

Question 1:

$(15/15) * (14/15) * (13/15) * (12/15) * (11/15) * (10/15) * (9/15) * (8/15) = 128128/1265625$
The probability is 128128/1265625.

Question 2:

First digit has 5 options (all odd numbers)

Second digit has 4 options (all odd numbers that aren't the first digit)

Fifth digit has 5 options (all even numbers)

Third digit has 7 options

Fourth digit has 6 options

$5 * 4 * 5 * 7 * 6 = 4200$ numbers satisfy criteria

$4200/10000 = 21/50$

The probability is 21/50.

Question 3:

$P(A|B) = 3/6 = 1/2$

$P(A)$:

LLL No

LLH No

LHL No

LHH Yes

HLL No

HLH Yes

HHL Yes

HHH Yes

$P(A) = 4/8 = 1/2$

They are independent.

Question 1 again for some reason:

$(52/52) * (12/52) * (11/52) * (10/52) * (9/52) = 1485/913952$

Expected value is = $913952/1485$

The expected number is $913952/1485$.

Question 2: Electric Boogaloo:

Let A = probability of superstar playing

Let B = probability of winning exactly 4/5 games

$P(A|B) = P(B|A) * P(A) / P(B)$

$P(A|B) = P(B|A) * P(A) / (P(B|A) * P(A) + P(B|!A) * P(!A))$

$P(A|B) = P(B|A) * P(A) / (P(B|A) * 3/4 + P(B|!A) * 1/4)$

$P(A|B) = P(B|A) * P(A) / (P(B|A) * 3/4 + P(B|!A) / 4)$

$P(A|B) = 4 * P(B|A) * P(A) / (P(B|A) * 3 + P(B|!A))$

$P(A|B) = 4 * P(B|A) * 3/4 / (P(B|A) * 3 + P(B|!A))$

$P(A|B) = 3 * P(B|A) / (3 * P(B|A) + P(B|!A))$

$P(B|A) = (5 \text{ choose } 4) * (7/10)^4 * (3/10) = 7203/20000$

$P(A|B) = 3 * (7203/20000) / (3 * (7203/20000) + P(B|!A))$

$P(A|B) = 21609/20000 / (3 * (7203/20000) + P(B|!A))$

$P(A|B) = 21609 / (20000 * (3 * (7203/20000) + P(B|!A)))$

$P(A|B) = 21609 / (20000 * (21609/20000 + P(B|!A)))$

$P(A|B) = 21609 / (20000 * 21609/20000 + 20000 * P(B|!A))$

$$P(A|B) = 21609 / (21609 + 20000 * P(B|\neg A))$$

$$P(B|\neg A) = P(B|A) = \binom{5}{4} * (1/2)^4 * (1/2) = 5/32$$

$$P(A|B) = 21609 / (21609 + 20000 * 5/32)$$

$$P(A|B) = 21609 / (21609 + 3125)$$

$$P(A|B) = 21609 / 24734$$

The probability is 21609/24734.