# Machine Learning - Mini Project 1

# Richard Campo in collaboration with Julia Klauss

```
import pandas as pd
import numpy as np
import os
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as sm
```

## Part 3: Preparing the Data

```
In [103...
          PATH = r"C:\Users\RichardCampo\Documents\GitHub\Machine-Learning\Mini Project 1"
          df_acs = pd.read_csv(os.path.join(PATH, "usa_00001.csv"))
          df_acs.head()
In [104...
Out[104...
             YEAR SAMPLE SERIAL
                                                  HHWT
                                                                CLUSTER STRATA GQ PERNUN
                                         CBSERIAL
              2022
                     202201
                              1295 2022010074840 10800.4 2022000012951
                                                                           80001
                                                                                    4
                     202201
              2022
                              4519 2022000095945 32078.8
                                                          2022000045191
                                                                           70001
              2022
                     202201
                              4519 2022000095945 32078.8 2022000045191
                                                                                    1
                                                                           70001
              2022
                     202201
                              4841 2022000116058
                                                   9672.0 2022000048411
                                                                           60001
```

 $5 \text{ rows} \times 26 \text{ columns}$ 

202201

2022

5002 2022000126199 94946.8 2022000050021

260001

1

Out[106		YEAR	SAMPLE	SERIAL	CBSERIAL	HHWT	CLUSTER	STRATA	GQ	PERNUN
	0	2022	202201	1295	2022010074840	10800.4	2022000012951	80001	4	
	1	2022	202201	6453	2022000222947	13218.4	2022000064531	180101	1	
	2	2022	202201	6937	2022000255097	10639.2	2022000069371	10001	1	
	3	2022	202201	11773	2022000589219	5642.0	2022000117731	250001	1	
	4	2022	202201	14513	2022000775583	15152.8	2022000145131	120101	1	
	5 rc	ows × 2	7 columns							
In [107	<pre># codes 62 - 100 include people with a HS degree but no BA  df_acs["HSDIP"] = np.where(         (62 &lt;= df_acs["EDUCD"]) &amp; (df_acs["EDUCD"] &lt;= 100),         1,         0 )  # codes 101 - 116 are college grads and higher, excluding 999 NAs  df_acs["COLDIP"] = np.where(         (101 &lt;= df_acs["EDUCD"]) &amp; (df_acs["EDUCD"] &lt;= 116),         1,         0 )</pre>									
In [108	df df df df	_acs["E _acs["H _acs["N _acs["N	BLACK"] = HISPANIC"; MARRIED"] FEMALE"] =	np.wher ] = np.wh = np.whe	e(df_acs["RACE" e(df_acs["RACE" here(df_acs["H] ere(df_acs["MAR re(df_acs["SEX" df_acs["VETSTAT	[] == 2, [SPAN"].i [ST"].isi [] == 2,	1, 0) sin([1, 2, 3, 4) n([1, 2]), 1, (1, 0)		))	
In [109				_	np.multiply(df_ np.multiply(df	_				
In [110	df <sub>.</sub>	_acs["/	AGESQ"] =	np.powe	r(df_acs["AGE"]	, 2)				
	#	drop vo	alues when	re INCWA	GE is 0 because	ln(0) i	s undefined			

# Part 4: Data Analysis

df\_acs = df\_acs[df\_acs["INCWAGE"] != 0]

df\_acs["INCWAGE\_LOG"] = np.log(df\_acs["INCWAGE"])

```
In [111...
columns = ["YEAR", "INCWAGE", "INCWAGE_LOG", "EDUCDC", "FEMALE", "AGE",
"AGESQ", "WHITE", "BLACK", "HISPANIC", "MARRIED", "NCHILD", "VET", "HSDIP",
"COLDIP", "HSDIP-EDUCDC", "COLDIP-EDUCDC"]
```

# df\_acs[columns].describe()

Out[111...

