

**Question: 34**

Find all the subgroups of  $\mathbb{Z}_3 \times \mathbb{Z}_3$ . Use this information to show that  $\mathbb{Z}_3 \times \mathbb{Z}_3 \not\cong \mathbb{Z}_9$ .

**Solution:** The subgroups of  $\mathbb{Z}_3 \times \mathbb{Z}_3$  are:

- $\mathbb{Z}_3 \times \mathbb{Z}_3$
- $\{(0, 0)\}$
- $\{(0, 0), (1, 0), (2, 0)\}$
- $\{(0, 0), (0, 1), (0, 2)\}$
- $\{(0, 0), (1, 1), (2, 2)\}$
- $\{(0, 0), (1, 2), (2, 1)\}$

The subgroups of  $\mathbb{Z}_9$  are:

- $\mathbb{Z}_9$
- $\{0\}$
- $\{0, 3, 6\}$

Since these groups have different sets of subgroups, they are not isomorphic. In other words,  $\mathbb{Z}_3 \times \mathbb{Z}_3 \not\cong \mathbb{Z}_9$ .

**Question: 35**

Find all the subgroups of the symmetry group of an equilateral triangle

**Solution:**

**Question: 41**

Prove that

$$G = \{a + b\sqrt{2} : a, b \in \mathbb{Q} \wedge a, b \neq 0\}$$

is a subgroup of  $\mathbb{R}^*$  under the group operation of multiplication.

**Solution:**

**Question: 45**

Prove that the intersection of two subgroups of a group  $G$  is also a subgroup of  $G$ .

**Solution:**

**Question: 46**

Prove or disprove: If  $H$  and  $K$  are subgroups of a group  $G$ , then  $H \cup K$  is a subgroup of  $G$ .

**Solution:**

**Question: 47**

Prove or disprove: If  $H$  and  $K$  are subgroups of a group  $G$ , then  $HK = \{hk : h \in H \wedge k \in K\}$  is a subgroup of  $G$ . What if  $G$  is abelian?

*Solution:*

**Question: 48**

Let  $G$  be a group and  $g \in G$ . Show that

$$Z(G) = \{x \in G : gx = xg \text{ for all } g \in G\}$$

is a subgroup of  $G$ . This subgroup is called the **center** of  $G$ .

*Solution:*

**Question: 53**

Let  $H$  be a subgroup of  $G$  and

$$C(H) = \{g \in G : gh = hg \text{ for all } h \in H\}.$$

Prove that  $C(H)$  is a subgroup of  $G$ . This group is called the **centralizer** of  $H$  in  $G$ .

*Solution:*

**Question: 54**

Let  $H$  be a subgroup of  $G$ . If  $g \in G$ , show that  $gHg^{-1} = \{ghg^{-1} : h \in H\}$  is also a subgroup of  $G$ .

*Solution:*