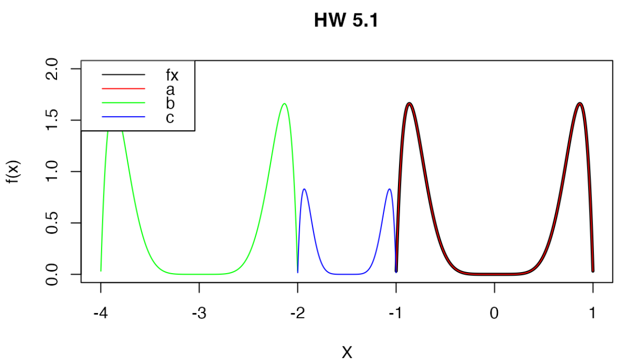
HW5

Thursday, October 9, 2025 9:27 AM

1. CB 3.36 $f_X = \frac{63}{4}(x^6 - x^8)$ $-1 < x < 1$ GRAPH: $\frac{1}{\sigma} f\left(\frac{x-\mu}{\sigma}\right)$ ON SAME AXIS

a. $\mu = 0$, $\sigma = 1$ b. $\mu = 3, \sigma = 1$ c. $\mu = 3$, $\sigma = 2$

Plot generated on R. Happy to share my code if needed. Note that the original (f(x)) and the plot of Part A are just the same graph, plotted on top of each other on the right side. Bit hard to see



2. $f_X(x|\mu,\sigma) = \frac{1}{\sigma\pi\left(1 + \left(\frac{x-\mu}{\sigma}\right)^2\right)}$

a. SHOW $P(X \geq \mu) = P(X \leq \mu) = \frac{1}{2}$

$P(X \geq \mu) = \int_\mu^\infty \frac{1}{\sigma\pi\left(1+\left(\frac{x-\mu}{\sigma}\right)^2\right)}\,dx = \frac{1}{\sigma\pi}\left[\sigma \cdot \operatorname{atan}\left(\frac{x-\mu}{\sigma}\right)\right]\Big|_\mu^\infty = \frac{1}{\pi}\left[\operatorname{atan}(\infty) - \operatorname{atan}(0)\right] = \frac{1}{\pi}\left[\frac{\pi}{2} - 0\right] = \frac{1}{2}$

$P(X \leq \mu) = \int_{-\infty}^{\mu} \cdots = \frac{1}{\pi}\left[\operatorname{atan}(0) - \operatorname{atan}(-\infty)\right] = \frac{1}{\pi}\left[0 - (-\pi/2)\right] = \frac{1}{2}$

b. SHOW $P(X \geq \mu + \sigma) = P(X \leq \mu - \sigma) = \frac{1}{4}$

LET $\mu = 0, \sigma = 1$: $P(X \geq \mu + \sigma) = \int_1^\infty \cdots = \frac{1}{\pi}\left[\operatorname{atan}(\infty) - \operatorname{atan}(1)\right] = \frac{1}{\pi}\left[\pi/2 - \pi/4\right] = \frac{1}{4}$

$P(X \leq \mu - 1) = \int_{-\infty}^{-1} \cdots = \frac{1}{\pi}\left[\operatorname{atan}(-1) - \operatorname{atan}(-\infty)\right] = \frac{1}{\pi}\left[-\pi/4 + \pi/2\right] = 1/4$

FROM 3.38,

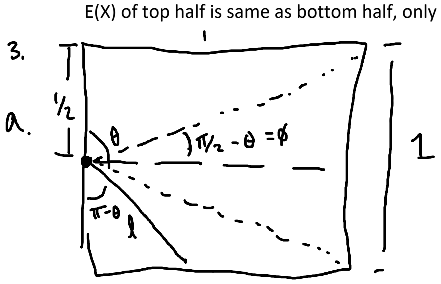
FOR PDF $P(Z \geq z_\alpha) = \alpha$, IF $x_\alpha = \sigma z_\alpha + \mu$ THEN $P(X \geq x_\alpha) = \alpha$

f_X DESCRIBED IN A MEET THESE REQUIREMENTS, WHERE NOW $x_\alpha = \mu + \sigma$. THUS $P(X \geq \mu + \sigma) = P(X \leq \mu - \sigma) = \frac{1}{4}$

E(X) of top half is same as bottom half, only need to evaluate on top half

3.

a.



