**CSc 245 Discrete Structures - Summer 2021**

# Homework #4

Due: July 2nd, 2021 by 11:59 p.m (MST).

**Instructions:**

1. **Homework assignments are to be completed individually, not in groups.**
2. If you need help, take advantage of Piazza and office hours.
3. Assignments are to be submitted in PDF form via Gradescope. They may be typed (which is preferable and strongly recommended) or handwritten with each page scanned or photographed and compiled into a single PDF.
4. If you choose to handwritten your assignments, please write neatly. Illegible assignments may not be graded.
5. Extra credit will be given for typed homework. To make this easier, a `Latex` template will be provided for each assignment.
6. Show your work (when appropriate) for partial credit!
7. In your proofs, be sure to do the following:
   - Start your proof with "Proof (style):" where style is the type of proof you are using.
   - State any assumptions you are making.
   - Clearly define any variables used.
   - Conclude with "Therefore," and then restate the conjecture that you proved.

1. Prove or disprove: If the average of 4 distinct integers is 10, at least one of the 4 integers must be greater than 12.
2. Prove the following conjecture: for all real numbers a,b, and c, max(a,max(b,c))=max(max(a,b),c).
3. You have 6 colors of socks in your drawer. Prove, using contradiction, that if you pick 19 socks, you must have at least two pairs of the same color.
4. Prove the following conjecture: For any two integers $m$ and $n$, $m-n$ is even if and only if $m^2-n^2$ is even. Because this is a biconditional proposition (if and only if), be sure to prove both directions.
5. Complete the following proof.

   **Proposition**: If $3|a^2$, then $3|a$.

   **Proof**: (By contradiction) Assume $3|a^2$ but $3 \nmid a$.

   By definition of the divides operator, $a^2 \bmod 3 = 0$. We know that when we divide an integer by 3, there are only three possible remainders: $0, 1$, or 2 (otherwise, we could have divided out an additional 3). Thus, $a\%3$ will be either $0, 1$, or 2. However, since $3 \nmid a$, $a\%3$ can only be 1 or 2.

   Assume $a\%3 = 1$. Thus, $a = 3n+1$. $a^2 = 9n^2 + 6n + 1$. [Complete the proof starting here.]

6. Prove, by contradiction, that $\sqrt{3}$ is irrational. (Hint: use the result of the proof in the previous question).

7. Prove, by contraposition, that if $x^3 - 1$ is even, then $x$ is odd.

8. Explain what is wrong with this proof.

   **Proposition**: If $n$ and $m$ are even integers, then $4|(n+m)$.

   **Proof**: (Direct) Assume $n$ and $m$ are even.
   By definition of an even number, $\exists k$ such that $n = 2k$. and $\exists k$ such that $m = 2k$.
   Thus, $n + m = 2k + 2k = 4k$.
   We know that $(n + m) \bmod 4 = 0$ because $n + m = 4k$ and $4k$ is clearly a multiple of 4
   Because $(n + m) \bmod 4 = 0$, we know from the definition of the divides operator that $4|(n + m)$.
   Therefore, if $n$ and $m$ are even integers, then $n + m$ is divisible by 4.