Question: 5

For each of the following rings R with ideal I, give an addition table and a multiplication table for R/I.

a. $R = \mathbb{Z}$ and $I = 6\mathbb{Z}$

b. $R = \mathbb{Z}_{12}$ and $I = \{0, 3, 6, 9\}$

Solution:

Question: 6

Find all the homomorphisms $\phi : \mathbb{Z}/6\mathbb{Z} \to \mathbb{Z}/15\mathbb{Z}$.

Solution:

Question: 7

Prove that $\mathbb{R}$ is not isomorphic to $\mathbb{C}$.

Solution:

Question: 8

Prove or disprove: The ring $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is isomorphic to the ring $\mathbb{Q}(\sqrt{3}) = \{a + b\sqrt{3} : a, b \in \mathbb{Q}\}$.

Solution:

Question: 25

Let $\{I_\alpha\}_{\alpha \in A}$ be a collection of ideals in a ring R. Prove that $\bigcap_{\alpha \in A} I_\alpha$ is also an ideal in R. Give an example to show that if I_1 and I_2 are ideals in R, then $I_1 \cup I_2$ may not be an ideal.

Solution:

Question: 26

Let R be an integral domain. Show that if the only ideals in R are $\{0\}$ and R itself, R must be a field.

Solution:

Question: 27

Let R be a commutative ring. An element a in R is nilpotent if $a^n = 0$ for some positive integer n. Show that the set of all nilpotent elements forms an ideal in R.

Solution:

Question: 37

An element x in a ring is called an idempotent if $x^2 = x$. Prove that the only idempotents in an integral domain are 0 and 1. Find a ring with a idempotent x not equal to 0 or 1.

Solution: