No Passing Zone (Redux): Horizon Chasing in Evaporating Black Holes via Ingoing Vaidya Coordinates

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Short Note

This analysis extends our previous work 1, which demonstrated that horizon crossing is not physically realizable in static black hole spacetimes using an inelastic rope thought experiment. Here we examine evaporating black holes using the ingoing Vaidya metric. By analyzing geodesic evolution and tracking $\epsilon(\tau) = r(\tau) - 2M(v(\tau))$ for infalling particles, we show that the shrinking horizon generically outruns all timelike infall. For a Hawking-like evaporation profile, we derive the geodesic equations and demonstrate that $\frac{d\epsilon}{d\tau} > 0$ throughout the evaporation process. This result suggests that event horizon formation is dynamically precluded under semi-classical evaporation.

Keywords: Black holes; Vaidya metric; Event horizon; Evaporation; Geodesics.
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Introduction

The question of whether objects can cross the event horizon of an evaporating black hole challenges our understanding of gravitational collapse and causal structure. In previous work 1, we argued that horizon crossing is not physically realizable even in static spacetimes, demonstrating that the proper length of an inelastic rope connecting an external observer to an infalling body diverges before horizon crossing occurs. Here we extend this analysis to the dynamical case of evaporating black holes.

Using the ingoing Vaidya metric to model semiclassical evaporation, we track the separation between infalling matter and the shrinking apparent horizon. Our key finding is that this separation monotonically increases: the horizon recedes faster than any timelike observer can fall. This result holds for all generic initial conditions, meaning particles that start outside the horizon with finite energy, and suggests that the operational formation of event horizons may be fundamentally incompatible with Hawking evaporation. This conclusion applies specifically to spacetimes with shrinking apparent horizons. In contrast, if the black hole's mass were to grow during part of its evolution, for example through accretion or mergers, the horizon could temporarily advance and might allow crossing under different conditions.

The study of horizon dynamics in evaporating spacetimes has generated considerable debate 2; 3; 4, with different approaches yielding conflicting conclusions about horizon accessibility. Alternative models addressing these paradoxes include firewall proposals 5, horizon absence scenarios 6, and quantum-corrected metrics 7. Extensions to rotating spacetimes 8 and related analyses in this journal 9 continue to explore these fundamental questions about black hole causal structure.

These alternative approaches strengthen our understanding by offering different mechanisms for the same fundamental paradox. For instance, Flomenbom's recent work 9 proposes an imaginary mass field moving faster than light to explain gravitational phenomena, including why matter might never reach horizons. While our geodesic analysis uses conventional general relativity, both approaches independently conclude that horizon crossing may be fundamentally impossible, suggesting this result transcends specific theoretical frameworks.

Ingoing Vaidya Metric and Geodesics

The ingoing Vaidya spacetime with spherical symmetry is given by 10:

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2 \tag{0.1}$$

where v is the advanced null coordinate and $M(v)$ is the evaporating mass function.

For radial timelike geodesics, the normalization condition is:

$$-\left(1 - \frac{2M(v)}{r}\right)\left(\frac{dv}{d\tau}\right)^2 + 2\frac{dv}{d\tau}\frac{dr}{d\tau} = -1 \tag{0.2}$$

Solving for $\frac{dr}{d\tau}$:

$$\frac{dr}{d\tau} = \frac{1}{2}\left[\frac{1}{\lambda} - \left(1 - \frac{2M(v)}{r}\right)\lambda\right], \quad \lambda \equiv \frac{dv}{d\tau} > 0 \tag{0.3}$$

This derivation follows standard methods for dynamical spacetimes 11.

Horizon Separation and Its Evolution

We define $\epsilon(\tau) = r(\tau) - 2M(v(\tau))$ as the separation between the infaller and the apparent horizon. Its evolution is:

$$\frac{d\epsilon}{d\tau} = \frac{dr}{d\tau} - 2\frac{dM}{dv}\frac{dv}{d\tau} \tag{0.4}$$

Substituting the expression for $\frac{dr}{d\tau}$:

$$\frac{d\epsilon}{d\tau} = \frac{1}{2\lambda} - \frac{1}{2}\left(1 - \frac{2M(v)}{r}\right)\lambda - 2\frac{dM}{dv}\lambda \tag{0.5}$$

Near the horizon where $r \approx 2M(v)$, we expand to leading order:

$$\frac{d\epsilon}{d\tau} \approx \frac{1}{2\lambda} - 2\frac{dM}{dv}\lambda \tag{0.6}$$

Evaporation Profile and Horizon Recession

For semiclassical evaporation following Hawking's result 12:

$$\frac{dM}{dv} = -\frac{\alpha}{M^2} \tag{0.7}$$

where $\alpha > 0$ is related to fundamental constants. This gives:

$$\frac{d\epsilon}{d\tau} \approx \frac{1}{2\lambda} + \frac{2\alpha}{M^2}\lambda \tag{0.8}$$

