

1 changedp Dynamic Programming Table

2 Pseudo-code

2.1 changegreedy

```
changegreedy(int c){
    int changeArray[4] = {0, 0, 0, 0};

    while( c >= 50){
        if(c-50 >= 0){
            changeArray[3] += 1;
            c = c-50;
        }
    }

    while( c >= 25){
        if(c-25 >= 0){
            changeArray[2] += 1;
            c = c-25;
        }
    }

    while( c >= 10){
        if(c-10 >= 0){
            changeArray[1] += 1;
            c = c-10;
        }
    }

    while( c > 0){
        if(c-1 >= 0){
            changeArray[0] += 1;
            c = c-1;
        }
    }

    return changeArray;
}
```

2.2 changedp

Make a list to fill with values, from 0 to change to be found + 1
handle the base case of change needed = 0

for index from 1 to change + 1{ <-this is the filling of the list
go through each coin in the list{
if the coin is greater than the current value to be found then skip it
else if the current minimum coins is zero or (the length of the current

```

    minimum - the current coin + 1) is greater than the length of the current
    minimum
        if the current minimum minus the current coin is not NULL{
            the current minimum is now the current minimum minus
            the current coin with the old coins still attached to
            the array / list add the current coin to the array / list
        }
    }
}
print the length of the last piece in the list (the last value being the change
needed) then print the coins that make it up (stored at the last value of the list)

```

3 Proofs of Correctness

3.0.1 changedp

Let $T[v]$ = is the minimum number of coins possible to make change for value v .

Base Case:

When $v = 0$ then $T[0] = 0$ which is the minimum number of coins to make change for zero. Thus $T[0]$ is arbitrarily correct. Likewise when $v = V[i]$, i is the minimum number of coins to make change for v .

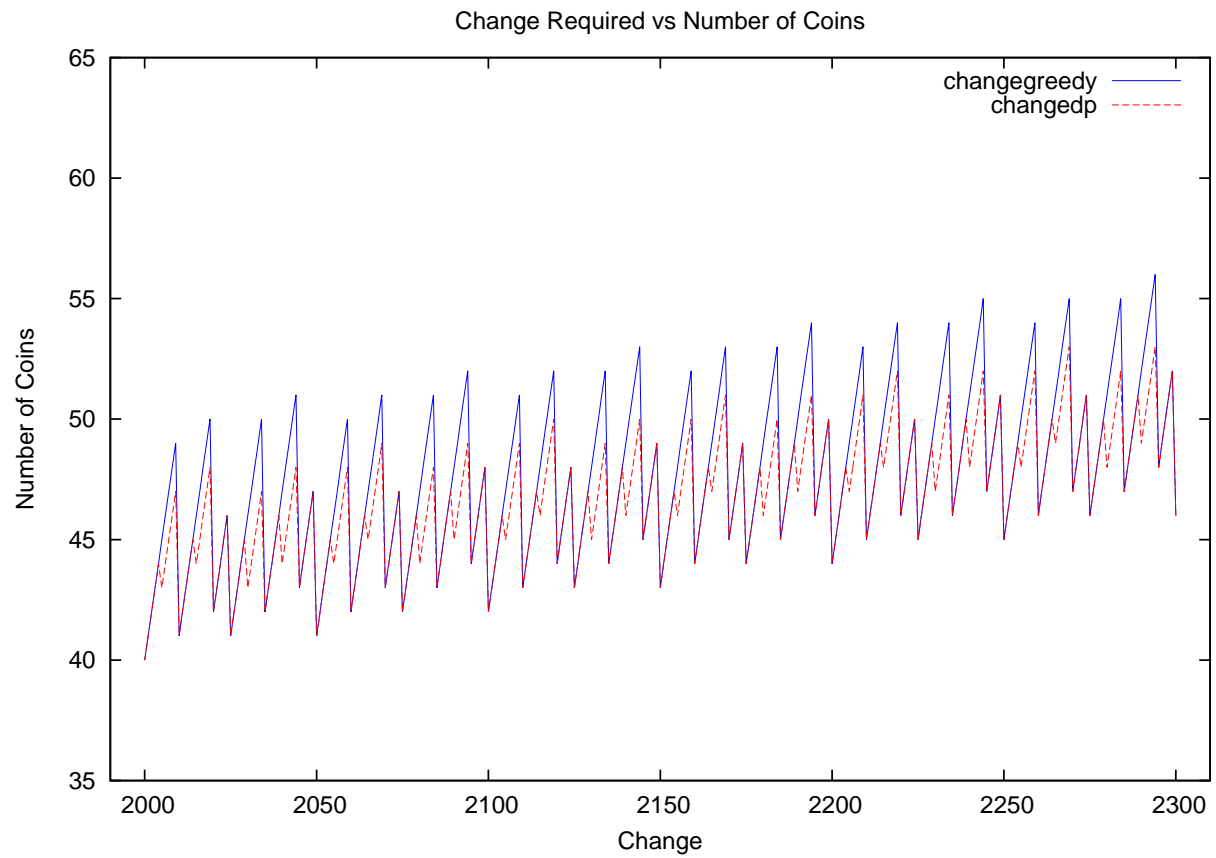
Inductive Hypothesis:

