

# BRI509: Introduction to Brain Signal Processing

## Assignment No. 2

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October 25, 2022

### 1. Explain the following terms.

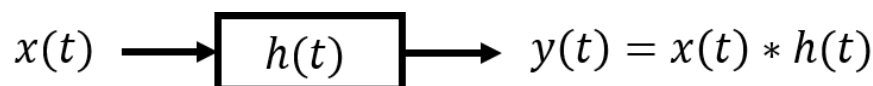
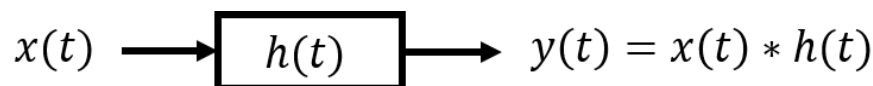
(a) Impulse response

#### **Meaning:**

The impulse response is the system's response to a unit impulse occurring at  $t = 0$ . If the input signal is  $x(t) = \delta(t)$ , then the impulse response is  $y(t) = h(t)$ .

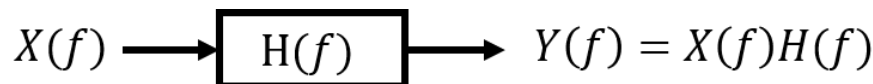
#### **Convolution**

The response of a system to an arbitrary input signal  $x(t)$  can be calculated by convolving the input signal with the impulse response, that is,  $y(t) = x(t) * h(t)$ .



#### **CTFT**

In the Fourier-domain, the response of a system to an arbitrary input signal  $x(t)$  can be calculated by taking the inverse Fourier transform of the product of the Fourier transforms of the input signal and the impulse response, that is,  $y(t) = \mathcal{F}^{-1}\{Y(f)\} = \mathcal{F}^{-1}\{X(f)H(f)\}$ .



(b) Harmonic functions in Fourier series

The Fourier series of  $x(t)$  is given by

$$x(t) = \sum_{k=0}^{+\infty} c_x[k] e^{\frac{j2\pi kt}{T}}, \quad (1)$$

where  $k = [0, +\infty)$  is the harmonic number,  $T$  is the period, and  $c_x[k]$  is the harmonic function. The **harmonic function** can be represented as

$$c_x[k] = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{-\frac{j2\pi kt}{T}} dt, \quad (2)$$

where  $t_0$  is any arbitrary time.

(c) Unit-sync function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t} \quad (3)$$

(d) How to approximate CTFT using DFT

CTFT can be approximated at discrete frequencies by

$$\begin{aligned} X(kf_s/N) &\cong T_s \sum_{n=0}^{N-1} x(nT_s) e^{-j2\pi kn/N} \\ &\cong T_s \times \mathcal{DFT}(x(nT_s)), \quad |k| \ll N \end{aligned} \quad (4)$$

where  $T_s = 1/f_s$  is chosen such that the signal  $x(t)$  does not change much with this amount of time, and  $N$  is chosen such that the signal energy of the signal  $x(t)$  can be covered within the time range 0 to  $NT_s$ .

(e) Graph the CTFT of cosine function  $\cos(2\pi f_0 t)$  and sine function  $\sin(2\pi f_0 t)$

$$\cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)] \quad (5)$$

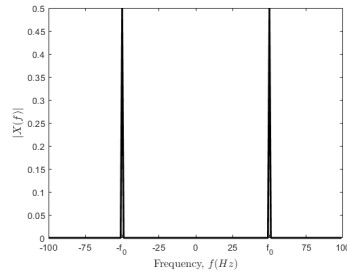


Figure 1: The CTFT of  $\cos(2\pi f_0 t)$

$$\sin(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{j}{2} [\delta(f + f_0) - \delta(f - f_0)] \quad (6)$$

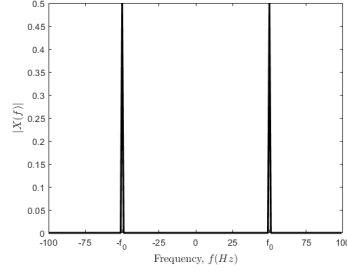


Figure 2: The CTFT of  $\sin(2\pi f_0 t)$

## 2. Solve the following problems.

- (a) Find the impulse response  $h[n]$  of the system described by the difference equation

$$5y[n] + 2y[n-1] - 3y[n-2] = x[n] \quad (7)$$

**Solution:**

- (i) Let  $x[n] = \delta[n] \rightarrow y[n] = h[n]$ . The difference equation now becomes

$$5h[n] + 2h[n-1] - 3h[n-2] = \delta[n]. \quad (8)$$

- (ii) **Homogeneous Solution:**

(1.) Let  $h[n] = K_h z^n$ .

(2.) Substituting this into the difference equation will give

$$\begin{aligned} 5h[n] + 2h[n-1] - 3h[n-2] &= 0 \\ 5K_h z^n + 2K_h z^{n-1} - 3K_h z^{n-2} &= 0 \\ K_h z^{n-2} (5z^2 + 2z - 3) &= 0 \\ 5z^2 + 2z - 3 &= 0 \end{aligned} \quad (9)$$

$$z_{1,2} = \frac{-2 \pm \sqrt{4 - 4(5)(-3)}}{2(5)}$$

$$\boxed{z_1 = \frac{-2 + 8}{10} = \frac{3}{5}} \quad \boxed{z_2 = \frac{-2 - 8}{10} = -1}$$

(3.) Therefore, the impulse response can now be represented as

$$h[n] = K_{h,1} \left(\frac{3}{5}\right)^n + K_{h,2} (-1)^n \quad (10)$$

(4.) Initial conditions:

