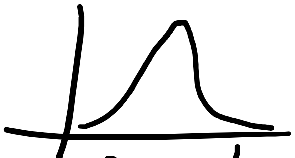
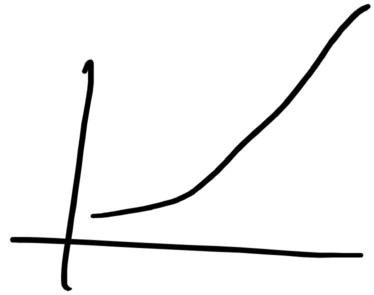
DSD - CO2 - 2024-07-18

18 July 2024 09:20

-> Central Limit Theorem : Axiomatic



Pop Sample



-> Degree of Freedom : (D-o-F)

D-o-F = No. of independent obs - No. of parameters estimated

Sal Age ~ N - 1

-> Chi-Square Dist $(\chi)^2$

↳ $x_1, x_2, \ldots x_n$ from $N(\mu, \sigma)$

$$\sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma}\right)^2$$

For n-1 D-o-F

$$\sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{\sigma}\right)^2 = \frac{n s^2}{\sigma^2}$$

S. Error

-> Estimating Population Variance (σ^2)

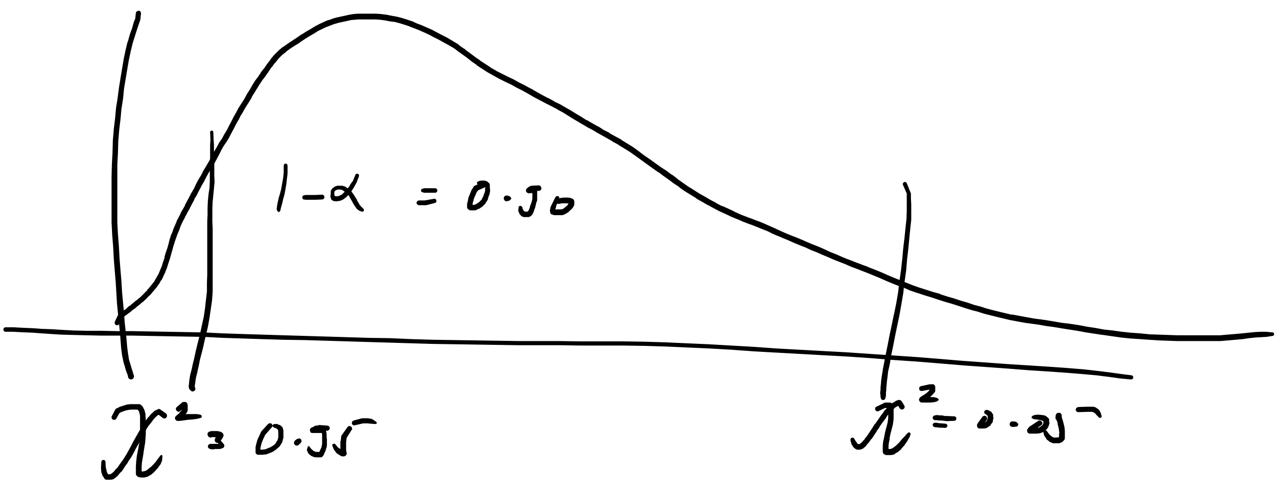
↳ $s^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$ } Equal in large samples

$$s_1^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$ large samples

→ Case I : Small Sample $(n<30)$

Dist Stat : $\dfrac{(n-1)s_1^2}{\sigma^2}$ with d-o-f $(n-1)$

$$\left(\frac{(n-1)s_1^2}{\chi^2_{\alpha/2}} , \frac{(n-1)s_1^2}{\chi^2_{1-\alpha/2}} \right)$$ Interval Estimate



→ $\chi^2_{\alpha/2} = 27.587$ & $\chi^2_{1-\alpha/2} = 8.672$

$$\left(\frac{(n-1)s_1^2}{\chi^2_{\alpha/2}} , \frac{(n-1)s_1^2}{\chi^2_{1-\alpha/2}} \right)$$

$$= \left(\frac{17 \times 1.62 \times 1.62}{27.587}, \frac{17 \times 1.62 \times 1.62}{8.672} \right)$$

$$(1.61724, 5.1446)$$

-> Case 2 : Large Sample ($n \geq 30$)

↳ Standard Error for $s_1^2 = \sigma^2 \sqrt{\frac{2}{(n-1)}}$

↳ This uses C.L.T $= \dfrac{s_1^2 - \sigma^2}{\sigma^2 \sqrt{2/(n-1)}}$

$$\to C.I = \left[\frac{s_1^2}{1 + Z_{\alpha/2} \sqrt{2/(n-1)}}, \frac{s_1^2}{1 - Z_{\alpha/2} \sqrt{2/(n-1)}} \right]$$

$$\left[\frac{7.93 \times 7.93}{1 + 1.96 \times \sqrt{2/45}}, \frac{7.93 \times 7.93}{1 - 1.96 \times \sqrt{2/45}} \right]$$

$$[45.04714, 105.1105)$$

→ Proportion : $\frac{x}{n} = \pi$

S.Error for proportion $= \sqrt{\pi(1-\pi)/n}$

Sampling Proportion $p = \frac{x}{n}$

$$Z = \frac{x - n\pi}{\sqrt{n\pi(1-\pi)}} = \frac{p - \pi}{\sqrt{\pi(1-\pi)/n}}$$

When $n \geq 30$ or $n\pi \geq 5$ as well as $n(1-\pi) \geq 5$