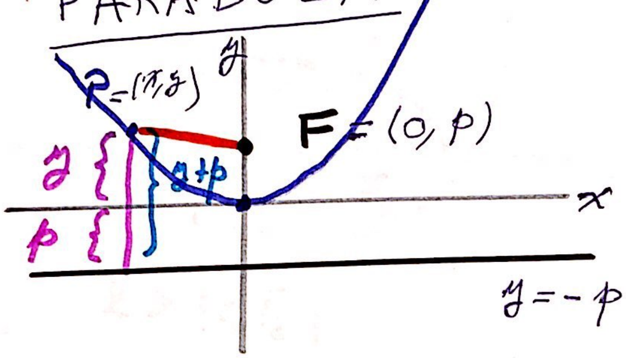
PARÁBOLA



La parábola de foco $F=(0,p)$ y directriz $\{y=-p\}$ consiste en todos los puntos $P=(x,y)$ tales que

$$\sqrt{x^2+(y-p)^2} = y+p$$

$$x^2 + (\cancel{y^2} - 2py + \cancel{p^2}) = \underbrace{(y+p)^2}_{\cancel{y^2}+2py+\cancel{p^2}}$$

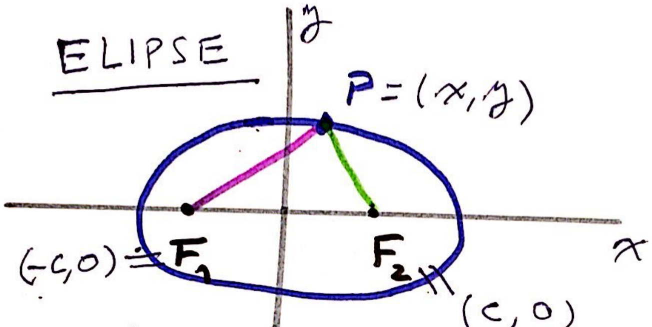
$$x^2 - 2py = 2py$$

$$x^2 = 4py$$

$$y = \underbrace{\frac{1}{4p}}_{=a} x^2$$

$$\begin{cases} x = t \\ y = a \cdot t^2 \end{cases}$$

ELIPSE



$$2a = \ell > |F_1F_2| = 2c$$

$$\underbrace{|PF_1|}_{\parallel} + \underbrace{|PF_2|}_{\parallel} = \ell$$

$$\sqrt{(x+c)^2+y^2} + \sqrt{(x-c)^2+y^2} = 2a$$

$$(x-c)^2 + \cancel{y^2} = \left(2a - \sqrt{(x+c)^2+y^2}\right)^2$$

$$\cancel{x^2} - 2cx + \cancel{c^2} + \cancel{y^2} = 4a^2 - 4a\sqrt{(x+c)^2+y^2} + \underbrace{(x+c)^2}_{\cancel{x^2}+2cx+\cancel{c^2}} + \cancel{y^2}$$

$$-2cx = 4a^2 - 4a\sqrt{(x+c)^2+y^2} + 2cx$$

$$\cancel{4}a\sqrt{(x+c)^2+y^2} = \cancel{4}a^2 + \cancel{4}cx$$

$$\underbrace{a^2(x^2+2cx+c^2+y^2)}_{a^2x^2+\cancel{2a^2cx}+a^2c^2+a^2y^2} = \underbrace{(a^2+cx)^2}_{a^4+\cancel{2a^2cx}+c^2x^2}$$

$$\underbrace{(a^2-c^2)}_{b^2}x^2 + a^2y^2 = \underbrace{a^4-a^2c^2}_{a^2\underbrace{(a^2-c^2)}_{b^2}}$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

$$a^2 - c^2 = b^2$$

$$c^2 = a^2 - b^2$$

$$|c| = \sqrt{a^2-b^2}$$

¿Cómo se parametriza una elipse?

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

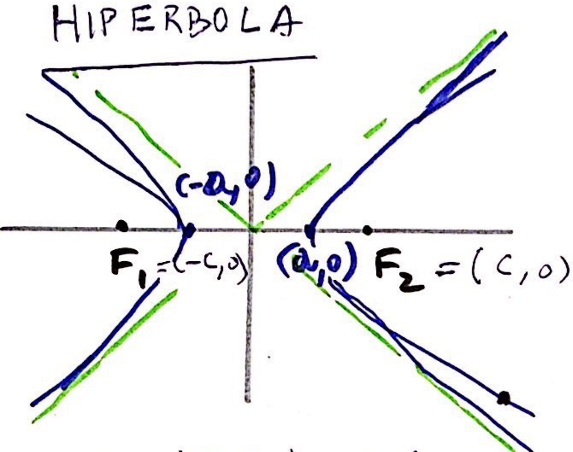
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$\theta \in [0, 2\pi)$

$$\frac{x}{a} = \cos\theta \qquad \begin{cases} x = a\cos\theta \\ y = b\,\mathrm{sen}\,\theta \end{cases}$$

$$\frac{y}{b} = \mathrm{sen}\,\theta$$

HIPÉRBOLA



$$\underset{2a}{\underset{\shortparallel}{\ell}} < |F_1F_2| = 2c$$

$$|PF_1| - |PF_2| = \pm 2a$$

$$\Rightarrow \boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

$$\boxed{c^2 = a^2 + b^2}$$

$$\frac{1}{a^2} - \frac{1}{b^2}\left(\frac{y}{x}\right)^2 = \frac{1}{x^2} \xrightarrow[x\to\infty]{} 0$$

$$\left(\frac{y}{x}\right)^2 \longrightarrow \frac{b^2}{a^2}$$

$$\frac{y}{x} \longrightarrow \pm\frac{b}{a}$$

$\boxed{y = \pm\frac{b}{a}x}$ son

asintotas.

$$x^2 = a^2 + \frac{a^2}{b^2}\cdot y^2$$

$$x = \pm\sqrt{a^2 + \frac{a^2}{b^2}\cdot y^2}$$
$$= \pm a\sqrt{1 + \frac{y^2}{b^2}}$$