2 REGIONS: $0 < \theta < \operatorname{atan}(2)$ (A)

$\operatorname{atan}(2) < \theta < \pi/2$ B)

REGION: A: $\ell(\theta) = \frac{1}{2\cos\theta}$ $\cos(\theta) = \frac{1/2}{\ell}$

B: $\ell(\theta) = \frac{1}{\cos(\pi/2 - \theta)}$ $\cos(\pi/2 - \theta) = 1/\ell$

$= \frac{1}{\cos\phi}$

atan(2)

$$P(A) = \frac{\operatorname{atan}(2)}{\pi/2} \qquad P(B) = \frac{\frac{\pi}{2} - \operatorname{atan}(2)}{\pi/2} = 1 - \frac{[illegible]}{\pi/2}$$

$$E(\theta \mid A) = \frac{1}{\operatorname{atan}2}\int_0^{\operatorname{atan}(2)} \frac{1}{2\cos\theta}\,d\theta = \overset{\frac{1}{\operatorname{atan}2}}{\frac{1}{2}}\Big[\ln(\sec\theta + \tan\theta)\Big]_0^{\operatorname{atan}(2)} = \overset{\frac{1}{\operatorname{atan}2}}{\frac{1}{2}}\Big[\ln(\sec(\operatorname{atan}(2)) + 2)\Big] - \frac{1}{2}\Big[\ln(\underbrace{1 + 0}_{0})\Big] \approx .652$$

$$E(\theta \mid B) = \frac{1}{\pi/2 - \operatorname{atan}2}\int_{\operatorname{atan}2}^{\pi/2} \frac{1}{\cos(\pi/2 - \theta)}\,d\theta = -\overset{1/(\pi/2 - \operatorname{atan}2)}{}\Big[\ln(\sec(\pi/2 - \theta) + \tan(\pi/2 - \theta)\Big]_{\operatorname{atan}2}^{\pi/2} = \overset{\frac{1}{\pi/2 - \operatorname{atan}2}}{}-\ln(\sec(0) + \tan(0)) + \ln(\sec(\pi/2 - \operatorname{atan}2) + \tan(\pi/2 - \operatorname{atan}2)) \approx 1.04$$

$$E(\theta) = E(\theta \mid A)P(A) + E(\theta \mid B)P(B) \approx .652\left(\frac{\operatorname{atan}2}{\pi/2}\right) + 1.04\left(1 - \frac{\operatorname{atan}(2)}{\pi/2}\right) \approx .766$$

b.

```r
#################
# Q3
#################
M = c(1000, 10000, 100000)

for (m in M) {
  theta <- seq(0, pi/2, length.out = m)
  gx <- numeric(m)
  cutoff = atan(2)

  for (i in 1:m) {
    ang <- theta[i]
    if (ang <= cutoff) {
      gx[i] <- 1 / (2 * cos(ang))
    }
    else {
      gx[i] <- 1 / cos((pi/2) - ang)
    }
  }
  cat("M:", m, "Mean:", mean(gx), "SE:", round(sd(gx)/sqrt(m), 4), "\n")
}
```

CONSOLE TERMINAL PROBLEMS OUTPUT PORTS DEBUG CONSOLE

~/BYU/stat230/Exam2

```
+       gx[i] <- 1 / (2 * cos(ang))
+     }
+     else {
+       gx[i] <- 1 / cos((pi/2) - ang)
+     }
+   }
+   cat("M:", m, "Mean:", mean(gx), "SE:", round(sd(gx)/sqrt(m), 4), "\n")
+ }
M: 1000 Mean: 0.7658567 SE: 0.0071
M: 10000 Mean: 0.7658708 SE: 0.0022
M: 1e+05 Mean: 0.7658722 SE: 7e-04
```

