

## Math 136 Homework #3

1. Use the undetermined coefficients method (in the proof of the Chinese remainder theorem) to solve the following simultaneous congruence:

$$3x \equiv 3 \pmod{37}$$

$$2x \equiv 15 \pmod{53}$$

2. Use the Euclidean algorithm to find the greatest common divisor (gcd) of 345118 and 6753 and use the algorithm to find the complete solution set to

$$6753x + 345118y = 1$$

(if a solution exists).

3. Recall that an integer  $c$  *admits an inverse modulo  $m$*  if there exists an integer  $x$  for which  $cx \equiv 1 \pmod{m}$ . Show that if  $c$  admits an inverse modulo  $m$ , then  $c$  and  $m$  are relatively prime. (Note: so far, we've shown the converse holds.)
4. In a certain city, mayoral elections occur every 5 years and last occurred 2 years ago. Dog-catcher elections, on the other hand, occur every 7 and occurred last year. If it is 2019, find the next year that will feature both mayoral and dog-catcher elections. (Hint: this involves the Chinese remainder theorem method used in problem 1.)
5. Using reduction modulo  $m$  techniques, show that the quadratic Diophantine equation

$$x^2 - 2y^2 + 12y = 224$$

admits no solutions. (Hint: reduce modulo 8.)