Question: 4

Prove or disprove: Any subring of a field F containing 1 is an integral domain.

Solution: Let $R \subseteq F$. Suppose $x, y \in R$ such that $xy = 0$. Since the 0 element is the same in R and F, either $x = 0$ or $y = 0$ and as such, R has no zero divisors and therefore, is an integral domain. 😄

Question: 6

Let F be a field of characteristic zero. Prove that F contains a subfield isomorphic to $\mathbb{Q}$.

Solution:

Question: 10

A field F is called a ***prime field*** if it has no proper subfields. If E is a subfield of F and E is a prime subfield of F:

a. Prove that every field contains a unique prime subfield.

b. If F is a field of characteristic 0, prove that the prime subfield of F is isomorphic to the field of rational numbers, $\mathbb{Q}$.

c. If F is a field of characteristic p, prove that the prime subfield of F is isomorphic to the field of integers modulo p, $\mathbb{Z}_p$.

Solution:

a. To convince ourselves that E is nonempty, we realize that $0, 1 \in E$. For any $a, b \in E$, $a, b \in L$, so ab, $a + b$, $a - b$, and a/b are all in L, and thus all in E. As such, E is a subfield.

b. Ok

c. Ok