4.

```r
#################
# Q4
#################
M <- 10000
x_a <- seq(0, 1, length.out = M)
fx_a <- 5 / (atan(5) * (1 + 25*x_a^2))
x_b <- abs(rcauchy(M, location = 0, scale = 1/5))
x_b <- x_cauchy/max(x_cauchy)
fx_b <- 5 / (atan(5) * (1 + 25*x_b^2))
w_b <- 1/(2*dcauchy(x_b, 0, 1/5))
x_c <- rbeta(M, 2, 1)
fx_c <- 5 / (atan(5) * (1 + 25*x_c^2))
w_c <- 1/dbeta(x_c, 2, 1)
x_d <- rbeta(M, 1, 2)
fx_d <- 5 / (atan(5) * (1 + 25*x_d^2))
w_d <- 1/dbeta(x_d, 1, 2)
cat("Unif - E(X):", mean(fx_a), "VAR:", var(fx_a), "\n")
cat("Cauchy - E(X):", mean(fx_b * w_b), "VAR:", var(fx_b), "\n")
cat("Beta(2,1) - E(X):", mean(fx_c * w_c), "VAR:", var(fx_c), "\n")
cat("Beta(1,2) - E(X):", mean(fx_d * w_d), "VAR:", var(fx_d), "\n")
```

CONSOLE TERMINAL PROBLEMS OUTPUT PORTS DEBUG CONSOLE

~/BYU/stat230/Exam2

```
+ x_d <- rbeta(M, 1, 2)
+ fx_d <- 5 / (atan(5) * (1 + 25*x_d^2))
+ w_d <- 1/dbeta(x_d, 1, 2)
+ cat("Unif - E(X):", mean(fx_a), "VAR:", var(fx_a), "\n")
+ cat("Cauchy - E(X):", mean(fx_b * w_b), "VAR:", var(fx_b), "\n")
+ cat("Beta(2,1) - E(X):", mean(fx_c * w_c), "VAR:", var(fx_c), "\n")
+ cat("Beta(1,2) - E(X):", mean(fx_d * w_d), "VAR:", var(fx_d), "\n")
Unif - E(X): 1.000089 VAR: 1.075568
Cauchy - E(X): 1.143728 VAR: 0.006772578
Beta(2,1) - E(X): 1.11683 VAR: 0.3109136
Beta(1,2) - E(X): 1.003995 VAR: 1.30164
```

The Answers to parts A-D are given in the screenshots.

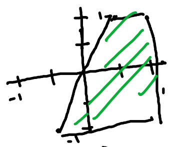
C: The Variance of the Cauchy is much lower because it has more data points concentrated where the function of interest is 'interesting' (ie. More points centered on the tall parts of the original function)

5. CB 4.1 $x \in [-1,1]$ $y \in [-1,1]$ $\quad f(x,y) = \frac{1}{4}$

a. $P(x^2+y^2<1)$ $\quad x^2+y^2 \Rightarrow$ AREA $= \pi$ $\quad$ TOTAL AREA $= (1--1) \times (1--1) = 2\times 2 = 4$

$P(x^2+y^2<1) = \pi/4$

b. $P(2x-y>0)$



$y<2x$

$2y<x<1 \quad -1<y<1$

$$\int_{-1}^{1}\int_{2y}^{1} \tfrac{1}{4}\,dx\,dy = \int_{-1}^{1} \tfrac{1}{4}x\Big|_{2y}^{1} dy = \int_{-1}^{1} \tfrac{1}{2} - \tfrac{1}{4}y\,dy = \tfrac{1}{2}y - \tfrac{1}{4}y^2\Big|_{-1}^{1} = \tfrac{1}{2} - \tfrac{1}{4} + \tfrac{1}{2} - \tfrac{1}{4} = \tfrac{1}{2}$$