	YEAR	INCWAGE	INCWAGE_LOG	EDUCDC	FEMALE	AGE	i
count	8376.0	8376.000000	8376.000000	8376.000000	8376.000000	8376.000000	8376.
mean	2022.0	66839.484241	10.611709	14.276982	0.484002	41.548352	1899.
std	0.0	80757.732310	1.133255	3.052954	0.499774	13.151301	1100.
min	2022.0	20.000000	2.995732	0.000000	0.000000	18.000000	324.
25%	2022.0	25000.000000	10.126631	12.000000	0.000000	31.000000	961.
50%	2022.0	47000.000000	10.757903	14.000000	0.000000	41.000000	1681.
75%	2022.0	80000.000000	11.289782	16.000000	1.000000	53.000000	2809.
max	2022.0	761000.000000	13.542389	22.000000	1.000000	65.000000	4225.

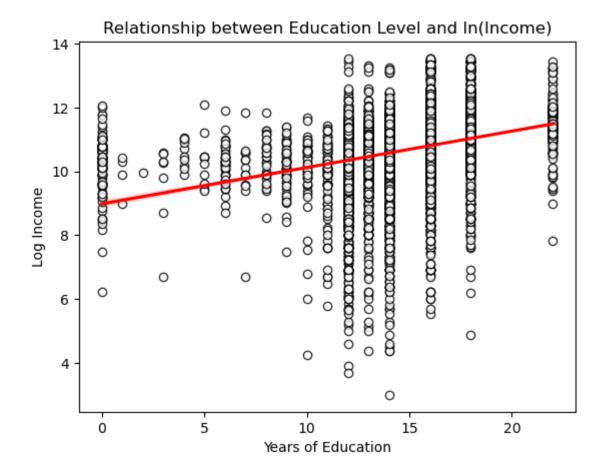
Q2

```
In [112... fig, ax = plt.subplots()

sns.regplot(
    df_acs,
    x = "EDUCDC",
    y = "INCWAGE_LOG",
    scatter_kws={'color': 'white', 'edgecolors': 'black'},
    line_kws={'color': 'red'},
    ax=ax
    )

ax.set_xlabel("Years of Education")
ax.set_ylabel("Log Income")
ax.set_title("Relationship between Education Level and In(Income)")
```

Out[112... Text(0.5, 1.0, 'Relationship between Education Level and ln(Income)')



OLS Regression Results							
Dep. V	ariable:	INCWAG	E_LOG	R-	squared:	0.	290
Model:			OLS	Adj. R-	0.	289	
IV	lethod:	Least Squares		F-	statistic:	34	41.2
	Date:	Tue, 23 Jai	n 2024	Prob (F-s	statistic):	(	0.00
	Time:	14	1:44:52	Log-Lik	elihood:	-115	500.
No. Observ	ations:		8376		AIC:	2.302e	+04
Df Res	siduals:		8365		BIC:	2.310e	+04
Df	Model:		10				
Covarianc	e Type:	non	robust				
	coef	std err		t P> t	[0.025	0.975]	
const	5.7254	0.125	45.94		5.481	5.970	
EDUCDC	0.0976	0.004	27.13	0.000	0.091	0.105	
FEMALE	-0.4060	0.021	-19.06	6 0.000	-0.448	-0.364	
AGE	0.1685	0.006	27.43	7 0.000	0.156	0.181	
AGESQ	-0.0018	7.27e-05	-24.43	2 0.000	-0.002	-0.002	
WHITE	-0.0307	0.029	-1.05	0.294	-0.088	0.027	
BLACK	-0.2140	0.047	-4.59	9 0.000	-0.305	-0.123	
HISPANIC	-0.0714	0.035	-2.02	3 0.043	-0.141	-0.002	
MARRIED	0.2009	0.025	8.12	6 0.000	0.152	0.249	
NCHILD	-0.0076	0.011	-0.69	4 0.488	-0.029	0.014	
VET	0.0660	0.054	1.22	7 0.220	-0.039	0.171	
Omn	nibus: 27	754.229	Durbin-	Watson:	1.9	00	
Prob(Omni					13819.3		
	Skew:	-1.507	-	rob(JB):		00	
Kur	tosis:	8.524		ond. No.			

#### Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 2.64e+04. This might indicate that there are strong multicollinearity or other numerical problems.

- (a.) We can see from the  $\mathbb{R}^2$  that the model explains 29% of the variation in In(income).
- (b.) The model predicts that an additional year of education will increase expected income by 9.76%, all else constant. We can see from the very low p-value that the expected effect of years of education on income is statistically significant at the 5% level. Thus, we can reject the null hypothesis in favor of the alternative hypothesis that the effect of an additional year of education on income is different from 0.

This amount is also practically significant since a 9.76% boost to income is meaningful for nearly everyone. For example, we predict that an additional year of education would raise the income of someone making 50,000to54,880, all else constant.

(c.) The model predicts that an individual will receive the highest wage at age:

$$rac{-\hat{eta}_{AGE}}{2\hat{eta}_{AGESQ}} = rac{-0.1685}{2(-0.0018)} = 46.8$$

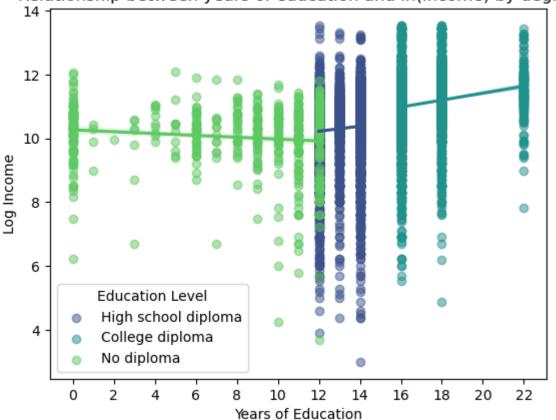
In other words, we expect someone in the sample to earn their highest wage at about age 47.

- (d.) All else equal, the model predicts that men will have higher wages because the estimated coefficient of the sex dummy variable "FEMALE" is negative. This could be because of discrimination, meaning women are paid less than men because employers choose to pay women less. Alternatively, this could be because of omitted variable bias, meaning there is a confounding variable that is correlated with being a woman that is a predictor of lower wages. Examples of possible confounding variables include industry, occupation, and number of hours worked. There is also an argument that we should not control for these factors since discrimination could push women toward less lucrative industries and occupations, and so the result we are getting is closer to the true wage penalty women receive due to discrimination.
- (e.) The estimated coefficient of "WHITE" is -0.0307, meaning we predict that being white decreases wages by about 3%. Since the p-value is 0.294 > 0.05, we fail to reject the null hypothesis that being white has no effect on expected income at a 5% significance level. As the hint in the problem set implies, this could be because many Hispanic Americans classify themselves as white since "Hispanic" is an ethnicity, not a race, according to the US Census. If we only looked at non-Hispanic white Americans, we might see a positive expected effect on wages due to discrimination in favor of white Americans.