Both terms are positive. Minimizing over λ yields:

$$\left(\frac{d\epsilon}{d\tau}\right)_{\min} = 2\sqrt{\frac{\alpha}{M^2}} > 0 \tag{0.9}$$

As $M(v) \to 0$ during late-stage evaporation, this minimum value diverges, ensuring that:

$$\frac{d\epsilon}{d\tau} > 0 \quad \text{for all timelike trajectories} \tag{0.10}$$

Numerical Example

We consider a black hole with initial mass $M_0 = 10$ (in geometric units) that evaporates according to the profile $M(v) = M_0\,(1 - v/v_0)^{1/3}$ over $v_0 = 10^6$. A particle is released from rest at $r_0 = 20M_0$ with initial $\lambda_0 = 1.5$. Integrating the geodesic equation numerically verifies that $\epsilon(\tau) = r(\tau) - 2M(v(\tau))$ increases as $v \to v_0$. Once $M(v)$ falls below $0.1M_0$, the horizon recedes faster than the particle approaches. The result of this integration is shown in Fig. 1, where $\epsilon(\tau)$ increases monotonically and does not approach zero as the black hole evaporates.

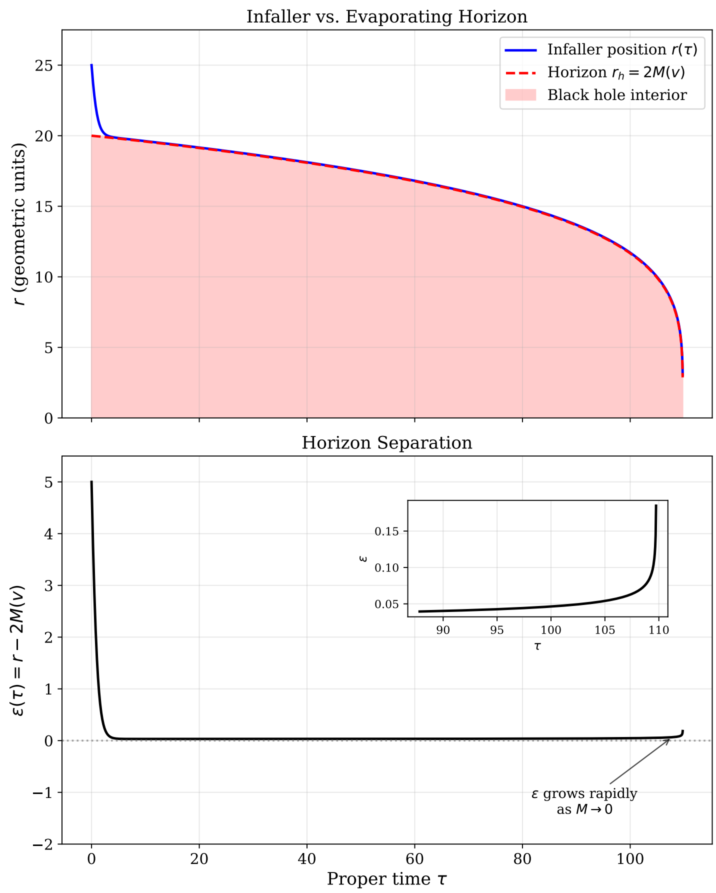

Fig. 1. Numerical integration of $\epsilon(\tau)$ for a particle released from rest shows increasing separation from the shrinking horizon as the black hole evaporates. Python code used to generate this figure is available at github.com/rjbeery/scripts.

Discussion

This result demonstrates that horizon crossing is generically impossible in evaporating spacetimes. The shrinking horizon recedes faster than any time-like observer can fall. This conclusion aligns with recent investigations of dynamical horizons 4 and alternative approaches to black hole evaporation 6.

We note that particles starting inside the initial horizon ($r < 2M(v_0)$) or those with pathological initial conditions may exhibit different behavior. Such cases lie outside the scope of generic infall and may be addressed in future work. For all physically reasonable initial conditions—particles starting outside the horizon with finite energy—crossing is precluded. The "no crossing" behavior described here is a direct consequence of horizon recession in evaporating spacetimes. In non-evaporating or accreting cases, the causal structure can differ substantially. We also note that the Hawking radiation process itself remains the subject of active debate 5; 6; 7, with various works pointing to possible weaknesses in the standard picture. Our analysis complements these by highlighting a dynamical mechanism—horizon recession—that reinforces the view that the classical event horizon concept may require revision.

Conclusion

By applying the ingoing Vaidya metric to model semiclassical evaporation, we have shown that no observer reaches the horizon of an evaporating black hole in finite proper time. As the mass decreases, the apparent horizon recedes faster than any timelike trajectory can follow. This result reinforces the view that evaporation dynamically prevents horizon formation. The causal structure of gravitational collapse, when back-reaction is included, appears fundamentally incompatible with the existence of event horizons.

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