$n$	$x[n]$	$y[n-2]$	$y[n-1]$	$y[n]$
0	$x[0] = 1$	$h[-2] = 0$	$h[-1] = 0$	$5h[0] + 2h[-1] - 3h[-2] = 1$ $h[0] = \frac{1}{5}$
1	$x[1] = 0$	$h[-1] = 0$	$h[0] = \frac{1}{5}$	$5h[1] + 2h[0] - 3h[-1] = 0$ $h[1] = \frac{-2}{25}$

(5.) Applying the initial conditions:

**Initial Condition 1:**

$$\begin{aligned} h[0] &= \frac{1}{5} = K_{h,1} \left(\frac{3}{5}\right)^0 + K_{h,2} (-1)^0 \\ \frac{1}{5} &= K_{h,1} + K_{h,2} \end{aligned} \tag{11}$$

**Initial Condition 2:**

$$\begin{aligned} h[1] &= \frac{-2}{25} = K_{h,1} \left(\frac{3}{5}\right)^1 + K_{h,2} (-1)^1 \\ \frac{-2}{25} &= K_{h,1} \left(\frac{3}{5}\right) + K_{h,2} (-1) \end{aligned} \tag{12}$$

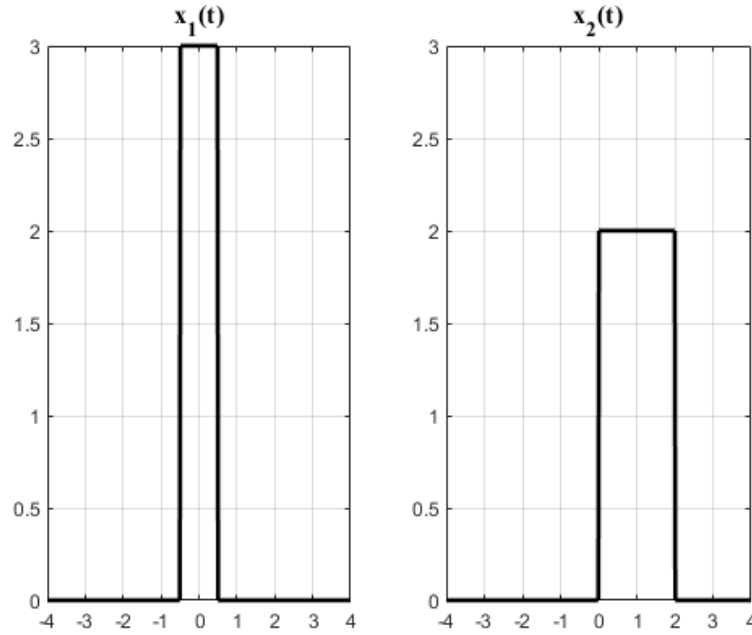
These system of equations can be written in matrix form as

$$\begin{aligned} \begin{bmatrix} \frac{1}{5} \\ \frac{-2}{25} \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ \frac{3}{5} & -1 \end{bmatrix} \begin{bmatrix} K_{h,1} \\ K_{h,2} \end{bmatrix} \\ \begin{bmatrix} K_{h,1} \\ K_{h,2} \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ \frac{3}{5} & -1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{5} \\ \frac{-2}{25} \end{bmatrix} \\ \begin{bmatrix} K_{h,1} \\ K_{h,2} \end{bmatrix} &= \begin{bmatrix} \frac{3}{40} \\ \frac{5}{40} \end{bmatrix} \end{aligned} \tag{13}$$

(iii.) Therefore, the impulse response is given by

$$\boxed{h[n] = \frac{3}{40} \left(\frac{3}{5}\right)^n + \frac{5}{40} (-1)^n} \tag{14}$$

(b) Find the convolution of the two functions  $x_1(t)$  and  $x_2(t)$ .



### Solution

(i.) The functions  $x_1(t)$  and  $x_2(t)$  can be represented as

$$x_1(t) = 3\text{rect}(t) = 3 \left[ u\left(t + \frac{1}{2}\right) - u\left(t - \frac{1}{2}\right) \right] \quad (15)$$

$$\begin{aligned} x_2(t) &= 2\text{rect}\left(\frac{t-1}{2}\right) = 2 \left[ u\left(\frac{t}{2}\right) - u\left(\frac{t-2}{2}\right) \right] \\ &= 2[u(t) - u(t-2)] \end{aligned} \quad (16)$$

(ii.) From the differentiation property of convolution,  $y'(t) = x'(t) * h(t)$  or  $y'(t) = x(t) * h'(t)$ . Therefore,

$$\begin{aligned} y(t) &= x_1(t) * x_2(t) \\ y'(t) &= x_1'(t) * x_2(t) \\ y''(t) &= x_1'(t) * x_2'(t) \\ y''(t) &= 2 \left[ \delta\left(t + \frac{1}{2}\right) - \delta\left(t - \frac{1}{2}\right) \right] * 3[\delta(t) - \delta(t-2)] \\ y''(t) &= 6 \left[ \delta\left(t + \frac{1}{2}\right) * \delta(t) - \delta\left(t + \frac{1}{2}\right) * \delta(t-2) - \delta\left(t - \frac{1}{2}\right) * \delta(t) \right. \\ &\quad \left. + \delta\left(t - \frac{1}{2}\right) * \delta(t-2) \right] \end{aligned} \quad (17)$$

$$\begin{aligned}
y''(t) &= 6 \left[ \delta \left( t + \frac{1}{2} \right) - \delta \left( t - \frac{3}{2} \right) - \delta \left( t - \frac{1}{2} \right) + \delta \left( t - \frac{5}{2} \right) \right] \\
y'(t) &= 6 \left[ u \left( t + \frac{1}{2} \right) - u \left( t - \frac{3}{2} \right) - u \left( t - \frac{1}{2} \right) + u \left( t - \frac{5}{2} \right) \right] \\
y(t) &= 6 \left[ \text{ramp} \left( t + \frac{1}{2} \right) - \text{ramp} \left( t - \frac{3}{2} \right) - \text{ramp} \left( t - \frac{1}{2} \right) + \text{ramp} \left( t - \frac{5}{2} \right) \right]
\end{aligned}
\tag{18}$$

