SEST - CO3 - 2024-08-02 - Apply Statistical Estimation and Significance Testing

02 August 2024 09:22

→ Hypothesis Testing for Difference of 2 proportions:

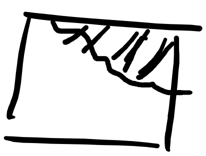
→ π_1 & π_2 ~ Population Proportions.

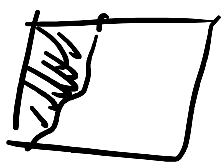
→ p_1 & p_2 ~ Sample Proportions

→ $H_0 : \pi_1 = \pi_2$ Against $H_1 : \pi_1 \neq \pi_2$

→ $H_0 : \pi_1 = \pi_2$ Against $H_1 : \pi_1 > \pi_2$

→ $H_0 : \pi_1 = \pi_2$ Against $H_1 : \pi_1 < \pi_2$





→ Test Statistic

$$Z_c = \frac{p_1 - p_2}{\sqrt{\hat{\pi}(1-\hat{\pi})\left(\frac{1}{n} + \frac{1}{m}\right)}}$$

→ $$\hat{\pi} = \frac{n \times p_1 + m \times p_2}{n + m}$$

→ Z_c is $N(0,1)$

→ Both sample sizes are large (Above 30)

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-> $H_0 = \pi_1 = \pi_2$ against $H_1 = \pi_1 \neq \pi_2$

$$p_1 = \frac{200}{400} = 0.5$$

$$p_2 = \frac{325}{600} = 0.5416$$

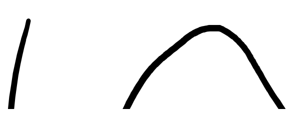
$$\hat{\pi} = \frac{n \times p_1 + m \times p_2}{n + m} = \frac{200 + 325}{400 + 600}$$

$$\hat{\pi} = \underline{\underline{0.525}}$$

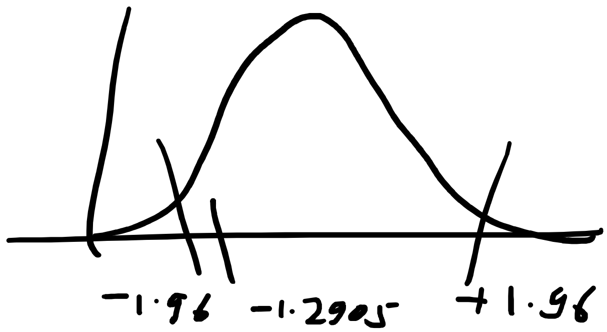
-> $$Z_c = \frac{p_1 - p_2}{\sqrt{\hat{\pi}(1-\hat{\pi})\left(\frac{1}{n} + \frac{1}{m}\right)}}$$

$$= \frac{0.5 - 0.5416}{\sqrt{0.525(1-0.525) \times \left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$$Z_c = \underline{\underline{-1.2905}}$$



α	Z_c
$\lvert -1.96 \rvert$	$\lvert -1.2905 \rvert$



$|-1.96| \quad |-1.2905|$

$1.96 > 1.2505$

$\therefore$ H_0 is accepted & we conclude that opinion of men & women is the same.

→ <u>Hypothesis Testing for diff of 2 variances:</u>

→ Snedecor's F-Distribution.

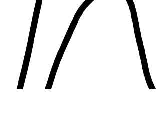
$H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$

$H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 > \sigma_2^2$

$H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 < \sigma_2^2$

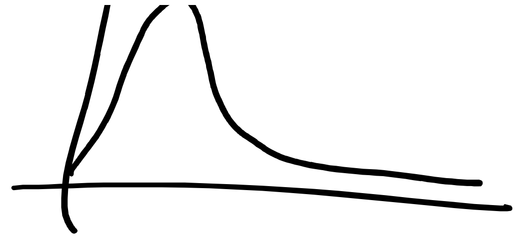
Test Statistic $F_c = \dfrac{s_1^2(x)}{s_1^2(y)} = \dfrac{\text{Sample variance of } 1^{st}}{\text{Sample variance of } 2^{nd}}$

→ DOF = $(n-1)$ & $(m-1)$



→ DOF = $(n-1)$ & $(m-1)$

F-dist = $(0, \infty)$



→ Is there a diff in variances (or risks) b/w the NYSE & NASDAQ? Test 2.5%

	NYSE	NASDAQ
No. of stocks	21	25
Sample Mean	3.27	2.53
Sample S.D	1.30	1.16

→ $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$

$$F_c = \frac{s_1^2(x)}{s_2^2(y)} = \frac{1.30 \times 1.30}{1.16 \times 1.16} = \underline{\underline{1.255}}$$

