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%%MATLAB EXPLORATION 12
%Rhea Jaxon

Part 1: first data set

data is given by an mx2 matrix with x-values in column 1 and y-values in column 2

```
data1=[ 1 8 ; 2 6 ; 4 5 ; 5 7 ; 8 16 ];  
% extracts the x-values in column 1  
x=data1(:,1);  
% extracts the y-values in column 2  
y=data1(:,2);
```

first model: linear $y=a_0+a_1x$

constructs the design matrix which is the coefficient matrix of the system $Ax=b$ obtained from plugging in data points into the model

```
A=[ 1 x(1) ;  
    1 x(2) ;  
    1 x(3) ;  
    1 x(4) ;  
    1 x(5) ]  
% b vector is just the y-values  
b=y;  
% gets xhat least squares solution using fact that A has linearly  
% independent columns so  $A^T A$  is invertible - note A' is MATLAB transpose  
xhat=inv(A'*A)*A'*b;  
% extracts alpha and beta values from the xhat vector  
a0=xhat(1);  
a1=xhat(2);  
% defines the function for plotting  
% WARNING: you must use .* for 'times' and .^ for 'exponent' or else  
% MATLAB will give an error code  
f1=@(x) a0+a1.*x;  
% compute the least squares error  $\|A*xhat-b\|$   
errorLine=sqrt((A*xhat-b)'*(A*xhat-b))  
% draws figure  
figure(1)  
fplot(f1,[0 8])
```

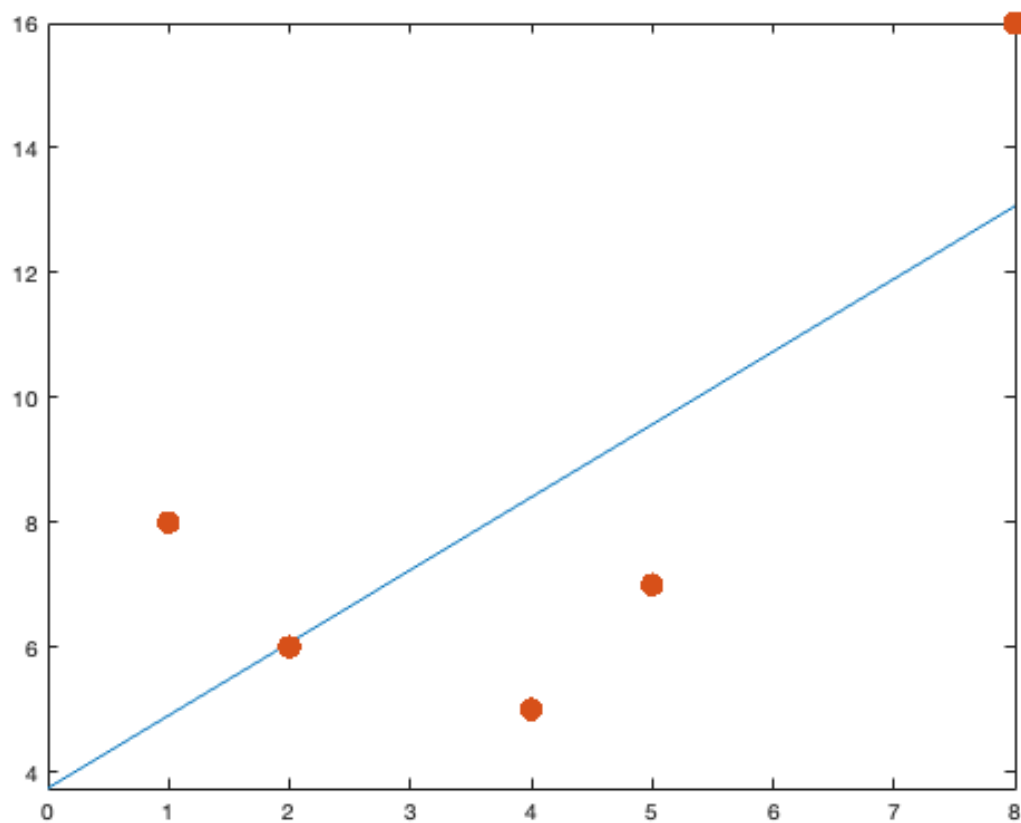
```
hold on
plot(x,y,'.','markersize',28)
hold off
```