$P(2x-y>0) = \frac{1}{2}$ $\quad$ *CAN ALSO SEE VISUALLY IT'S 1/2

c $P(|x+y|<2)$ $\quad |x+y| \in (0,2)$, THUS THIS COVERS THE FULL SUPPORT OF $|x+y|$, SO $P(|x+y|<2) = 1$

6. CB 4.4abc $\quad f_{xy} = C(x+2y) \quad 0<y<1,\ 0<x<2$

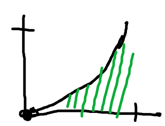
a. $\int_0^1\int_0^2 c(x+2y)\,dx\,dy = 1$ $\quad c\int_0^1 \frac{1}{2}x^2 + 2yx\Big|_0^2 dy = c\int_0^1 2+4y\,dy = c\left[2y+2y^2\right]_0^1 = c[2+2] = 4c = 1 \quad C = \frac{1}{4}$

b. $f_x = \int_0^1 \frac{1}{4}(x+2y)\,dy = \frac{1}{4}xy + y^2\Big|_0^1 = \frac{1}{4}x + 1$

c. $F(x,y) = \int_0^y \int_0^x \frac{1}{4}(a+2b)\,da\,db = \frac{1}{4}\int_0^y \left(\frac{1}{2}a^2+2ba\right)\Big|_0^x db = \frac{1}{4}\int_0^y \frac{1}{2}x^2 + 2bx\,db = \frac{1}{4}\left[\frac{1}{2}x^2b + b^2x\right]_0^y$

$= \frac{1}{4}\left[\frac{1}{2}x^2y + y^2x\right]$

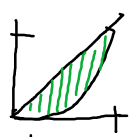
7. CB 4.5



a. $f(x,y) = x+y \quad 0<x<1 \quad 0<y<1$ $\quad$ FIND $P(x>\sqrt{y}) \Rightarrow P(x^2>y)$

$$P(x>\sqrt{y}) = \int_0^1\int_0^{x^2} x+y\,dy\,dx = \int_0^1 xy + \tfrac{1}{2}y^2\Big|_0^{x^2} dx = \int_0^1 x^3 + \tfrac{1}{2}x^4\,dx = \tfrac{1}{4}x^4 + \tfrac{1}{10}x^5\Big|_0^1 = \tfrac{1}{4} + \tfrac{1}{10} = \tfrac{7}{20}$$

b. $f(x,y) = 2x \quad 0<x<1,\ 0<y<1 \quad P(x^2<y<x)$

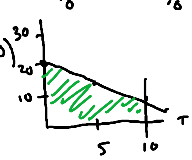


$$P(x^2<y<x) = \int_0^1\int_{x^2}^{x} 2x\,dy\,dx = \int_0^1 2xy\Big|_{x^2}^{x} dx = \int_0^1 2x^2 - 2x^3\,dx = \tfrac{2}{3}x^3 - \tfrac{1}{2}x^4\Big|_0^1 = \tfrac{2}{3} - \tfrac{1}{2} = \tfrac{1}{6}$$

8. CB 4.7 $\quad$ L = LEAVE : $0<L<30$ $\quad$ T = TRAVEL $0<T<10$ $\quad f_{(L,T)} = C(L+T+40) = A$

$$1 = \int_0^{30}\int_0^{10} C(L+T+40)\,dT\,dL = c\int_0^{30} LT + \tfrac{1}{2}T^2 + 40T\Big|_0^{10} dL = c\int_0^{30} 10L + 450\,dL = c\left[5L^2 + 450L\right]_0^{30} = c[900 + 13500] = 14400c \quad C = \tfrac{1}{14400}$$

$P(A<60) = P(L+T+40<60) = P(L+T<20)$



$$= \int_0^{10}\int_0^{20-T} \tfrac{1}{14400}(L+T+40)\,dL\,dT = c\int_0^{10} \tfrac{1}{2}L^2 + TL + 40L\Big|_0^{20-T} dT$$

