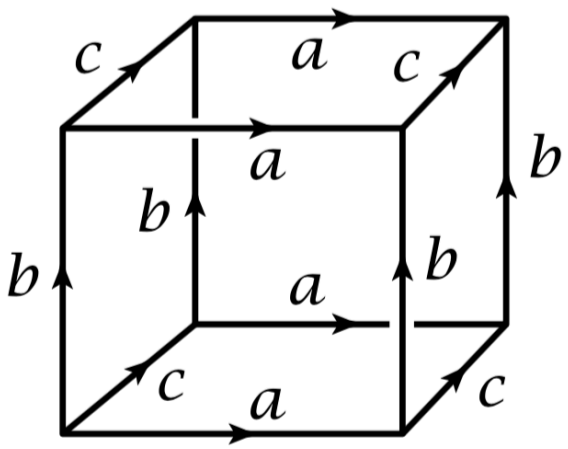
9. Simplicial Homology

1. Draw a Δ-complex for the annulus and
   (a) Calculate the integer homology groups of the simplicial annulus of Example 7.3.
   (b) Calculate the mod 2 homology groups of the simplicial annulus of Example 7.3.
2. Triangulate the closed interval $[0, 1]$ and calculate its homology groups with $\mathbb{Z}_2$ coefficient. Verify that the result does not change if you use a different triangulation.
3. Triangulate the cylinder $\mathbb{S}^1 \times [0, 1]$ and
   (a) Calculate its integral homology groups.
   (b) Calculate its homology groups with $\mathbb{Z}_2$ coefficient.
4. Draw a Δ-complex for the circle $\mathbb{S}^1$ and calculate the integral homology groups of $\mathbb{S}^1$.
   (a) Calculate its integral homology groups.
   (b) Calculate its homology groups with $\mathbb{Z}_2$ coefficient.
5. Draw a Δ-complex for sphere $\mathbb{S}^2$ and calculate its integral homology groups.
   (a) Calculate its integral homology groups.
   (b) Calculate its homology groups with $\mathbb{Z}_2$ coefficient.
6. Draw a Δ-complex for the torus T^2 and calculate its integral homology groups.
   (a) Calculate its integral homology groups.
   (b) Calculate its homology groups with $\mathbb{Z}_2$ coefficient.
7. Draw a Δ-complex for real projective plane $\mathbb{R}P^2$ and calculate its integral homology groups.
   (a) Calculate its integral homology groups.
   (b) Calculate its homology groups with $\mathbb{Z}_2$ coefficient.
8. Draw a Δ-complex for the Klein bottle and calculate its integral homology groups.
   (a) Calculate its integral homology groups.
   (b) Calculate its homology groups with $\mathbb{Z}_2$ coefficient.
9. Triangulate the Möbius band and
   (a) Calculate its integral homology groups.
   (b) Calculate its homology groups with $\mathbb{Z}_2$ coefficient.
   (c) Verify that the universal coefficient theorem holds for this space.

10. Take the simplicial torus of Example 7.8, and glue in another 2-simplex joining the three innermost edges. How does gluing in this 2-simplex change the homology? Compare the change in the homology with the change in the Euler characteristic.

11. A 3-dimensional torus $T^3 = \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$ can be constructed from a cube by identifying each pair of opposite square faces as in the following figure. Calculate the integral homology of a 3-dimensional torus.



12. Let X be the quotient space obtained from an 8-sided polygonal region P by pasting its edges together according to the labeling scheme $acadbcb^{-1}d$. Note that all vertices of P are mapped to the same point of the quotient space X by the pasting map.

    (a) Calculate the integral homology groups of X.

    (b) Calculate the homology groups of X with $\mathbb{Z}_2$ coefficient.

    (c) Verify that the universal coefficient theorem holds for X.