

1. NumPy – Introduction

NumPy is a Python package. It stands for 'Numerical Python'. It is a library consisting of multidimensional array objects and a collection of routines for processing of array.

Numeric, the ancestor of NumPy, was developed by Jim Hugunin. Another package Numarray was also developed, having some additional functionalities. In 2005, Travis Oliphant created NumPy package by incorporating the features of Numarray into Numeric package. There are many contributors to this open source project.

Operations using NumPy

Using NumPy, a developer can perform the following operations:

- Mathematical and logical operations on arrays.
- Fourier transforms and routines for shape manipulation.
- Operations related to linear algebra. NumPy has in-built functions for linear algebra and random number generation.

NumPy – A Replacement for MatLab

NumPy is often used along with packages like **SciPy** (Scientific Python) and **Matplotlib** (plotting library). This combination is widely used as a replacement for MatLab, a popular platform for technical computing. However, Python alternative to MatLab is now seen as a more modern and complete programming language.

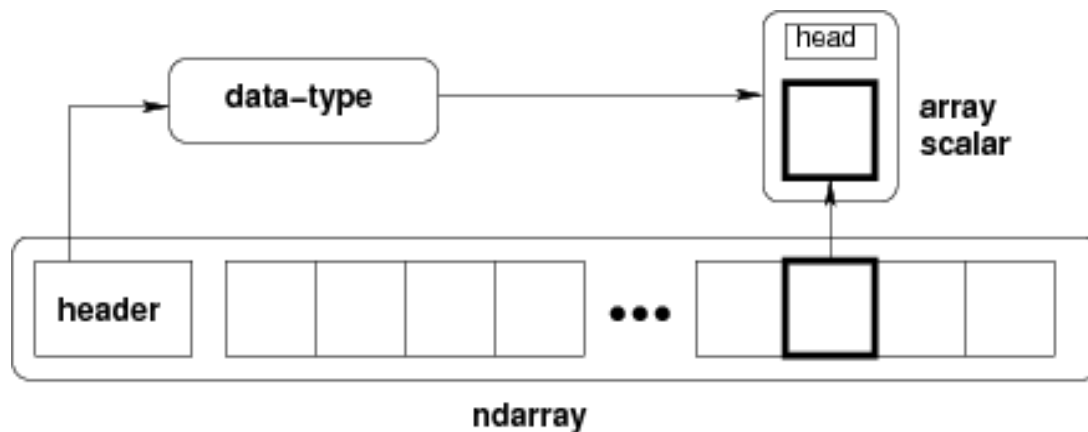
It is open source, which is an added advantage of NumPy.

3. NumPy – ndarray Object

The most important object defined in NumPy is an N-dimensional array type called **ndarray**. It describes the collection of items of the same type. Items in the collection can be accessed using a zero-based index.

Every item in an ndarray takes the same size of block in the memory. Each element in ndarray is an object of data-type object (called **dtype**).

Any item extracted from ndarray object (by slicing) is represented by a Python object of one of array scalar types. The following diagram shows a relationship between ndarray, data type object (dtype) and array scalar type:



An instance of ndarray class can be constructed by different array creation routines described later in the tutorial. The basic ndarray is created using an array function in NumPy as follows:

```
numpy.array
```

It creates an ndarray from any object exposing array interface, or from any method that returns an array.

```
numpy.array(object, dtype=None, copy=True, order=None, subok=False, ndmin=0)
```

The above constructor takes the following parameters:

object	Any object exposing the array interface method returns an array, or any (nested) sequence
dtype	Desired data type of array, optional
copy	Optional. By default (true), the object is copied
order	C (row major) or F (column major) or A (any) (default)
subok	By default, returned array forced to be a base class array. If true, sub-classes passed through
ndimin	Specifies minimum dimensions of resultant array

Take a look at the following examples to understand better.

Example 1

```
import numpy as np
a=np.array([1,2,3])
print a
```

The output is as follows:

```
[1, 2, 3]
```

Example 2

```
# more than one dimensions
import numpy as np
a = np.array([[1, 2], [3, 4]])
print a
```

The output is as follows:

```
[[1, 2]
 [3, 4]]
```

Example 3

```
# minimum dimensions
import numpy as np
a=np.array([1, 2, 3,4,5], ndmin=2)
print a
```

The output is as follows:

```
[[1, 2, 3, 4, 5]]
```

Example 4

```
# dtype parameter
import numpy as np
a = np.array([1, 2, 3], dtype=complex)
print a
```

The output is as follows:

```
[ 1.+0.j,  2.+0.j,  3.+0.j]
```

The **ndarray** object consists of contiguous one-dimensional segment of computer memory, combined with an indexing scheme that maps each item to a location in the memory block. The memory block holds the elements in a row-major order (C style) or a column-major order (FORTRAN or MatLab style).

