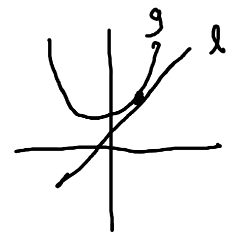
30 Sep 2025

Tuesday, September 30, 2025 1:59 PM

JENSEN'S : CONVEX: $E[g(x)] \geq g[E(x)]$

PASSES THROUGH $E(x), g(E(x))$

PROOF: LET $l(x)$ BE A TANGENT LINE TO $g(x)$ (g IS CONVEX)



$\therefore\ g(x) \geq l(x) = a + bx$

$E(g(x)) \geq E(a+bx) = a + bE(x) = l[E(x)] = g[E(x)]$

Characterizing Distributions

Thm. 2.3.11: Fx(x), Fy(y) are CDFs, all of whose moments exist.
A) If X and Y have bounded support, Fx(u) = Fy(u) for all u iff E[X^r] = E[Y^r], r = 0, 1, 2, 3, ...
B) If MGF exists Mx(t) = My(t) for some t in neighborhood of 0, then Fx(u) = Fy(u) for all u

Thm. 2.3.12: A sequence of r.v.s, {Xi, i = 1, 2, 3,...} each with an MGF M_xi (t). Further suppose lim(i->inf) M_xi (t) = M_x (t) for t in neighborhood of 0, and M_x(t) is also an MGF
Then: there is a unique CDF F_X(x) whose moments are determined by M_x (t) and lim(i->inf) F_xi (x) = F_x (x)

Ex. SHOW BIN(n,p) CAN BE APPROX. BY POISSON $(\lambda = np)$ WHEN n IS LARGE AND np IS SMALL ($\lambda = np$ IS CONSTANT)

LEMMA: LET $a_1, a_2, a_3 \ldots$ BE SEQ. OF #s S.T. $\lim_{n\to\infty} a_n = a$

THEN $\lim_{n\to\infty}\left(1 + \frac{a_n}{n}\right)^n = e^a$

NOTE: $Y \sim \text{POIS}(\lambda)$, $M_Y = e^{\lambda(e^t - 1)}$

$X \sim \text{BIN}(n,p)$, $M_X = [pe^t + (1-p)]^n$

$\lim_{n\to\infty} M_X = \lim [pe^t + (1-p)]^n = \lim [1 + p(e^t - 1)]^n$ $\quad \lambda = np$

$= \lim \left[1 + \frac{\lambda}{n}(e^t - 1)\right]^n = \lim \left[1 + \frac{\lambda(e^t - 1)}{n}\right]^n \Rightarrow e^{\lambda(e^t - 1)}$

Ex. FINDING EXP. VALUES

$X \sim N(\mu, \sigma^2)$ $\quad E[e^x] = E[e^{1x}] = M_X(1) = e^{\mu(1) + \frac{\sigma^2(1)^2}{2}} = e^{\mu + \sigma^2/2}$