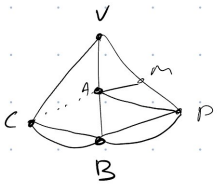
Question: 1

The radius of the base of a right circular cone is 1. The vertex of the cone is V, and P is a point on the circumference of the base. The length of PV is 6 and the midpoint of PV is M. A piece of string is attached to M and wound tightly twice round the cone finishing at P. What is the length of the string?

Solution:



PB along the circular arc $= 2\pi$

$\Rightarrow \angle PVB = \frac{\pi}{3} \quad \because VP = 6$

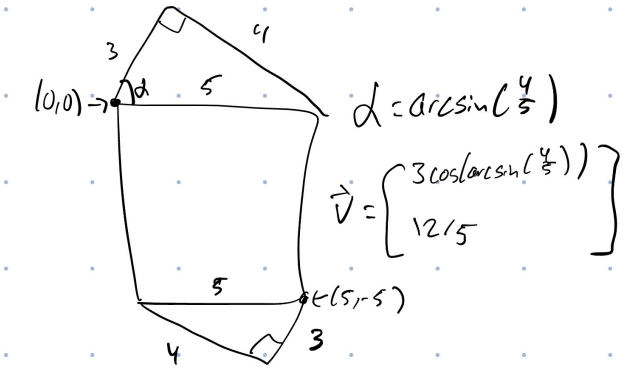
$\Rightarrow \angle PVC = \frac{2\pi}{3}$

$VC = 6 \qquad MC = \sqrt{3^2 + 6^2 - 2 \cdot 3 \cdot 6 \cdot \cos\left(\frac{2\pi}{3}\right)} = 3\sqrt{7}$

Question: 2

Take a 5×5 square, and put $3-4-5$ right triangles on its top and bottom sides, oriented such that they stick out of the square, and are 180-degree rotations of each other. Determine the distance between the right-angled corners of the $3-4-5$ triangles.

Solution: Start with the following diagram:



This let's us coord-bash the rest of the problem. The top corner can be expressed as the vector v that I wrote out, while the bottom right corner can be expressed as $\begin{bmatrix} 5 \\ -5 \end{bmatrix} - v$. Now we can just find the vector that describes one corner to the other, which is $v - \left(\begin{bmatrix} 5 \\ -5 \end{bmatrix} - v\right) = 2v - \begin{bmatrix} 5 \\ -5 \end{bmatrix}$. Calculating the magnitude of this vector, we get $7\sqrt{2}$ as the answer.