→ α $(n-1)$ & $(m-1)$

$(21-1)$ & $(25-1)$

α = DOF $(20, 24)$

α = DOF (20, 24)

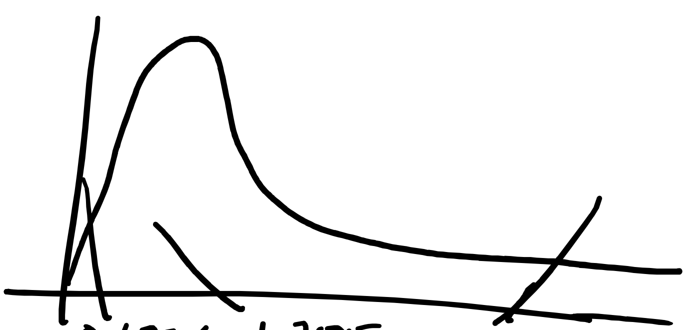
DF2	DF1 1	α = 0.025 2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	Inf
1	647.79	799.5	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63	976.71	984.87	993.1	997.25	1001.4	1005.6	1009.8	1014	1018.3
2	38.506	39	39.166	39.248	39.298	39.332	39.355	39.373	39.387	39.398	39.415	39.431	39.448	39.456	39.465	39.473	39.481	39.49	39.498
3	17.443	16.044	15.439	15.101	14.885	14.735	14.624	14.54	14.473	14.419	14.337	14.253	14.167	14.124	14.081	14.037	13.992	13.947	13.902
4	12.218	10.649	9.9792	9.6045	9.3645	9.1973	9.0741	8.9796	8.9047	8.8439	8.7512	8.6565	8.5599	8.5109	8.461	8.411	8.36	8.309	8.257
5	10.007	8.4336	7.7636	7.3879	7.1464	6.9777	6.8531	6.7572	6.6811	6.6192	6.5245	6.4277	6.3286	6.278	6.227	6.175	6.123	6.069	6.015
6	8.8131	7.2599	6.5988	6.2272	5.9876	5.8198	5.6955	5.5996	5.5234	5.4613	5.3662	5.2687	5.1684	5.1172	5.065	5.012	4.959	4.904	4.849
7	8.0727	6.5415	5.8898	5.5226	5.2852	5.1186	4.9949	4.8993	4.8232	4.7611	4.6658	4.5678	4.4667	4.415	4.362	4.309	4.254	4.199	4.142
8	7.5709	6.0595	5.416	5.0526	4.8173	4.6517	4.5286	4.4333	4.3572	4.2951	4.1997	4.1012	3.9995	3.9472	3.894	3.84	3.784	3.728	3.67
9	7.2093	5.7147	5.0781	4.7181	4.4844	4.3197	4.197	4.102	4.026	3.9639	3.8682	3.7694	3.6669	3.6142	3.56	3.505	3.449	3.392	3.333
10	6.9367	5.4564	4.8256	4.4683	4.2361	4.0721	3.9498	3.8549	3.779	3.7168	3.6209	3.5217	3.4185	3.3654	3.311	3.255	3.198	3.14	3.08
11	6.7241	5.2559	4.63	4.2751	4.044	3.8807	3.7586	3.6638	3.5879	3.5257	3.4296	3.3299	3.2261	3.1725	3.118	3.061	3.004	2.944	2.883
12	6.5538	5.0959	4.4742	4.1212	3.8911	3.7283	3.6065	3.5118	3.4358	3.3736	3.2773	3.1772	3.0728	3.0187	2.963	2.906	2.848	2.787	2.725
13	6.4143	4.9653	4.3472	3.9959	3.7667	3.6043	3.4827	3.388	3.312	3.2497	3.1532	3.0527	2.9477	2.8932	2.837	2.78	2.72	2.659	2.595
14	6.2979	4.8567	4.2417	3.8919	3.6634	3.5014	3.3799	3.2853	3.2093	3.1469	3.0502	2.9493	2.8437	2.7888	2.732	2.674	2.614	2.552	2.487
15	6.1995	4.765	4.1528	3.8043	3.5764	3.4147	3.2934	3.1987	3.1227	3.0602	2.9633	2.8621	2.7559	2.7006	2.644	2.585	2.524	2.461	2.395
16	6.1151	4.6867	4.0768	3.7294	3.5021	3.3406	3.2194	3.1248	3.0488	2.9862	2.889	2.7875	2.6808	2.6252	2.568	2.509	2.447	2.383	2.316
17	6.042	4.6189	4.0112	3.6648	3.4379	3.2767	3.1556	3.061	2.9849	2.9222	2.8249	2.723	2.6158	2.5598	2.502	2.442	2.38	2.315	2.247
18	5.9781	4.5597	3.9539	3.6083	3.382	3.2209	3.0999	3.0053	2.9291	2.8664	2.7689	2.6667	2.559	2.5027	2.445	2.384	2.321	2.256	2.187
