Math 136 Homework #6

1. Recall the Möbius function defined in class:

$$\mu(n) := \begin{cases} 0 & \text{if } n \text{ is not square-free} \\ (-1)^s & \text{if } n = p_1 \ldots p_s, \text{ where } p_i \text{ are distinct primes} \end{cases}$$

   Show that $\mu(n)$ is multiplicative (see definition in the notes.)

2. Factor the integer 645 and use the result given in class to determine if it can be expressed as a sum of two squares.

3. Use Legendre's theorem to determine if the homogeneous quadratic equation $3x^2 + 4y^2 = 5z^2$ admits a nontrivial integer solution (x, y, z) (where, "nontrivial" means that x, y, z are not all 0).

4. Does the quadratic equation $3x^2 + 7y^2 = 701$ admits an integer solution (x, y)?