1. **Fill in the Blank** (5000pts)
   *Note: Items 1-4 deal with Red-Black properties.*

   1. Every node is either red or black
   2. Color-wise, the root and leaves are black
   3. If a node is red, then its parent is black
   4. All simple paths from any node $x$ to a descendant leaf have the same number of black nodes
   5. The pop operation is used to remove an element from a stack
   6. The element removed from a queue is the first-most element added to the queue
   7. Dijkstra's algorithm produces shortest paths between nodes.
   8. If the next empty location is found in a squared number sequence, then you are using quadratic probing.

2. **Matching** (30,000 points)

| | | |
|---|---|---|
| a. | Comments | (D) Accessed by a parameter to main. |
| b. | Block | (C) In Java, integer, floating-point, Boolean, and character. |
| c. | Primitive Types | (B) A sequence of statements within braces. |
| d. | Command-line argument | (A) Make code easier for humans to read but have no semantic meaning. |
| e. | Null reference | (E) The value of an object reference that does not refer to any object. |
| f. | Mergesort | (G) A recursive instance that can be solved without recursion. |
| g. | Base Case | (F) A divide-and-conquer algorithm that obtains an O(N log N ) sort. |
| h. | Bellman–Ford algorithm | (I) An alg that is used to solve the positive-weighted, shortest-path proble |
| i. | Dijkstra's algorithm | (H) An alg that is used to solve the negative-weighted, shortest-path probl |

3. (A Single Point) Fill in the runtimes of each sorting algorithm.

| Sorting Algo | Big Oh | Theta |
|---|---|---|
| Selection Sort | O($n^2$) | Yes |
| Bubble Sort | O($n^2$) | No |
| Merge Sort | O(nlogn) | Yes |
| Heap Sort | O(nlogn) | No |
| Quick Sort | O(nlogn) | No |

4. (1600 pts) Give the expected Big-O runtime for the following operations on using the given data structures.

   **Red-Black Tree:** search, min, max, successor, and predecessor
   O(logn)

   **Bubble Sort:** Best, worst, and average case
   n, $n^2$, and $n^2$

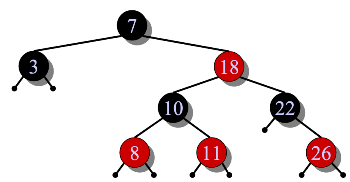
   **Selection Sort:** Best, worst, and average case
   All $n^2$

   **Heapsort:** Best, worst, and average case
   n (if all keys are distinct, nlogn), nlogn, nlogn

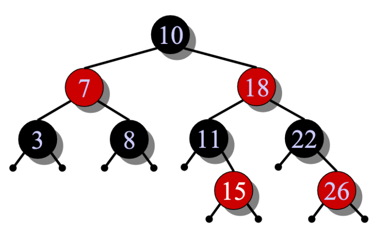
5. (12 pts) **Red-Black** Below is the adjacency matrix for a Red-Black tree. The outer edges indicate the value at a given node (the first value is the root, the next two are the node's children (left to right), and so on in an inorder traversal).
**Draw the red-black tree below, indicating black nodes as boxes and red nodes as circles.**

$$\begin{array}{cc} & \begin{array}{cccccccc} 7 & 3 & 18 & 10 & 22 & 8 & 11 & 26 \end{array} \\ \begin{array}{c} 7 \\ 3 \\ 18 \\ 10 \\ 22 \\ 8 \\ 11 \\ 26 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

*sorry, this one was a little bit of a disaster*




**Now, insert a node x=15 into the tree.** *Idea:* Recolor, moving violation up the tree. Right-Rotate(18), Left-Rotate(7), and recolor.



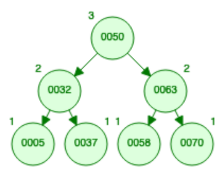
6. **Trees** (5! points)Perform a preorder traversal on this tree
   Remember: Preorder is root, left, right

```
         a
        / \
       c    g
      / \  / \
     x   m p   q
    /     \
   y       w
```

   a,c,x,y,m,g,p,w,q

7. **AVL** (4! points) Perform the AVL instructions listed below. Perform correct rotations.

   Remove 10: replace 10 with the largest value in its left subtree (8), remove 8, single right rotation.
   Insert 58:




8. **AVL** (6? points) Do that again. Perform correct rotations.
   **Here, make 2 the parent of 1 and 14 and the left child of 32.**

9. (10 or 18 points) **Heaps**

   (a) Describe the structure of a min-heap, how do each nodes compare to their children?
   The children in a min-heap will be larger than their parent.

   (b) Fill the runtimes of min-heap operations

| | |
|---|---|
| GetMin() | O(1) : returns the root element |
| RemoveMin() | O(logn) : remove min element (remove the root AND maybe heapify) |
| Insert() | O(logn) : add a new key, may have to traverse up to fix any errors |

   (c) Fill in the array index locations of each node

| | |
|---|---|
| The Root | A[0] |
| Node i's Left Child | A[(2*i) +1] |
| Node i's Right Child | A[(2*i)+2] |
| Node i's Parent | A[(i-1)/2] |

10. **Hash Slinging Slasher** ($\pi$ points) Insert the following into a hash table using linear probing with a hash function H(x) = k mod 7. $[76, 93, 40, 47, 10, 55]$

    **You should end up with this array:** $[47, 55, 93, 10, -, 40, 76]$

    What is quadratic probing? An open addressing method to avoid collisions. Uses a quadratic function to find the next available space for an item.

11. **Code** ($2^e$ points) Describe the runtime of the following psedocode functions.

```
def ILoveOwls(n):
      result = 0
      while n > 1:
            n //= 2
            result += 1
      return result
```

This runs in O(logn) time. The value of $n$ is halved on each iteration of the loop. If n = $2^x$, then logn = x.

```
def IReallyLoveOwls(n)
      result = 0
      for i in range (n):
            for j in range (i, n)
                  for k in range (n):
                        result += 1
      return result
```

O($n^3$) since we run through 3 for loops, each of size n.