DSD - CO2 - 2024-08-12 - Apply data and sampling distribution techniques

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$\rightarrow P(X=x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$

$\rightarrow \lambda = \mu$ & $\sigma^2 = \lambda$

$\rightarrow \sigma = \sqrt{\lambda}$

→ Plutonium -239

→ Let x represent the no of decays in a 2 second period.

$\lambda = 2.3 \times 2 = \underline{\underline{4.6}}$

$P(X=3) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{4.6^3 \cdot e^{-4.6}}{3!} = \underline{\underline{0.163}}$

So 16.3 % probability of decays happening in 3 second.

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \dfrac{4.6^0 \cdot e^{-4.6}}{0!} + \dfrac{4.6^1 \cdot e^{-4.6}}{1!} + \dfrac{4.6^2 \cdot e^{-4.6}}{2!} + \dfrac{4.6^3 \cdot e^{-4.6}}{3!}$

$= \underline{\underline{0.325}}$

∴ The probability of having no more than 3

$\therefore$ The probability of having no more than 3 radio-active decays is $\underline{\underline{32.5\%}}$.

$\rightarrow \lambda = 12, \quad x = 8$

$$P(X=8) = \frac{\lambda^{x} \cdot e^{-\lambda}}{x!} = \frac{12^{8} \cdot e^{-12}}{8!} = 0.0655$$

$\therefore$ the probability of having 8 customers in one day is $\underline{\underline{6.55\%}}$

$\rightarrow \lambda = 7, \quad x = 9$

$$P(X=9) = \frac{7^{9} \cdot e^{-7}}{9!} = 0.1014 \approx \underline{\underline{10.14\%}}$$

$\therefore$ The probability of recieving 9 messages in a 2 hour frame is $\underline{\underline{10.14\%}}$.

$\rightarrow \lambda_1 = 7 \xrightarrow{4x} \lambda_2 = 28 \qquad t_1 = 2hrs \xrightarrow{4x} t_2 = 8hrs$

$\rightarrow P(X=14) = 28^{24} \cdot e^{-28} \quad = \quad 0.010051 \approx 1.01\%$

$\rightarrow P(X=24) = \dfrac{28^{24} \cdot e^{-28}}{24!} = 0.060095 \approx \underline{\underline{6.01\%}}$

$\therefore$ The probability of recieving 24 text messages in a frame of 8 hours is $\underline{\underline{6.01\%}}$

$\rightarrow \lambda = 8 \; ; \; x = 7$

$\therefore \dfrac{8^7 \cdot e^{-8}}{7!} = 0.13959 \approx \underline{\underline{13.95\%}}$

$\therefore$ The probability of recieving 7 calls in an hour is $\underline{\underline{13.95\%}}$

$$\rightarrow P(x \leq 5) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$$

$$= \frac{8^0 \cdot e^{-8}}{0!} + \frac{8^1 \cdot e^{-8}}{1!} + \frac{8^2 \cdot e^{-8}}{2!} + \frac{8^3 \cdot e^{-8}}{3!} + \frac{8^4 \cdot e^{-8}}{4!} + \frac{8^5 \cdot e^{-8}}{5!}$$

$$= e^{-8} \left[\frac{8^0}{0!} + \frac{8^1}{1!} + \frac{8^2}{2!} + \frac{8^3}{3!} + \frac{8^4}{4!} + \frac{8^5}{5!} \right]$$

$$= e^{-8}\left[\frac{8^0}{0!} + \frac{8^1}{1!} + \frac{8^2}{2!} + \frac{8^3}{3!} + \frac{8^4}{4!} + \frac{8^5}{5!}\right]$$

$$= 0.1912 \approx \underline{\underline{19.12\%}}$$

$\therefore$ The probability of recieving $\underline{\text{atmost}}$ 5 calls in one hour time-frame is $\underline{\underline{19.12\%}}$.

$$\rightarrow P(x > 6) = 1 - P(x \leq 6)$$

$$= 1 - e^{-8}\left[\frac{8^0}{0!} + \frac{8^1}{1!} + \frac{8^2}{2!} + \frac{8^3}{3!} + \frac{8^4}{4!} + \frac{8^5}{5!} + \frac{8^6}{6!}\right]$$

$$= 0.686626 \approx \underline{\underline{68.7\%}}$$

$\therefore$ the probability of recieving more than 6 calls in an one hour time-frame is $\underline{\underline{68.7\%}}$.