$= c\int_0^{10} 1000 - 40T\,dT = c\left[1000T - 20T^2\right]_0^{10}$

$$= c \int_0 \tfrac{1}{2}(400 - 40T + T^2) + 20T - T^2 + 800 - 40T \, dT = c \int_0 200 - \ldots$$

$$= c\left[10000 - 2080\right] = \frac{8000}{14400} = \frac{80}{144} = \frac{5}{9}$$

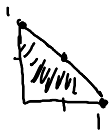
9. $f_{(x,y)} = \frac{6}{7}(x+y)^2 = \frac{6}{7}(x^2+2xy+y^2) \quad 0<x<1 \quad 0<y<1$

a. i. $P(X>Y)$

$$\int_0^1\int_0^x \frac{6}{7}(x^2+2xy+y^2)\,dy\,dx = \frac{6}{7}\int_0^1 x^2y + xy^2 + \tfrac{1}{3}y^3\Big|_0^x dx = \frac{6}{7}\int_0^1 x^3 + x^3 + \tfrac{1}{3}x^3\,dx = \frac{6}{7}\int_0^1 \tfrac{7}{3}x^3\,dx$$

$$= 2\cdot\tfrac{1}{4}x^4\Big|_0^1 = \frac{1}{2}$$

ii. $P(X+Y\le 1)$

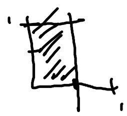


$$\int_0^1\int_0^{1-x} \frac{6}{7}(x^2+2xy+y^2)\,dy\,dx = \frac{6}{7}\int_0^1 x^2y + xy^2 + \tfrac{1}{3}y^3\Big|_0^{1-x} dx = \frac{6}{7}\int_0^1 x^2 - x^3 + x(1-2x+x^2) + \tfrac{1}{3}(1-2x+x^2)(1-x)$$

$$= \frac{6}{7}\int_0^1 x^2 - \cancel{x^3} + x - 2x^2 + \cancel{x^3} + \tfrac{1}{3}(1-2x+x^2-x+2x^2-x^3)\,dx = \frac{6}{7}\int_0^1 x - x^2 + \tfrac{1}{3} - x + x^2 - \tfrac{1}{3}x^3\,dx$$

$$= \frac{6}{7}\int_0^1 \tfrac{1}{3} - \tfrac{1}{3}x^3\,dx = \frac{2}{7}\left(x - \tfrac{1}{4}x^4\right)\Big|_0^1 = \frac{2}{7}\left(1-\tfrac{1}{4}\right) = \frac{2}{7}\left(\tfrac{3}{4}\right) = \frac{1}{7}\left(\tfrac{3}{2}\right) = \frac{3}{14}$$

iii. $P(X\le\frac{1}{2})$



$$\int_0^{1/2}\int_0^1 \frac{6}{7}(x^2+2xy+y^2)\,dy\,dx = \frac{6}{7}\int_0^{1/2} x^2y + xy^2 + \tfrac{1}{3}y^3\Big|_0^1 dx = \frac{6}{7}\int_0^{1/2} x^2 + x + \tfrac{1}{3}\,dx$$

$$= \frac{6}{7}\left(\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + \tfrac{1}{3}x\right)\Big|_0^{1/2} = \frac{6}{7}\left(\tfrac{1}{3}\left(\tfrac{1}{8}\right) + \tfrac{1}{8} + \tfrac{1}{6}\right) = \frac{6}{7}\left(\tfrac{1}{24} + \tfrac{3}{24} + \tfrac{4}{24}\right) = \frac{6}{7}\left(\tfrac{1}{3}\right) = \frac{2}{7}$$

b. $f_X = \int_0^x \frac{6}{7}x^2+2xy+y^2\,dy = \frac{6}{7}\left(x^2y + xy^2 + \tfrac{1}{3}y^3\right)_0^x = \frac{6}{7}\left(x^3+x^3+\tfrac{1}{3}x^3\right) = 2x^3$

