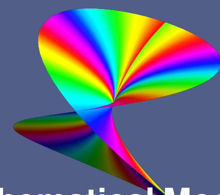
UCRIVERSIDE
Department of Mathematics

MATH 165B - Introduction to Complex Variables

Worksheet 11



Topics: Applications of Conformal Mappings: Two Dimensional Mathematical Models: Steady State Temperatures

Readings from handout from Howell and Matthews: Section 10.4

10.4 Two Dimensional Mathematical Models

Consider the Theorem 10.4 from the handout:

Theorem 10.4 (Orthogonal Families of Level Curves)

Let $\phi(x,y)$ be harmonic in a domain D. Let $\psi(x,y)$ be the harmonic conjugate, and let

$$f(z) = \phi(x,y) + i\psi(x,y)$$

be the corresponding analytic function (called the complex potential). Then the two families of level curves F_1 and F_2 given by

$$F_1 = \{\phi(x,y) = K_1 : K_1 \text{ is a real constant }\} \tag{1}$$

$$F_2 = \{\psi(x,y) = K_2 : K_2 \text{ is a real constant}\} \tag{2}$$

are orthogonal in the sense that if (a,b) is a point common to the two curves $\phi(x,y) = K_1$, and $\psi(x,y) = K_2$ and if $f'(a+ib) \neq 0$, then these two curves intersect orthogonally.

The complex potential and the corresponding level curves defined in this theorem have have many physical interpretations. We will see a few of them and some of you are considering others for your project.

- Give a harmonic function $\phi(x,y)$ and find its harmonic conjugate $\psi(x,y)$. Prove Theorem 10.4 for these functions.
- Draw the level curves for your choice $\phi(x,y)$ and the $\psi(x,y)$ that you computed.
- (P) Read Table 10.1 of the handout. Have you encountered any of these applications in other courses? Did you notice the properties of the theorem between the level curves? Find a picture online where you can observe the orthogonality between these corresponding family of curves.

HOMEWORK PROBLEMS

- Let $T(x,y) = T_1 + \frac{T_2 - T_1}{b-a}(y-a)$ where T_1, T_2, a and b are positive real constants
  1. Find the harmonic conjugate of T and the corresponding families of curves F_1 and F_2 defined in Theorem 10.4.
  2. Draw a few curves of the families F_1 and F_2 for $b = 3, a = 1, T_2 = 20$, and $T_1 = 15$