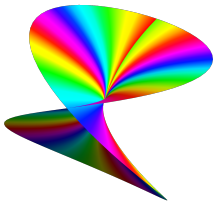
UC RIVERSIDE
UNIVERSITY OF CALIFORNIA
Department of Mathematics

MATH 165B - Introduction to Complex Variables

Midterm Exam



Prob. #	Points	Score
1	25 points	
2	25 points	
3	25 points	
4	25 points	
Extra Credit	20 points	
Total	100 points	

Show your work

Problem 1: In each case, write the principal part of the function at its isolated singular point and determine whether that point is a pole, a removable singular point, or an essential singular point:

(a) $\exp\left(\dfrac{1}{z^2}\right)$

(b) $\dfrac{z^3}{1-z}$

(c) $\dfrac{\sin 2z}{z}$

(d) $\dfrac{\cos z - 1}{z^2}$

(e) $\dfrac{1}{(1-z)^3}$

Problem 2: Find

(a) The residue of $f_1(z) = \frac{\pi}{z-z^2}$ at $z = 0$

(b) The residue of $f_2(z) = z\cos\left(\frac{1}{z}\right)$ at $z = 0$

(c) The residue of $f_3(z) = \frac{z-\sin z}{2z}$ at $z = 0$

(d) A function f_4 with a simple pole at $z = 0$ such that the residue of f_4 at $z = 0$ is π.

(e) A function f_5 with a pole of order 3 at $z = 0$ such that the residue of f_5 at $z = 0$ is 17.

Problem 3: Consider the integral

$$\int_C \frac{2z^3+3}{(z+1)(z^2+4)}dz$$

taken counterclockwise around the curve C.

(a) Find the value of the integral when the curve C is the circle $|z-1|=2$

(b) Find the value of the integral when the curve C is the circle $|z|=4$

(c) Give a curve C such that the value of the integral is 0.

Problem 4: Show that the image of the right half plane $\mathrm{Re}(z) > \frac{1}{2}$, under the mapping $w = \dfrac{1}{z}$, is the disk $|w-1| < 1$.

Extra Credit Problem

Show that all four zeros of the polynomial $g(z) = z^4 - 7z - 1$ lie in the disk $|z| < 2$

Extra Credit STAR PROBLEM

Show that the parabola $2x = 1 - y^2$ is mapped onto the cardioid $\rho = 1 + \cos\phi$ by the reciprocal transformation $w = \dfrac{1}{z}$.