The estimated coefficient of "BLACK" is -0.2140, meaning we predict that being black decreases wages by about 21%. Since the p-value is nearly 0 < 0.05, we can reject the null hypothesis in favor of the alternative hypothesis that the expected effect of being black on wages is different from 0. This could be because of discrimination against black workers or omitted variables such as parents' income, wealth, and access to social networks. Likely it's some combination of both.

```
In [114...
          fig, ax = plt.subplots()
          colors = sns.color_palette("viridis", 3)
          sns.regplot(
              data=df_acs[df_acs['HSDIP'] == 1],
              x="EDUCDC",
              y="INCWAGE_LOG",
              scatter_kws={'alpha': 0.5, 'color': colors[0]},
              line_kws={'color': colors[0]},
              ax=ax,
              label='High school diploma'
          sns.regplot(
              data=df_acs[df_acs['COLDIP'] == 1],
              x="EDUCDC",
              y="INCWAGE_LOG",
              scatter_kws={'alpha': 0.5, 'color': colors[1]},
              line_kws={'color': colors[1]},
              ax=ax,
              label='College diploma'
              )
          sns.regplot(
              data=df_acs[(df_acs['HSDIP'] == 0) & (df_acs['COLDIP'] == 0)],
              x="EDUCDC",
              y="INCWAGE_LOG",
              scatter_kws={'alpha': 0.5, 'color': colors[2]},
              line_kws={'color': colors[2]},
              ax=ax,
              label='No diploma'
          ax.set_xlabel("Years of Education")
          ax.set_ylabel("Log Income")
          # https://stackoverflow.com/a/12608937
          ax.set_xticks(
              range(int(min(df_acs['EDUCDC'])), int(max(df_acs['EDUCDC'])) + 1, 2)
          ax.set_title("Relationship between years of education and ln(income) by degree")
          ax.legend(title="Education Level", loc='best')
          plt.show()
```





## Q5

A model that will allow the predicted In(income) to vary by degree is:

$$\begin{split} \ln(\text{INCWAGE}) &= \beta_0 + \beta_1 \text{EDUCDC} + \beta_2 \text{FEMALE} + \beta_3 \text{AGE} \\ &+ \beta_4 \text{AGESQ} + \beta_5 \text{WHITE} + \beta_6 \text{BLACK} + \beta_7 \text{HISPANIC} \\ &+ \beta_8 \text{MARRIED} + \beta_9 \text{NCHILD} + \beta_{10} \text{VET} + \beta_{11} \text{HSDIP} \\ &+ \beta_{12} \text{COLDIP} + \beta_{13} \text{HSDIP-EDUCDC} + \beta_{14} \text{COLDIP-EDUCDC} \\ &+ \epsilon \end{split}$$

This model allows for both differential intercepts and differential slopes by diploma achieved by including both the dummy variables and their interaction effects. The coefficients of HSDIP and COLDIP allow the intercepts for people with a high school diploma or college diploma to vary, where  $\beta_0$  is the intercept for people with no diploma,  $\beta_0 + \beta_{11}$  is the intercept for people with a high school diploma, and  $\beta_0 + \beta_{12}$  is the intercept for people with a college diploma. This way, each level of degree can shift predicted income up, reflecting the effect of receiving a diploma.

The differential slopes are achieved thanks to the interaction effects. The slope of EDUCDC for people with no high school diploma is  $\beta_1$ , the slope for people with a high school diploma is  $\beta_1 + \beta_{13}$ , and the slope for people with a college diploma is  $\beta_1 + \beta_{14}$ . This way, each level of degree can have a steeper slope reflecting the increasing returns to higher levels of education.

You could argue that this is the best possible model of how the world works if the true relationships between years of education and log income are linear. This way, we know we know our model is closely fitting the data, so we are accurally predicing log income. If the true relationship is linear, then adding additional polynomial terms could cause overfitting, which would result in poor predictions on a different sample.

