The direct product of two groups G and H is denoted by $G \times H$ and consists of all ordered pairs (g, h) with $g \in G$ and $h \in H$, where the group operation is defined componentwise.

The class equation of a finite group G is the equation that relates the order of G to the sum of the orders of its conjugacy classes. More formally, if $g \in G$ and C_g denotes the conjugacy class of g in G, then the class equation of G is:

$$|G| = \sum_{g \in G} |C_g|$$

Now, to write the class equation for the direct product of A_4 and Z_2, we need to find the conjugacy classes of the elements in $A_4 \times Z_2$.

First, note that Z_2 is the cyclic group of order 2 and has only two elements: the identity element and a non-identity element.

Next, recall that A_4 is the alternating group of degree 4 and has 12 elements. We can list its elements as:

$$\begin{aligned} A_4 = \{&(1), (12)(34), (13)(24), (14)(23), \\ &(123), (132), (124), (142), (134), (143), (234), (243)\} \end{aligned}$$

Now, we can form the elements of $A_4 \times Z_2$ by taking the direct product of each element in A_4 with both the identity element and the non-identity element of Z_2. This gives us 24 elements, which we can list as:

$$\begin{aligned} &(1, 0), (1, 1), \\ &((12)(34), 0), ((12)(34), 1), \\ &((13)(24), 0), ((13)(24), 1), \\ &((14)(23), 0), ((14)(23), 1), \\ &((123), 0), ((123), 1), \\ &((132), 0), ((132), 1), \\ &((124), 0), ((124), 1), \\ &((142), 0), ((142), 1), \\ &((134), 0), ((134), 1), \\ &((143), 0), ((143), 1), \\ &((234), 0), ((234), 1), \\ &((243), 0), ((243), 1) \end{aligned}$$

To find the conjugacy classes of $A_4 \times Z_2$, we need to determine which elements are conjugate to each other under the group operation. Two elements (g, h) and (g', h') are conjugate if there exists an element $(x, y) \in A_4 \times Z_2$ such that $(g, h) = (x, y)(g', h')(x^{-1}, y^{-1})$. Since the group operation in $A_4 \times Z_2$ is defined componentwise, we can write this condition as:

$$(gxg^{-1}, hyh^{-1}) = (g', h')(x, y)(g'^{-1}, h'^{-1})$$

This gives us two conditions: $gxg^{-1} = g'$ and $hyh^{-1} = h'$. Using these conditions, we can form the following conjugacy classes:

Therefore, the class equation for $A_4 \times Z_2$ is:

$$|A_4 \times Z_2| = 1 + 2 + 2 + 4 + 4 + 1 + 2 + 2 + 4 + 4 = 24$$

Note that each conjugacy class has either one or two elements, and the sum of the orders of the conjugacy classes equals the order of the group, as expected.

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