4. NumPy – Data Types

NumPy supports a much greater variety of numerical types than Python does. The following table shows different scalar data types defined in NumPy.

Data Types	Description
bool_	Boolean (True or False) stored as a byte
int_	Default integer type (same as C long; normally either int64 or int32)
intc	Identical to C int (normally int32 or int64)
intp	Integer used for indexing (same as C ssize_t; normally either int32 or int64)
int8	Byte (-128 to 127)
int16	Integer (-32768 to 32767)
int32	Integer (-2147483648 to 2147483647)
int64	Integer (-9223372036854775808 to 9223372036854775807)
uint8	Unsigned integer (0 to 255)
uint16	Unsigned integer (0 to 65535)
uint32	Unsigned integer (0 to 4294967295)
uint64	Unsigned integer (0 to 18446744073709551615)
float_	Shorthand for float64
float16	Half precision float: sign bit, 5 bits exponent, 10 bits mantissa
float32	Single precision float: sign bit, 8 bits exponent, 23 bits mantissa
float64	Double precision float: sign bit, 11 bits exponent, 52 bits mantissa
complex_	Shorthand for complex128
complex64	Complex number, represented by two 32-bit floats (real and imaginary components)
complex128	Complex number, represented by two 64-bit floats (real and imaginary components)

NumPy numerical types are instances of dtype (data-type) objects, each having unique characteristics. The dtypes are available as np.bool_, np.float32, etc.

Data Type Objects (dtype)

A data type object describes interpretation of fixed block of memory corresponding to an array, depending on the following aspects:

- Type of data (integer, float or Python object)
- Size of data
- Byte order (little-endian or big-endian)
- In case of structured type, the names of fields, data type of each field and part of the memory block taken by each field
- If data type is a subarray, its shape and data type

The byte order is decided by prefixing '<' or '>' to data type. '<' means that encoding is little-endian (least significant is stored in smallest address). '>' means that encoding is big-endian (most significant byte is stored in smallest address).

A dtype object is constructed using the following syntax:

```
numpy.dtype(object, align, copy)
```

The parameters are:

- **Object:** To be converted to data type object
- **Align:** If true, adds padding to the field to make it similar to C-struct
- **Copy:** Makes a new copy of dtype object. If false, the result is reference to built-in data type object

Example 1

```
# using array-scalar type
import numpy as np
dt=np.dtype(np.int32)
print dt
```

The output is as follows:

```
int32
```

Example 2

```
#int8, int16, int32, int64 can be replaced by equivalent string 'i1',
'i2','i4', etc.
import numpy as np
dt = np.dtype('i4')
```

```
print dt
```

The output is as follows:

```
int32
```

Example 3

```
# using endian notation
import numpy as np
dt = np.dtype('>i4')
print dt
```

The output is as follows:

```
>i4
```

The following examples show the use of structured data type. Here, the field name and the corresponding scalar data type is to be declared.

Example 4

```
# first create structured data type
import numpy as np
dt = np.dtype([('age', np.int8)])
print dt
```

The output is as follows:

```
[('age', 'i1')]
```

Example 5

```
# now apply it to ndarray object
import numpy as np
dt = np.dtype([('age', np.int8)])
a = np.array([(10,), (20,), (30,)], dtype=dt)
print a
```

The output is as follows:

```
[(10,) (20,) (30,)]
```

Example 6

```
# file name can be used to access content of age column
import numpy as np
dt = np.dtype([('age', np.int8)])
a = np.array([(10,),(20,),(30,)], dtype=dt)
print a['age']
```

The output is as follows:

```
[10 20 30]
```

Example 7

The following examples define a structured data type called **student** with a string field 'name', an **integer field** 'age' and a **float field** 'marks'. This dtype is applied to ndarray object.

```
import numpy as np
student=np.dtype([('name','S20'), ('age', 'i1'), ('marks', 'f4')])
print student
```

The output is as follows:

```
[('name', 'S20'), ('age', 'i1'), ('marks', '<f4')])
```

Example 8

```
import numpy as np
student=np.dtype([('name','S20'), ('age', 'i1'), ('marks', 'f4')])
a = np.array([('abc', 21, 50),('xyz', 18, 75)], dtype=student)
print a
```

The output is as follows:

```
[('abc', 21, 50.0), ('xyz', 18, 75.0)]
```

Each built-in data type has a character code that uniquely identifies it.

- **'b'**: boolean
- **'i'**: (signed) integer
- **'u'**: unsigned integer
- **'f'**: floating-point
- **'c'**: complex-floating point
- **'m'**: timedelta
- **'M'**: datetime
- **'O'**: (Python) objects
- **'S', 'a'**: (byte-)string
- **'U'**: Unicode
- **'V'**: raw data (void)

5. NumPy – Array Attributes

In this chapter, we will discuss the various array attributes of NumPy.

ndarray.shape

This array attribute returns a tuple consisting of array dimensions. It can also be used to resize the array.

