Question: 2

Which of the following multiplication tables defined on the set $G = \{a, b, c, d\}$ form a group? Support your answer in each case.

(a)

$\circ$	a	b	c	d
a	a	c	d	a
b	b	b	c	d
c	c	d	a	b
d	d	a	b	c

(c)

$\circ$	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

(b)

$\circ$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

(d)

$\circ$	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	b	a	d
d	d	d	b	c

Solution:

a. This is not a group because it doesn't have an identity element, e, such that $g \circ e = e \circ g = g$ for all $g \in G$.

b. This table does have an identity element, $e = a$. Now we check for invertibility. In this table, each element is it's own inverse because $g \circ g = e = a$ for all $g \in G$.

c. d

d.

Question: 7

Let $S = \mathbb{R} \setminus \{-1\}$ and define a binary operation on S by $a * b = a + b + ab$. Prove that $(S, *)$ is an abelian group.

Solution:

Question: 17

Give an example of three different groups with eight elements. Why are the groups different?

Solution:

Question: 25

Let a and b be elements in a group G. Prove that $ab^n a^{-1} = (aba^{-1})^n$ for $n \in \mathbb{Z}$.

Solution:

Question: 31

Show that if $a^2 = e$ for all elements a in a group G, then G must be abelian.

Solution:

Question: 32

Show that if G is a finite group of even order, then there is an $a \in G$ such that a is not the identity and $a^2 = e$.

Solution:

Question: 33

Let G be a group and suppose that $(ab)^2 = a^2b^2$ for all a and b in G. Prove that G is an abelian group.

Solution: