Question: 1

Solution:

Question: 4

Show that there is a rearrangement of the fractions $-\frac{1}{1}, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \ldots$ such that the sum of the rearranged series is π.

Solution: By the Riemann series theorem, this is possible. I will now devise an algorithm to make this apparent.

Question: 8

Evaluate

$$\lim_{x\to\infty} (2x)^{1+1/2x} - (x)^{1+1/x} - x$$

Solution: We start by factoring out an x and realizing that $x^{1/x} = e^{\ln(x)/x}$. Doing this, we get:

$$\lim_{x\to\infty} x(2(2x)^{1/2x} - (x)^{1/x} - 1)$$

$$\lim_{x\to\infty} x(2e^{\ln(2x)/2x} - e^{\ln(x)/x} - 1)$$

$$\lim_{x\to\infty} x\left(2 + \frac{2\ln(2x)}{2x} + O\left(\frac{\ln(x)^2}{x^2}\right) - \left(1 + \frac{\ln(x)}{x} + O\left(\frac{\ln(x)^2}{x^2}\right)\right) - 1\right)$$

$$\lim_{x\to\infty} x\left((2-1-1) + \left(\frac{\ln(2x)}{x} - \frac{\ln(x)}{x}\right) + \left(O\left(\frac{\ln(x)^2}{x^2}\right) - O\left(\frac{\ln(x)^2}{x^2}\right)\right)\right)$$

$$\lim_{x\to\infty} x\left(\frac{\ln(2)}{x}\right) = \ln(2)$$