Math 172 homework 8, due June 5, 2020

1. (Book 50.17) Let E be a finite extension of F. Show that if E is a splitting field, then it is a splitting field of a single polynomial $f(x) \in F[x]$.

2. (Book 50.20) Show that the automorphism group $Aut(\mathbb{Q}(2^{(1/3)}/\mathbb{Q})$ is the trivial group. (Remember that $Aut(E/F)$ is the group of automorphisms of $\varphi : E \to E$ such that $\varphi(a) = a$ for all $a \in F$.)

3. (Book 51.3) Find an $\alpha \in \mathbb{Q}(\sqrt{2}, \sqrt{3}) =: E$ so that $E = \mathbb{Q}(\alpha)$. Compute explicitly how to write $\sqrt{2}$ and $\sqrt{3}$ in terms of α.

4. (Book 51.17) Let K be a finite normal extension of F and let $G = Aut(K/F)$. Define the *norm* and *trace* of an element $\alpha \in K$ as follows:

$$N(\alpha) := \prod_{\sigma \in G} \sigma(\alpha), \qquad Tr(\alpha) := \sum_{\sigma \in G} \sigma(\alpha)$$

   Show that $N(\alpha)$ and $Tr(\alpha)$ are elements of F. (Hint: what is $\sigma(N(\alpha))$ for $\sigma \in G$? Then, look at part 2 of Theorem 53.6 in the special case $E = F$.)

5. (Book 51.18 (a,b,e,f)) Let $K = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Using the definition in the previous problem, compute the norm and trace of $\alpha = \sqrt{2}$ and $\beta = \sqrt{2} + \sqrt{3}$.