(iii.) Table of values

$t$	$y(t)$
$t = -\frac{1}{2}$	$y \left( -\frac{1}{2} \right) = 6 (0 - 0 - 0 + 0) = 0$
$t = 0$	$y(0) = 6 \left( \frac{1}{2} - 0 - 0 + 0 \right) = 3$
$t = \frac{1}{2}$	$y \left( \frac{1}{2} \right) = 6 (1 - 0 - 0 + 0) = 6$
$t = 1$	$y(1) = 6 \left( \frac{3}{2} - 0 - \frac{1}{2} + 0 \right) = 6$
$t = \frac{3}{2}$	$y \left( \frac{3}{2} \right) = 6 (2 - 0 - 1 + 0) = 6$
$t = 2$	$y(2) = 6 \left( \frac{5}{2} - \frac{1}{2} - \frac{3}{2} + 0 \right) = 3$
$t = \frac{5}{2}$	$y \left( \frac{5}{2} \right) = 6 (3 - 1 - 2 + 0) = 0$

(iv.) Plot of the convolution of  $x_1(t)$  and  $x_2(t)$ :

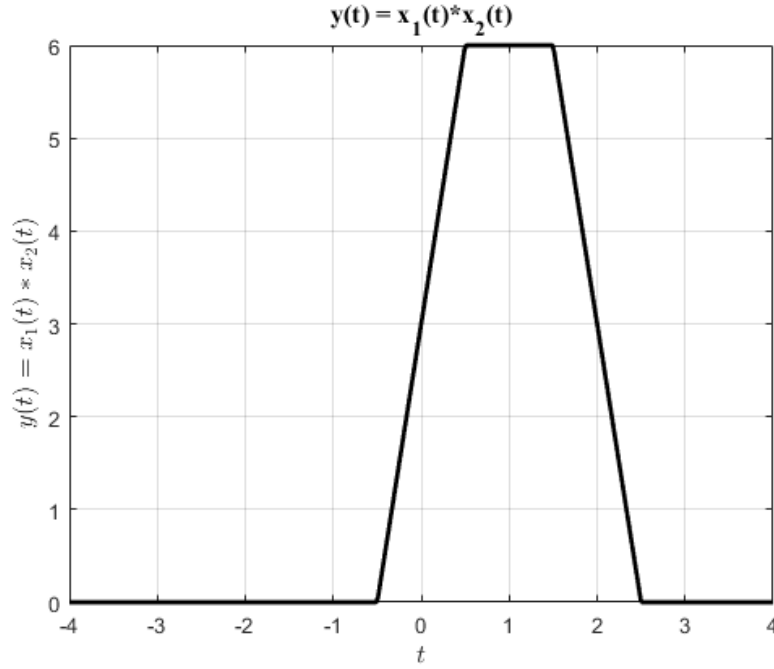


Figure 3: Plot of the convolution of  $x_1(t) = 3\text{rect}(t)$  and  $x_2(t) = 2\text{rect}(\frac{t-1}{2})$

(c) Find the complex CTFS harmonic function of  $x(t) = 10\text{rect}(t/2) * \delta_4(t)$  using

$$c_x[k] = \frac{1}{T} \int_T x(t) e^{-\frac{j2\pi kt}{T}} dt \quad (19)$$

**Solution**

(i.) From  $x(t) = 10\text{rect}(t/2) * \delta_4(t) = 10\text{rect}(t/2) * \delta_4(t)$ , the period can be written as  $T = 4$ . Also, the width of the rectangle function  $\text{rect}(t/2)$  is  $w = 2$ .

(ii.) Thus, the harmonic function can be calculated as:

$$\begin{aligned} c_x[k] &= \frac{1}{T} \int_T x(t) e^{-\frac{j2\pi kt}{T}} dt \\ &= \frac{1}{T} \int_{-T/2}^{T/2} 10\text{rect}(t/2) * \delta_4(t) e^{-\frac{j2\pi kt}{T}} dt \\ &= \frac{10}{4} \int_{-4/2}^{4/2} \text{rect}(t/2) e^{-\frac{j2\pi kt}{4}} dt \\ &= \frac{5}{2} \int_{-1}^1 e^{-\frac{j2\pi kt}{4}} dt \\ &= \frac{5}{2} \left( \frac{e^{-\frac{j2\pi kt}{4}}}{-j2\pi k/4} \right) \Big|_{-1}^1 \\ &= \frac{5}{2} \left( \frac{e^{-j2\pi k/4} - e^{j2\pi k/4}}{-j2\pi k/4} \right) \\ &= \frac{5}{2} \left( \frac{\sin(2\pi k/4)}{\pi k/4} \right) = \frac{5}{2} \left( \frac{\sin(\pi k/2)}{\pi k/4} \right) \cdot \frac{2}{2} \\ &= 5 \left( \frac{\sin(\pi k/2)}{\pi k/2} \right) \\ \boxed{c_x[k] = 5\text{sinc}(k/2) \cdot \delta_1[k]} \end{aligned} \quad (20)$$

(d) Find the CTFT of  $x_t = 24 \cos(100\pi t) \sin(10,000\pi t)$ .

