Question: 4

i dont know how to type set this.

Solution: First realization is that the determinant is a function of the different independent x_i. And we also see that since if $x_i = x_j$, then the determinant is zero, we can write the determinant as a product of linear factors of the form $(x_i - x_j)$ for $1 \leqslant i < j \leqslant n$. But since all the k_i are positive, this determinant is divisible by $x_1 x_2 \ldots x_n$. So, it suffices to show that $n!$ divides the following value:

$$x_1 x_2 \ldots x_n \prod_{1 \leqslant i < j \leqslant n} (x_i - x_j)$$

A proof with induction trivially shows this.

Question: 7

Let A be a square matrix. Prove that there exists B such that $ABA = A$.

Solution: Rephrasing the problem for invertible matrices, we have to prove that there exists B such that either $AB = I$ or $BA = I$. We constructively find the solutions for B here, trivially showing that B can either be the right or left inverse of A.

For the noninvertible case, we have the trivial result of the Moore-Penrose inverse.