CSc 245 Discrete Structures - Summer 2021

Homework #5

Due: July 16th, 2021 by 11:59 p.m (MST).
(64 points)

Instructions:

1. **Homework assignments are to be completed individually, not in groups.**
2. If you need help, take advantage of Piazza and office hours.
3. Assignments are to be submitted in PDF form via Gradescope. They may be typed (which is preferable and strongly recommended) or handwritten with each page scanned or photographed and compiled into a single PDF.
4. If you choose to handwritten your assignments, please write neatly. Illegible assignments may not be graded.
5. Extra credit will be given for typed homework. To make this easier, a `Latex` template will be provided for each assignment.
6. Show your work (when appropriate) for partial credit!

1. (6 points) Determine if each of the following relations are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.
   (a) $\{(a, b) \mid a$ is taking fewer courses than b, $a, b \in$Students$\}$
   (b) $\{(x, y) \mid x * y > 0,\ x \in \mathbb{Z}\}$
   (c) $\{(x, y) \mid x = 2,\ x, y \in \mathbb{Z}\}$
2. (6 points) Let $R = \{(2, 1), (3, 1), (2, 2), (1, 3)\}$ on $\{1, 2, 3\}$ and $S = \{(1, 2), (3, 3), (2, 1), (1, 3), (4, 1)\}$ on $\{1, 2, 3, 4\}$. Perform the following operations:
   (a) $R \cap S$
   (b) $R - S$.
   (c) $R \circ S$
3. (6 points) Given the following relations, perform each specified operation.
   $P = \{(x, y) \mid x = 2y\ x, y \in \mathbb{Z}\}$,
   $Q = \{(x, y) \mid x = 2y + 1\ x, y \in \mathbb{Z}\}$,
   $R = \{(x, y) \mid xy > 0\ x, y \in \mathbb{R}\}$, and
   $S = \{(x, y) \mid xy < 0\ x, y \in \mathbb{R}\}$
   (a) $P \cap Q$
   (b) $P \cup Q$
   (c) $R \circ S$

4. (4 points) For each of the following relations, give the matrix representation for that relation. Then, using the matrix, determine if the relation is reflexive, symmetric, antisymmetric, and/or transitive. Show your work.

   (a) $R = \{(1,1),(1,2),(2,2),(3,1),(3,3)\}$

   (b) $R = \{(x,y) \mid x\%y = 0,\ x,y \in H\}$ on the set $H = \{1,2,3,4\}$.

5. (6 points) In lecture we learned how to determine properties of relations using matrices. We are also able to use matrices to find the union, intersection and composite of two relations. These three operations on relations correspond to the join, meet, and boolean product operators of zero-one matrices. For two relations A and B, we can find their union by using the "join" operator on their corresponding matrices ($M_A \wedge M_B$), their intersection by using the "meet" operator on their corresponding matrices ($M_A \vee M_B$), and their composite ($A \circ B$) by taking the boolean product of their corresponding matrices ($M_B \odot M_A$, note the order of the matrices). Let R and S be the relations on the set $\{a,b,c\}$ corresponding to the below matrices. Use the below matrices to perform the following operations on the relations R and S.

   $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $M_S = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

   (a) $R \cup S$

   (b) $R \cap S$

   (c) $R \circ S$

6. (5 points) Let $R = \{(x,y), \mid xy > 0\ x,y \in H\}$ on $H = \{-2,-1,0,1,2\}$.

   (a) Draw the graph of the following relation.

   (b) Using the graph, determine if the relation is reflexive, irreflexive, symmetric, antisymmetric, and/or transitive. Briefly explain your answer.

7. (6 points) Describe how we can use the graphs of two relations R and S to create a graph corresponding to each of the following operations.

   (a) Intersection

   (b) Composite

   (c) Inverse

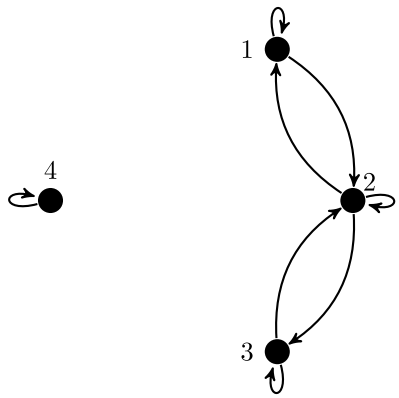
8. (9 points) For each of the following, determine if it is an equivalence relation. If it is an equivalence relation, show that it satisfies all required properties. If it is not, state all properties of an equivalence relation that it lacks and explain why it does not satisfy them.

   (a) $\{(a,b) \mid a$ and b have the same birthday, $a,b \in$People$\}$

   (b) $\{(x,y) \mid xy > 0,\ x \in \mathbb{R}\}$

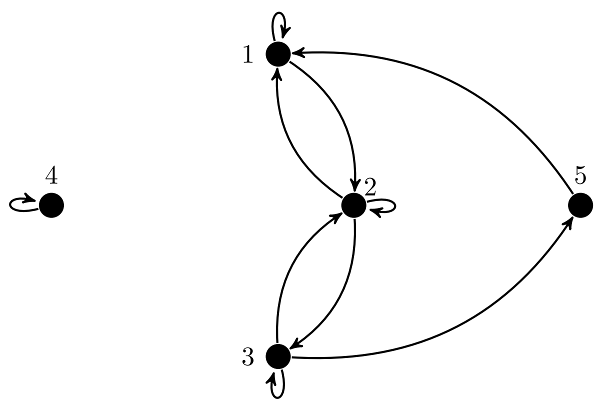
   (c) $\{(x,y) \mid x - y > 0\ x,y \in \mathbb{Z}\}$

9. (4 points) Use the graph below to answer the following questions.



   (a) The above graph represents a relation on the set $\{1, 2, 3, 4\}$. What are the equivalence classes of the equivalence relation in the above graph?

   (b) We now modify our relation to expand its domain and codomain to be the set $\{1, 2, 3, 4, 5\}$ and adds some new pairs to the relation. Below is our updated graph. What edges must we add to this graph to make it an equivalence relation?

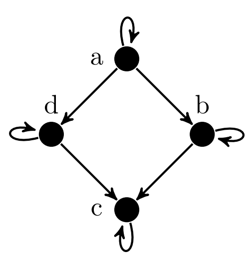


10. (12 points) For each of the following, determine if it is (i) a weak partial ordering, (ii) a strict partial ordering and/or (iii) a total ordering. For each type of relation that it is, show that it satisfies the required properties. For any type that it is not, specify the properties it fails to satisfy and explain why it does not satisfy them.

   (a) $\{(a, b) \mid a \text{ is older than } b\ a, b \in \text{People}\}$ on the set of all people in the world.

   (b) $\{(x, y) \mid x \leq y,\ x, y \in \mathbb{Z}\}$ on the set $\mathbb{Z}$

   (c) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$



   (d)