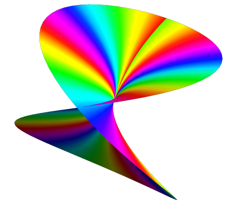
UC RIVERSIDE
UNIVERSITY OF CALIFORNIA
Department of Mathematics

MATH 165B - Introduction to Complex Variables

Final Exam



Prob. #	Points	Score
1	30 points	
2	30 points	
3	30 points	
4	30 points	
5	40 points	
6	40 points	
Total	200 points	

Show your work!

The Exam will be centered around the following 3 functions (the domain considered is the largest set in the complex function where the function can be defined).

$F_1(z) = z + \frac{1}{z}$ (The Joukowsky transformation that some of you have studied in the project)

$$F_2(z) = z + \frac{1}{z} + \frac{1}{z^2}$$

$$F_3(z) = z + z^2 + \frac{1}{z}$$

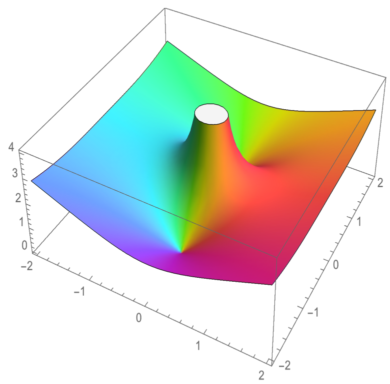
Problem 1:

1. Determine number and order of the zeros of F_1, F_2 and F_3.
2. Can you approximate the values of these zeros?(Mathematica or other software is allow)
3. Find the singular points of F_1, F_2 and F_3 and classify them.

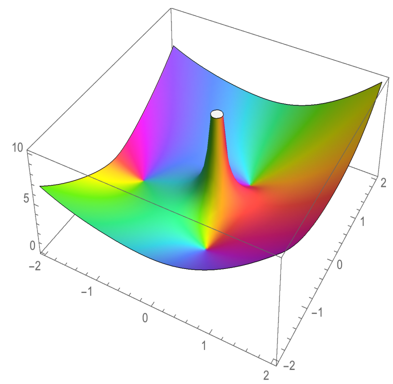
Problem 2

The following pictures correspond to the graph of the modulus of the functions F_i, $i = 1,2,3$ (i.e. $t = |F_i(z)|$) and color according to the argument of the corresponding F_i.

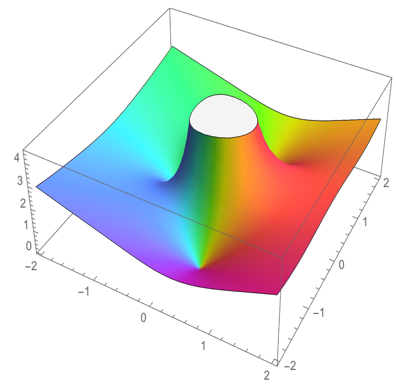
Plot A



Plot B



Plot C



Match the plot with the corresponding F_i, $i = 1,2,3$ and justify your reasoning.

- **Plot A:**____
  Justification:

- **Plot B:**____
  Justification:

- **Plot C:**____
  Justification:

Problem 3: Find

(a) The residue of $F_k(z)$ at $z = 0$, $k = 1, 2, 3$

(b) The residue of $F_k(z)$ at $z = i$, $k = 1, 2, 3$

(c) The residue of $\frac{1}{F_k(z)}$ at $z = i$, $k = 1, 2, 3$

Problem 4: Consider the integral

$$\int_C \left(F_1(z) + \frac{1}{F_1(z)} \right) dz$$

taken counterclockwise around the curve C.

(a) Find the value of the integral when the curve C is the circle $|z| = \frac{1}{2}$.

(b) Find the value of the integral when the curve C is the circle $|z| = 4$.

(c) Give a curve C such that the value of the integral is 0.

Problem 5: Find the image of the following curves under the Joukowsky transformation $F_1(z) = z + \frac{1}{z}$

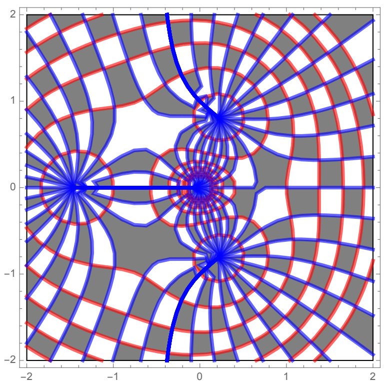
(a) The unit circle $|z| = 1$

(b) A circle with its center at a point $x_0\,(0 < x_0 < 1)$ on the x axis and passing through the point $z = -1$. This is a special case of the profile of a Joukowsky airfoil. Give a the profile by mapping some points that the image of the circle and that points exterior to the circle map onto points exterior to the profile.

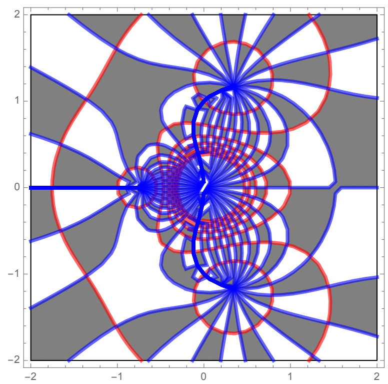
Problem 6: Conformal Mappings

I. The following pictures correspond to the graph of the level curves of the functions $Re(F_i)$ (in red) and $Im(F_i)$ (in blue), $i = 1, 2, 3$. The shading is done to emphasize where the mapping is conformal.

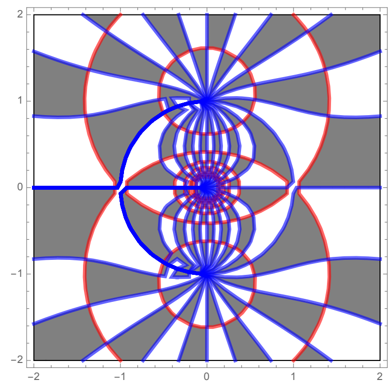
Plot D



Plot E



Plot F



Match the plot with the corresponding F_i, $i = 1, 2, 3$ and justify your reasoning.

- **Plot D:**____

  Justification:

- **Plot E:**____

  Justification:

- **Plot F:**____

  Justification:

II. Determine the points in the plane where F_1 is conformal. What can you say about the functions F_2 and F_3.