# ENSC 21
# CREATIVE OUTPUT

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Chosen topic:


# METHOD OF UNDETERMINED COEFFICIENT

Problem:

$$y'' - 4y' + 6y = e^{2x} + \cos(x)$$

# Solution

1) Manual Solution

2) Creative Solution

# 1) Manual Solution

$y'' - 4y' + 6y = e^{(2x)} + \cos(x)$

non-homogeneous, $g(x) \neq 0$

- solve for $y_c$ by finding roots:

$D^2y - 4Dy + 6y = 0$

$m^2 - 4m + 6 = 0$

$D = dy/dx$

$$m = [-(-4) \pm \sqrt{(-4)^2 - 4(1)(6)}\ ] / 2(1)$$

$$m = (4 \pm \sqrt{-8}) / 2$$

$$m = (4 \pm 2\sqrt{-2}) / 2$$

$$m = 2 \pm \sqrt{-1} \times 2$$

$$\boxed{\begin{aligned} m &= 2 + \sqrt{2}\, i \\ m &= 2 - \sqrt{2}\, i \end{aligned}}$$

Roots are solved!!!

$$y_c = e^{ax}(c_1^* \cos bx + c_2^* \sin bx)$$

complex & distinct roots

RECALL!

$a \pm bi$

$2 \pm \sqrt{2}\, i$

$y_c = e^{(2x)}\,(C_1\cos\sqrt{(2)}x + C_2\sin\sqrt{(2)}x)$

- solve for $y_p$

$g(x) = e^{(2x)} + \cos(x)$

| | |
|---|---|
| $\alpha \cos kx$ | $A \cos kx + B \sin kx$ |
| $\alpha \sin kx$ | |
| $\alpha e^{kx}$ | $Ae^{kx}$ |

$y_p = Ae^{(2x)} + B\cos(x) + C\sin(x)$

$y_p' = 2Ae^{(2x)} - B\sin(x) + C\cos(x)$

$y_p'' = 4Ae^{(2x)} - B\cos(x) - C\sin(x)$

- substitute the $y_p$ values to the given:

$y'' - 4y' + 6y = e^{2x} + \cos(x)$

$\rightarrow 4Ae^{2x} - B\cos(x) - C\sin(x) - 4[2Ae^{2x} - B\sin(x) + C\cos(x)] + 6[Ae^{2x} + B\cos(x) + C\sin(x)] = e^{2x} + \cos(x)$

$\rightarrow e^{2x}(4A-8A+6A) + \cos(x)(-B-4C+6B) + \sin(x)(-C+4B+6C) = e^{2x} + \cos(x)$

$2A = 1 \rightarrow A = 1/2$ (e^2x)

2 equations 2 unknowns:

$-4C + 5B = 1$ (cos x)

$5C + 4B = 0$ (sin x)

$$(-4C+5B = 1)\ 5/4$$
$$5C+4B = 0$$

$$
\begin{array}{rl}
 & -5C+25/4\,B = 5/4 \\
+ & \underline{5C+4B = 0} \\
 & 0 + 41/4\,B = 5/4 \\
 & B = 5/41
\end{array}
$$

substitute B for C

$$5C+4(5/41) = 0$$
$$5C = -20/41$$
$$C = -4/41$$

- substitute the A, B, C values to y

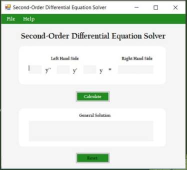
$y_p = 1/2e^{(2x)} + 5/41\cos(x) - 4/41\sin(x)$

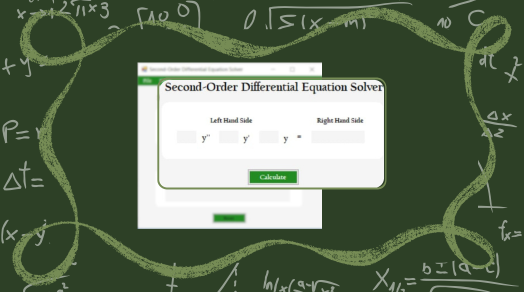
$y = y_c + y_p$

$y = e^{(2x)}\,[C_1\cos\sqrt{(2)}x + C_2\sin\sqrt{(2)}x + 1/2] + 5/41\cos(x) - 4/41\sin(x)$

# 2) Creative Solution

self-made
code to solve 2nd
order DE (MUC)




Second-Order Differential Equation Solver
Left Hand Side
Right Hand Side
y''
y'
y
=
Calculate

$$y = e^{(2x)} [C \cos\sqrt{(2)}x + C \sin\sqrt{(2)}x + 1/2] + 5/41\cos(x) - 4/41\sin(x)$$

**General Solution**

y(x) = e^2x [C1 cos (1.4142135623731)x + C2 sin (1.4142135623731)x]+0.5e^(2x)+0.121951219512195cos(x)+-0.0975609756097561sin(x)

Final Answer

$$y(x) = e^{2x}(C_1 \cos(\sqrt{2}x) + C_2 \sin(\sqrt{2}x)) + \frac{1}{2}e^{2x} + \frac{5}{41}\cos x - \frac{4}{41}\sin x$$

# THANK YOU FOR LISTENING!

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