Assignment 2

BRI 509 Introduction to Brain Signal Processing

due date : 2020.4.27

Name : ____________________________

Studendt ID # : __________________________________

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1. Explain the following terms (1 point).

(a) Impulse response

- meaning

- convolution

- CTFT

(b) Harmonic functions in Fourier Series

(c) Unit-sync function

(d) How to approximate CTFT using DFT.

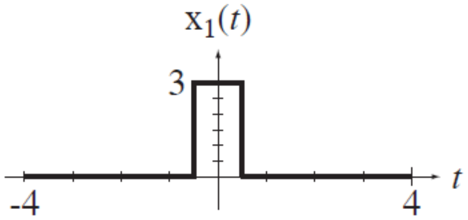
(e) Graph the CTFT of the cosine function $\cos(2\pi f_0 t)$ and sine function $\sin(2\pi f_0 t)$

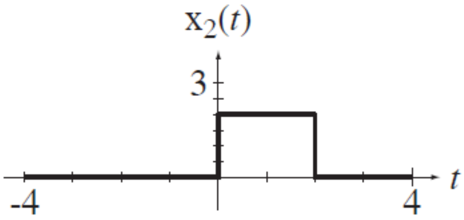
2. Solve the following problems (2 points).

(a) Find the impulse response $h[n]$ of the system described by the difference equation

$$5y[n] + 2y[n-1] - 3y[n-2] = x[n].$$

(b) Find the convolution of the two functions $x_1(t)$ and $x_2(t)$.





(c) Find the complex CTFS harmonic function of $x(t)$=10rect$(t/2)$∗δ_4(t)

using $c_x[k] = \frac{1}{T}\int_T x(t)\, e^{-j2\pi kt/T}\, dt.$ 5

(d) Find the CTFT of $x(t) = 24\cos(100\pi t)\sin(10{,}000\pi t)$.

Table 6.4 More Fourier transform pairs

$\delta(t) \xleftrightarrow{\mathcal{F}} 1$	$1 \xleftrightarrow{\mathcal{F}} \delta(f)$
$\mathrm{sgn}(t) \xleftrightarrow{\mathcal{F}} 1/j\pi f$	$\mathrm{u}(t) \xleftrightarrow{\mathcal{F}} (1/2)\delta(f) + 1/j2\pi f$
$\mathrm{rect}(t) \xleftrightarrow{\mathcal{F}} \mathrm{sinc}(f)$	$\mathrm{sinc}(t) \xleftrightarrow{\mathcal{F}} \mathrm{rect}(f)$
$\mathrm{tri}(t) \xleftrightarrow{\mathcal{F}} \mathrm{sinc}^2(f)$	$\mathrm{sinc}^2(t) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(f)$
$\delta_{T_0}(t) \xleftrightarrow{\mathcal{F}} f_0\delta_{f_0}(f),\ f_0 = 1/T_0$	$T_0\delta_{T_0}(t) \xleftrightarrow{\mathcal{F}} \delta_{f_0}(f),\ T_0 = 1/f_0$
$\cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} (1/2)[\delta(f - f_0) + \delta(f + f_0)]$	$\sin(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} (j/2)[\delta(f + f_0) - \delta(f - f_0)]$

(e) Find and graph the inverse DTFT of $X(F) = \left[\text{rect}\left(50\left(F - \frac{1}{4}\right)\right) + \text{rect}\left(50\left(F + \frac{1}{4}\right)\right)\right] * \delta_1(F)$

Table 7.5 More DTFT pairs

$$\delta[n] \xleftrightarrow{\mathcal{F}} 1$$

$\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \dfrac{1}{1-e^{-j2\pi F}} + (1/2)\delta_1(F),$	$\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \dfrac{1}{1-e^{-j\Omega}} + \pi\delta_1(\Omega)$
$\mathrm{sinc}(n/w) \xleftrightarrow{\mathcal{F}} w\,\mathrm{rect}(wF) * \delta_1(F),$	$\mathrm{sinc}(n/w) \xleftrightarrow{\mathcal{F}} w\,\mathrm{rect}(w\Omega/2\pi) * \delta_{2\pi}(\Omega)$
$\mathrm{tri}(n/w) \xleftrightarrow{\mathcal{F}} w\,\mathrm{drcl}^2(F,w),$	$\mathrm{tri}(n/w) \xleftrightarrow{\mathcal{F}} w\,\mathrm{drcl}^2(\Omega/2\pi,w)$
$1 \xleftrightarrow{\mathcal{F}} \delta_1(F),$	$1 \xleftrightarrow{\mathcal{F}} 2\pi\delta_{2\pi}(\Omega)$
$\delta_{N_0}[n] \xleftrightarrow{\mathcal{F}} (1/N_0)\delta_{1/N_0}(F),$	$\delta_{N_0}[n] \xleftrightarrow{\mathcal{F}} (2\pi/N_0)\delta_{2\pi/N_0}(\Omega)$
$\cos(2\pi F_0 n) \xleftrightarrow{\mathcal{F}} (1/2)[\delta_1(F-F_0)+\delta_1(F+F_0)],$	$\cos(\Omega_0 n) \xleftrightarrow{\mathcal{F}} \pi[\delta_{2\pi}(\Omega-\Omega_0)+\delta_{2\pi}(\Omega+\Omega_0)]$
$\sin(2\pi F_0 n) \xleftrightarrow{\mathcal{F}} (j/2)[\delta_1(F+F_0)-\delta_1(F-F_0)],$	$\sin(\Omega_0 n) \xleftrightarrow{\mathcal{F}} j\pi[\delta_{2\pi}(\Omega+\Omega_0)-\delta_{2\pi}(\Omega-\Omega_0)]$

$$\mathrm{u}[n-n_0]-\mathrm{u}[n-n_1] \xleftrightarrow{\mathcal{Z}} \frac{e^{j2\pi F}}{e^{j2\pi F}-1}\left(e^{-j2\pi n_0 F}-e^{-j2\pi n_1 F}\right) = \frac{e^{-j\pi F(n_0+n_1)}}{e^{-j\pi F}}(n_1-n_0)\,\mathrm{drcl}(F,n_1-n_0)$$

$$\mathrm{u}[n-n_0]-\mathrm{u}[n-n_1] \xleftrightarrow{\mathcal{Z}} \frac{e^{j\Omega}}{e^{j\Omega}-1}\left(e^{-jn_0\Omega}-e^{-jn_1\Omega}\right) = \frac{e^{-j\Omega(n_0+n_1)/2}}{e^{-j\Omega/2}}(n_1-n_0)\,\mathrm{drcl}(\Omega/2\pi,n_1-n_0)$$

3. MATLAB coding (2 points).

(a) Using the DFT, find the approximate CTFT of

$$x(t) = \begin{Bmatrix} t(1-t), & 0 < t < 1 \\ 0, & otherwise \end{Bmatrix} = t(1-t)\mathrm{rect}(t - 1/2)$$

(b) Graph the CTFT of 523.25/2Hz(C4), 587.33/2Hz(D4), 659.26/2Hz(E4), 698.46/2Hz(F4), 784/2Hz(G4), 440Hz(A4), 493.8Hz(B4), 523.25Hz(C5).

- Source code

- Plots

- Make an MP3 file containing C4, D4, E4, F4, G4, A4, B4, C5.