

**Question: 12**

If  $ghg^{-1} \in H$  for all  $g \in G$  and  $h \in H$ , show that the left cosets are identical to the right cosets. That is, show  $gH = Hg$  for all  $g \in G$ .

**Solution:**

**Question: 17**

Suppose that  $[G : H] = 2$ . If  $a$  and  $b$  are not in  $H$ , show that  $ab \in H$ .

**Solution:**

**Question: 19**

Let  $H$  and  $K$  be subgroups of group  $G$ . Prove that  $gH \cap gK$  is a coset of  $H \cap K$  in  $G$ .

**Solution:**

**Question: 20**

Let  $H$  and  $K$  be subgroups of group  $G$ . Define a relation  $\sim$  on  $G$  by  $a \sim b$  if there exists an  $h \in H$  and a  $k \in K$  such that  $hak = b$ . Show that this relation is an equivalence relation. The corresponding equivalence classes are called **double cosets**. Compute the double cosets of  $H = \{(1), (1\ 2\ 3), (1\ 3\ 2)\}$  in  $A_4$ .

**Solution:**