

$$X = \text{Diagram 1} = \text{Diagram 2} \cup \text{Diagram 3}$$

Diagram 1: Two spheres joined at a point. The left sphere has a red vertical line labeled U . The right sphere has a blue vertical line labeled V .

Diagram 2: Two spheres joined at a point. The left sphere has a blue vertical line labeled U . The right sphere has a red vertical line labeled V .

Diagram 3: Two spheres joined at a point. The left sphere has a red vertical line labeled U . The right sphere has a blue vertical line labeled V .

$U \cap V = \{\bullet\}$ is path connected.

U and V are open in the subspace topology and both contain \bullet and are path connected.

$$\text{Diagram 2} \sim S^1 \sim \text{Diagram 3}$$

Diagram 2: Two spheres joined at a point. The left sphere has a blue vertical line labeled U . The right sphere has a red vertical line labeled V .

Diagram 3: Two spheres joined at a point. The left sphere has a red vertical line labeled U . The right sphere has a blue vertical line labeled V .

because the point that's not on the spheres is contractible.

So by Seifert-Van Kampen Theorem

$$\begin{aligned} \pi_1(X) &\cong \frac{\pi_1(U) * \pi_1(V)}{\pi_1(\{\bullet\})} \cong \frac{\pi_1(S^1) * \pi_1(S^1)}{0} \\ &\cong \frac{0 * 0}{0} \cong 0 \end{aligned}$$