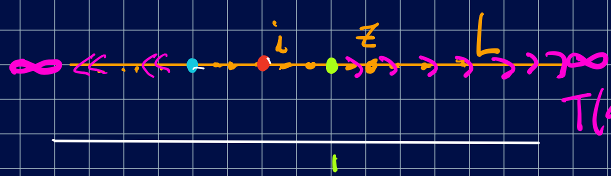
$w = 1/z$ maps "circles" to "circle"

(a circle or a line)

Example



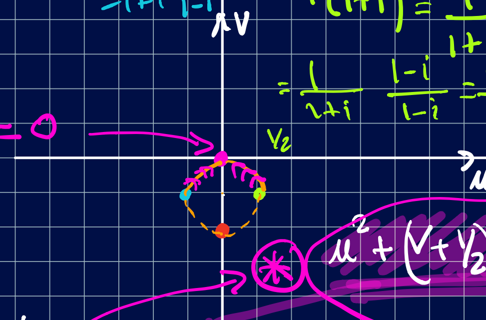
$z = t + i \quad t \in \mathbb{R}$

$T(i) = \frac{1}{i} = -i$

$T(-1+i) = \frac{1}{-1+i}\,\frac{-1-i}{-1-i} = \frac{-1-i}{2}$

$T(1+i) = \frac{1}{1+i} = \frac{1}{1+i}\,\frac{1-i}{1-i} = \frac{1-i}{2}$

$T(\infty) = 0$



$⊛ \quad u^2 + (v + 1/2)^2 = \frac{1}{4}$

Derived using conceptual understanding of $1/z$ and 3 points

We can also find the equation of the circle by doing the following computations:

$$T(t+i) = \frac{1}{t+i} = \frac{1}{t+i}\,\frac{t-i}{t-i} =$$

$$= \frac{t-i}{t^2+1} = \frac{t}{t^2+1} + i\,\frac{-1}{t^2+1}$$

$$= u + iv$$

$z = t + i$, $t \in \mathbb{R}$ $\rightsquigarrow$ $\boxed{u = \frac{t}{t^2+1} \qquad v = \frac{-1}{t^2+1}}$ A circle?

$$u^2 + \left(v + \frac{1}{2}\right)^2 = \left(\frac{t}{t^2+1}\right)^2 + \left(\frac{-1}{t^2+1} + \frac{1}{2}\right)^2 =$$

$$= \frac{t^2}{(t^2+1)^2} + \frac{(-2 + t^2 + 1)^2}{2^2(t^2+1)^2} =$$

$$= \frac{1}{4}\left[\frac{4t^2+(t^2-1)^2}{(t^2+1)^2}\right] =$$

$$= \frac{1}{4}\left[\frac{\cancel{4}^{2}t^2+t^4-\cancel{2t^2}+1}{(t^2+1)^2}\right] =$$

$$= \frac{1}{4}\ \underbrace{\cancel{\frac{(t^2+1)^2}{(t^2+1)^2}}}_{=1} = 1/4$$

Yes!

$\therefore$ $\boxed{u^2+(v+1/2)^2 = 1/4\text{, i.e. the circle of center } (0,-1/2) \text{ and radius } 1/2}$

See also the file "transl'overz.pdf" with detailed Mathematica® pictures of the image of $[-2,2]\times[-2,2]$ under $T(z)=\frac{1}{z}$.