Assume $T[k]$ is correct for some positive integer k .

Inductive Step:

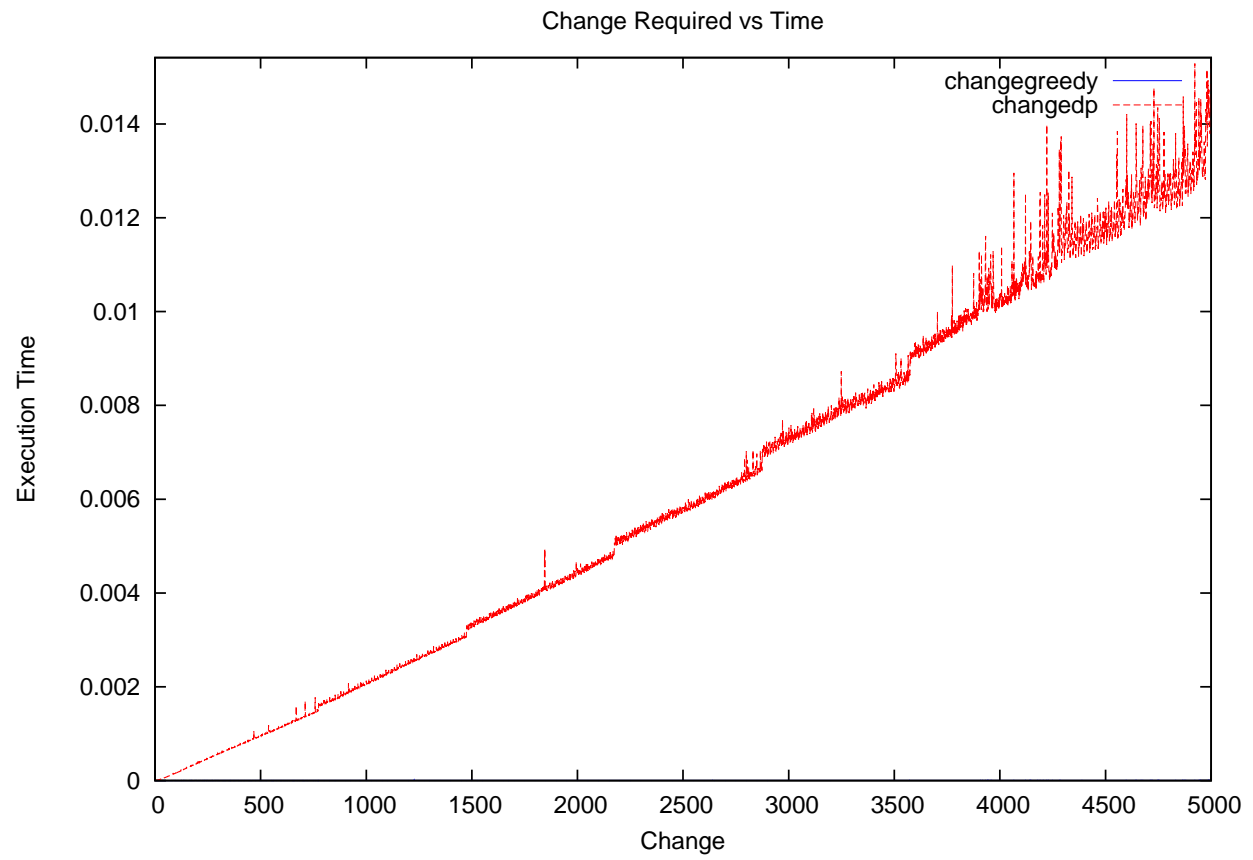
For the $k+1$ will either be $T[k+1] = \min \{T[k - V[i]] + 1\} + 1$ where the minimum solution for k will be the minimum solution for $k+1$ with the addition of another coin. Or $k+1 < \min \{T[k - V[i]] + 1\}$ where $k+1$ has a minimum solution that was less than the minimum of k .

