$|\sin 0x| = 0 = 0|\sin x|$ and $|\sin 1x| = 1|\sin x|$

Assume,

$$\exists n \in \mathbb{N} : \forall k \in \mathbb{N} : k \leq n \implies |\sin kx| \leq k|\sin x|$$

Then,

$$\begin{aligned}
|\sin(n+1)x| = |\sin(nx+x)| = |\sin nx \cos x + \cos(nx)\sin x| \\
\leq |\sin nx \cos x| + |\cos(nx)\sin x| = |\sin nx||\cos x| + |\cos(nx)||\sin x| \\
\forall n \in \mathbb{N}, |\cos nx| \leq 1 \implies |\sin(n+1)x| \leq |\sin nx| + |\sin x| \\
|\sin nx| \leq n|\sin x| \implies |\sin(n+1)x| \leq n|\sin x| + |\sin x| = (n+1)|\sin x|
\end{aligned}$$

Therefore the inequality $|\sin nx| \leq n|\sin x|$ is valid for every natural number and zero.

Note,

$$0.001 \approx |\sin 0.001| > 0.001|\sin(1)| \approx 0.0008417$$