**Table 6.4** More Fourier transform pairs

$\delta(t) \xleftrightarrow{\mathcal{F}} 1$	$1 \xleftrightarrow{\mathcal{F}} \delta(f)$
$\text{sgn}(t) \xleftrightarrow{\mathcal{F}} 1/j\pi f$	$u(t) \xleftrightarrow{\mathcal{F}} (1/2)\delta(f) + 1/j2\pi f$
$\text{rect}(t) \xleftrightarrow{\mathcal{F}} \text{sinc}(f)$	$\text{sinc}(t) \xleftrightarrow{\mathcal{F}} \text{rect}(f)$
$\text{tri}(t) \xleftrightarrow{\mathcal{F}} \text{sinc}^2(f)$	$\text{sinc}^2(t) \xleftrightarrow{\mathcal{F}} \text{tri}(f)$
$\delta_{T_0}(t) \xleftrightarrow{\mathcal{F}} f_0 \delta_{f_0}(f), f_0 = 1/T_0$	$T_0 \delta_{T_0}(t) \xleftrightarrow{\mathcal{F}} \delta_{f_0}(f), T_0 = 1/f_0$
$\cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} (1/2)[\delta(f - f_0) + \delta(f + f_0)]$	$\sin(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} (j/2)[\delta(f + f_0) - \delta(f - f_0)]$

### Solution

(i.)  $\cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)]$ . Therefore,

$$\begin{aligned} 24 \cos(100\pi t) &\xleftrightarrow{\mathcal{F}} \frac{24}{2} [\delta(f - 50) + \delta(f + 50)] \\ &= 12 [\delta(f - 50) + \delta(f + 50)] \end{aligned} \quad (21)$$

(ii.)  $\sin(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{j}{2} [\delta(f + f_0) - \delta(f - f_0)]$ . Therefore,

$$\sin(10,000\pi t) \xleftrightarrow{\mathcal{F}} \frac{j}{2} [\delta(f + 5,000) - \delta(f - 5,000)] \quad (22)$$

(iii.)  $g(t)h(t) \xleftrightarrow{\mathcal{F}} G(f) * H(f)$ . Therefore,

$$\begin{aligned} 24 \cos(100\pi t) \sin(10,000\pi t) &\xleftrightarrow{\mathcal{F}} 12 [\delta(f - 50) + \delta(f + 50)] * \frac{j}{2} [\delta(f + 5,000) \\ &\quad - \delta(f - 5,000)] \\ &= j6 [\delta(f - 50) * \delta(f + 5,000) \\ &\quad - \delta(f - 50) * \delta(f - 5,000) \\ &\quad + \delta(f + 50) * \delta(f + 5,000) \\ &\quad - \delta(f + 50) * \delta(f - 5,000)] \\ 24 \cos(100\pi t) \sin(10,000\pi t) &\xleftrightarrow{\mathcal{F}} j6 [\delta(f + 4950) - \delta(f - 5050) \\ &\quad + \delta(f + 5050) - \delta(f - 4950)] \end{aligned} \quad (23)$$



(e) Find and graph the inverse DTFT of

$$X(F) = \left[ \text{rect} \left( 50 \left( F - \frac{1}{4} \right) \right) + \text{rect} \left( 50 \left( F + \frac{1}{4} \right) \right) \right] * \delta_1(F) \quad (24)$$

**Table 7.5 More DTFT pairs**

$\delta[n] \xleftrightarrow{\mathcal{F}} 1$	
$u[n] \xleftrightarrow{\mathcal{F}} \frac{1}{1 - e^{-j2\pi F}} + (1/2)\delta_1(F),$	$u[n] \xleftrightarrow{\mathcal{F}} \frac{1}{1 - e^{-j\Omega}} + \pi\delta_1(\Omega)$
$\text{sinc}(n/w) \xleftrightarrow{\mathcal{F}} w \text{rect}(wF) * \delta_1(F),$	$\text{sinc}(n/w) \xleftrightarrow{\mathcal{F}} w \text{rect}(w\Omega/2\pi) * \delta_{2\pi}(\Omega)$
$\text{tri}(n/w) \xleftrightarrow{\mathcal{F}} w \text{drcl}^2(F, w),$	$\text{tri}(n/w) \xleftrightarrow{\mathcal{F}} w \text{drcl}^2(\Omega/2\pi, w)$
$1 \xleftrightarrow{\mathcal{F}} \delta_1(F),$	$1 \xleftrightarrow{\mathcal{F}} 2\pi\delta_{2\pi}(\Omega)$
$\delta_{N_0}[n] \xleftrightarrow{\mathcal{F}} (1/N_0)\delta_{1/N_0}(F),$	$\delta_{N_0}[n] \xleftrightarrow{\mathcal{F}} (2\pi/N_0)\delta_{2\pi/N_0}(\Omega)$
$\cos(2\pi F_0 n) \xleftrightarrow{\mathcal{F}} (1/2)[\delta_1(F - F_0) + \delta_1(F + F_0)],$	$\cos(\Omega_0 n) \xleftrightarrow{\mathcal{F}} \pi[\delta_{2\pi}(\Omega - \Omega_0) + \delta_{2\pi}(\Omega + \Omega_0)]$
$\sin(2\pi F_0 n) \xleftrightarrow{\mathcal{F}} (j/2)[\delta_1(F + F_0) - \delta_1(F - F_0)],$	$\sin(\Omega_0 n) \xleftrightarrow{\mathcal{F}} j\pi[\delta_{2\pi}(\Omega + \Omega_0) - \delta_{2\pi}(\Omega - \Omega_0)]$
$u[n - n_0] - u[n - n_1] \xleftrightarrow{\mathcal{Z}} \frac{e^{j2\pi F}}{e^{j2\pi F} - 1} (e^{-j2\pi n_0 F} - e^{-j2\pi n_1 F}) = \frac{e^{-j\pi F(n_0 + n_1)}}{e^{-j\pi F}} (n_1 - n_0) \text{drcl}(F, n_1 - n_0)$	
$u[n - n_0] - u[n - n_1] \xleftrightarrow{\mathcal{Z}} \frac{e^{j\Omega}}{e^{j\Omega} - 1} (e^{-j\Omega n_0} - e^{-j\Omega n_1}) = \frac{e^{-j\Omega(n_0 + n_1)/2}}{e^{-j\Omega/2}} (n_1 - n_0) \text{drcl}(\Omega/2\pi, n_1 - n_0)$	

