$$\frac{\sin x}{x} = \frac{1}{x}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!} = 1 + \sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}$$

So,

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 + \lim_{x\to 0}\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!} = 1$$

Also,

$$\lim_{x\to \frac{\pi}{2}}\frac{\sin x}{x} = \frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$$

Both, $\sin x$ and $\frac{1}{x}$ are continuous on $(0, \frac{\pi}{2})$. So, $\frac{\sin x}{x}$ is also continuous there, by the intermediate value theorem,

$$\forall x \in \left(0, \frac{\pi}{2}\right) \quad \frac{2}{\pi} < \frac{\sin x}{x} < 1$$