Dep. Variable	: INCW	AGE_LOG	R	-square	ed:	0.312
Model		OLS	Adj. R	-square	ed:	0.311
Method	Leas	t Squares	F	-statist	ic:	271.0
Date	: Tue, 23	Jan 2024	Prob (F-	statisti	c):	0.00
Time	:	14:45:02	Log-Li	kelihoo	od: -	11365.
No. Observations	:	8376		Α	IC: 2.27	'6e+04
Df Residuals	:	8361		В	IC: 2.28	37e+04
Df Model	:	14				
Covariance Type	: n	onrobust				
	coef	std err	t	P> t	[0.025	0.975]
const	6.8402	0.144	47.435	0.000	6.558	7.123
EDUCDC	-0.0095	0.009	-1.044	0.297	-0.027	0.008
FEMALE	-0.4175	0.021	-19.854	0.000	-0.459	-0.376
AGE	0.1565	0.006	25.684	0.000	0.145	0.168
AGESQ	-0.0016	7.2e-05	-22.771	0.000	-0.002	-0.001
WHITE	-0.0039	0.029	-0.134	0.893	-0.060	0.053
BLACK	-0.1592	0.046	-3.466	0.001	-0.249	-0.069
HISPANIC	-0.0515	0.035	-1.479	0.139	-0.120	0.017
MARRIED	0.1810	0.024	7.421	0.000	0.133	0.229
NCHILD	-0.0040	0.011	-0.373	0.709	-0.025	0.017
VET	0.0859	0.053	1.620	0.105	-0.018	0.190
HSDIP	-0.7817	0.214	-3.647	0.000	-1.202	-0.362
COLDIP	-0.3085	0.207	-1.494	0.135	-0.713	0.096
HSDIP-EDUCDC	0.0904	0.018	5.083	0.000	0.056	0.125
COLDIP-EDUCDC	0.0809	0.014	5.598	0.000	0.053	0.109
Omnibus:	2904.504	Durbin	-Watson:		1.956	
Prob(Omnibus):	0.000		Bera (JB):			
Skew:	-1.592	•	Prob(JB):		0.00	
Kurtosis:	8.701	(	Cond. No.	4.99	e+04	

#### Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 4.99e+04. This might indicate that there are strong multicollinearity or other numerical problems.

(a.)

```
In [116...
          hs_predict_dat = df_acs.query(
              "AGE == 22 and "
              "FEMALE == 1 and "
              "WHITE == 0 and "
              "BLACK == 0 and "
              "HISPANIC == 0 and "
              "MARRIED == 0 and "
              "NCHILD == 0 and "
              "VET == 0 and "
              "HSDIP == 1 and "
              "EDUCDC == 12"
          )[predictors]
          # https://stackoverflow.com/a/36533181
          hs_predict_dat = sm.add_constant(hs_predict_dat, has_constant="add")
          hs_prediction = model.get_prediction(hs_predict_dat)
          hs_prediction.summary_frame(alpha=0.05)[:1]
```

Out[116...

	mean	mean_se	mean_ci_lower	mean_ci_upper	obs_ci_lower	obs_ci_upper
1133	9.26157	0.039964	9.183231	9.339909	7.415901	11.107239

The model predicts that a 22 year old woman who is neither black, white, nor Hispanic, is not married, has no children, and is not a veteran with a high school diploma will earn  $e^{9.26157}$ , or about \$10,526 annually.

We could also calculate this manually:

```
6.8402 + 12(-0.0095 + 0.0904) - 0.4175 + (22)(0.1565) - (22^2)(0.0016) - 0.7817 = 9.2 or about e^{9.2804} equals $10,726 annually.
```

The difference between the predictions may be due to rounding.

```
In [117...
col_predict_dat = df_acs.query(
          "AGE == 22 and "
          "FEMALE == 1 and "
          "WHITE == 0 and "
          "BLACK == 0 and "
          "HISPANIC == 0 and "
          "MARRIED == 0 and "
```

```
"NCHILD == 0 and "
"VET == 0 and "
"COLDIP == 1 and "
"EDUCDC == 16"
)[predictors]

col_predict_dat = sm.add_constant(col_predict_dat, has_constant="add")

col_prediction = model.get_prediction(col_predict_dat)

col_prediction.summary_frame(alpha=0.05)[:1]
```

Out[117...

	mean	mean_se	mean_ci_lower	mean_ci_upper	obs_ci_lower	obs_ci_upper
4649	9.905595	0.039356	9.828447	9.982744	8.059976	11.751214

The model predicts that a 22 year old woman who is neither black, white, nor Hispanic, is not married, has no children, and is not a veteran with a college diploma will earn  $e^{9.905595}$ , or about \$20,042 annually.