### Solution

(i.)  $\text{sinc}\left(\frac{n}{w}\right) \xleftrightarrow{\mathcal{F}} w \text{rect}(wF) * \delta_1(F)$ . Therefore, if  $w = 50$ , then

$$\frac{1}{50} \text{sinc}\left(\frac{n}{50}\right) \xleftrightarrow{\mathcal{F}} \frac{1}{50} \cdot 50 \text{rect}(50F) * \delta_1(F) = \text{rect}(50F) * \delta_1(F) \quad (25)$$

(ii.) The frequency shifting property is given by  $e^{j2\pi F_0 n} x[n] \xleftrightarrow{\mathcal{F}} X(F - F_0)$ . Therefore,  
 $F_0 = \frac{1}{4}$ :

$$e^{j2\pi n/4} \cdot \frac{1}{50} \text{sinc}\left(\frac{n}{50}\right) \xleftrightarrow{\mathcal{F}} \text{rect}\left(50\left(F - \frac{1}{4}\right)\right) * \delta_1(F) \quad (26)$$

$$F_0 = -\frac{1}{4}:$$

$$e^{-j2\pi n/4} \cdot \frac{1}{50} \text{sinc}\left(\frac{n}{50}\right) \xleftrightarrow{\mathcal{F}} \text{rect}\left(50\left(F + \frac{1}{4}\right)\right) * \delta_1(F) \quad (27)$$

(iii.) Adding eqs. (26) and (27) gives

$$\begin{aligned} (e^{j2\pi n/4} + e^{-j2\pi n/4}) \frac{1}{50} \text{sinc}\left(\frac{n}{50}\right) &\xleftrightarrow{\mathcal{F}} \left[ \text{rect}\left(50\left(F - \frac{1}{4}\right)\right) \right. \\ &\quad \left. + \text{rect}\left(50\left(F + \frac{1}{4}\right)\right) \right] * \delta_1(F) \end{aligned} \quad (28)$$

Therefore, the inverse DTFT of

$$X(F) = \left[ \text{rect}\left(50\left(F - \frac{1}{4}\right)\right) + \text{rect}\left(50\left(F + \frac{1}{4}\right)\right) \right] * \delta_1(F) \quad (29)$$

is:

$$\begin{aligned}
 x[n] &= (e^{j2\pi n/4} + e^{-j2\pi n/4}) \frac{1}{50} \text{sinc}\left(\frac{n}{50}\right) \\
 &= 2 \cos\left(\frac{2\pi n}{4}\right) \cdot \frac{1}{50} \text{sinc}\left(\frac{n}{50}\right)
 \end{aligned} \tag{30}$$

$$\boxed{x[n] = \frac{1}{25} \cos\left(\frac{\pi n}{2}\right) \text{sinc}\left(\frac{n}{50}\right)}$$

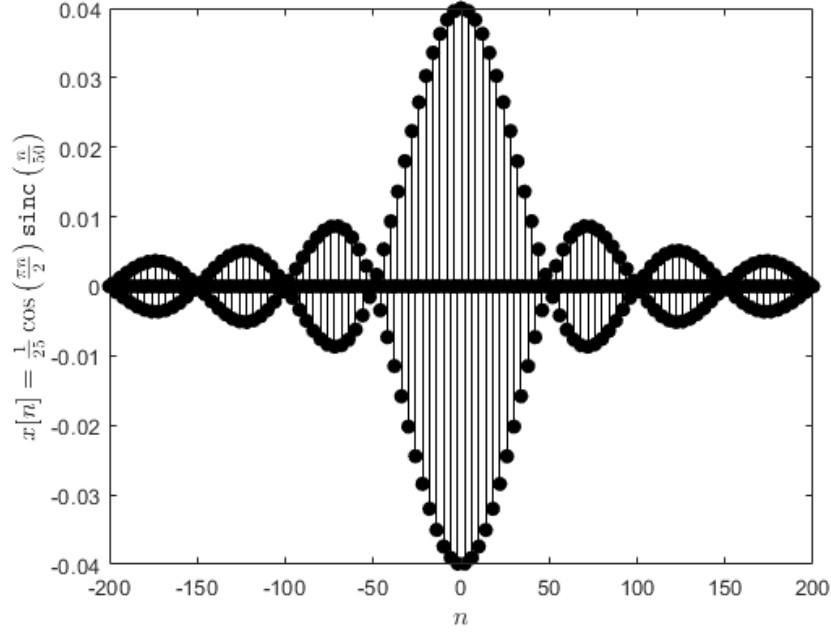


Figure 4: The inverse DTFT of  $X(F) = [\text{rect}(50(F - \frac{1}{4})) + \text{rect}(50(F + \frac{1}{4}))] * \delta_1(F)$