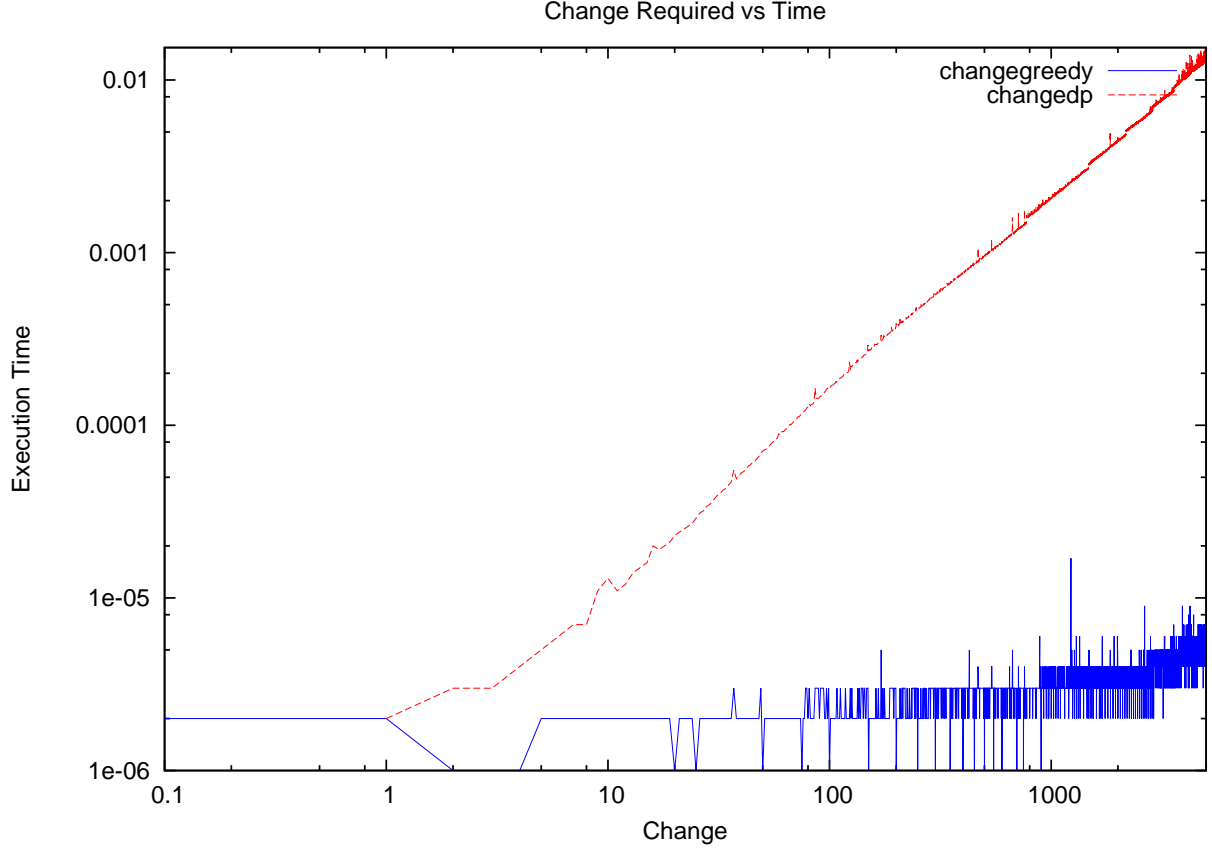
4 Coins Required for Values of C



There are times where both functions find the same number of coins, and there are time where the functions differ by at most 3. But they both have a similar increasing and decreasing slope over the same values of C.

5 Experimental Runtime





It appears that `changedp` has a linear slope on the loglog plot while `changegreedy` has a constant slope in the loglog plot.