$f_Y = \int_0^y \frac{6}{7}\left(x^2+2xy+y^2\right)dy = \frac{6}{7}\left(\tfrac{1}{3}x^3 + x^2y + y^2x\right)_0^y = \frac{6}{7}\left(\tfrac{7}{3}y^3\right) = 2y^3$

10. CB 4.31 $\quad Y \sim \text{BIN}(n,x)$

a. BINOMIAL DIST, WE KNOW $E(Y) = \frac{1}{x} \quad \text{VAR}(Y) = nx(1-x)$

b. $f(x,y) = \binom{n}{k}x^k(1-x)^{n-k}$ AGAIN, BECAUSE BINOM. DIST.

c. $f_Y = \frac{1}{N}\sum_{j=1}^{N}\binom{n}{k}x_j^k(1-x_j)^{n-k} = \frac{\binom{n}{k}}{N}\sum_{j=1}^{N}x_j^k(1-x_j)^{n-k}$

11. FIND $f_X(x) \quad f(x|y) = \frac{f(x,y)}{f_Y} \quad f(x,y) = f(x|y)\cdot f_Y \quad f_X = \int_{-\infty}^{\infty} f(x,y)\,dy$

a. $X|Y \sim \text{BIN}(n,Y)$, $Y\sim \text{BETA}(a,b) \quad f(x,y) = \binom{n}{x}y^x(1-y)^{n-x}\cdot\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\cdot y^{(a-1)}\cdot(1-y)^{b-1}$

$$= \binom{n}{x}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\cdot y^{(x+a-1)}\cdot(1-y)^{n+b-(x+1)}$$

$$f_X = \binom{n}{x}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^1 y^{x+a-1}(1-y)^{n-x+b-1}\,dy$$

$$= \binom{n}{x}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\frac{\Gamma(x+a)\Gamma(n-x+b)}{\Gamma(a+n+b)}\underbrace{\int_0^1 \frac{\Gamma(a+n+b)}{\Gamma(x+a)\Gamma(n-x+b)}y^{(x+a)-1}(1-y)^{(n-x+b)-1}\,dy}_{1}$$

$$= \binom{n}{x}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\frac{\Gamma(x+a)\Gamma(n-x+b)}{\Gamma(a+n+b)} \quad x\in 0,1,2,\ldots$$

This is probably the kernel of some function or something. I'm not sure

b. $X|Y\sim \text{Exp}(Y) \quad Y\sim\Gamma(a,b) \quad f(x,y) = ye^{-yx}\cdot Y \quad f_Y = \frac{1}{\Gamma(a)b^a}y^{a-1}e^{-yb}$

$$f_x = \int_0^\infty \frac{1}{\Gamma(a) b^a} y^a e^{-y(x+b)} \, dy = \frac{\Gamma(a+1)}{(x+b)^{a+1}\Gamma(a) b^a} \underbrace{\int_0^\infty \frac{(x+b)^{a+1}}{\Gamma(a+1)} y^{(a+1)-1} e^{-y(x+b)} \, dy}_{1}$$

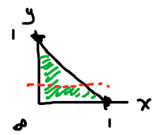
$$f_x = \frac{\Gamma(a+1)\, b^a}{(x+b)^{a+1}\Gamma(a)} = \frac{a b^a}{(x+b)^{a+1}} \quad x > 0$$

12. $f(X|Y) = \frac{f_{x,y}}{f_Y}$ $\quad f_{x,y} = f(y|x) f_x = \frac{y}{2x^2} \cdot 24x^2, \quad \begin{matrix} 0 < x < 1/2 \\ 0 < y < 2x \end{matrix}$ $\quad f_y = \int_0^{1/2} 12y \, dx = 6y$

