Sample final: short answer

(a) (*6 points*) Use the law of quadratic reciprocity to determine whether or not 12 is a square modulo 31.

(b) (*6 points*) Use induction to show that $11^n \equiv 1 + 10n \pmod{100}$ for all integers $n \geq 1$.

(c) (*6 points*) Suppose that $f : \mathbb{N} \to \mathbb{R}$ is a multiplicative function, p is a prime and m is a positive integer for which

$$f(p^m) = \begin{cases} m & \text{if } m \text{ is odd} \\ -m & \text{if } m \text{ is even} \end{cases}$$

Compute $F(56)$, where $F(n) = \sum_{d \mid n} f(d)$.

(d) (*6 points*) Show that if $n > 1$, then $12^n - 1$ is not prime.