CSc 245 - Introduction to Discrete Structures
Summer 2020

Name: ____

Midterm

Released July 9th, Due July 10th by 5pm

Directions

1. Write your name at the top
2. This exam is open book, closed internet. Looking for answers online or asking for/using answers written by other people is a violation of academic integrity.
3. Exams must be completed **individually**. You may not discuss questions with other students or on Piazza.
4. Clarifications may be asked on Piazza, but please make them private. We will decide if they should be made public.
5. Response may be typed or handwritten. Make sure your responses are legible!
6. Make your answers as precise and concise and to the point as possible, while still answering the questions asked.
7. Show your work, where appropriate, for potential partial credit. Vague, incomplete, and/or ambiguous answers will not receive full credit.
8. Submit your responses on Gradescope by Friday (7/10) at 5pm MST (Tucson time).

1. (6 points) Math Review

   (a) Give 3 *different* integers that are congruent to 23 modulo 5.

   (b) Evaluate $36|6$ and $54|9$. Explain your answers.

   (c) Convert 110101 to Decimal. Show your work.

2. (6 points) Complete parts (a) and (b) using the given propositions below.

   b : my bread is under-baked. p : my bread is under-proved.
   t : I start baking before noon g : the bread is good.

   (a) Convert to Logic: If my bread is under-baked or under-proved, then the bread is not good.

   (b) Convert to English: $\neg t \rightarrow ((b \vee p) \wedge \neg g)$

3. (3 points) For each of these statements, write the antecedent and consequent in the spaces.

   (a) I will bake bread unless I run out of time.

   Antecedent: ______________________ Consequent: ______________________

   (b) James will ride his bike only if it is warm outside.

   Antecedent: ______________________ Consequent: ______________________

   (c) I will make pizza provided that I have all of the ingredients.

   Antecedent: ______________________ Consequent: ______________________

4. (4 points) Given $P(x, y) : x\%y = 0$ state the truth values of the following. Briefly justify your answer.

   (a) $\exists x \forall y\, P(x, y)$.

   (b) $\forall x \exists y\, P(x, y)$.

5. (3 points) What rule of inference is used in each of the following arguments? Note: these statements each correspond to a *single* rule, you simply need to state the name of the rule.

   (a) French bread contains yeast. Therefore some breads contain yeast.

   (b) All harry potter books are good. Therefore Harry potter and the Prisoner of Azkaban is good.

   (c) I wash the dishes or read my book. I don't wash the dishes. Therefore, I read my book.

6. (4 points) Give a counterexample to disprove the conjecture: given n is even and k is an integer, if nk is even, then k must be even.

7. (2 points) Given $A = \begin{bmatrix} 1 & 2 \\ 4 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 0 \\ 5 & 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 7 & 1 \end{bmatrix}$

   (a) What pairs of sets can be multiplied together? (note: order matters!)

   (b) What pairs can be summed together?

8. (4 points) Given $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, evaluate the following:

(a) $A \wedge B$

(b) $A \vee B$

(c) $A \odot B$

(d) $A \cdot B$

9. (4 points) Let A be the set of all positive integers less than 6. Let $B = \{4, 5, 7, 8\}$

(a) Express A in set builder notation.

(b) What is $|A|$?

(c) What is $A \cap B$

(d) Draw the Venn Diagram of A and B.

10. (15 points) Consider the following compound proposition: $\neg q \wedge (p \vee q) \rightarrow \neg(q \wedge \neg p)$

(a) Show that $\neg q \wedge (p \vee q) \rightarrow \neg(q \wedge \neg p)$ is a tautology using truth tables.

(b) Now show that $\neg q \wedge (p \vee q) \rightarrow \neg(q \wedge \neg p)$ is a tautology using equivalences.

11. (15 points) Use the following predicates to complete parts (a)-(c).

$G(x)$: x has a garden $\quad$ $P(x, y)$: x has y planted in their garden
$V(y)$: y is a vegetable $\quad$ $H(y)$: y is an herb.

(a) Convert to Logic

i. Everyone with a garden has a vegetable in their garden.

ii. Ian has a garden with the herb Rosemary planted in it.

