SEST - CO3 - 2024-08-01 - Apply Statistical Estimations and Significance Testing

01 August 2024 09:17

→ Case 1 : Both samples are independent of each other & both population variances σ_x^2 & σ_y^2 are known

→ $\bar{x}$ & $\bar{y}$ are sample means

→ Using CLT, distribution of $\frac{T - E(T)}{S.E(T)}$ is $N(0,1)$

T = Sample Statistic

$E(T)$ = Expected Value (Mean)

$S.E(T)$ = Standard Error

→ Parameter of Interest - $(\mu_x \times \mu_y)$

→ $E(\bar{x} - \bar{y}) = \mu_x \times \mu_y$

→ $S.E(\bar{x} - \bar{y}) = \sqrt{\frac{\sigma_x^2}{n}} + \sqrt{\frac{\sigma_y^2}{m}}$

→ $Z_c = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}}$ ~ 2 Samples for means.

Sup-A ≈ x

$\bar{x}$, $\bar{y}$

Sup-A ≈ x

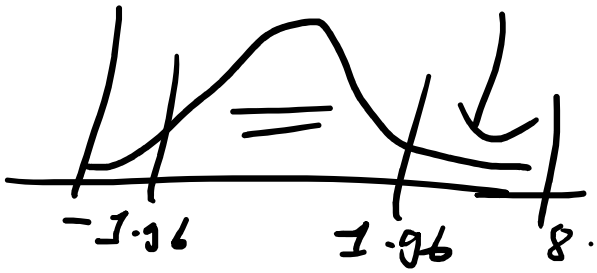
Sup-B ≈ y

→ $H_0 : \mu_x = \mu_y$ against $H_1 = \mu_x \neq \mu_y$

→ $Z_c = \dfrac{\bar{x} - \bar{y}}{\sqrt{\dfrac{\sigma_x^2}{n} + \dfrac{\sigma_y^2}{m}}} = \dfrac{250 - 220}{\sqrt{\dfrac{40 \times 40}{400} + \dfrac{55 \times 55}{400}}}$

$Z_c = 8.8225$

→ $(-1.96, 1.96) \approx \alpha$ for 5% S.L.



$Z_c > \alpha$

$8.8225 > 1.96$

∴ We reject H_0 & conclude that both the women shoppers are not equal.

→ Case 2: Both samples are independent & both population variances are unknown.

→ Student's t-Dist

↳ Both populations have normal distribution

↳ Both populations have normal distribution

↳ Both population variances are equal.

→ Pooled Sample Variance!

$$s^2 = \frac{1}{m+n-2}\left[\sum_{i=1}^{n}(x_i-\bar{x})^2 + \sum_{j=1}^{m}(y_j-\bar{y})^2\right]$$

→ Test Statistic: $t_c = \dfrac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n}+\frac{1}{m}}}$

→

	NYSE	NASDAQ
No of Stocks	21	25
Sample Mean	3.27	2.53
Sample S.D	1.30	1.16

→ $H_0: \mu_x = \mu_y$ against $H_1: \mu_x \neq \mu_y$

Modification in formula!

$$s^2 = \frac{1}{m+n-2}\left[(m-1)s_m^2 + (n-1)s_n^2\right]$$

$$s = \frac{1}{m+n-2}\left[(m-1)s_m + (n-1)s_n\right]$$

For pooled $= \frac{1}{21+25-2}\left[20\times 1.30\times 1.30 + 24\times 1.16\times 1.16\right]$

$$s^2 = 1.502$$

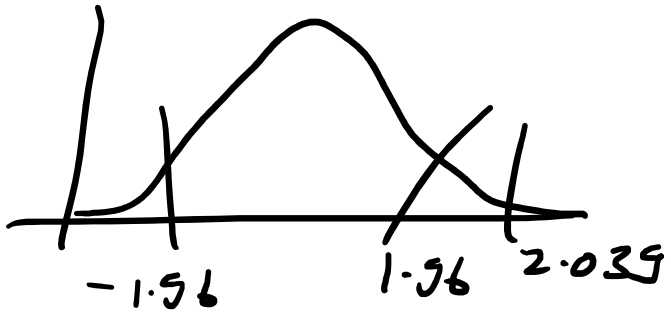
$$\rightarrow \quad t_c = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{3.27 - 2.53}{\sqrt{1.502}\sqrt{\frac{1}{21} + \frac{1}{25}}}$$

$$T_c = 2.039$$

$\Rightarrow$ D-o-f $(m+n-2)$

$$(21+25)-2 = 44$$

5% $(-1.96, 1.96) \approx \alpha$



∴ Reject H_0 & conclude that there is a difference b/w the 2 dividends.

→ Case 3: Both samples are related.

→ Samples are not independent but related.

Samples $n = m$

Diff $d_i = x_i - d_i \quad (i = 1, 2, 3 \ldots n)$

Using the difference

$$\bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_i$$

$$S.D = S_d = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (d_i - \bar{d})^2}$$

Test Statistic $= \dfrac{\bar{d}}{S_d / \sqrt{n}}$

D-O-F $(n-1)$

-> Salesperson Before After

A	6	4
B	20	6
C	3	2
D	0	0
E	4	0

-> Conduct the test assuming normal dist at 1% α

-> $H_0 : \mu_x = \mu_y$ against $H_1 : \mu_x \neq \mu_y$

-> $d_i = |-2|-14|-1|0|-4|$

$\therefore \bar{d} = -4.2$

$\therefore Sd = 5.67$

$T_c = \dfrac{\bar{d}}{Sd/\sqrt{n}} = \dfrac{-4.2}{5.67/\sqrt{5}} = -1.65$

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924

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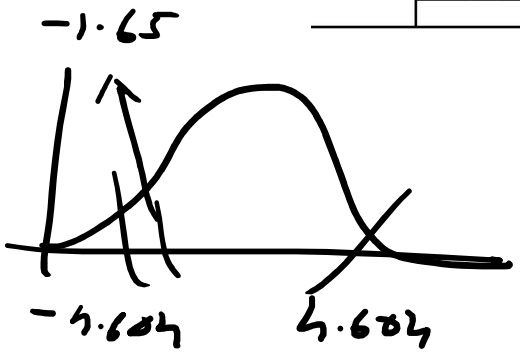
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

(-4.604 to +4.604)



t_c α

$|-1.65| \quad [-4.604]$

$1.65 < 4.604$

→ Accept H_0 & conclude there is no change after training of the subjects.