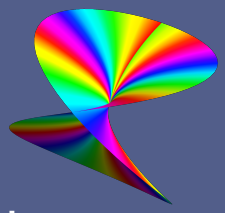
UC RIVERSIDE
Department of Mathematics

MATH 165B - Introduction to Complex Variables

Worksheet 9



Topics: Conformal Mappings: Preservation of Angles and Scale Factors

Readings from Brown & Churchill: Sections 101 and 102

The following picture shows how conformal mappings could be found everywhere:

101. Preservations of Angles

Dictionary Definition:
conformal: (adjective) *(of a map projection or a mathematical mapping) preserving the correct angles between directions within small areas, though distorting distances. Also called orthomorphic.*

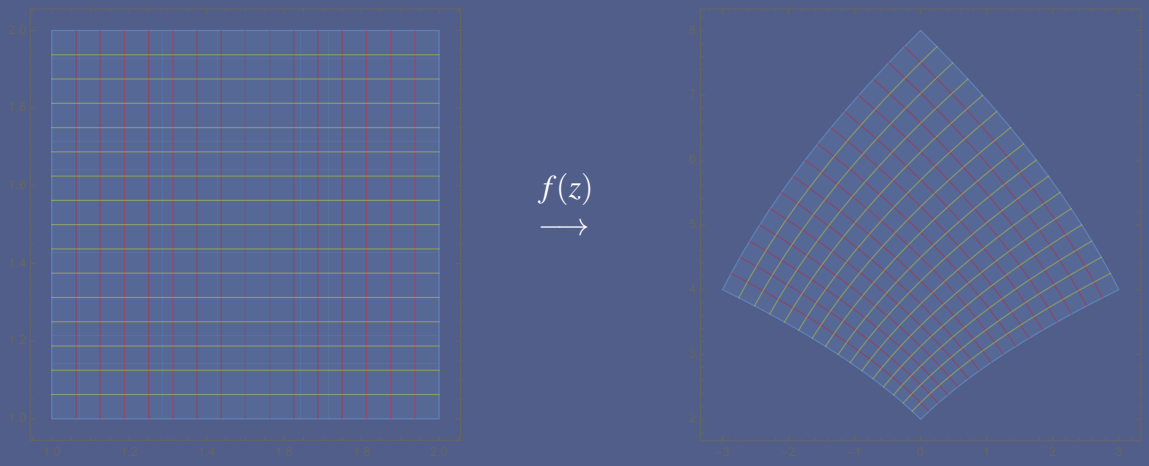
Complex Analysis Definition:
A transformation $w = f(z)$ is said to be conformal at a point z_0 if f is analytic there and $f'(z_0) \neq 0$.

In this section we will see that the complex analysis and the dictionary definition are consistent.

- Example of a conformal mapping:

$$f : [1,2] \times [1,2] \to \mathbb{C}$$
$$f(z) = z^2$$

.



- Wolfram Demonstration Project: Conformal Maps:

  `https://demonstrations.wolfram.com/ConformalMaps/`

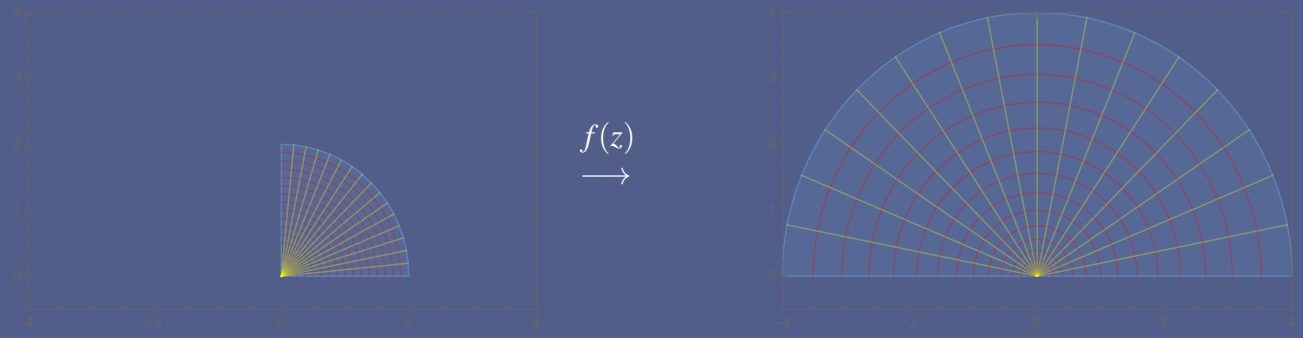
- Read Examples 1, 2 & 3
- (P) Give three examples of conformal mappings
- Reproduce the argument of the preservation of angles given in the book for the mapping $f(z) = z^2$ at the point $z_0 = 1 + i$.

102. Scale Factors

- Example of the scale factor in a conformal mapping:

$$f : \{z = re^{it} : 0 < r < 2,\ 0 < t < \frac{\pi}{4}\}$$

$$f(z) = z^2$$



- (P) Give an example with scale factor 3 at the point $z_0 = 1 + i$.
- Read Example about $f(z) = z^2$ in the textbook.

- (P) Give an example with scale factor 3 at the point $z_0 = 1 + i$.

HOMEWORK PROBLEMS FOR SECTION 101 and 102

1. Page 362: #1, #3, #4, and #5
2. Star Problems: Page 363: #10

The Star Problems are intended for students who are interested in challenging problems, they can substitute regular problems in the assignment.