Example 1

```
import numpy as np
a=np.array([[1,2,3],[4,5,6]])
print a.shape
```

The output is as follows:

```
(2, 3)
```

Example 2

```
# this resizes the ndarray
import numpy as np
a=np.array([[1,2,3],[4,5,6]])
a.shape=(3,2)
print a
```

The output is as follows:

```
[[1 2]
 [3 4]
 [5 6]]
```

Example 3

NumPy also provides a reshape function to resize an array.

```
import numpy as np
a = np.array([[1,2,3],[4,5,6]])
b = a.reshape(3,2)
print b
```

The output is as follows:

```
[[1 2]
 [3 4]
 [5 6]]
```

ndarray.ndim

This array attribute returns the number of array dimensions.

Example 4

```
# an array of evenly spaced numbers
import numpy as np
a = np.arange(24)
print a
```

The output is as follows:

```
[0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23]
```

Example 5

```
# this is one dimensional array
import numpy as np
a = np.arange(24)
a.ndim

# now reshape it
b = a.reshape(2,4,3)
print b
# b is having three dimensions
```

The output is as follows:

```
[[[ 0,  1,  2]
  [ 3,  4,  5]
  [ 6,  7,  8]
  [ 9, 10, 11]]

 [[12, 13, 14]
  [15, 16, 17]
  [18, 19, 20]]
```

```
[21, 22, 23]]]
```

numpy.itemsize

This array attribute returns the length of each element of array in bytes.

Example 6

```
# dtype of array is int8 (1 byte)
import numpy as np
x = np.array([1,2,3,4,5], dtype=np.int8)
print x.itemsize
```

The output is as follows:

```
1
```

Example 7

```
# dtype of array is now float32 (4 bytes)
import numpy as np
x = np.array([1,2,3,4,5], dtype=np.float32)
print x.itemsize
```

The output is as follows:

```
4
```

numpy.flags

The ndarray object has the following attributes. Its current values are returned by this function.

C_CONTIGUOUS (C)	The data is in a single, C-style contiguous segment
F_CONTIGUOUS (F)	The data is in a single, Fortran-style contiguous segment
OWNDATA (O)	The array owns the memory it uses or borrows it from another object
WRITEABLE (W)	The data area can be written to. Setting this to False locks the data, making it read-only
ALIGNED (A)	The data and all elements are aligned appropriately for the hardware
UPDATEIFCOPY (U)	This array is a copy of some other array. When this array is deallocated, the base array will be updated with the contents of this array

Example 8

The following example shows the current values of flags.

```
import numpy as np
x = np.array([1,2,3,4,5])
print x.flags
```

The output is as follows:

```
C_CONTIGUOUS : True
F_CONTIGUOUS : True
OWNDATA : True
WRITEABLE : True
ALIGNED : True
UPDATEIFCOPY : False
```

6. NumPy – Array Creation Routines

A new **ndarray** object can be constructed by any of the following array creation routines or using a low-level ndarray constructor.

numpy.empty

It creates an uninitialized array of specified shape and dtype. It uses the following constructor:

```
numpy.empty(shape, dtype=float, order='C')
```

The constructor takes the following parameters.

Shape	Shape of an empty array in int or tuple of int
Dtype	Desired output data type. Optional
Order	'C' for C-style row-major array, 'F' for FORTRAN style column-major array

Example 1

The following code shows an example of an empty array.

```
import numpy as np
x = np.empty([3,2], dtype=int)
print x
```

The output is as follows:

```
[[22649312    1701344351]
 [1818321759    1885959276]
 [16779776     156368896]]
```

Note: The elements in an array show random values as they are not initialized.

numpy.zeros

Returns a new array of specified size, filled with zeros.

```
numpy.zeros(shape, dtype=float, order='C')
```

The constructor takes the following parameters.

Shape	Shape of an empty array in int or sequence of int
Dtype	Desired output data type. Optional
Order	'C' for C-style row-major array, 'F' for FORTRAN style column-major array

Example 2

```
# array of five zeros. Default dtype is float
import numpy as np
x = np.zeros(5)
print x
```

The output is as follows:

```
[ 0.  0.  0.  0.  0.]
```

Example 3

```
import numpy as np
x = np.zeros((5,), dtype=np.int)
print x
```

Now, the output would be as follows:

```
[0 0 0 0 0]
```

Example 4

```
# custom type
import numpy as np
x = np.zeros((2,2), dtype=[('x', 'i4'), ('y', 'i4')])
print x
```

It should produce the following output:

```
[[ (0, 0) (0, 0) ]
 [ (0, 0) (0, 0) ]]
```

numpy.ones

Returns a new array of specified size and type, filled with ones.

```
numpy.ones(shape, dtype=None, order='C')
```

The constructor takes the following parameters.