$= 12y$

$$f(X|Y) = \frac{12y}{6y} = 2$$

13. $f_{x,y} = 24xy \quad x > 0,\ y > 0,\ 0 < x + y < 1$



$P(X < \frac{1}{2} | Y = \frac{1}{4})$ $\quad f(x|y) = \frac{f_{x,y}}{f_y}$ $\quad f_y = \int_{-\infty}^{\infty} f_{x,y} \, dx = \int_0^{1-y} 24xy \, dx = 12x^2 y \Big|_0^{1-y} = 12y(1 - 2y + y^2)$

$$= \frac{24x(1/4)}{12(1/4)(1 - \frac{1}{2} + \frac{1}{4})} = \frac{6x}{3(3/4)} = \frac{6x}{9/4} = \frac{24x}{9} = \frac{8}{3}x$$

$$1 - \int_0^{1/2} \frac{8}{3} x \, dx = \frac{4}{3} x^2 \Big|_0^{1/2} = 1 - \frac{4}{3}\left(\frac{1}{2}\right)^2 = \frac{2}{3}$$

$$P(X < \tfrac{1}{2} | Y = \tfrac{1}{4}) = \frac{2}{3}$$

14. $X|Y \sim \text{PARETO}(1, Y)$, $Y \sim \text{GAMMA}(\alpha, \beta)$ ← RATE $\quad$ FIND f_x $\quad f_x = \int_{-\infty}^{\infty} f_{x,y}$

$f_{x,y} = f(x|y) \cdot f_y$ $\quad f_y = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y}$ $\quad f_{x|y} = \frac{y}{x^{y+1}}$

$$f_{xy} = \frac{\frac{\beta^\alpha}{\Gamma(\alpha)} y^{(\alpha+1)-1} e^{-\beta y}}{x^y x} = \frac{\beta^\alpha}{\Gamma(\alpha) x} y^{(\alpha+1)-1} e^{-\beta y} \left(\frac{1}{x}\right)^y$$

$$f_x = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty y^{(\alpha+1)-1} e^{-\beta y} \underbrace{x^{-(y+1)}}_{e^{\ln(x^{-(y+1)})}} dy = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty y^{(\alpha+1)-1} e^{-\beta y} e^{-(y+1)\ln(x)} dy = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty y^{(\alpha+1)-1} e^{-\beta y - y\ln x} e^{-\ln x} dy$$

$$= \frac{\beta^\alpha}{\Gamma(\alpha)} e^{-\ln x} \int_0^\infty y^{(\alpha+1)-1} e^{-y(\beta + \ln x)} dy \qquad \begin{matrix} A = \alpha + 1 \\ B = \beta + \ln x \end{matrix}$$

$$= \frac{\beta^\alpha}{\Gamma(\alpha)} e^{-\ln x} \frac{\Gamma(\alpha+1)}{(\beta + \ln x)^{\alpha+1}} \underbrace{\int_0^\infty \frac{(\beta + \ln x)^{\alpha+1}}{\Gamma(\alpha+1)} y^{(\alpha+1)-1} e^{-y(\beta + \ln x)} dy}_{1}$$

$$= \frac{\beta^\alpha \alpha e^{-\ln x}}{(\beta + \ln x)^{\alpha+1}} = \frac{\beta^\alpha \alpha}{x(\beta + \ln x)^{\alpha+1}} \quad x > 1$$

15. CB 3.24 (a, b, d, ONLY FIND PDF)

a. $X \sim \text{EXP}(\beta)$ $\quad Y = X^{1/\gamma}$ $\quad F_x = 1 - e^{-\beta x}$ $\quad F_y = P(X^{1/\gamma} \le y) = P(X \le y^\gamma) = 1 - e^{-\beta y^\gamma}$ ← $\beta = \frac{1}{\lambda}$

