CSc 245 — Introduction to Discrete Structures (McCann)

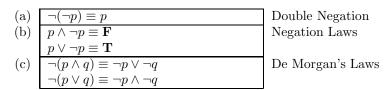
Last Revised: May 2014

The Page O' Logical Equivalences ("POLE")

<u>Table I</u>: Some Equivalences using AND (\wedge) and OR (\vee):

(a)	$ \begin{cases} p \land p \equiv p \\ p \lor p \equiv p \end{cases} $	Idempotent Laws
	$p \lor p \equiv p$	
(b)	$p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination Laws
	$p \lor \mathbf{T} \equiv \mathbf{T}$	
(c)	$p \wedge \mathbf{T} \equiv p$	Identity Laws
	$p \lor \mathbf{F} \equiv p$	
(d)	$p \wedge q \equiv q \wedge p$	Commutative Laws
	$p \vee q \equiv q \vee p$	
(e)	$(p \land q) \land r \equiv p \land (q \land r)$	Associative Laws
	$(p \lor q) \lor r \equiv p \lor (q \lor r)$	
(f)	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive Laws
	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	
(g)		Absorption Laws
/	$ \begin{array}{l} p \wedge (p \vee q) \equiv p \\ p \vee (p \wedge q) \equiv p \end{array} $	_
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<u>Table II</u>: Some More Equivalences (adding Negation (\neg)):



<u>Table III</u>: Still More Equivalences (adding Implication (\rightarrow)):

(a) $p \rightarrow q \equiv \neg p \lor q$ (b) $p \rightarrow q \equiv \neg q \rightarrow \neg p$ (c) $\mathbf{T} \rightarrow p \equiv p$ (d) $p \rightarrow \mathbf{F} \equiv \neg p$ (e) $p \rightarrow p \equiv \mathbf{T}$ (f) $p \rightarrow q \equiv (p \land \neg q) \rightarrow \mathbf{F}$ (g) $\neg p \rightarrow q \equiv p \lor q$ (h) $\neg (p \rightarrow q) \equiv p \land \neg q$ (i) $\neg (p \rightarrow q) \equiv p \land \neg q$ (j) $(p \rightarrow q) \lor (q \rightarrow p) \equiv \mathbf{T}$ (k) $(p \land q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$ (l) $(p \land q) \rightarrow r \equiv (p \rightarrow r) \lor (q \rightarrow r)$ (m) $(p \lor q) \rightarrow r \equiv (p \rightarrow r) \land (q \rightarrow r)$ (n) $p \rightarrow (q \land r) \equiv (p \rightarrow q) \land (p \rightarrow r)$ (o) $p \rightarrow (q \lor r) \equiv (p \rightarrow q) \lor (p \rightarrow r)$ (p) $p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \rightarrow r)$ Law of the Contrapositive "Law of the Contrapositive "Law of the True Antecedent" Self-implication (a.k.a. Reflexivity) Reductio Ad Absurdum Exportation Law (a.k.a. Currying)

<u>Table IV</u>: Yet More Equivalences (adding Exclusive OR (\oplus) and Biimplication $(\leftrightarrow)):$

(a) $p \leftrightarrow q \equiv (p \to q) \land (q \to p)$ Definition of Biimplication (b) $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$ $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$ Definition of Exclusive Or (c) $p \oplus q \equiv \neg (p \leftrightarrow q)$ Definition of Exclusive Or

Notes:

- 1. p, q, and r represent arbitrary logical expressions. They may represent equivalent expressions (e.g., if $p \equiv q$, then by Absorption $p \land (p \lor p) \equiv p$).
- 2. **T** and **F** represent the logical values True and False, respectively.
- 3. These tables show many of the common and/or useful logical equivalences; this is not an exhaustive collection!