Response Letter to Associate Editor

Quantifying Roughness in SAR Imagery with the Rényi Entropy (GRSL-00722-2025)

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Dear Editors

We are grateful to the reviewers for the valuable comments and the time spent on checking our manuscript. We have addressed their comments towards improving the paper contents and presentation. Below we detail the changes and updates incorporated into the manuscript in this round of review.

Comment 1

This manuscript proposes a statistically grounded method for quantifying roughness in SAR imagery using a non-parametric estimator of Rényi entropy, refined via bootstrap correction, and developed into a hypothesis test that distinguishes fully developed speckle and heterogeneous clutter. The paper is technically sound and well-presented. However, the following comments should be carefully addressed in the revised version to enhance clarity, applicability, and completeness:
1. While the paper is generally well-written and structured, several mathematical expressions—particularly Equations (5) and (8)—may be difficult to interpret for practitioners or applied researchers. The authors are encouraged to provide more intuitive explanations or illustrative interpretations alongside the formulas.

Response

We have revised the manuscript to clarify their meaning and practical relevance, as detailed below.

Equation (5): Rényi entropy of the $\mathcal{G}_I^0$ model. This expression presents the closed-form Rényi entropy of the heterogeneous $\mathcal{G}_I^0$ distribution as the sum of two components:

$$H_\lambda(\mathcal{G}_I^0) = H_\lambda(\Gamma_{\text{SAR}}) + \left[\text{extra term depending on } \alpha\right].$$

In intuitive terms:

- $H_\lambda(\Gamma_{\text{SAR}})$ represents the "baseline entropy" of fully developed speckle (homogeneous clutter).
- The additional term captures how much entropy increases due to texture (heterogeneity), caused by the roughness parameter α.

When $\alpha \to -\infty$ (fully developed speckle), the excess term vanishes and (5) collapses to the homogeneous case in (4). This motivates our test: we check whether the *excess* is statistically different from zero.

> To improve clarity, we expanded the explanation following Equation (5) and clarified that Figure 1 illustrates the behavior of the excess entropy term as α decreases, confirming its convergence to the homogeneous case.

Equation (8): test statistic. The test statistic is simply defined as:

$$S_{\widetilde{H}_\lambda}(Z; L) = \text{“estimated entropy”} - \text{“theoretical entropy under } H_0\text{”}.$$

Its purpose is to detect deviations from homogeneity:

- If the data are homogeneous, both terms match closely $\Rightarrow$ $S \approx 0$.
- If the data are heterogeneous, the estimated entropy exceeds the theoretical one $\Rightarrow$ $|S|$ increases and the p-value becomes small.

This formulation avoids parameter estimation for the $\mathcal{G}_I^0$ model, offering a simple, interpretable, and statistically grounded test.

> We added a brief explanation below Equation (8), and clarified in the main text (related to Figure 4) that the centered distribution confirms correct behavior under H_0.

Why Eq.(5) benefits practitioners. Closed-form Rényi entropy for $\mathcal{G}_I^0$ is absent from the SAR literature; Having it allows others to incorporate an analytical feature into algorithms and applications.

> These additions are included in Sections II-B and III-D of the revised manuscript.
>
> Note: Due to space limitations, the derivation of Equation (5), which was previously included in the Appendix, has been omitted in the revised version.

Comment 2

2. The current methodology assumes that the number of looks, L, is known a priori. In practice, this parameter may be unknown or inaccurately estimated from SAR metadata. The authors should discuss the sensitivity of the proposed test to variations or inaccuracies in L, and optionally suggest techniques for estimating L directly from the data or include robustness analysis.

Response

We thank the reviewer for this important observation.
In our experiments, we used the number of looks L reported in the metadata of each SAR image, computed as the product of azimuth and range looks provided by the Sentinel Application Platform (SNAP). However, we recognize that this value may be imprecise in practice due to preprocessing steps such as multilooking.
To validate its reliability, we manually selected homogeneous patches from each image and estimated the Equivalent Number of Looks (ENL) using the classical estimator based on the coefficient of variation ($\widehat{L} = 1/\mathrm{CV}^2$). The results were consistent with the metadata; for example, in the Dublin image, we obtained $\widehat{L} \approx 16.3$ from water regions, which closely matches the metadata value $L = 16$.
We also tested the sensitivity of the proposed hypothesis test to slight under- and overestimations of L, and found that the resulting p-value maps and binary decisions remained stable. These findings indicate that the test is robust to moderate inaccuracies in L.

> A brief explanation have been included in Section IV-A of the revised manuscript.

Comment 3

3. Figures (Figs. 6–8) are informative but could benefit from quantitative evaluation. Add comparative metrics beyond visual comparison.

Response

We thank the reviewer for the suggestion. We have added a quantitative performance evaluation to the revised manuscript to complement the visual comparison.
Specifically, we manually selected eight regions of interest (ROIs) in each SAR image: four clearly homogeneous areas (e.g., water, croplands) and four clearly heterogeneous areas (e.g., urban centers). Each ROI was labeled as class 0 (blue) or class 1 (red), as illustrated in Figs. 5a, 6a, and 7a (corresponding to Figs. 6a, 7a, and 8a in the original submission).
These ROIs were rasterized to create a partial ground truth mask. Binary decision maps were obtained by thresholding the p-value maps at a 0.05 significance level, assigning a value of 1 to pixels with $p < 0.05$ (heterogeneous), and 0 otherwise (homogeneous). These predictions were compared to the ground truth labels, and standard metrics—F1-score, kappa coefficient, and overall accuracy—were computed. The results are reported in Table III.
To further support the evaluation, we computed ROC curves and the corresponding AUC values for each method, as shown in Fig. 9.

These additions are included in Section IV-C of the revised manuscript.

Comment 4

The manuscript convincingly shows that Rényi-based entropy measures offer advantages over classical Shannon entropy for heterogeneity detection. However, no comparisons are made to machine learning-based SAR classifiers or segmentation models, which are now widely adopted. Justification is required.

Response

While machine learning-based classifiers and segmentation models are widely used in SAR analysis, our focus in this study is on interpretable, statistically grounded methods that provide analytical significance measures in the form of p-values. In contrast to data-driven ML approaches, which typically require extensive labeled datasets, hyperparameter tuning, and large training efforts, our entropy-based tests offer lightweight, unsupervised alternatives that are easy to deploy and statistically explainable. These properties make them attractive for tasks such as rapid change detection or preliminary scene screening where data availability is limited or interpretability is essential. Moreover, our objective is not to outperform ML classifiers, but to compare the effectiveness of two statistical entropy measures under controlled conditions using SAR data.

We have clarified this distinction in the revised conclusions of the manuscript.