Noninteracting, Zero temperature

$\hat{h}_j$ operator: $\hat{h}_j = -J(\hat{c}_j^\dagger \hat{c}_{j+1} + \hat{c}_{j+1}^\dagger \hat{c}_j)$

Initial Hamiltonian: $\hat{P} = \sum_{j=0}^{L-1} j\hat{h}_j = -J\sum_{j=0}^{L-1} j(\hat{c}_j^\dagger \hat{c}_{j+1} + \hat{c}_{j+1}^\dagger \hat{c}_j)$

Post-quench Hamiltonian: $\hat{H} = \sum_{j=0}^{L-1} \hat{h}_j = \sum_{j=0}^{L-1} -J(\hat{c}_j^\dagger \hat{c}_{j+1} + \hat{c}_{j+1}^\dagger \hat{c}_j)$

Emergent Hamiltonian: $\mathcal{H}(t) = \hat{P} + t\hat{Q} = \sum_{j=0}^{L-1} \left[j\hat{c}_j^\dagger \hat{c}_j - t(i\hat{c}_{j+1}^\dagger \hat{c}_j - i\hat{c}_j^\dagger \hat{c}_{j+1}) \right]$

Analytical single particle correlation solution ($m \neq n$): $C_{mn} = i^{n-m} \frac{t(J_m(2t)J_{n+1}(2t) - J_{m+1}(2t)J_n(2t))}{n-m}$

Analytical single particle correlation solution ($m = n$):

Effective single particle correlation solution: $C_{mn} = \sum_{j=1}^{N} \phi_{q_j}^*(m)\phi_{q_j}(n)$ ($\{\phi_q\}$ is the set of $N = 1$ energy eigenstates)

Noninteracting, Infinite Temperature

Initial Hamiltonian, current operator, emergent Hamiltonian are the same as in the $T = 0$ case.

Initial (mixed) state:
The full set of computational basis states contains each arrangement of N particles along length $L = 4N$ lattice. Extract basis states in which all N particles are confined to the middle half of the lattice. Initialize initial state $|\psi_0\rangle$ as en empty list. Convert these binary strings into normalized lists and append each to $|\psi_0\rangle$.

Effective (mixed) state:
Diagonalize emergent Hamiltonian. For each normalized computational basis state in initial state $|\psi_0\rangle$, for each site entry in this basis state, if the entry is not zero, append the eigenstate (list) from the emergent Hamiltonian corresponding to that entry to a list. Append that 2d list to the mixed state list.

Interacting (t-V)

$\hat{h}_j$ operator: $\hat{h}_j = -J(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + V(c_j^\dagger c_j - 1/2)(c_{j+1}^\dagger c_{j+1} - 1/2)$

Initial Hamiltonian:

$$\begin{aligned}\hat{P} &= \sum_{j=0}^{L-1} j\hat{h}_j \\ &= \sum_{j=0}^{L-1} j\left(J(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + V(c_j^\dagger c_j - 1/2)(c_{j+1}^\dagger c_{j+1} - 1/2)\right)\end{aligned}$$

Current operator:

$$\hat{Q} = \sum_j \left[iJ^2(c_{j+2}^\dagger c_j - c_j^\dagger c_{j+2}) - iV\left((c_{j+1}^\dagger c_j - c_j^\dagger c_{j+1})(n_{j+2} - 1/2) + (c_{j+2}^\dagger c_{j+1} - c_{j+1}^\dagger c_{j+2})(n_j - 1/2)\right)\right]$$

Post-quench Hamiltonian:

$$\begin{aligned}\hat{H} &= \sum_{j=0}^{L-1} \hat{h}_j \\ &= \sum_{j=0}^{L-1} \left[-J(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + V(c_j^\dagger c_j - 1/2)(c_{j+1}^\dagger c_{j+1} - 1/2)\right]\end{aligned}$$

Emergent Hamiltonian: $\mathcal{H}(t) = \hat{P} + t\hat{Q}$

Exact single particle correlation solution: $C_{ij} = \sum_{\alpha,\beta}^{\binom{L}{N}} \sum_{m,n}^{\binom{L}{N}} e^{it(E_\alpha - E_\beta)} \langle\alpha|\psi_0\rangle\langle\psi_0|\beta\rangle \alpha^*(m)\beta(n)\langle m|\hat{c}_i^\dagger \hat{c}_j|n\rangle$

$|\alpha\rangle, |\beta\rangle$ denote energy eigenstates obtained via exact diagonalization of the post-quench Hamiltonian

Effective single particle correlation solution: $C_{ij} = \sum_{m,n}^{\binom{L}{N}} \phi(m)\phi^*(n)\langle m|\hat{c}_i^\dagger \hat{c}_j|n\rangle$