Math Review Summary

CSc 245, Summer 2020

This is a summary of important math concepts from the math review appendix from Dr. McCann's book. For a more detailed review, please read the appendix (on the course webpage).

1 Fractions

Common Fraction Equalities

(a) $\frac{x}{z} + \frac{y}{z} = \frac{x+y}{z}$ (b) $\frac{x}{z} - \frac{y}{z} = \frac{x-y}{z}$ (c) $\frac{x}{z}\frac{y}{z} = \frac{xy}{z^2}$ (d) $\frac{\frac{x}{z}}{\frac{y}{z}} = \frac{x}{y}$

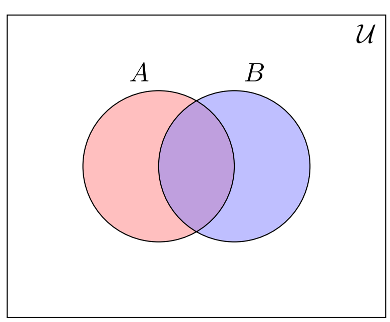
(e) $\frac{x}{w} + \frac{y}{z} = \frac{xz+yw}{wz}$ (f) $\frac{x}{w} - \frac{y}{z} = \frac{xz-yw}{wz}$ (g) $\frac{x}{w}\frac{y}{z} = \frac{xy}{wz}$ (h) $\frac{\frac{x}{w}}{\frac{y}{z}} = \frac{xz}{wy}$

2 Rational Numbers

Rational Number: A value that can be expressed as the ratio of two integers

3 Set Basics

- **Set**: an unordered collection of unique objects $S = \{x_1, x_2, \ldots\}$
- **Notation**:
  - $s \in S$ s is a member of S
  - $\emptyset$ is the empty set ($S = \{\}$)
  - { variables | constraints for membership } ("variables such that they satisfy the constraints for membership")
  - $\mathcal{U}$ is the universal set (all objects that could possibly be in the set)
- **Operators**:
  - **Union**: $A \cup B$, all objects in A or B (or both)
  - **Intersection**: $A \cap B$, all objects in both A and B
  - **Difference**: $A - B$, all objects in both A that are not also in B
  - **Complement**: $\overline{A}$, all objects in $\mathcal{U}$ that are not in A ($\mathcal{U} - A$)
  - **Cardinality**: $|A|$, the number of objects in A
- **Venn Diagram**:



- **Notations of Sets of Numbers**:
  - $\mathbb{Z}$: All integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
  - $\mathbb{Z}^+$: Positive integers $\{1, 2, 3, \ldots\}$
  - $\mathbb{Z}^0$: Non-negative integers $\{0, 1, 2, 3, \ldots\}$
  - $\mathbb{Z}^-$: Negative integers $\{\ldots, -3, -2, -1\}$
  - $\mathbb{Z}^{even}$: Even integers $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$
  - $\mathbb{Z}^{odd}$: Odd integers $\{\ldots, -3, -1, 1, 3, \ldots\}$
  - $\mathbb{Q}$: Rational numbers
  - $\overline{\mathbb{Q}}$: Irrational numbers
  - $\mathbb{R}$: all real numbers

4 Associative, Commutative, Distributive, and Transitive properties

- **Associative**: An operation $\diamond$ is associative if $a \diamond (b \diamond c) = (a \diamond b) \diamond c$
- **Commutative**: An operation $\diamond$ is commutative if $a \diamond b = b \diamond a$
- **Distributive**: Operations $\diamond$ and $\Box$ are distributive if:

  $a\Box(b \diamond c) = (a\Box b) \diamond (a\Box c)$ ($\Box$ is left-distributive over $\diamond$) and

  $(b \diamond c)\Box a = (b\Box a) \diamond (c\Box a)$ ($\Box$ is right-distributive over $\diamond$)
- **Transitive**: An relationship $\circ$ is transitive if whenever $a \circ b$ and $b \circ c$, then $a \circ c$ (e.g. $a < b$ and $b < c$ implies $a < c$).

5 Properties of Inequalities

- **Addition:** If $a < b$, then $a + c < b + c$. This holds for $\leq, >, \geq$.
- **Multiplication ($c > 0$):** If $a < b$, then $ac < bc$. This holds for $\leq, >, \geq$.
- **Multiplication ($c < 0$):** If $a < b$, then $ac > bc$. This holds for $\leq, >, \geq$ (the sign flips).
- Subtraction follows the rules of addition. Division follows the rules of multiplication.

6 Summation and Product Notations

- **Summation Notation:** In $\sum_{i=0}^{k} s(i)$, i is the *index*, $i = 0$ is the *lower limit*, k is the *upper limit*, and $s(i)$ is the sequence we are summing.
- **Product Notation:** In $\prod_{i=0}^{k} s(i)$, everything is the same as summation, except we use π to indicate that we multiply the sequence.

7 Integer Division

- **Modulo** - Denoted by % or mod , the modulus operator gives the remainder of an integer division. E.g. $10 \% 4 = 2$
- **Congruency** - a is congruent to b modulo m (denoted $a \equiv b(\bmod m)$), if $a \% m = b \% m$ or $(a - b) \% m = 0$
- **Divides:** - The "divides" operator, denoted $a|b$, returns `True` if $b \% a = 0$ and `False` otherwise.

8 Evens and Odds

- **Even** - An integer, n is even if there exists an integer k such that $n = 2k$ (or $2|n$, $n \% 2 = 0$, $n \equiv 0 \bmod 2$)
- **Odd** - An integer, n is odd if there exists an integer k such that $n = 2k + 1$ (or $2 \nmid n$, $n \% 2 = 1$, $n \equiv 1 \bmod 2$)

9 Logarithms and Exponents

Laws of Exponents and Logarithms:

(a) $w^{x+y} = w^x w^y$	(b) $(w^x)^y = w^{xy}$	(c) $v^x w^x = (vw)^x$
(d) $\frac{w^x}{w^y} = w^{x-y}$	(e) $\frac{v^x}{w^x} = (\frac{v}{w})^x$	(f) $\log_b(x^y) = y \log_b x$
(g) $\log_b(xy) = \log_b x + \log_b y$	(h) $\log_b(\frac{x}{y}) = \log_b x - \log_b y$	(i) $b^{\log_b x} = x$
(j) $\log_a x = \frac{\log_b x}{\log_b a}$	(k) If $b^y = x$, then $\log_b x = y$	

10 Quadratic Equations

- **Quadratic Equation**: Equation of the form $ax^2 + bx + c$ where $a \neq 0$
- **Factoring Quadratics**: $(fx + d)(gx + e) = (fg)x^2 + (gd + fe)x + de$
- **Quadratic Formula**: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

11 Number Systems

- Binary: Base 2, Digits 0,1
- Decimal: Base 10, Digits 0-9
- Octal: Base 8, Digits 0-7
- Hexadecimal: Base 16, Digits 0-9,A-F