Rohan Jain

sorry i got lazy

Question: 2

Let $f(1) = 3$ and $f(n+1) = 3^{f(n)}$. Find the last digit of $f(2012)$.

Solution: We claim that 3 raised to any odd number is equal to $1 \pmod 4$. This is very easy to show with modular arithmetic. As such, we take the exponent $\pmod 4$ to find the last digit. We know that the exponent has to end in a 7 and that it is equal to $3 \pmod 4$, which means that the exponent $\pmod 4$ is 3. Thus, we have that $f(2012) \equiv 3^3 \equiv 27 \equiv 7 \pmod{10}$.

Question: 3

Let $f(1) = 3$ and $f(n+1) = 3^{f(n)}$. Find the last two digits of $f(2012)$.

Solution: We claim that $3^{20} = 1 \pmod{100}$, which is trivially shown by Fermat's or Euler's. As such, we take the exponent $\pmod{20}$ to find the last two digits. We know that the exponent has to end in a 7 and that it is equal to $3 \pmod 4$, which means that the exponent $\pmod{20}$ is 7. Thus, we have that $f(2012) \equiv 3^7 \equiv 2187 \equiv 87 \pmod{100}$.