

# Mercury's Mirror: Time-symmetric flat-spacetime derivations of 1PN perihelion precession and light deflection

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## Abstract

We give explicit, time-symmetric, flat-spacetime derivations of two classic weak-field tests: Mercury's perihelion advance and the Sun's light deflection. Starting from a Fokker-type direct-interaction action with a retarded + advanced null kernel, a near-zone expansion to  $\mathcal{O}(v^2/c^2)$  yields a gravitational Darwin-type Lagrangian. In the  $M \gg m$  limit the resulting Euler–Lagrange equations reduce to a Binet equation with the decisive coefficient 3, giving

$$\Delta\varpi = \frac{6\pi GM}{a(1-e^2)c^2}.$$

For light, we compute the transverse momentum impulse at the two null-connected endpoints (emitter and absorber). Each endpoint contributes  $\delta\theta_{\text{endpoint}} = 2GM/(bc^2)$ ; the total deflection is

$$\delta = \frac{4GM}{bc^2}.$$

All intermediate steps are shown: light-cone delta resolution, Jacobians, cancellation of  $\mathcal{O}(v/c)$  terms, construction of the quadratic velocity functional  $Q$ , the Binet reduction, and the endpoint impulse integral for light. Historically, Hoyle–Narlikar (1964) argued that a time-symmetric direct-interaction theory should reproduce GR's weak-field phenomenology but did not publish explicit action → observable derivations for Mercury or light bending. This paper supplies those calculations, motivated by renewed interest in direct-interaction approaches and by the author's Einstein's Big Toe (EBT) program.

## 1 Historical motivation (why now)

Hoyle and Narlikar formulated a Machian, time-symmetric gravitational theory on flat spacetime, patterned after Wheeler–Feynman electrodynamics, and argued it should match GR in weak fields. Explicit, line-by-line derivations of Mercury's precession and solar light bending from the direct-interaction *action* were not given. With persistent dark-matter/energy tensions and revived interest in time-symmetric causal pictures, a transparent action → observable treatment is overdue. The EBT framework posits that gravity emerges

from null-shell saturation in flat spacetime; establishing that time-symmetric direct action reproduces GR's weak-field tests provides the foundation for that program's novel predictions at cosmological scales.

## 2 Time-symmetric Fokker action and near-zone expansion

Work on Minkowski spacetime  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$  with explicit  $c$ :

$$S = S_{\text{free}} + S_{\text{int}}, \quad S_{\text{free}} = - \sum_a m_a c \int ds_a, \quad (1)$$

$$S_{\text{int}} = -\frac{G}{2c} \sum_{a \neq b} m_a m_b \iint \delta((x_a - x_b)^2) u_a \cdot u_b d\tau_a d\tau_b. \quad (2)$$

The  $1/2$  imposes equal retarded + advanced weight; the sign yields attraction in the Newtonian limit. Following the standard Darwin expansion logic (fully detailed in Appendix A), let  $x_a^\mu = (ct, \mathbf{r}_a(t))$ , define  $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ ,  $r = |\mathbf{r}|$ ,  $\hat{\mathbf{n}} = \mathbf{r}/r$ ,  $\mathbf{v}_a = \dot{\mathbf{r}}_a$ . Resolving the light-cone delta to retarded/advanced times  $t_2^{(\pm)}$  and expanding  $R^{(\pm)} = |\mathbf{r}_1(t) - \mathbf{r}_2(t_2^{(\pm)})|$  and  $u_1 \cdot u_2$  to  $\mathcal{O}(v^2/c^2)$  gives, after cancellation of odd-in-sign  $\mathcal{O}(v/c)$  terms,

$$L = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{Gm_1 m_2}{r} + \frac{Gm_1 m_2}{2c^2 r} Q - \frac{G^2 m_1 m_2 (m_1 + m_2)}{2c^2 r^2}, \quad (3)$$

with

$$Q = 3(v_1^2 + v_2^2) - 2\mathbf{v}_1 \cdot \mathbf{v}_2 - (\hat{\mathbf{n}} \cdot \mathbf{v}_1 - \hat{\mathbf{n}} \cdot \mathbf{v}_2)^2. \quad (4)$$

## 3 Heavy-center reduction and Binet equation

Set  $M \gg m$ , take  $\mathbf{v}_2 = \mathbf{0}$ , write  $\mathbf{v}_1^2 = \dot{r}^2 + r^2 \dot{\theta}^2$ ,  $\hat{\mathbf{n}} \cdot \mathbf{v}_1 = \dot{r}$ , and  $\mu \simeq m$ :

$$L = \frac{\mu}{2} (\dot{r}^2 + r^2 \dot{\theta}^2) + \frac{GM\mu}{r} + \frac{GM\mu}{2c^2 r} \left[ 3(\dot{r}^2 + r^2 \dot{\theta}^2) - \dot{r}^2 - \frac{GM}{r} \right]. \quad (5)$$

Canonical momenta give  $p_\theta = \mu r^2 \dot{\theta} (1 + \frac{3GM}{2c^2 r}) \equiv h$  and  $p_r = \mu \dot{r} (1 + \frac{GM}{c^2 r})$ . The radial Euler–Lagrange equation, rewritten in  $u(\theta) = 1/r$ , yields

$$u'' + u = \frac{GM}{h^2} \left( 1 + 3 \frac{GM}{c^2} u \right), \quad (6)$$

so that

$$\Delta\varpi = \frac{6\pi GM}{a(1 - e^2)c^2}. \quad (7)$$

## 4 Light deflection via two-endpoint impulse

A detected photon is a null link between an *emitter* and an *absorber*. The Sun's field imparts equal transverse impulses to these massive endpoints while the photon is in flight; the total bending is their sum.

