

**Question: 2**

Prove that  $\mathbb{C}^*$  is isomorphic to the subgroup of  $GL_2(\mathbb{R})$  consisting of matrices of the form

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix}.$$

**Solution:**

**Question: 13**

Let  $\omega = \text{cis}(2\pi/n)$  be the primitive  $n$ th root of unity. Prove that matrices

$$A = \begin{pmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

generate a multiplicative group isomorphic to  $D_n$ .

**Solution:**

**Question: 18**

Prove that the subgroup of  $\mathbb{Q}^*$  consisting of elements of the form  $2^m 3^n$  for  $m, n \in \mathbb{Z}$  is an internal direct product isomorphic to  $\mathbb{Z} \times \mathbb{Z}$ .

**Solution:**

**Question: 23**

Prove or disprove the following assertion. Let  $G, H$ , and  $K$  be groups. If  $G \times K \cong H \times K$  then  $G \cong H$ .

**Solution:**

**Question: 29**

Show that  $S_n$  is isomorphic to a subgroup of  $A_{n+2}$ .

**Solution:**

**Question: 36**

Prove that  $A \mapsto B^{-1}AB$  is an automorphism of  $SL_2(\mathbb{R})$  for all  $B$  in  $GL_2(\mathbb{R})$ .

**Solution:**