In Class: Feb 10

RJ Cass

$X_i \overset{\text{iid}}{\sim} Pareto(\theta_1, \theta_2) : f(x|\theta) = \frac{\theta_1 \theta_2^{\theta_1 - 1}}{x^{\theta_1+1}}; x > \theta_2; \theta_1, \theta_2 > 0$

Find the sufficient statistic for $\theta = (\theta_1, \theta_2)$.

$$\begin{aligned}
L(\theta|x) &= \prod_{i=1}^{n} \frac{\theta_1 \theta_2^{\theta_1 - 1}}{x_i^{\theta_1+1}} I(x > \theta_2) \\
&= \theta_1^n \theta_2^{n(\theta_1 - 1)} \prod \frac{1}{x_i^{\theta_1+1}} I(x > \theta_2) \\
&= \theta_1^n \theta_2^{n(\theta_1 - 1)} e^{ln(\sum x_i^{-(\theta_1+1)} I(x > \theta_2))} \\
&= \theta_1^n \theta_2^{n(\theta_1 - 1)} e^{-(\theta_1+1) \sum ln(x_i) + I(x > \theta_2)}
\end{aligned}$$

Using what we talked about in class of the $T(x)$ being in the exponent, we have the two pieces: $\sum ln(x_i)$ and $x > \theta_2$. The second term can be simplifed to just needing the minimum of x to be greater than θ_2:

$\therefore T(\theta) = (\sum ln(x_i), x_{(1)})$