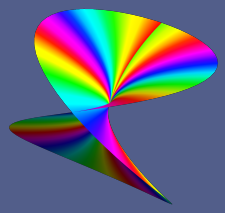
UC RIVERSIDE
UNIVERSITY OF CALIFORNIA
Department of Mathematics

MATH 165B - Introduction to Complex Variables

Worksheet 6



Topics: Linear Transformations and the Transformation $w = \dfrac{1}{z}$

Readings from Brown & Churchill: Sections 90, 91 and 92

- **Introduction**

  Watch the short movie: *Möbius Transformations Revealed*. *This movie is the work of by Douglas Arnold and Jonathan Rogness which "...depicts the beauty of Möbius transformations and shows how moving to a higher dimension reveals their essential unity..."*

  https://www.youtube.com/watch?v=0z1fIsUNhO4t=12s

  You can learn more about this movie at

  http://www-users.math.umn.edu/ arnold/moebius/

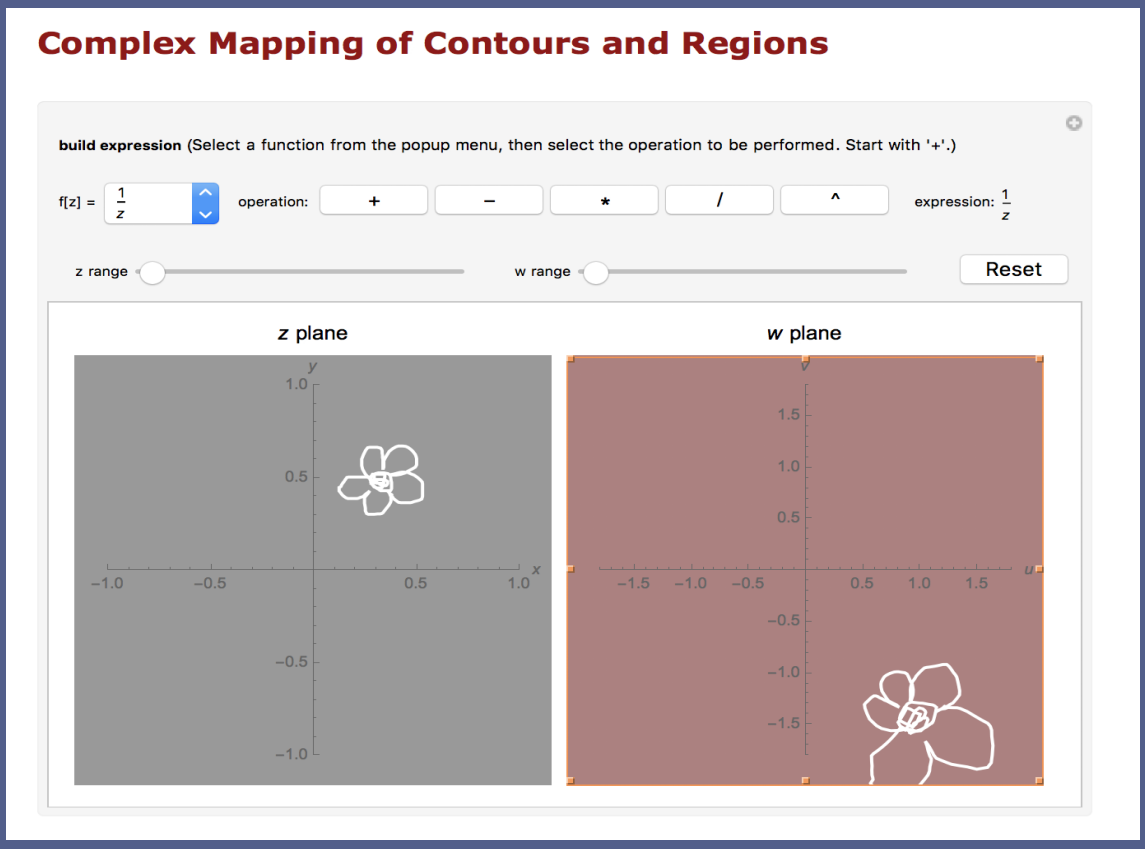
- (P) Write a brief "review" of the *Möbius Transformations Revealed* movie addressing how it is related to what you have learned of complex analysis in Math 165A-B.

90. Linear Transformations

- Find the images of the triangle with vertices $P_1 = -1, P_2 = i,$ and $P_3 = 1$ under three examples of linear transformations $L(z) = Az + B$ with
  - $|A| > 1$
  - $|A| = 1$
  - $|A| < 1$

91-92. Transformation $w = \dfrac{1}{z}$

- Write a list of all the properties of the transformation $w = \dfrac{1}{z}$ from sections 91 and 92. Give concrete examples of these properties.
- The following picture is a screen capture from the demonstration Complex Mapping of Contours and Regions from the Wolfram Demonstration Project. I drew a flower with the cursor in the z-plane and the program drew its image under $w = \dfrac{1}{z}$



  - (P) Describe and justify the effect of $w = \dfrac{1}{z}$ on the flower using the list of properties of the transformation that you summarized from the textbook.
  - Star Option: Define a mathematical flower consisting of a circle (the center) and five circles or ellipses (the petals) in the first quadrant and inside the unit circle. Find its image under the transformation $w = \dfrac{1}{z}$.

- If you have 1GB of extra space in your computer, you can go to the Wolfram Demonstration Project and download Wolfram player and the demonstration Complex Mapping of Contours and Regions:

  https://demonstrations.wolfram.com/ComplexMappingOfContoursAndRegions/

  Notice that the player is not Mathematica. It is a collection of demonstrations that the community has done using Mathematica. You could download the codes of the demonstrations to learn how to write them.

  There are many other complex analysis demonstrations that you could explore:

  https://demonstrations.wolfram.com/topic.html?topic=Complex+Analysislimit=20

HOMEWORK PROBLEMS FOR SECTION 90, 91 and 92

1. Page 318: #2, #4, #5 #7, #8, #9, and #11
2. Star Problems: Page 318: #10, #13, #14

The Star Problems are intended for students who are interested in challenging problems, they can substitute regular problems in the assignment.