If we calculate the prediction manually:

```
6.8402 + 16(-0.0095 + 0.0809) - 0.4175 + (22)(0.1565) - (22^2)(0.0016) - 0.3085 = 9.9 or about e^{9.9252} equals $20,439 annually.
```

The difference between the predictions again may be due to rounding.

(b.)

We can tell the president that people with college degrees do have higher predicted wages than people without college degrees, all else constant. Using our model's predictions, from part (a), a 22 year old woman who is neither black, white, nor Hispanic, is not married, has no children, and is not a veteran with a college diploma will earn 20,042-10,526=\$9,516 more than a similar woman with only a high school diploma.

If we look back to the regression output, we can see that college graduates have a higher intercept than high school graduates but a slightly shallower slope, though this difference is likely not statistically significant since the confidence intervals have substantial overlap. The higher intercept for college graduates indicates that they have higher salaries on average than high school graduates, all else constant, though a large portion of their higher earnings comes from simply having 4 more years of education than high school graduates.

(c.)

I don't think we have enough evidence from this model to say whether increasing student loan subsidies would be a good idea or not, because we still have omitted variable bias. People in our sample who go to college are likely fundamentally different from people who do not go to college in terms of ambition, motivation, ability, and other hard to measure

variables. Thus, we cannot say for sure that getting a college degree has a causal effect on wages. In other words, people who go to college might have made more money than people who do not even if they had not gone to college.

I would advise the president that a more thorough analysis using methods that allow for causal inference would be needed to determine whether increasing student loan subsidies is the correct policy decision.

(d.)

The model explains 31.2% of the variation in log wages according to the  $R^2$  value. This is a bit higher than the 29.0% of variation explained in the first model without interaction terms, but we still are only explaining a fraction of the total variation in log wages. Ideally, we would want to add more variables such as years of experience, industry, and occupation to get a better fit.

(e.)

The multiple regression above is pretty simple, so I would not take the predictions too seriously. If we want to make better predictions, we should include more variables so we can explain more of the variation in log income, and we would want to test out different functional forms with additional polynomial terms and interaction terms. Making these additions could improve our adjusted  $\mathbb{R}^2$ , meaning we are explaining more of the variation in log income. This way, our model can fit the data better and we can make more accurate predictions.

```
In [134...
          df_acs["NCHILD-MARRIED"] = np.multiply(df_acs["NCHILD"], df_acs["MARRIED"])
          df_acs["COLDIP-VET"] = np.multiply(df_acs["COLDIP"], df_acs["VET"])
          df_acs["FEMALE-HSDIP"] = np.multiply(df_acs["FEMALE"], df_acs["HSDIP"])
          df_acs["NCHILD-FEMALE"] = np.multiply(df_acs["NCHILD"], df_acs["FEMALE"])
          df acs["HSDIP-BLACK"] = np.multiply(df_acs["HSDIP"], df_acs["BLACK"])
          df_acs["HSDIP-HISPANIC"] = np.multiply(df_acs["HSDIP"], df_acs["HISPANIC"])
          df_acs["AGE-NCHLT5"] = np.multiply(df_acs["AGE"], df_acs["NCHLT5"])
          df_acs["FEMALE-WHITE"] = np.multiply(df_acs["FEMALE"], df_acs["WHITE"])
          df_acs["EDUCDCSQ"] = np.power(df_acs["EDUCDC"], 2)
          df_acs["EDUCDCCUB"] = np.power(df_acs["EDUCDC"], 3)
          predictors = ["EDUCDC", "FEMALE", "AGE", "AGESQ", "WHITE", "BLACK", "HISPANIC",
          "MARRIED", "NCHILD", "VET", "HSDIP", "COLDIP", "HSDIP-EDUCDC", "COLDIP-EDUCDC",
          "NCHLT5", "NCHILD-MARRIED", "COLDIP-VET", "FEMALE-HSDIP", "EDUCDCSQ",
          "EDUCDCCUB", "NCHILD-FEMALE", "HSDIP-BLACK", "HSDIP-HISPANIC",
          "AGE-NCHLT5", "FEMALE-WHITE"]
          X = df_acs[predictors]
          X = sm.add_constant(X)
          y = df_acs["INCWAGE_LOG"]
```

```
model = sm.OLS(y, X).fit()
model.summary()
```