### 3. MATLAB coding

(a) Using the DFT, find the approximate CTFT of

$$x(t) = \begin{cases} t(1-t), & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases} = t(1-t)\text{rect}\left(t - \frac{1}{2}\right)$$

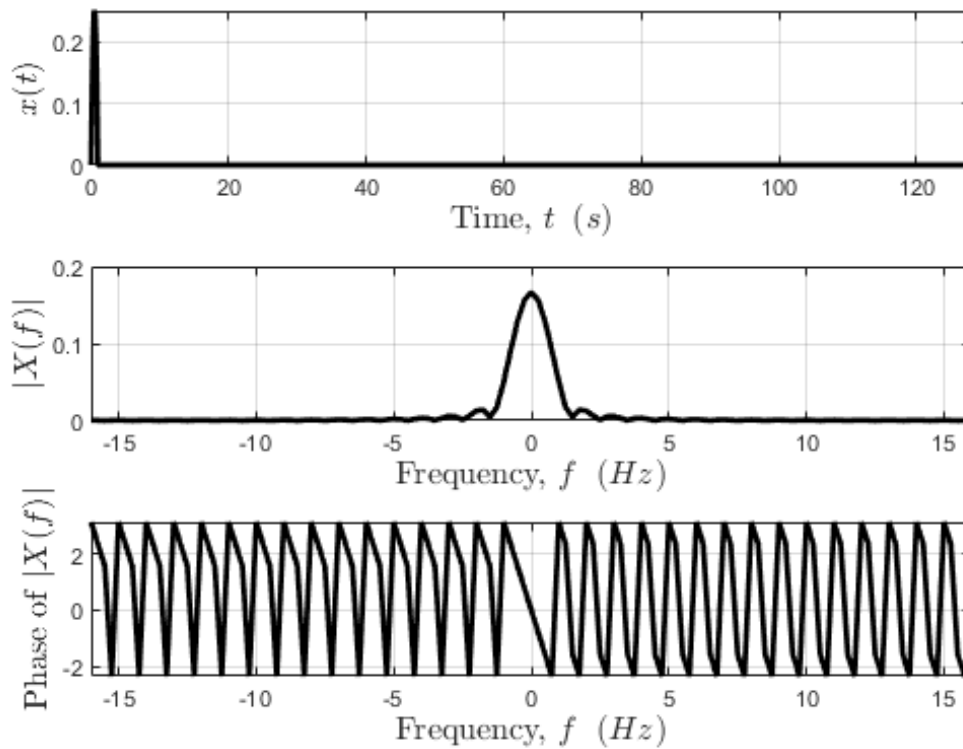
#### Source Code

```
1 % BRI509 (Introduction to Brain Signal Processing)
2 % Assignment # 2
3 % Author: Raymart Jay E. Canoy
4 % Student ID #: 2020021376
5
6 %% Problem 3a
7 N = 64;          % Sample 128 times
8 Ts = 2/N;        % Sampling interval
9
10 fs = 1/Ts;       % Sampling rate
11 df = fs/N;       % Frequency-domain resolution
12
13 n = [0: N-1]; % Indices
14
15 t = n*Ts;        % Time vector
16 x = t.*(1-t).*prob3a_rect(t - 0.5); % x(t)
17
18 num = 2*N;
19 X = fftshift(Ts*fft(x, num, 2)); % X(f) with zero-padding
20
21 k = [-N/2:N/(num):N/2-N/(num)]';
22
23 figure;
24 subplot(3,1,1);
25 p = plot([t, linspace(3, num-1, num-1)], [x, zeros(1, num-1)], 'k');
26 set(p, 'LineWidth', 2);
27 grid on
28 xlim([0 128])
29 xlabel('Time, $t$ \;\; (s)$', 'Interpreter', 'Latex', 'FontName', 'Times', 'FontSize', 12);
30 ylabel('$x(t)$', 'Interpreter', 'Latex', 'FontName', 'Times', 'FontSize', 12)
31 subplot(3,1,2)
32 p = plot(k*df, abs(X), 'k')
33 set(p, 'LineWidth', 2);
34 grid on
35 xlim([-16 16])
36 xlabel('Frequency, $f$ \;\; (Hz)$', 'Interpreter', 'Latex',
```

```

        'FontName', 'Times', 'FontSize', 12);
37 ylabel('$|X(f)|$', 'Interpreter', 'Latex', 'FontName', '
    Times', 'FontSize', 12)
38 subplot(3,1,3)
39 p = plot(k*df, angle(X), 'k')
40 set(p, 'LineWidth', 2);
41 grid on
42 xlim([-16 16])
43 xlabel('Frequency, $f \\\; (Hz)$', 'Interpreter', 'Latex',
    'FontName', 'Times', 'FontSize', 12);
44 ylabel('Phase of $|X(f)|$', 'Interpreter', 'Latex', '
    FontName', 'Times', 'FontSize', 12)

```



- (b.1) Graph the CTFT of of C4(523.25/2Hz), D4 (587.33/2Hz), E4 (659.26/2Hz), F4 (698.46/2Hz), G4 (784/2Hz), A4 (440Hz), B4 (493.8Hz), C5 (523.25Hz).

