Suppose that f is a differentiable real function in an open set $E \subset R^n$, and that f has a local maximum at a point $\mathrm{x} \in E$. Prove that $f'(\mathrm{x}) = 0$

slu.

Fix $\mathrm{y} \in \mathbb{R}^n$,

Let $\gamma : [-1,1] \longrightarrow \mathbb{R}^m; t \mapsto \mathrm{x} + t\mathrm{y}$

Let $\phi : [-1,1] \longrightarrow \mathbb{R}; \phi = f \circ \gamma$

Since γ is differentiable, $(-1,1) \subset [-1,1]$ is open, and f maps $\gamma(-1,1)$ into $\mathbb{R}$, it follows by Theorem 9.15 that ϕ is differentiable on $(-1,1)$. And formula (22) holds for $-1 < t < 1$,

$$\phi'(t) = f'(\gamma(t))\gamma'(t) \tag{1}$$

Since, x is a local maximum of f. It follows that 0 is a local maximum of ϕ, since $\phi(0) = f(\mathrm{x})$.

Since ϕ has local maximum at $t = 0$, it follows from Theorem 5.8 that $\phi'(0) = 0$, then

$$f'(\gamma(0))\gamma'(0) = f'(\mathrm{x})\mathrm{y} = 0 \tag{2}$$

Since y was an arbitrary non zero vector it follows that,

$$f'(\mathrm{x}) = 0 \quad \blacksquare$$