$-\frac{\beta^2 y^2}{2}$ ← $\beta = \frac{1}{\sigma}$

b. $X \sim Exp(\beta)$ $Y = \left(\frac{2X}{\beta}\right)^{1/2}$ $F_x = 1 - e^{-\beta x}$ $F_Y = P\left(\left(\frac{2X}{\beta}\right)^{1/2} \le y\right) = P\left(X \le \frac{\beta}{2} y^2\right)$

d. $X \sim \Gamma(3/2, \beta)$ $Y = (X/\beta)^{1/2}$ $f_x = \frac{\beta^{3/2}}{\Gamma(3/2)} x^{1/2} e^{-\frac{3}{2}x}$ $f_Y = \left(\frac{\beta^{1/2}}{\Gamma(3/2)}\right)^{1/2} x^{1/4} e^{-3/4 x}$

16. $f_{X_i} = 3x^2 \quad 0 < x < 1$ For X_1, X_2, X_3 $P(X_1 \ge 1/2) = 1 - P(X_1 \le 1/2) = 1 - \int_0^{1/2} 3x^2\,dx = 1 - x^3\Big|_0^{1/2} = 1 - \frac{1}{8} = \frac{7}{8}$

THEY ARE IND, SO OPTIONS ARE: YYN, YNY, NYY $= \left(\frac{7}{8}\right)^2\left(\frac{1}{8}\right)\cdot 3 = \frac{49 \cdot 3}{512} = \frac{147}{512}$

$P(Y) = \frac{7}{8}$ $P(N) = \frac{1}{8}$

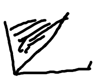
17. $f_{(x,y)} = e^{-(x+y)} \quad 0 < x < \infty, 0 < y < \infty$

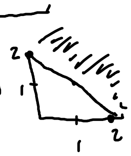
a. $f_x = \int_0^x e^{-(x+y)}\,dy = -e^{-(x+y)}\Big|_0^x = e^{-x} - e^{-2x}$ $\quad f_y = \int_0^y e^{-(x+y)}\,dx = e^{-y} - e^{-2y}$

b. $F_{(x,y)} = \int_0^x \int_0^y e^{-(a+b)}\,db\,da = \int_0^x -e^{-(a+b)}\Big|_0^y da = \int_0^x e^{-a} - e^{-(a+y)}\,da = -e^{-a} + e^{-(a+y)}\Big|_0^x$

$= \left[-e^{-x} + e^{-(x+y)} + e^0 - e^{-y}\right] = 1 + e^{-(x+y)} - e^{-x} - e^{-y}$

c. $P(X > 2) = \int_2^\infty f_x\,dx = \int_2^\infty e^{-x} - e^{-2x}\,dx = -e^{-x} + \frac{1}{2}e^{-2x}\Big|_2^\infty = \left[-e^{-\infty} + \frac{1}{2}e^{-2\infty} + e^{-2} - \frac{1}{2}e^{-2(2)}\right] = \frac{1}{e^2} - \frac{1}{2e^4}$

d. $P(X<Y)$  $= \int_0^\infty \int_x^\infty e^{-(x+y)}\,dy\,dx = \int_0^\infty -e^{-(x+y)}\Big|_x^\infty dx = \int_0^\infty -e^{-\infty} + e^{-2x}\,dx = -\frac{1}{2}e^{-2x}\Big|_0^\infty = \frac{1}{2}$

e. $P(X+Y>2)$ 

$= 1 - P(X+Y<2) = 1 - \int_0^2 \int_0^{2-x} e^{-(x+y)}\,dy\,dx = 1 - \int_0^2 -e^{-(x+y)}\Big|_0^{2-x} dx = 1 - \int_0^2 -e^{-2} + e^{-x}\,dx$

$= 1 - \left[-\frac{x}{e^2} - e^{-x}\right]_0^2 = 1 - \left[-\frac{2}{e^2} - \frac{1}{e^2} + 0 + 1\right] = 1 - \left[1 - \frac{3}{e^2}\right] = \frac{3}{e^2}$

f. IND. IFF $f(x,y) = g(x)\,h(x)$ $\quad f(x,y) = e^{-x-y} = e^{-x}e^{-y} = g(x)\,h(y)$

YES, THEY ARE INDEPENDENT