Math 136 Homework #1

1. Show that, for all positive integers n, $3 \mid 7^n - 1$.
2. Do there exist integers x, y satisfying $48x - 156y = 66$?
3. Compute the remainder of 37^{102} divided by 12.
4. Show that for an integer n, n^2 cannot be of the form $3k + 2$ (for k an integer).
5. (Challenge problem) Prove that among any $n + 1$ integers (where n is a natural number), there are at least two whose difference is divisible by n. (Hint: think about the remainders these integers give when divided by n.)