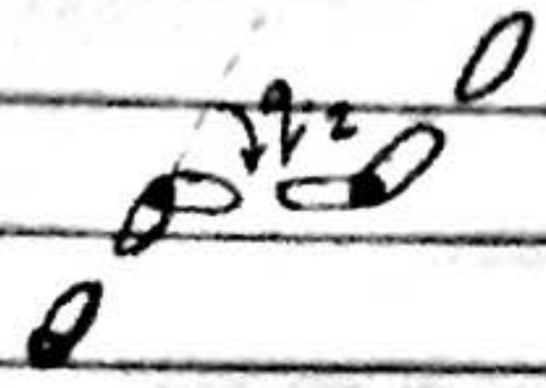

For spring/damper fitting, we use the following: $q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$ $\quad V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$ $\quad V = \dot{q}$

We want to fit a linear model:

$$\tau = Kq + Cv$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$= \begin{bmatrix} q_1 & 0 & 0 & q_2 & 0 & 0 & q_3 & 0 & 0 & v_1 & 0 & & 0 & 0 \\ 0 & q_1 & 0 & 0 & q_2 & 0 & 0 & q_3 & 0 & 0 & v_1 & \cdots & v_3 & 0 \\ 0 & 0 & q_1 & 0 & 0 & q_2 & 0 & 0 & q_3 & 0 & 0 & & 0 & v_3 \end{bmatrix} \begin{bmatrix} K_{11} \\ K_{21} \\ \vdots \\ K_{33} \\ C_{11} \\ \vdots \\ C_{23} \\ C_{33} \end{bmatrix}$$

$$= \left(\begin{bmatrix} q_1 & q_2 & q_3 & v_1 & v_2 & v_3 \end{bmatrix} \otimes I \right) \begin{bmatrix} vec(K) \\ vec(C) \end{bmatrix}$$

Stack k timesteps:

$$\begin{bmatrix} \tau_1^{(1)} \\ \tau_2^{(1)} \\ \vdots \\ \tau_2^{(k)} \\ \tau_3^{(k)} \end{bmatrix} = \left(\begin{bmatrix} q_1^{(1)} & q_2^{(1)} & q_3^{(1)} & v_1^{(1)} & v_2^{(1)} & v_3^{(1)} \\ & & & \vdots & & \\ q_1^{(k)} & q_2^{(k)} & q_3^{(k)} & v_1^{(k)} & v_2^{(k)} & v_3^{(k)} \end{bmatrix} \otimes I \right) \begin{bmatrix} vec(K) \\ vec(C) \end{bmatrix}$$

Define: $F = \begin{bmatrix} | & & | \\ \tau^{(1)} & \cdots & \tau^{(k)} \\ | & & | \end{bmatrix}$ $\quad Q = \begin{bmatrix} | & & | \\ q^{(1)} & \cdots & q^{(k)} \\ | & & | \end{bmatrix}$ $\quad V = \begin{bmatrix} | & & | \\ v^{(1)} & \cdots & v^{(k)} \\ | & & | \end{bmatrix}$

Rewrite stacked setup:

$$\underbrace{vec(F)}_{y} = \underbrace{\left(\begin{bmatrix} Q^T & V^T \end{bmatrix} \otimes I \right)}_{A} \underbrace{\begin{bmatrix} vec(K) \\ vec(C) \end{bmatrix}}_{x}$$

Standard least squares: minimize $\|y - Ax\|^2$

Regularized least squares: minimize $\|y - Ax\|^2 + x^T W x$

PSD constraint: minimize $\|y - Ax\|^2 + x^T W x$

subject to $x = \begin{bmatrix} vec(-K) \\ vec(-C) \end{bmatrix}$

$K \succeq 0$ and $C \succeq 0$