#### Source Code

```
1 % BRI509 (Introduction to Brain Signal Processing)
2 % Assignment # 2
3 % Author: Raymart Jay E. Canoy
4 % Student ID #: 2020021376
5
6 % Notes:
7 % "The frequencies 440 Hz and 880 Hz both correspond to
8 % the musical note A,
9 % but one octave apart. In the western musical scale,
10 % there are 12 notes in
11 % every octave. These notes are evenly distributed (
12 % geometrically), so the
13 % next note above A, which is B flat, has frequency (440 x
14 % beta), where
15 % beta is the twelfth root of two."
16 % Source: https://ptolemy.berkeley.edu/eecs20/week8/scale.
17 % html
18
19 %% Part I: CTFT of the musical scales C4,D4,E4,F4,G4,A4,A5
20 % ,B5,C5
21
22 %% Having the container for the musical scales
23 clear; clc;
24
25 fs = 1200; % Sampling Frequency
26 T = 1/fs; % Sampling Period
27 dur = 1; % duration, s
28 L = dur*fs; % Length of signal
29 t = (0:L-1)*T; % One-second duration
30
31 % Sinusoid Signal
32 % y(t) = sin(2*pi*f*t), where f = 440*beta^(index), beta =
33 % 2^(1/12), and
34 % n is the index, i.e., n = [1, 13]
35 beta = 2^(1/12);
36 beta_exponents = [-9, -7, -5, -4, -2, 0, 2, 3];
37 note = @(index) sin(2*pi*(440*(beta^(index)))*t); % a
38 % sinusoid function
39
40 % musical_scale_keys: Names of the notes
41 musical_scale_keys = {'C4', 'D4', 'E4', 'F4', 'G4', 'A4',
42 % 'B4', 'C5'};
43 % musical_scale_values: An empty 13x1 cell, and will hold
44 % the notes' values
45 musical_scale_values = cell(length(musical_scale_keys), 1)
46 ;
47
48 % Assigns the values of each note to musical_scale_values
49 for ind = 1 : length(musical_scale_keys)
50 musical_scale_values{ind} = note(beta_exponents(ind));
51 end
```

```

41
42 % Dictionary-type container -- keys: notes's names; values
   : notes' values
43 music_octave = containers.Map(musical_scale_keys,
   musical_scale_values);
44
45 %% Fourier Transform of the musical scales
46 musical_scale_CTFT_values = cell(length(musical_scale_keys
   ),1);
47
48 n = 2*(2^nextpow2(L)); % So that I will get an amplitude
   of 0.5
49
50 for ind = 1 : length(musical_scale_keys)
51     x = music_octave(musical_scale_keys{ind});
52     X = fftshift(fft(x, n, 2));
53     P2 = abs(X/L);
54     musical_scale_CTFT_values{ind} = P2;
55 end
56
57 % Dictionary-type container -- keys: notes's names; values
   : CTFT values
58 music_scale_CTFT = containers.Map(musical_scale_keys,
   musical_scale_CTFT_values);
59
60
61 for set_num = 1 : 2
62     figure;
63     disp(['set = ' num2str(set_num)])
64     for ind = (floor(length(musical_scale_keys)/2))*(
   set_num - 1) + 1 : set_num*floor(length(
   musical_scale_keys)/2)
65         disp(['ind = ' num2str(ind)])
66         subplot(floor(length(musical_scale_keys)/4), 2,
   ind - (set_num - 1)*floor(length(musical_scale_keys)
   /2))
67         p = plot([-fs/2:fs/n:(fs/2)-(fs/n)],
   music_scale_CTFT(musical_scale_keys{ind}), 'k');
68         set(p, 'LineWidth', 1.5);
69         ylim([0 0.5])
70         yticks([0 0.1 0.2 0.3 0.4 0.5]);
71         xticks([-600 -500 -400 -300 -200 -100 0 100 200
   300 400 500 600])
72         xlabel('Frequency, $f \backslash; \backslash; (Hz)$', 'Interpreter',
   'Latex', 'FontName', 'Times', 'FontSize', 12);
73         ylabel('$|X(f)|$', 'Interpreter', 'Latex', '
   FontName', 'Times', 'FontSize', 12)
74         title([musical_scale_keys{ind}, '(f = ', num2str
   (440*(beta^(beta_exponents(ind))))', ' Hz)'], 'FontName
   ', 'Times', 'FontSize', 12);
75     end
76 end