Shape	Shape of an empty array in int or tuple of int
Dtype	Desired output data type. Optional
Order	'C' for C-style row-major array, 'F' for FORTRAN style column-major array

Example 5

```
# array of five ones. Default dtype is float
import numpy as np
x = np.ones(5)
print x
```

The output is as follows:

```
[ 1.  1.  1.  1.  1.]
```

Example 6

```
import numpy as np
x = np.ones([2,2], dtype=int)
print x
```

Now, the output would be as follows:

```
[[1 1]
 [1 1]]
```


7. NumPy – Array from Existing Data

In this chapter, we will discuss how to create an array from existing data.

numpy.asarray

This function is similar to `numpy.array` except for the fact that it has fewer parameters. This routine is useful for converting Python sequence into ndarray.

```
numpy.asarray(a, dtype=None, order=None)
```

The constructor takes the following parameters.

a	Input data in any form such as list, list of tuples, tuples, tuple of tuples or tuple of lists
dtype	By default, the data type of input data is applied to the resultant ndarray
order	C (row major) or F (column major). C is default

The following examples show how you can use the **asarray** function.

Example 1

```
# convert list to ndarray
import numpy as np
x = [1,2,3]
a = np.asarray(x)
print a
```

Its output would be as follows:

```
[1  2  3]
```

Example 2

```
# dtype is set
import numpy as np
x = [1,2,3]
```

```
a = np.asarray(x, dtype=float)
print a
```

Now, the output would be as follows:

```
[ 1.  2.  3.]
```

Example 3

```
# ndarray from tuple
import numpy as np
x = (1,2,3)
a = np.asarray(x)
print a
```

Its output would be:

```
[1  2  3]
```

Example 4

```
# ndarray from list of tuples
import numpy as np
x = [(1,2,3),(4,5)]
a = np.asarray(x)
print a
```

Here, the output would be as follows:

```
[(1, 2, 3) (4, 5)]
```

numpy.frombuffer

This function interprets a buffer as one-dimensional array. Any object that exposes the buffer interface is used as parameter to return an **ndarray**.

```
numpy.frombuffer(buffer, dtype=float, count=-1, offset=0)
```

The constructor takes the following parameters.

buffer	Any object that exposes buffer interface
dtype	Data type of returned ndarray. Defaults to float
count	The number of items to read, default -1 means all data
offset	The starting position to read from. Default is 0

Example 5

The following examples demonstrate the use of **frombuffer** function.

```
import numpy as np
s = 'Hello World'
a = np.frombuffer(s, dtype='S1')
print a
```

Here is its output:

```
['H' 'e' 'l' 'l' 'o' ' ' 'W' 'o' 'r' 'l' 'd']
```

numpy.fromiter

This function builds an **ndarray** object from any iterable object. A new one-dimensional array is returned by this function.

```
numpy.fromiter(iterable, dtype, count=-1)
```

Here, the constructor takes the following parameters.

iterable	Any iterable object
dtype	Data type of resultant array
count	The number of items to be read from iterator. Default is -1 which means all data to be read

The following examples show how to use the built-in **range()** function to return a list object. An iterator of this list is used to form an **ndarray** object.

Example 6

```
# create list object using range function
import numpy as np
list = range(5)
print list
```

Its output is as follows:

```
[0, 1, 2, 3, 4]
```

Example 7

```
# obtain iterator object from list
import numpy as np
list = range(5)
it = iter(list)

# use iterator to create ndarray
x = np.fromiter(it, dtype=float)
print x
```

Now, the output would be as follows:

```
[0.  1.  2.  3.  4.]
```

8. NumPy – Array from Numerical Ranges

In this chapter, we will see how to create an array from numerical ranges.

numpy.arange

This function returns an **ndarray** object containing evenly spaced values within a given range. The format of the function is as follows:

```
numpy.arange(start, stop, step, dtype)
```

The constructor takes the following parameters.

start	The start of an interval. If omitted, defaults to 0
stop	The end of an interval (not including this number)
step	Spacing between values, default is 1
dtype	Data type of resulting ndarray. If not given, data type of input is used

The following examples show how you can use this function.

Example 1

```
import numpy as np
x = np.arange(5)
print x
```

Its output would be as follows:

```
[0  1  2  3  4]
```

Example 2

```
import numpy as np
# dtype set
x = np.arange(5, dtype=float)
```

```
print x
```

Here, the output would be:

```
[0.  1.  2.  3.  4.]
```

Example 3

```
# start and stop parameters set
import numpy as np
x = np.arange(10,20,2)
print x
```

Its output is as follows:

```
[10  12  14  16  18]
```

numpy.linspace

This function is similar to **arange()** function. In this function, instead of step size, the number of evenly spaced values between the interval is specified. The usage of this function is as follows:

```
numpy.linspace(start, stop, num, endpoint, retstep, dtype)
```

The constructor takes the following parameters.

start	The starting value of the sequence
stop	The end value of the sequence, included in the sequence if endpoint set to true
num	The number of evenly spaced samples to be generated. Default is 50
endpoint	True by default, hence the stop value is included in the sequence. If false, it is not included
retstep	If true, returns samples and step between the consecutive numbers
dtype	Data type of output ndarray

The following examples demonstrate the use **linspace** function.