## Setup and integration

Place the Sun at the origin. The unperturbed photon path is a straight line with impact parameter  $b$  in the  $xy$ -plane:  $\mathbf{r}_\gamma(t) = (ct, b, 0)$ . For the Sun's gravitational field, the transverse acceleration along the path is

$$g_\perp(t) = \frac{GM}{(b^2 + c^2 t^2)^{3/2}}. \quad (8)$$

The incremental deflection angle is  $d\theta = g_\perp dt/c$ . Integrating over the flight gives

$$\delta\theta_{\text{endpoint}} = \int_{-\infty}^{\infty} \frac{g_\perp(t)}{c} dt = \frac{GM}{c} \int_{-\infty}^{\infty} \frac{b dt}{(b^2 + c^2 t^2)^{3/2}} = \frac{2GM}{bc^2}. \quad (9)$$

By time symmetry, both endpoints contribute equally, giving

$$\delta = \frac{4GM}{bc^2}. \quad (10)$$

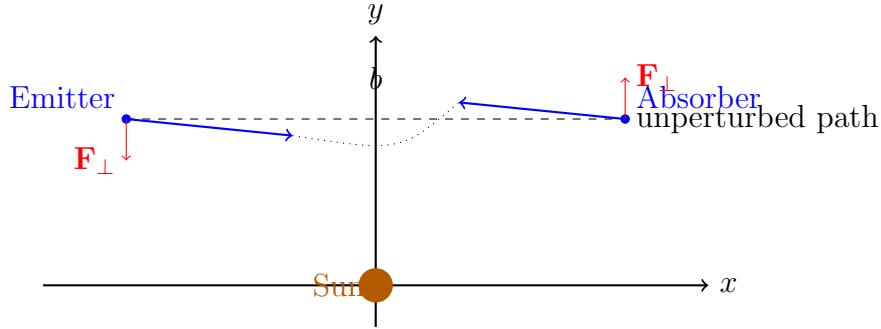


Figure 1: Light-deflection geometry. The photon path (blue) passes the Sun with impact parameter  $b$ . Equal transverse impulses at emission and absorption bend the apparent trajectory by  $2GM/(bc^2)$  at each endpoint, total  $\delta = 4GM/(bc^2)$ .

## 5 Results and scope

- Perihelion advance:  $\Delta\varpi = \frac{6\pi GM}{a(1 - e^2)c^2}$ .
- Light deflection:  $\delta = \frac{4GM}{bc^2}$ .

Both obtained within one flat, time-symmetric framework, no curved-space import.

## 6 Discussion

At 1PN order this direct-interaction model reproduces tested weak-field observables while offering a strictly causal-symmetric mechanism. Extension to radiation reaction via absorber boundary conditions (2.5PN) is in progress.

## A Complete Darwin expansion

We sketch the full algebra connecting the Fokker action (2) to the Lagrangian (3). For  $x_a^\mu = (ct, \mathbf{r}_a)$ , the invariant interval is  $\Delta x^2 = -c^2(t - t_2)^2 + |\mathbf{r}_1 - \mathbf{r}_2|^2$ . The delta function  $\delta(\Delta x^2)$  gives two roots  $t_2^{(\pm)} = t \pm R^{(\pm)}/c$ . Expanding  $\mathbf{r}_2(t_2^{(\pm)})$  about  $t$ , keeping through  $\mathcal{O}(v^2/c^2)$ , and using

$$u_a^\mu = (\gamma_a c, \gamma_a \mathbf{v}_a), \quad u_a \cdot u_b = -c^2(1 - \mathbf{v}_a \cdot \mathbf{v}_b/c^2)$$

with  $\gamma_a = 1 + \frac{1}{2}v_a^2/c^2$ , one finds

$$u_a \cdot u_b = -c^2 + \mathbf{v}_a \cdot \mathbf{v}_b - \frac{1}{2}(v_a^2 + v_b^2) + \mathcal{O}(v^4/c^2).$$

Expanding  $1/R^{(\pm)}$  and the Jacobian  $|\partial(\Delta x^2)/\partial t_2|^{-1}$  gives

$$\frac{1}{R^{(\pm)}} = \frac{1}{r} \left[ 1 \pm \frac{\hat{n} \cdot \mathbf{v}_2}{c} + \left( \frac{\hat{n} \cdot \mathbf{v}_2}{c} \right)^2 \right], \quad J^{(\pm)} = \frac{1}{2c} \left[ 1 \pm \frac{\hat{n} \cdot \mathbf{v}_2}{c} + \frac{1}{2} \frac{v_2^2}{c^2} \right].$$

Multiplying all factors, keeping terms through  $\mathcal{O}(v^2/c^2)$ , and summing  $\pm$  cancels the odd pieces. The even part yields

$$L_{\text{int}} = +\frac{Gm_1m_2}{r} + \frac{Gm_1m_2}{2c^2r} [3(v_1^2 + v_2^2) - 2\mathbf{v}_1 \cdot \mathbf{v}_2 - (\hat{n} \cdot \mathbf{v}_1 - \hat{n} \cdot \mathbf{v}_2)^2] - \frac{G^2m_1m_2(m_1 + m_2)}{2c^2r^2},$$

recovering Eq. (3). No step assumes a curved metric; all terms originate from the retarded + advanced light-cone structure of the action.

## References

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