For this problem, we employ the notations of 9.26 on the textbook. Let $f : \mathbb{R}^{n+m} \to \mathbb{R}^n$ be a continuously differentiable function. Suppose that $f(a,b) = 0$ for some $(a,b) \in \mathbb{R}^{n+m}$ and for all $(h,k) \in \mathbb{R}^{n+m}$, $[f'(h,k)]_x$ is invertible. Is there a function $g : \mathbb{R}^m \to \mathbb{R}^n$ such that $f(g(y), y) = 0$ for all $y \in \mathbb{R}^m$?

slu.

Consider $f : \mathbb{R}^2 \to \mathbb{R}; (x,y) \mapsto e^x - e^y \cos(y)$

$f(0,0) = 0$

$[f'(x,y)] = \begin{pmatrix} e^x & e^y \sin(y) - e^y \cos(y) \end{pmatrix}$

$\det([f'(x,y)]_x) = e^x \neq 0$

Since, $\cos(y)$ is periodic f is not injective.

f is not surjective since e^x is non-negative, and $-1 < -e^y \cos(y) < 1$, so $\forall (x,y) \in \mathbb{R}^2 - 1 < f(x,y)$.

So, f cannot define a in implicit function $g : \mathbb{R} \to \mathbb{R}$ such that g satisfies, $f(g(y), y) = 0, \quad \forall y \in \mathbb{R}$ ♦