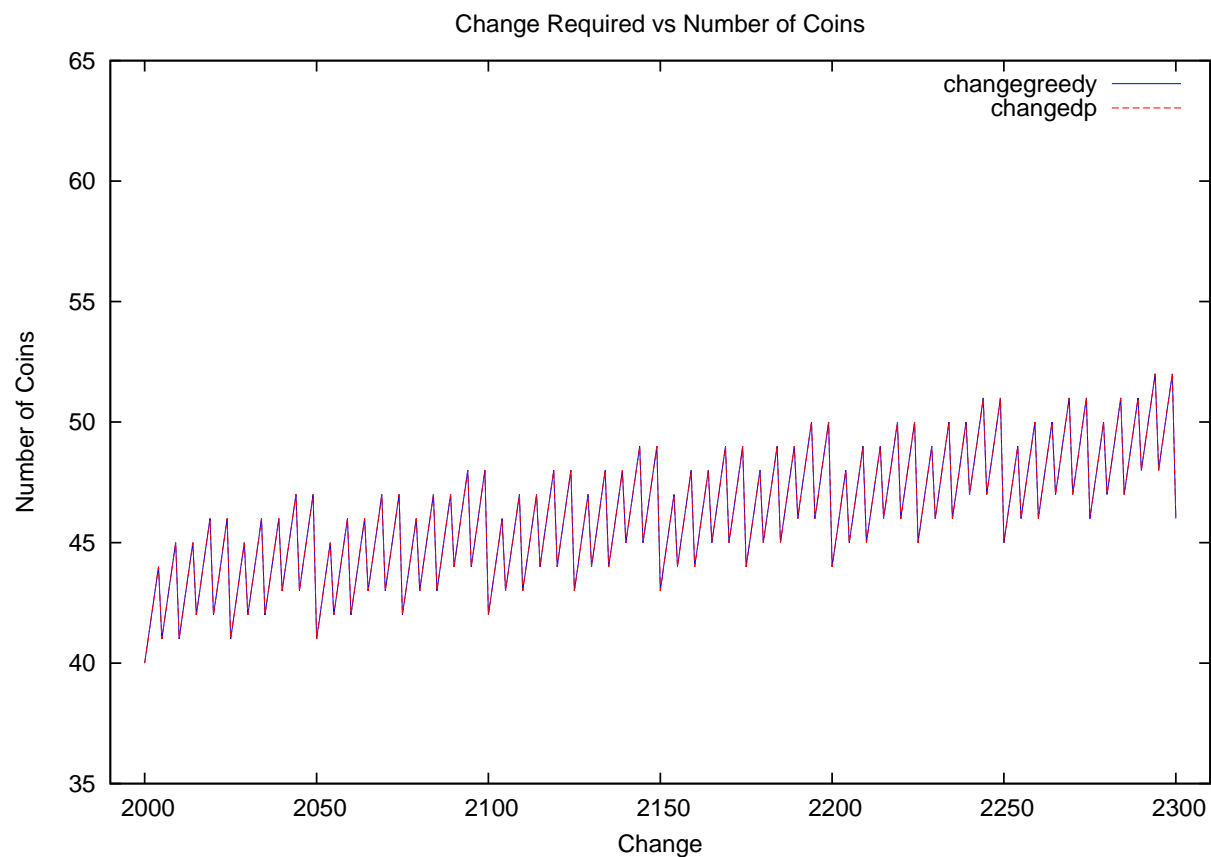
Using 2 data points from the timing results in `changedp` (0.000011s, 9 and 0.015415s, 5000) and plugging them into the equation:

$$Slope = m = \frac{\log(\frac{F_1}{F_0})}{\log(\frac{x_1}{x_0})} \quad (1)$$

results in $m = 1.1464$ which means that `changedp` is at least $O(n^{1.1464})$.

For `changegreedy`, the runtime goes from 0.000001s for change size 1 to 0.000007 for change size 5000. this means that `changegreedy` is roughly cn with $c < 1$. This mean that the asymptotic runtime of our implementation of `changegreedy` is $O(n)$

6 Coins Required for Values of C



In this case, both approaches find the the same amount of coins to make change for each value.

7 Coins as Values of P

The only thing that would change in this example is the base for each coin that you would use to divide the amount into.

8 Coins for which changegreedy fails but changedp does not?

Yes. Let $V = [10, 25]$ and $C = 90$. Greedy would get stuck because it would use 325 options, and 110 option. This would leave 5 left over, but no way to make change for it. The changedp however, would find 910 and be able to make correct change.s