Example 4

```
import numpy as np
x = np.linspace(10,20,5)
print x
```

Its output would be:

```
[10.  12.5  15.  17.5  20.]
```

Example 5

```
# endpoint set to false
import numpy as np
x = np.linspace(10,20, 5, endpoint=False)
print x
```

The output would be:

```
[10.  12.  14.  16.  18.]
```

Example 6

```
# find retstep value
import numpy as np
x = np.linspace(1,2,5, retstep=True)
print x
# retstep here is 0.25
```

Now, the output would be:

```
(array([ 1. ,  1.25,  1.5 ,  1.75,  2. ]), 0.25)
```

numpy.logspace

This function returns an **ndarray** object that contains the numbers that are evenly spaced on a log scale. Start and stop endpoints of the scale are indices of the base, usually 10.

```
numpy.logspace(start, stop, num, endpoint, base, dtype)
```

Following parameters determine the output of **logspace** function.

start	The starting point of the sequence is $\text{base}^{\text{start}}$
stop	The final value of sequence is $\text{base}^{\text{stop}}$
num	The number of values between the range. Default is 50
endpoint	If true, stop is the last value in the range
base	Base of log space, default is 10
dtype	Data type of output array. If not given, it depends upon other input arguments

The following examples will help you understand the **logspace** function.

Example 7

```
import numpy as np
# default base is 10
a = np.logspace(1.0, 2.0, num=10)
print a
```

Its output would be as follows:

```
[ 10.          12.91549665   16.68100537   21.5443469    27.82559402
 35.93813664   46.41588834   59.94842503   77.42636827  100.         ]
```

Example 8

```
# set base of log space to 2
import numpy as np
a = np.logspace(1,10,num=10, base=2)
print a
```

Now, the output would be:

```
[ 2.    4.    8.   16.   32.   64.  128.  256.  512. 1024.]
```


9. NumPy – Indexing & Slicing

Contents of ndarray object can be accessed and modified by indexing or slicing, just like Python's in-built container objects.

As mentioned earlier, items in ndarray object follows zero-based index. Three types of indexing methods are available: **field access**, **basic slicing** and **advanced indexing**.

Basic slicing is an extension of Python's basic concept of slicing to **n** dimensions. A Python slice object is constructed by giving **start**, **stop**, and **step** parameters to the built-in **slice** function. This slice object is passed to the array to extract a part of array.

Example 1

```
import numpy as np
a = np.arange(10)
s = slice(2,7,2)
print a[s]
```

Its output is as follows:

```
[2  4  6]
```

In the above example, an **ndarray** object is prepared by **arange()** function. Then a slice object is defined with start, stop, and step values 2, 7, and 2 respectively. When this slice object is passed to the ndarray, a part of it starting with index 2 up to 7 with a step of 2 is sliced.

The same result can also be obtained by giving the slicing parameters separated by a **colon :** (start:stop:step) directly to the **ndarray** object.

Example 2

```
import numpy as np
a = np.arange(10)
b = a[2:7:2]
print b
```

Here, we will get the same output:

```
[2  4  6]
```

If only one parameter is put, a single item corresponding to the index will be returned. If **a :** is inserted in front of it, all items from that index onwards will be extracted. If two parameters (with **:** between them) is used, items between the two indexes (not including the stop index) with default step one are sliced.

Example 3

```
# slice single item
import numpy as np
a = np.arange(10)
b = a[5]
print b
```

Its output is as follows:

```
5
```

Example 4

```
# slice items starting from index
import numpy as np
a = np.arange(10)
print a[2:]
```

Now, the output would be:

```
[2 3 4 5 6 7 8 9]
```

Example 5

```
# slice items between indexes
import numpy as np
a = np.arange(10)
print a[2:5]
```

Here, the output would be:

```
[2 3 4]
```

The above description applies to multi-dimensional **ndarray** too.

Example 6

```
import numpy as np
a = np.array([[1,2,3],[3,4,5],[4,5,6]])
print a

# slice items starting from index
```

```
print 'Now we will slice the array from the index a[1:]'
print a[1:]
```

The output is as follows:

```
[[1 2 3]
 [3 4 5]
 [4 5 6]]
```

Now we will slice the array from the index a[1:]

```
[[3 4 5]
 [4 5 6]]
```

Slicing can also include ellipsis (...) to make a selection tuple of the same length as the dimension of an array. If ellipsis is used at the row position, it will return an ndarray comprising of items in rows.