19	5.9216	4.5075	3.9034	3.5587	3.3327	3.1718	3.0509	2.9563	2.8801	2.8172	2.7196	2.6171	2.5089	2.4523	2.394	2.333	2.27	2.203	2.133
20	5.8715	4.4613	3.8587	3.5147	3.2891	3.1283	3.0074	2.9128	2.8365	2.7737	2.6758	2.5731	2.4645	2.4076	2.349	2.287	2.223	2.156	2.085
21	5.8266	4.4199	3.8188	3.4754	3.2501	3.0895	2.9686	2.874	2.7977	2.7348	2.6368	2.5338	2.4247	2.3675	2.308	2.246	2.182	2.114	2.042
22	5.7863	4.3828	3.7829	3.4401	3.2151	3.0546	2.9338	2.8392	2.7628	2.6998	2.6017	2.4984	2.389	2.3315	2.272	2.21	2.145	2.076	2.003
23	5.7498	4.3492	3.7505	3.4083	3.1835	3.0232	2.9023	2.8077	2.7313	2.6682	2.5699	2.4665	2.3567	2.2989	2.239	2.176	2.111	2.041	1.968
24	5.7166	4.3187	3.7211	3.3794	3.1548	2.9946	2.8738	2.7791	2.7027	2.6396	2.5411	2.4374	$\boxed{2.3273}$	2.2693	2.209	2.146	2.08	2.01	1.935
25	5.6864	4.2909	3.6943	3.353	3.1287	2.9685	2.8478	2.7531	2.6766	2.6135	2.5149	2.411	2.3005	2.2422	2.182	2.118	2.052	1.981	1.906
26	5.6586	4.2655	3.6697	3.3289	3.1048	2.9447	2.824	2.7293	2.6528	2.5896	2.4908	2.3867	2.2759	2.2174	2.157	2.093	2.026	1.954	1.878
27	5.6331	4.2421	3.6472	3.3067	3.0828	2.9228	2.8021	2.7074	2.6309	2.5676	2.4688	2.3644	2.2533	2.1946	2.133	2.069	2.002	1.93	1.853
28	5.6096	4.2205	3.6264	3.2863	3.0626	2.9027	2.782	2.6872	2.6106	2.5473	2.4484	2.3438	2.2324	2.1735	2.112	2.048	1.98	1.907	1.829
29	5.5878	4.2006	3.6072	3.2674	3.0438	2.884	2.7633	2.6686	2.5919	2.5286	2.4295	2.3248	2.2131	2.154	2.092	2.028	1.959	1.886	1.807
30	5.5675	4.1821	3.5894	3.2499	3.0265	2.8667	2.746	2.6513	2.5746	2.5112	2.412	2.3072	2.1952	2.1359	2.074	2.009	1.94	1.866	1.787
40	5.4239	4.051	3.4633	3.1261	2.9037	2.7444	2.6238	2.5289	2.4519	2.3882	2.2882	2.1819	2.0677	2.0069	1.943	1.875	1.803	1.724	1.637
60	5.2856	3.9253	3.3425	3.0077	2.7863	2.6274	2.5068	2.4117	2.3344	2.2702	2.1692	2.0613	1.9445	1.8817	1.815	1.744	1.667	1.581	1.482
120	5.1523	3.8046	3.2269	2.8943	2.674	2.5154	2.3948	2.2994	2.2217	2.157	2.0548	1.945	1.8249	1.7597	1.69	1.614	1.53	1.433	1.31
Inf	5.0239	3.6889	3.1161	2.7858	2.5665	2.4082	2.2875	2.1918	2.1136	2.0483	1.9447	1.8326	1.7085	1.6402	1.566	1.484	1.388	1.268	1

(20, 24)

$$F_U = F(0.025) = \underline{\underline{2.3273}}$$

$$F_L = F(0.975) = \frac{1}{F_U} = \frac{1}{F(0.025)} = \frac{1}{2.3273}$$

$$F_L = \underline{\underline{0.4256}}$$



∴ Accept the null hypothesis & we conclude that the risks (variances) of 2 indexes that the risks (variances) of [illegible] are not different.

→ Type I & Type 2 Errors:

	Actual Situation	
Decision	H_0 True	H_0 False
Accept H_0	No Error	Type II Error
Reject H_0	Type I Error	No Error

→ Chi-Square Test:

$$\chi_c^2 = \sum \frac{(O-E)^2}{E}$$

O = Observed Value   E = Expected Value