Let $f(x) = x^3 - \sin^2(x)\tan(x)$ and $g(x) = 2x^2 - \sin^2 x - x\tan x$

Since as $x \to \pi/2$ both f and g go to $-\infty$, we only need to show if $\exists \quad 0 < x < \pi/2$ such that f or g are positive.

```
var('x')
f = x^3 -1*sin(x)^2*tan(x)
g = 2*x^2 -1*sin(x)*sin(x)-1*x*tan(x)
def fderivatives (h,m):
    return ",".join(["f^{(%d)}(0) =" % n + latex(h.derivative(x,n).subs(x=0)) for n in range(m
def gderivatives (h,m):
    return ",".join(["g^{(%d)}(0) =" % n + latex(h.derivative(x,n).subs(x=0)) for n in range(m
```

The computation above shows that, $f^{(0)}(0) = 0, f^{(1)}(0) = 0, f^{(2)}(0) = 0, f^{(3)}(0) = 0, f^{(4)}(0) = 0$ and

$f^{(5)}(x) = -120 \sin(x)^2 \tan(x)^6 - 240 \cos(x)\sin(x)\tan(x)^5 - 120 \cos(x)^2 \tan(x)^4 - 120 \sin(x)^2 \tan(x)^4 - 240 \cos(x)\sin(x)\tan(x)^3 - 120 \cos(x)^2 \tan(x)^2 - 16 \sin(x)^2 \tan(x)^2 - 32 \cos(x)\sin(x)\tan(x) - 16 \sin(x)^2$

$= -(120 \sin(x)^2 \tan(x)^6 + 240 \cos(x)\sin(x)\tan(x)^5 + 120 \cos(x)^2 \tan(x)^4 + 120 \sin(x)^2 \tan(x)^4$

$+ 240 \cos(x)\sin(x)\tan(x)^3 + 120 \cos(x)^2 \tan(x)^2 + 16 \sin(x)^2 \tan(x)^2 + 32 \cos(x)\sin(x)\tan(x) + 16 \sin(x)^2)$

Since $\sin(x)\cos(x)\tan(x) = \sin^2(x)$, we have,

$f^{(5)}(x) = -(120 \sin(x)^2 \tan(x)^6 + 240 \sin^2(x)\tan(x)^4 + 120 \cos(x)^2 \tan(x)^4 + 120 \sin(x)^2 \tan(x)^4$

$+ 240 \sin^2(x)\tan(x)^2 + 120 \cos(x)^2 \tan(x)^2 + 16 \sin(x)^2 \tan(x)^2 + 48 \sin^2(x))$

All of the terms inside the parentheses are positive, so the fifth derivative is negative always. So, f never changes sign.

Also, $g^{(0)}(0) = 0, g^{(1)}(0) = 0, g^{(2)}(0) = 0, g^{(3)}(0) = 0$, and

$g^{(4)} = -24\,x\tan(x)^5 - 40\,x\tan(x)^3 - 24 \tan(x)^4 + 8 \cos(x)^2 - 8 \sin(x)^2 - 16\,x\tan(x) - 32 \tan(x)^2 - 8$

$= -(24\,x\tan(x)^5 + 40\,x\tan(x)^3 + 24 \tan(x)^4 - 8 \cos(x)^2 + 8 \sin(x)^2 + 16\,x\tan(x) + 32 \tan(x)^2 + 8)$

Since $x\tan x$ is positive in $(0, \pi/2)$ and $8 - 8\cos^2(x) = 8\sin^2(x)$ it follows $g^{(4)}$ is also negative. So, g never changes sign. So, g is negative.