```

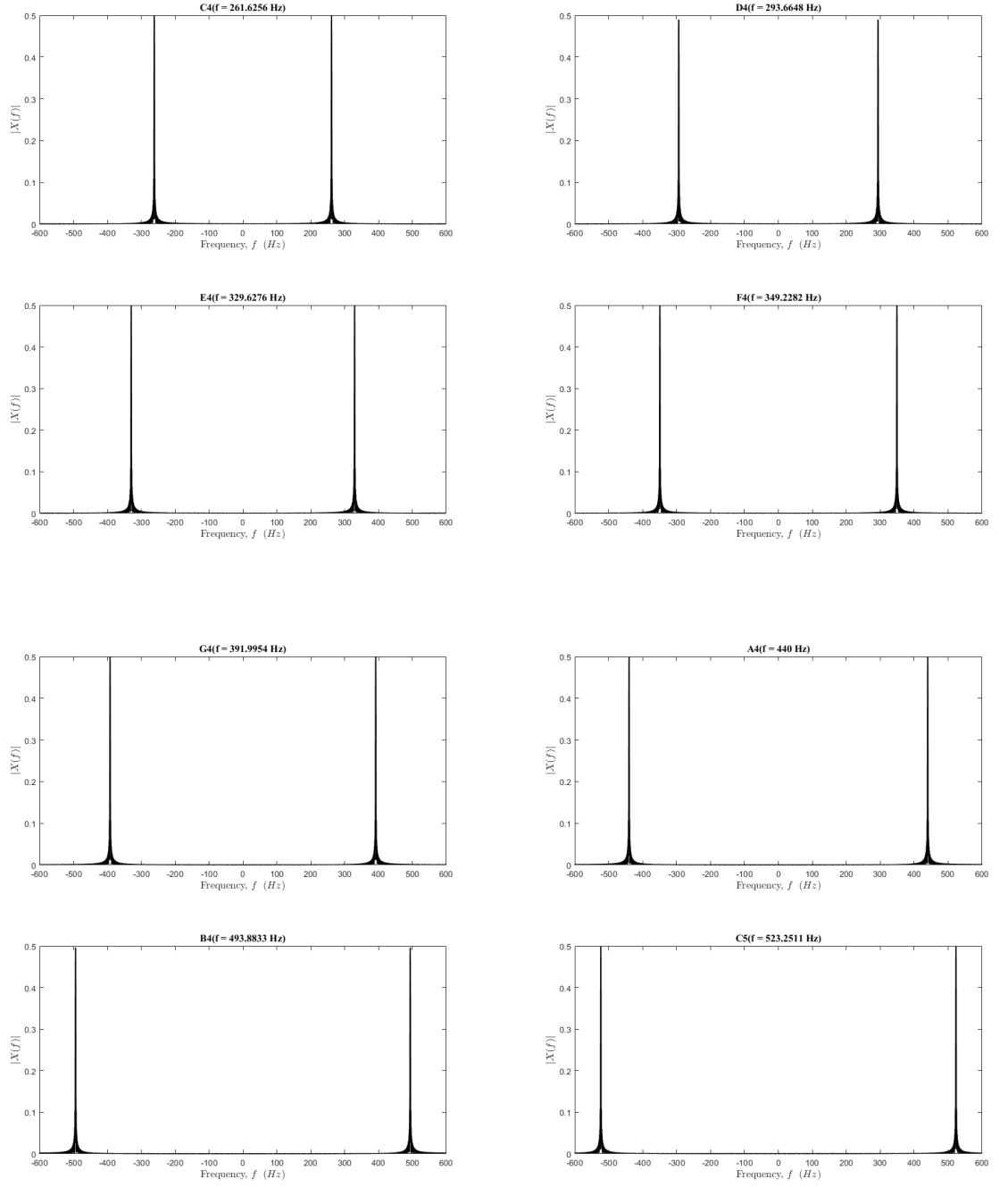


Figure 5: CTFT of the solfeges of C Major.

(b.2) Make an MP3 file containing C4, D4, E4, F4, G4, A4, B4, C5.

## Source Code

```

1 % BRI509 (Introduction to Brain Signal Processing)
2 % Assignment # 2
3 % Author: Raymart Jay E. Canoy
4 % Student ID #: 2020021376
5
6 % Notes:
7 % "The frequencies 440 Hz and 880 Hz both correspond to
8 % the musical note A,
9 % but one octave apart. In the western musical scale,
10 % there are 12 notes in
11 % every octave. These notes are evenly distributed (
12 % geometrically), so the
13 % next note above A, which is B flat, has frequency (440 x
14 % beta), where
15 % beta is the twelfth root of two."
16 % Source: https://ptolemy.berkeley.edu/eecs20/week8/scale.html
17
18 %% Part II: MP4 file containing the musical scales C4,D4,
19 % E4,F4,G4,A4,A5,B5,C5 one second each
20
21 clear; clc;
22
23 fs = 48000; % Sampling Frequency
24 T = 1/fs; % Sampling Period
25 dur = 1; % duration, s
26 L = dur*fs; % Length of signal
27 t = (0:L-1)*T; % One-second duration
28
29 % Sinusoid Signal
30 % y(t) = sin(2*pi*f*t), where f = 440*beta^(n - 1), beta =
31 % 2^(1/12), and
32 % n is the index, i.e., n = [1, 13]
33 beta = 2^(1/12);
34 beta_exponents = [-9, -7, -5, -4, -2, 0, 2, 3];
35 note = @(index) sin(2*pi*(440*(beta^(index)))*t); % a
36 % sinusoid function
37
38 % musical_scale_keys: Names of the notes
39 musical_scale_keys = {'C4', 'D4', 'E4', 'F4', 'G4', 'A4',
40 % 'B4', 'C5'};
41
42 % musical_scale_values: An empty 13x1 cell, and will hold
43 % the notes' values
44 musical_scale_values = cell(length(musical_scale_keys), 1)
45 % ;
46
47 % Assigns the values of each note to musical_scale_values
48 for ind = 1 : length(musical_scale_keys)
49     musical_scale_values{ind} = note(beta_exponents(ind));
50 end
51
52 % Dictionary-type container -- keys: notes's names; values

```



```

        : notes' values
41 music_octave = containers.Map(musical_scale_keys,
    musical_scale_values);
42
43 mp3_file = [];
44 filename = 'prob3b.mp4';
45 for ind = 1 : length(musical_scale_keys)
46     mp3_file = [mp3_file; [music_octave(musical_scale_keys
        {ind}) zeros(1, 100)]];
47 end
48
49 soundsc(mp3_file, fs)
50 audiowrite(filename, mp3_file, fs)

```

### Note: All the files were uploaded on GitHub

All the files in this document were uploaded on Github, and can be accessed at:

[https://github.com/recanoy/KoreaUniversity\\_BRI509](https://github.com/recanoy/KoreaUniversity_BRI509)

If there are errors in the solution or codes kindly email,  
recanoy@korea.ac.kr.