Example 7

```
# array to begin with
import numpy as np
a = np.array([[1,2,3],[3,4,5],[4,5,6]])
print 'Our array is:'
print a
print '\n'

# this returns array of items in the second column
print 'The items in the second column are:'
print a[...,1]
print '\n'

# Now we will slice all items from the second row
print 'The items in the second row are:'
print a[1,...]
print '\n'

# Now we will slice all items from column 1 onwards
print 'The items column 1 onwards are:'
print a[...,1:]
```

The output of this program is as follows:

Our array is:

```
[[1 2 3]
 [3 4 5]
 [4 5 6]]
```

The items in the second column are:

```
[2 4 5]
```

The items in the second row are:

```
[3 4 5]
```

The items column 1 onwards are:

```
[[2 3]
 [4 5]
 [5 6]]
```

10. NumPy – Advanced Indexing

It is possible to make a selection from ndarray that is a non-tuple sequence, ndarray object of integer or Boolean data type, or a tuple with at least one item being a sequence object. Advanced indexing always returns a copy of the data. As against this, the slicing only presents a view.

There are two types of advanced indexing: **Integer** and **Boolean**.

Integer Indexing

This mechanism helps in selecting any arbitrary item in an array based on its N-dimensional index. Each integer array represents the number of indexes into that dimension. When the index consists of as many integer arrays as the dimensions of the target ndarray, it becomes straightforward.

In the following example, one element of specified column from each row of ndarray object is selected. Hence, the row index contains all row numbers, and the column index specifies the element to be selected.

Example 1

```
import numpy as np
x = np.array([[1, 2], [3, 4], [5, 6]])
y = x[[0,1,2], [0,1,0]]
print y
```

Its output would be as follows:

```
[1  4  5]
```

The selection includes elements at (0,0), (1,1) and (2,0) from the first array.

In the following example, elements placed at corners of a 4X3 array are selected. The row indices of selection are [0, 0] and [3,3] whereas the column indices are [0,2] and [0,2].

Example 2

```
import numpy as np
x = np.array([[ 0,  1,  2],[ 3,  4,  5],[ 6,  7,  8],[ 9, 10, 11]])
print 'Our array is:'
print x
print '\n'
rows = np.array([[0,0],[3,3]])
```

```
cols = np.array([[0,2],[0,2]])
y = x[rows,cols]
print 'The corner elements of this array are:'
print y
```

The output of this program is as follows:

```
[[0  2]
 [9 11]]
```

The resultant selection is an ndarray object containing corner elements.

Advanced and basic indexing can be combined by using one slice (:) or ellipsis (...) with an index array. The following example uses slice for row and advanced index for column. The result is the same when slice is used for both. But advanced index results in copy and may have different memory layout.

Example 3

```
import numpy as np
x = np.array([[ 0,  1,  2],[ 3,  4,  5],[ 6,  7,  8],[ 9, 10, 11]])
print 'Our array is:'
print x
print '\n'

# slicing
z = x[1:4,1:3]
print 'After slicing, our array becomes:'
print z
print '\n'

# using advanced index for column
y = x[1:4,[1,2]]
print 'Slicing using advanced index for column:'
print y
```

The output of this program would be as follows:

```
Our array is:
```

```
[[ 0  1  2]
 [ 3  4  5]
 [ 6  7  8]
 [ 9 10 11]]
```

```
After slicing, our array becomes:
```

```
[[ 4  5]
 [ 7  8]
 [10 11]]
```

```
Slicing using advanced index for column:
```

```
[[ 4  5]
 [ 7  8]
 [10 11]]
```

Boolean Array Indexing

This type of advanced indexing is used when the resultant object is meant to be the result of Boolean operations, such as comparison operators.

Example 4

In this example, items greater than 5 are returned as a result of Boolean indexing.

```
import numpy as np
x = np.array([[ 0,  1,  2],[ 3,  4,  5],[ 6,  7,  8],[ 9, 10, 11]])
print 'Our array is:'
print x
print '\n'

# Now we will print the items greater than 5
print 'The items greater than 5 are:'
print x[x>5]
```

The output of this program would be:

```
Our array is:
```

```
[[ 0  1  2]
 [ 3  4  5]
 [ 6  7  8]
 [ 9 10 11]]
```

```
The items greater than 5 are:
```

```
[ 6  7  8  9 10 11]
```

Example 5

In this example, NaN (Not a Number) elements are omitted by using `~` (complement operator).

```
import numpy as np
a = np.array([np.nan, 1,2,np.nan,3,4,5])
print a[~np.isnan(a)]
```

Its output would be:

```
[ 1.  2.  3.  4.  5.]
```

Example 6

The following example shows how to filter out the non-complex elements from an array.

```
import numpy as np
a = np.array([1, 2+6j, 5, 3.5+5j])
print a[np.iscomplex(a)]
```

Here, the output is as follows:

```
[2.0+6.j  3.5+5.j]
```


11. NumPy – Broadcasting

The term **broadcasting** refers to the ability of NumPy to treat arrays of different shapes during arithmetic operations. Arithmetic operations on arrays are usually done on corresponding elements. If two arrays are of exactly the same shape, then these operations are smoothly performed.