(b) Convert to English: $H(\text{Thyme}) \wedge \forall y(G(x) \rightarrow P(x,\text{Thyme}))$, $x \in$ People .

(c) Express the negation of the statement in part (b) in:

i. Logic

ii. English

12. (10 points) Use rules of inference to show that conclusion follows from the given premises. First identify and label all predicates. Then convert the premises and conclusion to their corresponding logic statements using those predicates. Then apply rules of inference to the premises to reach the given conclusion. Note: The given premises alone do not give the conclusion, you must apply rules of inference to them to reach the conclusion.

    Everyone who drives a car has a license.

    Tina is a UA student and drives a car.

    $\therefore$ A UA student has a license.

13. (14 points) Given that n be an odd integer and k be an integer. Prove that an integer k is even **if and only if** kn is even

14. (10 points) Let A,B, and C be sets. Prove that if $A \cup B = B \cup C$ and $A \cap C = B \cap C$, then $A \subseteq B$ (Hint: it may be easier to not use a direct proof!)

CSc 245 — Introduction to Discrete Structures
(McCann)

Last Revised: May 2014

The Page O' Logical Equivalences ("POLE")

Table I: Some Equivalences using AND ($\wedge$) and OR ($\vee$):

(a)	$p \wedge p \equiv p$ $p \vee p \equiv p$	Idempotent Laws
(b)	$p \wedge \mathbf{F} \equiv \mathbf{F}$ $p \vee \mathbf{T} \equiv \mathbf{T}$	Domination Laws
(c)	$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity Laws
(d)	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutative Laws
(e)	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative Laws
(f)	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive Laws
(g)	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$	Absorption Laws

Table II: Some More Equivalences (adding Negation ($\neg$)):

(a)	$\neg(\neg p) \equiv p$	Double Negation
(b)	$p \wedge \neg p \equiv \mathbf{F}$ $p \vee \neg p \equiv \mathbf{T}$	Negation Laws
(c)	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws

Table III: Still More Equivalences (adding Implication ($\rightarrow$)):

(a)	$p \rightarrow q \equiv \neg p \vee q$	Law of Implication
(b)	$p \rightarrow q \equiv \neg q \rightarrow \neg p$	Law of the Contrapositive
(c)	$\mathbf{T} \rightarrow p \equiv p$	"Law of the True Antecedent"
(d)	$p \rightarrow \mathbf{F} \equiv \neg p$	"Law of the False Consequent"
(e)	$p \rightarrow p \equiv \mathbf{T}$	Self-implication (a.k.a. Reflexivity)
(f)	$p \rightarrow q \equiv (p \wedge \neg q) \rightarrow \mathbf{F}$	Reductio Ad Absurdum
(g)	$\neg p \rightarrow q \equiv p \vee q$	
(h)	$\neg(p \rightarrow q) \equiv p \wedge \neg q$	
(i)	$\neg(p \rightarrow \neg q) \equiv p \wedge q$	
(j)	$(p \rightarrow q) \vee (q \rightarrow p) \equiv \mathbf{T}$	Totality
(k)	$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$	Exportation Law (a.k.a. Currying)
(l)	$(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$	
(m)	$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$	
(n)	$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$	
(o)	$p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$	
(p)	$p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \rightarrow r)$	Commutativity of Antecedents

Table IV: Yet More Equivalences (adding Exclusive OR ($\oplus$) and Biimplication ($\leftrightarrow$)):

(a)	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Definition of Biimplication
(b)	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	
(c)	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$	
(d)	$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$	Definition of Exclusive Or
(e)	$p \oplus q \equiv \neg(p \leftrightarrow q)$	
(f)	$p \oplus q \equiv p \leftrightarrow \neg q \equiv \neg p \leftrightarrow q$	

Notes:

1. p, q, and r represent arbitrary logical expressions. They may represent equivalent expressions (e.g., if $p \equiv q$, then by Absorption $p \wedge (p \vee p) \equiv p$).
2. $\mathbf{T}$ and $\mathbf{F}$ represent the logical values True and False, respectively.
3. These tables show many of the common and/or useful logical equivalences; this is not an exhaustive collection!