A =

1	1
1	2
1	4
1	5
1	8

errorLine =

6.0305



second model: full quadratic $y=a_0+a_1*x+a_2*x^2$

[STUDENT FILL IN:] student should construct the design matrix which is the coefficient matrix of the system $Ax=b$ obtained from plugging in data points into the model $y=a_0+a_1*x+a_2*x^2$

```

A=[ 1 x(1) x(1)^2;
    1 x(2) x(2)^2;
    1 x(3) x(3)^2;
    1 x(4) x(4)^2;
    1 x(5) x(5)^2]
% copy-pasted/modified code from part 1
b=y;
xhat=inv(A'*A)*A'*b;
a0=xhat(1);
a1=xhat(2);
a2=xhat(3);
% defines the function for plotting
% WARNING: you must use .* for 'times' and .^ for 'exponent' or else
% MATLAB will give an error code
f2=@(x) a0+a1.*x+a2.*x.^2;
% least squares error ||A*xhat-b||
errorQuad=sqrt((A*xhat-b)'*(A*xhat-b))
% draws figure
figure(2)
fplot(f2,[0 8])
hold on
plot(x,y,'.','markersize',28)
hold off

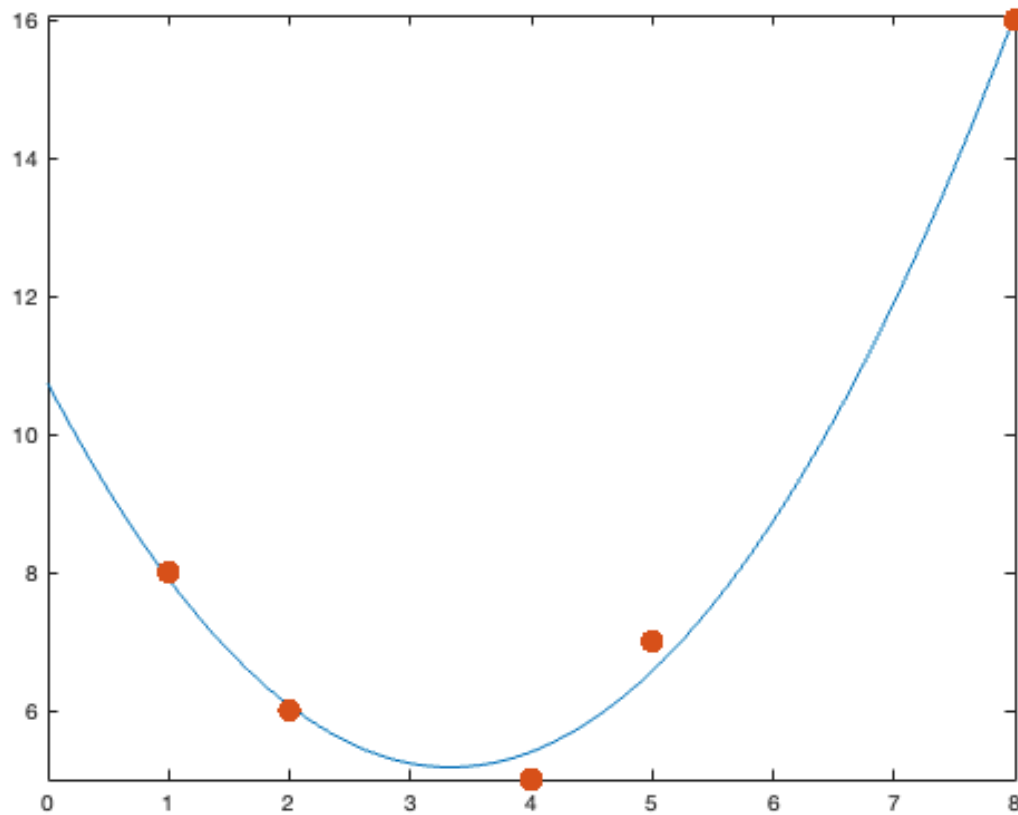
```

