2}} \right)$$

$$= \frac{1}{2\pi}\left(2\pi\frac{(-1)^n}{in} \right) = \frac{(-1)^n}{in} = \frac{-i(-1)^n}{n}$$

$$= \frac{i(-1)^{n+1}}{n}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |x|^2 dx = \frac{1}{2\pi}\int_{-\pi}^{0} (-x)^2 dx + \frac{1}{2\pi}\int_{0}^{\pi} x^2 dx$$

$$= \frac{1}{2\pi}\left(\frac{x^3}{3}\Big|_{-\pi}^{0} + \frac{x^3}{3}\Big|_{0}^{\pi} \right)$$

$$= \frac{1}{2\pi}\left(-\frac{(-\pi)^3}{3} + \frac{\pi^3}{3} \right)$$

$$= \frac{1}{2\pi}\left(\frac{2\pi^3}{3} \right)$$

$$= \frac{\pi^2}{3} = 2\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$= \sum_{n=1}^{\infty}\frac{1}{(-n)^2} + \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$= \sum_{-\infty}^{\infty}\frac{1}{n^2}$$

$$= \sum_{-\infty}^{\infty}\frac{|i|^2|(-1)^{n+1}|^2}{n^2}$$

$$= \sum_{-\infty}^{\infty}\left|\frac{i(-1)^{n+1}}{n}\right|^2$$

Thus,

$$\zeta(2) = \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$$