

Math 136 Homework #2

1. The cancellation property for arithmetic modulo m does **NOT** hold. In other words, the statement:

$$\text{For } a, b, c, m \text{ integers, } ca \equiv cb \pmod{m} \Rightarrow a \equiv b \pmod{m}$$

is false. Show that the statement is false by obtaining a counterexample (i.e., find some integers a, b, c, m for which the statement is false).

2. Find the smallest positive integer n (greater than 1) such that $4^n \equiv 4 \pmod{13}$. Use this to find all the (finitely many) possible values of $4^m \pmod{13}$ (where m ranges over all positive integers).
3. Use modular arithmetic to show that $x^2 + 19y^2 = 431$ does not admit integer solutions x and y .
4. If $4x^2 - 3y^2 = 1$ for some integers x and y , show that x and y are relatively prime.
5. Show that $\gcd(n, 2n + 1) = 1$.