

1. Expected utility of agent at $T = 0$:

$$U(c_1, c_2) = tu(c_1) + (1 - t)\rho u(c_2)$$

where...

$$u'(0) = \infty$$

$$u'(\infty) = 0$$

$$-\frac{u''(c)}{u'(c)} > 1$$

2. Optimal allocation:

$$\max_{x, c_1, c_2} [tu(c_1) + (1 - t)\rho u(c_2)]$$

...subject to resource constraints for periods 1 and 2:

$$tc_1 \leq x$$

$$(1 - t)c_2 \leq (1 - x)R$$

The resource constraints will be satisfied with equality since the function is strictly increasing. Therefore:

$$\max \left[tu \frac{x}{t} + (1 - t)\rho u \frac{(1 - x)R}{1 - t} \right]$$

FOC...

$$u' \left(\frac{x}{t} \right) = \rho R u' \left(\frac{(1 - x)R}{1 - t} \right)$$

or

$$u'(c_1) = \rho R u'(c_2)$$

$$\Rightarrow 0 \leq x \leq 1 \text{ at the optimum, so } c_1 > 0$$

$$\Rightarrow \text{Since } \rho R \geq 0 \text{ and the function is strictly concave: } c_2 \geq c_1$$

Therefore, it is possible to say that this allocation is efficient and incentive compatible.

3. The optimum:

$$c_1^* > 1$$

$$c_2^* < R$$