Example 1

```
import numpy as np
a = np.array([1,2,3,4])
b = np.array([10,20,30,40])
c = a*b
print c
```

Its output is as follows:

```
[10  40  90 160]
```

If the dimensions of two arrays are dissimilar, element-to-element operations are not possible. However, operations on arrays of non-similar shapes is still possible in NumPy, because of the broadcasting capability. The smaller array is **broadcast** to the size of the larger array so that they have compatible shapes.

Broadcasting is possible if the following rules are satisfied:

- Array with smaller **ndim** than the other is prepended with '1' in its shape.
- Size in each dimension of the output shape is maximum of the input sizes in that dimension.
- An input can be used in calculation, if its size in a particular dimension matches the output size or its value is exactly 1.
- If an input has a dimension size of 1, the first data entry in that dimension is used for all calculations along that dimension.

A set of arrays is said to be **broadcastable** if the above rules produce a valid result and one of the following is true:

- Arrays have exactly the same shape.
- Arrays have the same number of dimensions and the length of each dimension is either a common length or 1.
- Array having too few dimensions can have its shape prepended with a dimension of length 1, so that the above stated property is true.

The following program shows an example of broadcasting.

Example 2

```
import numpy as np
a = np.array([[ 0.0, 0.0, 0.0],[10.0,10.0,10.0],
              [20.0,20.0,20.0],[30.0,30.0,30.0]])
b = np.array([1.0,2.0,3.0])

print 'First array:'
print a
print '\n'

print 'Second array:'
print b
print '\n'

print 'First Array + Second Array'
print a+b
```

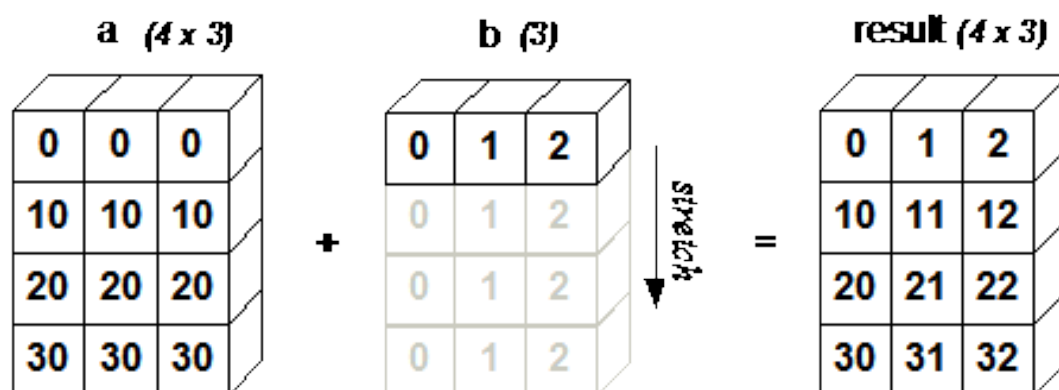
The output of this program would be as follows:

```
First array:
[[ 0.  0.  0.]
 [10. 10. 10.]
 [20. 20. 20.]
 [30. 30. 30.]]

Second array:
[ 1.  2.  3.]

First Array + Second Array
[[ 1.  2.  3.]
 [11. 12. 13.]
 [21. 22. 23.]
 [31. 32. 33.]]
```

The following figure demonstrates how array **b** is broadcast to become compatible with **a**.



12. NumPy – Iterating Over Array

NumPy package contains an iterator object **numpy.nditer**. It is an efficient multi-dimensional iterator object using which it is possible to iterate over an array. Each element of an array is visited using Python's standard Iterator interface.

Let us create a 3X4 array using `arange()` function and iterate over it using **nditer**.

