HW1

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1)

Setup:

$P(p), P(n)$: Probability of positive/negative result

$P(C) = .001, P(nC) = .999$ Probability of cancer/no cancer

$P(p|C) = .98, P(n|nC) = .96 \implies P(n|C) = .02, P(p|nC) = .04$

$P(p) = P(p|C) * P(C) + P(p|nC) * P(nC) = .98(.001) + .02(.04) = 0.00178$

a) $P(C|p) = \frac{P(p|C)P(C)}{P(p)} = \frac{.98(.001)}{.00178} = .551$

b) Given the likelihood of having cancer given a positive result is only around 1/2, I would suggest to the researchers that they either repeat the test multiple times on a subject (useful if test is cheap/fast) or find a way to reduce the chance of a positive result when the patient doesn't have cancer (given the current values, that will have a larger impact than increasing positive results if they do have cancer).

2)

a) We will assume that we pick door 1, and that Monty will open door 2. The probability that the car is behind door 1 is 1/3. There are 3 possibilities for Monty opening door 2:

P1: P(Monty opens door 2 | Car behind door 1) = 1/2

P2: P(Monty opens door 2 | Car behind door 2) = 0

P3: P(Monty opens door 2 | Car behind door 3) = 1

Thus, P(Monty opens door 2) = P(B) = $P1 * 1/3 + P2 * 1/3 + P3 * 1/3 = 1/6 + 1/3 = 1/2$

Thus, P(Car behind door 1 | Monty opens door 2) = P(A|B) = $\frac{P(B|A)P(A)}{P(B)} = \frac{(1/2)(1/3)}{1/2} = 1/3$

This means the probability of the car being behind door 1 (the original choice) is only 1/3, where as if we switch the probability is 2/3.

b)

I ran a simulation with 10^4 trials, which resulted in 3351 success when NOT switching, and 6649 successes when switching, roughly the same 1/3 vs. 2/3 ratio.