7. The Euler Number

1. Give an example of space build of two vertices and two edges which is homeomorphic to the simplicial circle but not a simplicial complex.

2. (a) Draw a triangulation of a closed interval $[a, b]$.
   (b) Calculate the Euler number of the closed interval $[a, b]$.

3. (a) Draw a triangulation of the cylinder.
   (b) Calculate the Euler number of the cylinder.

4. (a) Draw a triangulation of the Möbius band.
   (b) Calculate the Euler number of the Möbius band.

5. (a) Draw a triangulation of the torus $\mathbb{S}^2$.
   (b) Calculate the Euler number of the torus $\mathbb{S}^2$.

6. (a) Draw a triangulation of the torus $\mathbb{T}^2$.
   (b) Calculate the Euler number of the torus $\mathbb{T}^2$.

7. (a) Draw a triangulation of the surface of genus two, Σ_2.
   (b) Calculate the Euler number of the surface of genus two, Σ_2.

8. (*)
   (a) Draw a triangulation of the Klein bottle.
   (b) Calculate the Euler number of the Klein bottle.

9. Show that every triangulable space is Hausdorff.

10. Show directly that any two triangulations of the circle $\mathbb{S}^1$ have the same Euler number. Which other spaces can you give such a direct proof for?

11. For each positive integer n, find a simplicial complex with Euler number n. For each positive integer n, try to find a connected simplicial complex with Euler number n.

12. Which integers (positive or negative) can occur as the Euler number of a one-dimensional simplicial complex? Which integers can occur as the Euler number of a connected one-dimensional simplicial complex?

13. Give an example of a *"non-Hausdorff surface"* i.e., a topological space X which is not Hausdorff, but which has the property that every point has an open neighborhood homeomorphic with an open disc in $\mathbb{R}^2$.