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RJ Acuna

(862079740)

$\forall G, G' : [G \times G', G \times G'] = [G, G] \times [G', G']$

pf.

Since, $\forall$ groups Z, $[Z, Z]$ is subgroup, so in particular it is a group. And the direct product of groups is a group by the operation taken componentwise. It follows that, $\forall$ groups X, Y, $[X \times Y, X \times Y]$ and $[X, X] \times [Y, Y]$ have compatible group structures. Therefore, it is enough to show containment both ways.

Let $(x_1, y_1)(x_2, y_2)(x_1, y_1)^{-1}(x_2, y_2)^{-1} \in [X \times Y, X \times Y]$, and $(x_1 x_2 x_1^{-1} x_2^{-1}, y_1 y_2 y_1^{-1} y_2^{-1}) \in [X, X] \times [Y, Y]$.

$$\begin{aligned}[X \times Y, X \times Y] \ni (x_1, y_1)(x_2, y_2)(x_1, y_1)^{-1}(x_2, y_2)^{-1} &= (x_1, y_1)(x_2, y_2)(x_1^{-1}, y_1^{-1})(x_2^{-1}, y_2^{-1}) \\ &= (x_1 x_2 x_1^{-1} x_2^{-1}, y_1 y_2 y_1^{-1} y_2^{-1}) \in [X, X] \times [Y, Y]\end{aligned}$$

Since they are both arbitrary elements the conclusion follows by letting $X = G$, and $Y = G'$ ■

Section 15

13 Find both the center $Z(D_4)$ and the commutator subgroup C of the group D_4 of symmetries of the square in table 8.12.

pf.

Going along the slant diagonals of the table we can eliminate the diagonals that don't have symmetry about the main diagonal. Therefore, the only elements that commute with everything else are ρ_0 and ρ_2. So, $Z(D_4) = \{\rho_0, \rho_2\}$.

We know that if D_4/N is abelian then $[D_4, D_4] \leq N$.

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Section 35

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Additional Exercise(s)

1 Prove that every nilpotent group is solvable.

2 Determine the composition factors of $\mathbb{Z}_{48}$ and $\mathbb{Z}_{60}$ (with multiplicity).