

Math Review

Monday, September 8, 2025 9:42 PM

1. DERIVATIVES:

a) $4(x^2+3)^3 \sin(5\pi x) \Rightarrow 4a^3 \sin(5\pi x) \Rightarrow u v \Rightarrow u'v + u v'$
 $u' = 12(x^2+3)^2 \cdot 2x \quad v' = 5\pi \cos(5\pi x)$

$$f'(x) = 24x(x^2+3)^2 \sin(5\pi x) + 20\pi(x^2+3)^3 \cos(5\pi x)$$

b) $f(x) = \frac{x^2-1}{2x+3} \Rightarrow (x^2-1) \cdot (2x+3)^{-1} \Rightarrow u v \Rightarrow f'(x) = u'v + u v'$
 $u' = 2x \quad v' = -2(2x+3)^{-2}$

$$f'(x) = \frac{2x}{2x+3} - \frac{x^2-1}{2(2x+3)^2}$$

2. INTEGRATE

$$\int (x^2-1)^4 2x dx = \int 2x(x^4-2x^2+1)^2 dx = \int 2x(x^8-4x^6+4x^4-2x^2+1) dx$$

$$= \int (2x^9-8x^7+8x^5-4x^3+2x) dx = \frac{x^{10}}{5} - x^8 + \frac{4x^6}{3} - x^4 + x^2$$

3. SOLVE FOR X

a) $\ln(x) = 2 \quad x = e^2$

b) $\ln((2x+1)(x+2)) = \ln(x+2) = \ln((x+2)^2) \Rightarrow (2x+1)(x+2) = (x+2)^2 \Rightarrow$
 $2x+1 = x+2 \Rightarrow x = 1$

4. EVALUATE

a) $\int_6^{\infty} x e^{-2x} dx \Rightarrow u = x \quad dv = e^{-2x} dx$
 $du = 1 dx \quad v = -\frac{e^{-2x}}{2} \Rightarrow u v - \int v du \Rightarrow -\frac{x e^{-2x}}{2} \Big|_6^{\infty} + \int_6^{\infty} \frac{e^{-2x}}{2} dx$
 $\Rightarrow -0 + 0 + \left(\frac{e^{-2x}}{2^2} \right) \Big|_6^{\infty} = -0 + \frac{1}{2^2} = \frac{1}{2^2}$

b) $\int_1^2 x^3 \ln x dx \quad u = \ln x \quad dv = x^3$
 $du = \frac{1}{x} dx \quad v = \frac{x^4}{4} \Rightarrow \frac{x^4 \ln x}{4} \Big|_1^2 - \int_1^2 \frac{x^3}{4} dx \Rightarrow$
 $\frac{2^4 \ln(2)}{4} - \frac{1^4 \ln(1)}{4} - \frac{x^4}{16} \Big|_1^2 = 4 \ln(2) - 0 - \left(\frac{2^4}{16} - \frac{1^4}{16} \right) = 4 \ln(2) - 1 + \frac{1}{16} = 4 \ln(2) - \frac{15}{16}$

5. FIND:

$\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2} \Rightarrow \infty \Rightarrow \infty$

$$6. \frac{\partial}{\partial x} (e^{xy} + \ln(x^2 + y)) = \frac{1}{y} e^{xy} + \frac{2x}{x^2 + y}$$

$$7. \int \int \sqrt{xy} dx dy = \int \left(\frac{2}{3} x^{3/2} \right) y \Big|_0^y dy = \int \frac{2}{3} (y^{3/2} - y^3) dy = \frac{2}{3} \left(\frac{2}{7} y^{7/2} - \frac{y^5}{5} \right) \Big|_0^1 = \frac{4}{21} - \frac{1}{5}$$

$$8. a) x^2 - 3x + 2 = 0 \quad (x-2)(x-1) = 0$$

$$x = 2, x = 1$$

$$b) 9x^2 + 18x - 17 = 0 \quad \frac{-18 \pm \sqrt{324 + 612}}{18}$$

$$= -1 \pm \frac{\sqrt{936}}{18}$$

$$= -1 \pm \sqrt{\frac{52}{18}}$$

$$9. ax^2 + bx + c = 0 \text{ REAL } x \text{ IFF } \sqrt{b^2 - 4ac} \geq 0 \therefore b^2 - 4ac \geq 0 \therefore b^2 \geq 4ac \text{ AND } a \neq 0$$

10. LIMITS as $n \rightarrow \infty$

$$a) x^{1/n} \text{ For } x > 0 \Rightarrow 0 \text{ IF } x > 1, 1 \text{ IF } x \leq 1,$$

$$b) x^n \text{ For } |x| > 1 \Rightarrow 0 \text{ IF } |x| < 1, \left(\frac{1}{x}\right)^n = 0$$

$$c) \frac{1}{n^a} \text{ For } a > 0 \Rightarrow 0 \quad \frac{1}{n^a} \text{ ALWAYS STRONGER}$$

$$d) \left(1 + \frac{x}{n}\right)^n \Rightarrow \infty \quad (n \text{ POWER GROWS FASTER: } \left(1 + \frac{x}{n}\right)^n)$$

$$11. a) \text{ TOTAL: } 30 \quad \text{DICK: } 5 \quad \binom{30}{5} = 142506$$

$$b) 3 \text{ MEN, } 2 \text{ WOMEN: } \binom{17}{3} \binom{13}{2} = 53640$$

$$c) 1 \text{ required man: } 1 \binom{16}{2} \binom{13}{2} = 43680$$

$$d) \text{ MOST PEEK 3 LEADERS: } 30 \cdot 29 \cdot 28 = 24360$$

$$12. a) 7 \text{ choose } 3: \binom{7}{3} = 35$$

$$b) \text{ ORDER: } 7 \cdot 6 \cdot 5 = 21 = 5040$$

$$13. a) \text{ POSSIBLE VALUES: } 26 + 10 = 36 \quad \text{TOTAL PASSWORDS: } 36^8$$

$$b) \text{ FROM TOTAL, REMOVE THOSE THAT DON'T MEET CRITERIA: } 36^8 - 26^8$$

$$14. a) i: \{(111111), (11111), (1111), (111), (11), (1), (0)\} \dots$$

$$ii: [0, \infty) \text{ INTEGERS}$$

$$iii: [0, \infty)$$

$$iv: (0, \infty)$$

$$v: [0, 1]$$