A =

1	1	1
1	2	4
1	4	16
1	5	25
1	8	64

errorQuad =

0.6055



third model: [STUDENT FILL IN:] cubic

$$y=a_0+a_1x+a_2x^2+a_3x^3$$

[STUDENT FILL IN:] student should construct the design matrix which is the coefficient matrix of the system $Ax=b$ obtained from plugging in data points into the model

```
A=[1 x(1) x(1)^2 x(1)^3;
    1 x(2) x(2)^2 x(2)^3;
    1 x(3) x(3)^2 x(3)^3;
    1 x(4) x(4)^2 x(4)^3;
    1 x(5) x(5)^2 x(5)^3]
b=y;
xhat=inv(A'*A)*A'*b;
% [STUDENT FILL IN:] you may need to add/delete variables depending on how
% many terms your model has, e.g. three terms needs up to a2=xhat(3)
a0=xhat(1);
a1=xhat(2);
a2=xhat(3);
a3=xhat(4);
% [STUDENT FILL IN:] define the function for plotting
% WARNING: you must use .* for 'times' and .^ for 'exponent' or else
% MATLAB will give an error code
```

```

f3=@(x) a0+a1.*x+a2.*x.^2+a3.*x.^3;
% least squares error is ||A*xhat-b||
errorStudentModelforData1=sqrt((A*xhat-b)'*(A*xhat-b))
% draws figure
figure(3)
fplot(f3,[0 8])
hold on
plot(x,y, '.', 'markersize',28)
hold off

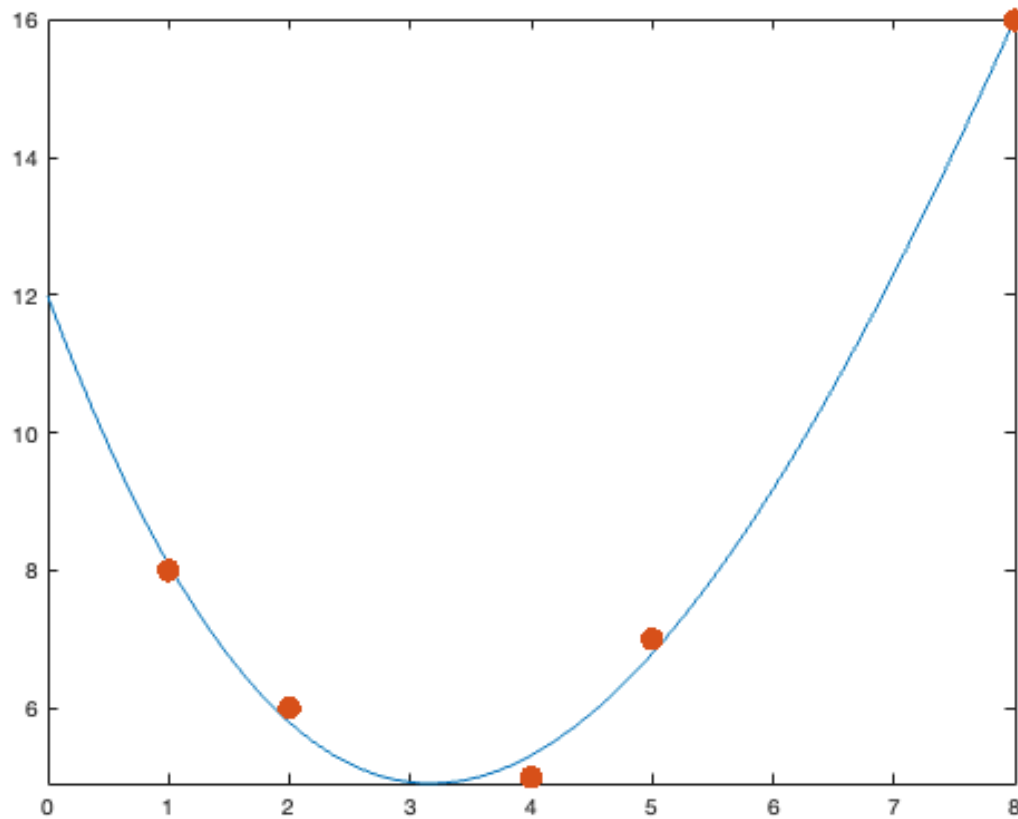
```

A =

1	1	1	1
1	2	4	8
1	4	16	64
1	5	25	125
1	8	64	512

errorStudentModelforData1 =

0.4407



answer questions: [STUDENT FILL IN:] answer questions as comments

Q1: which model has the best error (smallest error value)? Ans1: Quadratic Q2: what does the value of the error $|A\hat{a} - b|$ represent geometrically? Ans2: The standard deviation from the actual data points on the graph.

