Putnam Seminar Homework 7

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## Question: 1

Prove that for  $n \ge 1$ , you can tile a  $2^n \times 2^n$  checkerboard with L-shaped triominoes.

**Solution:** For n = 1, this solution is trivial. Now, assume the statement is true for any k = n - 1. Then we can divide a  $2^n \times 2^n$  board into its four quadrants. For one of the quadrants, we can tile it with L-shaped trionminoes except one of it's corners (this will be where the one missing tile is). For the other three, do the same thing but rotate each quadrants such that their empty corners meet at the center of the board. This is where we will place another L-shaped triomino. This completes the proof.

## Question: 3

Find the limit of  $F_{n+1}/F_n$ .

**Solution:** We have that

$$\begin{split} F_{n+1} &= F_n + F_{n-1} \\ \frac{F_{n+1}}{F_n} &= 1 + \frac{F_{n-1}}{F_n} = 1 + 1 / \frac{F_n}{F_{n-1}} \end{split}$$

If the limit is x, then we have that x = 1 + 1/x. Solve this means that the limit is equal to  $\varphi$ .