

PRODUCTO INTERNO  
PRODUCTO ESCALAR  
PRODUCTO PUNTO.

$$a, b \in \mathbb{R}^2 \text{ o } \mathbb{R}^3$$

$$a \cdot b$$

$$a = \langle x_1, y_1, z_1 \rangle$$

$$b = \langle x_2, y_2, z_2 \rangle$$

$$a \cdot b = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2$$

PROPS

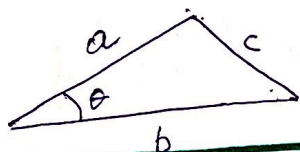
$$(1) a \cdot a = |a|^2$$

$$(2) a \cdot b = b \cdot a$$

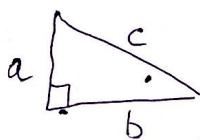
$$(3) a \cdot (b + c) = a \cdot b + a \cdot c$$

$$(4) (t \cdot a) \cdot b = t(a \cdot b)$$

# Teorema del Coseno



$$c^2 = a^2 + b^2 - 2ab \cos \theta$$



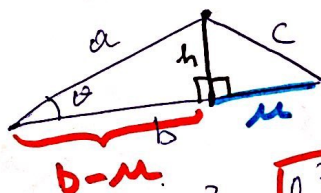
Pitagoras

$$c^2 = a^2 + b^2$$



$$c^2 = a^2 + b^2 - 2ab \cos \theta$$

Dem



Pitagoras:  $c^2 = h^2 + u^2$   
 $a^2 = h^2 + (b-u)^2$

$$\begin{aligned} \rightarrow h^2 &= a^2 - (b-u)^2 \\ &= a^2 - (b^2 - 2bu + u^2) \\ h^2 &= a^2 - b^2 + 2bu - u^2 \end{aligned}$$

$$c^2 = a^2 - b^2 + 2bu - \cancel{u^2} + \cancel{u^2}$$

$$\cos \theta = \frac{b-u}{a}$$

$$a \cos \theta = b - u$$

$$u = b - a \cos \theta$$

$$\begin{aligned} c^2 &= a^2 - b^2 + 2b(b - a \cos \theta) \\ &= a^2 - b^2 + 2b^2 - 2ba \cos \theta \\ &= a^2 + b^2 - 2ab \cos \theta \end{aligned}$$

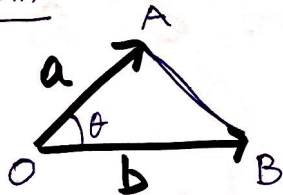
Teorema 2 (versión geométrica  
del prod. interno)

$a, b \in \mathbb{R}^3$  (o  $\mathbb{R}^2$ ), entonces

$$a \cdot b = |a| |b| \cos \theta$$

donde  $\theta$  es el ángulo entre  $a$  y  $b$

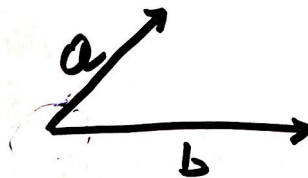
Dem



Aplicamos el Teorema  
del coseno.

$$|\vec{AB}|^2 = |a|^2 + |b|^2 - 2|a||b|\cos\theta$$

$$\begin{aligned} |\vec{AB}|^2 &= |a-b|^2 = (a-b) \cdot (a-b) \\ &= |a|^2 - a \cdot b - b \cdot a + |b|^2 \\ &= |a|^2 + |b|^2 - 2a \cdot b \end{aligned}$$



$$\cancel{-2|a||b|\cos\theta}$$

||

$$\cancel{-2a \cdot b}$$



### COROLARIOS

(1) si  $a, b \neq 0$

$$\cos \theta = \frac{a \cdot b}{|a| |b|}$$

Ej: si  $a = \langle 2, 2, -1 \rangle$   
 $b = \langle 5, -3, 2 \rangle$

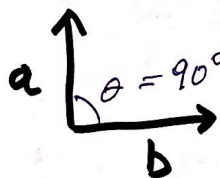
$$\Rightarrow |a| = \sqrt{2^2 + 2^2 + (-1)^2} = 3$$

$$|b| = \sqrt{5^2 + (-3)^2 + 2^2} = \sqrt{38}$$

$$a \cdot b = 2 \cdot 5 + 2 \cdot (-3) + (-1) \cdot 2 = 2$$

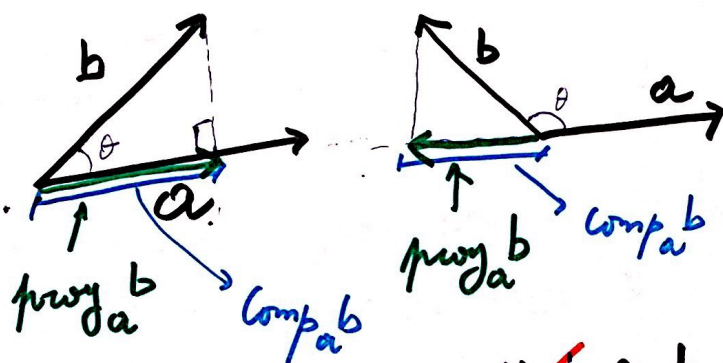
$$\Rightarrow \cos \theta = \frac{2}{3\sqrt{38}} \Rightarrow \theta \approx 84^\circ$$

(2)  $a \perp b \iff a \cdot b = 0$



$\theta = 90^\circ$   $\cos 90^\circ = 0$

## PROYECCIONES



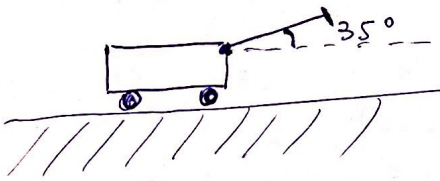
$$\text{comp}_a b = |b| \cos \theta = \cancel{|b|} \frac{a \cdot b}{|a| \cancel{|b|}}$$

$$= \frac{a \cdot b}{|a|}$$

$$\text{prwy}_a b = \text{comp}_a b \cdot \frac{a}{|a|}$$

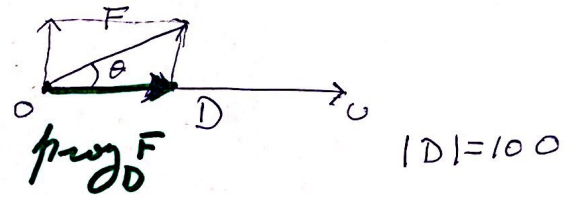
$$= \left( \frac{a \cdot b}{|a|^2} \right) a$$

## APLICACIÓN



Tiro del carrito por 100 m  
ejerciendo una fuerza constante  
de 70 N a un ángulo de  $35^\circ$   
sobre la horizontal.

¿cuál es el trabajo que realiza  
la fuerza?



$$W = \text{Comp}_D F \cdot |D|$$
$$= \frac{F \cdot D \cdot |D|}{|D|}$$

$$W = F \cdot D$$

$$W = F \cdot D = |F| |D| \cos \theta$$
$$= 70 \cdot 100 \cdot \cos 35$$
$$\approx 5734 \frac{\text{N} \cdot \text{m}}{\text{J}}$$

Joules