Part 2: student created data set must have at least 6 data points

[STUDENT FILL IN:] create a data2 matrix where the data is given by an $m \times 2$ matrix with $m > 5$ (at least 6 data points) with x-values in column 1 and y-values in column 2

```
data2=[2 2; 5 7; 4 3; 1 3; 7 4; 9 3];  
% extracts the x-values in column 1  
x=data2(:,1);  
% extracts the y-values in column 2  
y=data2(:,2);
```

fourth model: [STUDENT FILL IN:] cosine + exponential $y = a_0 + a_1 \cos(x) + a_2 \exp(x)$

[STUDENT FILL IN:] student should construct the design matrix which is the coefficient matrix of the system $Ax=b$ obtained from plugging in data points into the model

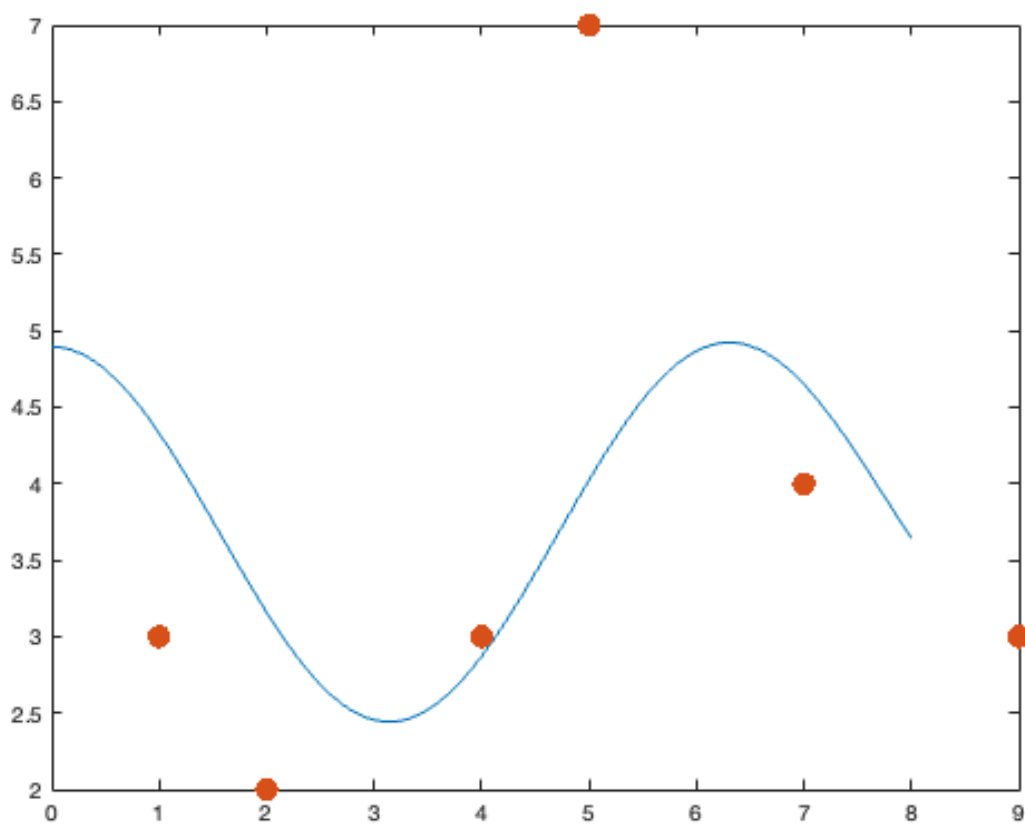
```
A=[1 cos(x(1)) exp(x(1));
    1 cos(x(2)) exp(x(2));
    1 cos(x(3)) exp(x(3));
    1 cos(x(4)) exp(x(4));
    1 cos(x(5)) exp(x(5));
    1 cos(x(6)) exp(x(6))];
b=y;
xhat=inv(A'*A)*A'*b;
% [STUDENT FILL IN:] you may need to add/delete variables depending on how
% many terms your model has
a0=xhat(1);
a1=xhat(2);
a2=xhat(3);
% [STUDENT FILL IN:] define the function for plotting
% WARNING: you must use .* for 'times' and .^ for 'exponent' or else
% MATLAB will give an error code
f4=@(x) a0+a1.*cos(x)+a2.*exp(x);
% least squares error is ||A*xhat-b||
errorStudentModelforData2=sqrt((A*xhat-b)'*(A*xhat-b))
% draws figure
% [STUDENT FILL IN:] You may need to modify the range of the fplot second
% argument [0 8] if your x-values go outside the range 0 <= x <= 8
figure(4)
fplot(f4,[0 8])
hold on
plot(x,y, '.', 'markersize',28)
hold off
```

A =

```
1.0e+03 *
    0.0010    -0.0004    0.0074
    0.0010     0.0003    0.1484
    0.0010    -0.0007    0.0546
    0.0010     0.0005    0.0027
    0.0010     0.0008    1.0966
    0.0010    -0.0009    8.1031
```

errorStudentModelforData2 =

```
3.5229
```



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