

## 9.2. Limitations of Homology Mod 2

$S^1$ , simplicial square,  $S^2$ ,  $T^2$ , rabbit ears have the same homology.

$$H_*(S^1) \approx H_*(\text{Annulus})$$

but this is not a surprise since  $S^1 \sim \text{Annulus}$ .

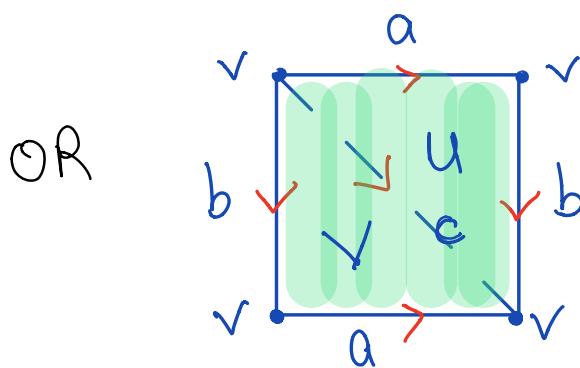
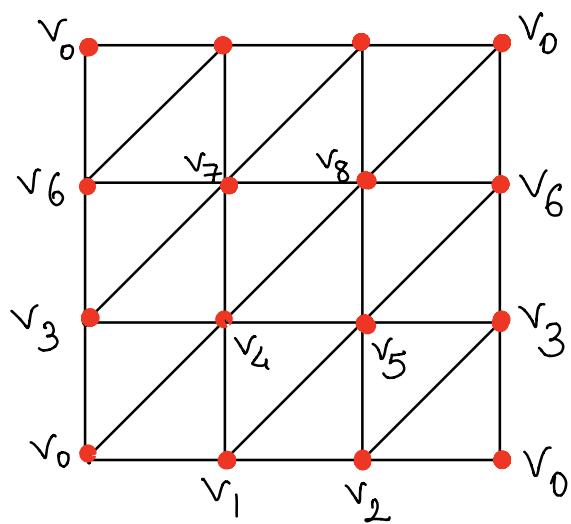
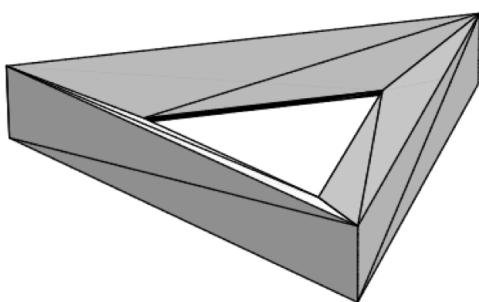
$$X \sim Y \Rightarrow H_*(X) \approx H_*(Y)$$

However homology is not as powerful as it could be.

Ex:  $H_*(T^2; \mathbb{Z}_2) \approx H_*(KB; \mathbb{Z}_2)$

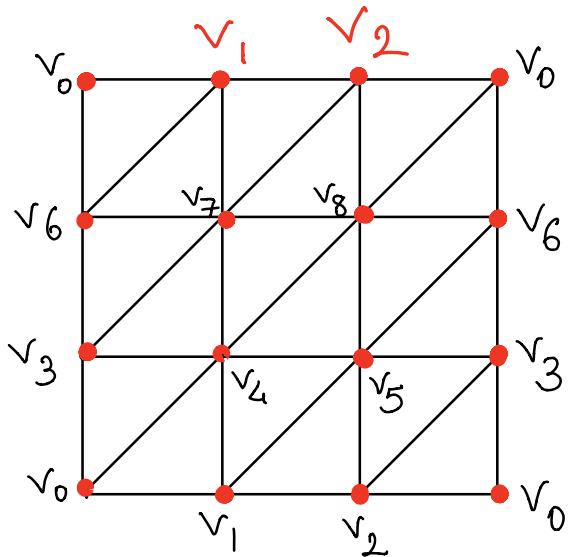
but  $T^2 \not\sim KB$

To understand the similarities and differences, we imagine the triangulation of the torus as coming from a triangulation of the square as below.

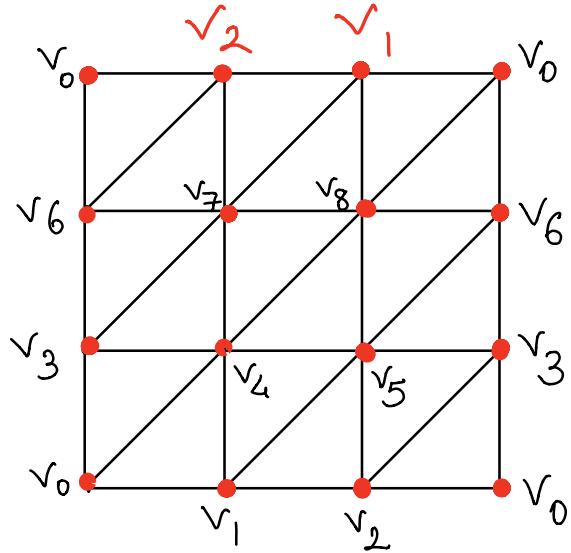


OR

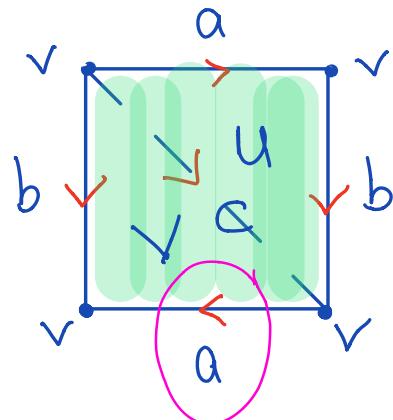
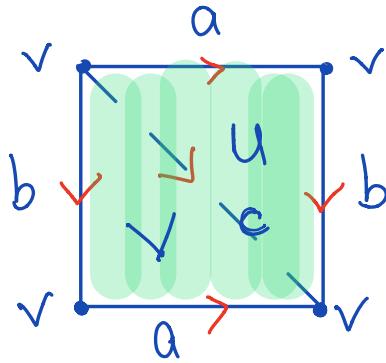
The vertex labels reflect the way that several vertices are identified with each other when we form the torus.



$T^2$



KB



$$\partial_2(v) = b + a - c$$

$$\partial_2(v) = b - a - c$$

This way we might be able to distinguish between these two chains and, hence between torus and KB.

In general, an  $n$ -simplex has  $(n+1)!$  orderings, but each can be turned into any other by a number of swaps.

Two orderings are considered to have the same orientation if they differ by an even number of swaps, and to have the opposite orientation if they differ by an odd number of swaps.

**Definition:** An **oriented simplex** is a set of orderings of the vertex list of a simplex s.t. all these orderings have the same orientation, and any other ordering having the same orientation is in the set.