## **OLS Regression Results**

Dep. Variable:	INCWAGE_LOG	R-squared:	0.321
Model:	OLS	Adj. R-squared:	0.319
Method:	Least Squares	F-statistic:	157.8
Date:	Tue, 23 Jan 2024	Prob (F-statistic):	0.00
Time:	14:56:59	Log-Likelihood:	-11312.
No. Observations:	8376	AIC:	2.268e+04
Df Residuals:	8350	BIC:	2.286e+04
Df Model:	25		

Covariance Type: nonrobust

	coef	std err	t	P> t	[0.025	0.975]
const	6.7162	0.151	44.491	0.000	6.420	7.012
EDUCDC	-0.0276	0.045	-0.620	0.536	-0.115	0.060
FEMALE	-0.1766	0.045	-3.896	0.000	-0.265	-0.088
AGE	0.1586	0.006	25.516	0.000	0.146	0.171
AGESQ	-0.0017	7.29e-05	-22.781	0.000	-0.002	-0.002
WHITE	0.0734	0.036	2.049	0.041	0.003	0.144
BLACK	-0.2727	0.067	-4.068	0.000	-0.404	-0.141
HISPANIC	-0.1321	0.050	-2.626	0.009	-0.231	-0.033
MARRIED	0.2192	0.029	7.617	0.000	0.163	0.276
NCHILD	0.1091	0.024	4.531	0.000	0.062	0.156
VET	0.1888	0.067	2.829	0.005	0.058	0.320
HSDIP	-1.3961	0.465	-3.000	0.003	-2.308	-0.484
COLDIP	-2.4913	0.951	-2.619	0.009	-4.356	-0.627
HSDIP-EDUCDC	0.1405	0.041	3.393	0.001	0.059	0.222
COLDIP-EDUCDC	0.2311	0.070	3.293	0.001	0.094	0.369
NCHLT5	0.3772	0.135	2.803	0.005	0.113	0.641
NCHILD-MARRIED	-0.0972	0.024	-4.115	0.000	-0.144	-0.051
COLDIP-VET	-0.2969	0.109	-2.736	0.006	-0.510	-0.084
FEMALE-HSDIP	-0.1009	0.042	-2.403	0.016	-0.183	-0.019
EDUCDCSQ	0.0051	0.005	0.991	0.322	-0.005	0.015

EDUCDCCUB	-0.0003	0.000	-2.045	0.041	-0.001	-1.3e-05
NCHILD-FEMALE	-0.1191	0.019	-6.261	0.000	-0.156	-0.082
HSDIP-BLACK	0.1810	0.080	2.266	0.024	0.024	0.338
HSDIP-HISPANIC	0.1453	0.060	2.422	0.015	0.028	0.263
AGE-NCHLT5	-0.0086	0.004	-2.192	0.028	-0.016	-0.001
FEMALE-WHITE	-0.1611	0.043	-3.709	0.000	-0.246	-0.076

Omnibus:	2881.226	Durbin-Watson:	1.962
Prob(Omnibus):	0.000	Jarque-Bera (JB):	14636.132
Skew:	-1.581	Prob(JB):	0.00
Kurtosis:	8.652	Cond. No.	4.13e+05

#### Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 4.13e+05. This might indicate that there are strong multicollinearity or other numerical problems.

By adding many additional interaction terms to allow for more differential slopes and adding polynomial terms to model a nonlinear relationship between years of education and log income, I was able to increase the adjusted  $R^2$  from 0.311 to 0.319, which is a modest improvement of 0.08 percentage points. My more flexible model now explains 32.1% of the variation in log income rather than only 31.2% of the variation.