Example 1

```
import numpy as np
a = np.arange(0,60,5)
a = a.reshape(3,4)
print 'Original array is:'
print a
print '\n'

print 'Modified array is:'
for x in np.nditer(a):
    print x,
```

The output of this program is as follows:

```
Original array is:
[[ 0  5 10 15]
 [20 25 30 35]
 [40 45 50 55]]

Modified array is:
0 5 10 15 20 25 30 35 40 45 50 55
```

Example 2

The order of iteration is chosen to match the memory layout of an array, without considering a particular ordering. This can be seen by iterating over the transpose of the above array.

```
import numpy as np
a = np.arange(0,60,5)
a = a.reshape(3,4)
print 'Original array is:'
```

```

print a
print '\n'

print 'Transpose of the original array is:'
b = a.T
print b
print '\n'

print 'Modified array is:'
for x in np.nditer(b):
    print x,

```

The output of the above program is as follows:

```

Original array is:
[[ 0  5 10 15]
 [20 25 30 35]
 [40 45 50 55]]

Transpose of the original array is:
[[ 0 20 40]
 [ 5 25 45]
 [10 30 50]
 [15 35 55]]

Modified array is:
0 5 10 15 20 25 30 35 40 45 50 55

```

Iteration Order

If the same elements are stored using F-style order, the iterator chooses the more efficient way of iterating over an array.

Example 3

```

import numpy as np
a = np.arange(0,60,5)
a = a.reshape(3,4)
print 'Original array is:'
print a

```

```

print '\n'

print 'Transpose of the original array is:'
b = a.T
print b
print '\n'

print 'Sorted in C-style order:'
c = b.copy(order='C')
print c
for x in np.nditer(c):
    print x,

print '\n'
print 'Sorted in F-style order:'
c = b.copy(order='F')
print c
for x in np.nditer(c):
    print x,

```

Its output would be as follows:

```

Original array is:
[[ 0  5 10 15]
 [20 25 30 35]
 [40 45 50 55]]

Transpose of the original array is:
[[ 0 20 40]
 [ 5 25 45]
 [10 30 50]
 [15 35 55]]

Sorted in C-style order:
[[ 0 20 40]
 [ 5 25 45]
 [10 30 50]
 [15 35 55]]

```

```

0 20 40 5 25 45 10 30 50 15 35 55

Sorted in F-style order:
[[ 0 20 40]
 [ 5 25 45]
 [10 30 50]
 [15 35 55]]
0 5 10 15 20 25 30 35 40 45 50 55

```

Example 4

It is possible to force **nditer** object to use a specific order by explicitly mentioning it.

```

import numpy as np
a = np.arange(0,60,5)
a = a.reshape(3,4)
print 'Original array is:'
print a
print '\n'

print 'Sorted in C-style order:'
for x in np.nditer(a, order='C'):
    print x,

print '\n'
print 'Sorted in F-style order:'
for x in np.nditer(a, order='F'):
    print x,

```

Its output would be:

```

Original array is:
[[ 0  5 10 15]
 [20 25 30 35]
 [40 45 50 55]]

Sorted in C-style order:
0 5 10 15 20 25 30 35 40 45 50 55

```

Sorted in F-style order:

0 20 40 5 25 45 10 30 50 15 35 55

Modifying Array Values

The **nditer** object has another optional parameter called **op_flags**. Its default value is read-only, but can be set to read-write or write-only mode. This will enable modifying array elements using this iterator.

Example 5

```
import numpy as np
a = np.arange(0,60,5)
a = a.reshape(3,4)
print 'Original array is:'
print a
print '\n'

for x in np.nditer(a, op_flags=['readwrite']):
    x[...] = 2*x

print 'Modified array is:'
print a
```

Its output is as follows:

```
Original array is:
[[ 0  5 10 15]
 [20 25 30 35]
 [40 45 50 55]]

Modified array is:
[[ 0 10 20 30]
 [40 50 60 70]
 [80 90 100 110]]
```

External Loop

The **nditer** class constructor has a '**flags**' parameter, which can take the following values:

c_index	C_order index can be tracked
f_index	Fortran_order index is tracked
multi-index	Type of indexes with one per iteration can be tracked
external_loop	Causes values given to be one-dimensional arrays with multiple values instead of zero-dimensional array

Example 6

In the following example, one-dimensional arrays corresponding to each column is traversed by the iterator.

```
import numpy as np
a = np.arange(0,60,5)
a = a.reshape(3,4)
print 'Original array is:'
print a
print '\n'

print 'Modified array is:'
for x in np.nditer(a, flags=['external_loop'], order='F'):
    print x,
```

The output is as follows:

```
Original array is:
[[ 0  5 10 15]
 [20 25 30 35]
 [40 45 50 55]]

Modified array is:
[ 0 20 40] [ 5 25 45] [10 30 50] [15 35 55]
```

Broadcasting Iteration

If two arrays are **broadcastable**, a combined **nditer** object is able to iterate upon them concurrently. Assuming that an array **a** has dimension 3X4, and there is another array **b** of dimension 1X4, the iterator of following type is used (array **b** is broadcast to size of **a**).

Example 7

```

import numpy as np
a = np.arange(0,60,5)
a = a.reshape(3,4)
print 'First array is:'
print a
print '\n'

print 'Second array is:'
b = np.array([1, 2, 3, 4], dtype=int)
print b

print '\n'
print 'Modified array is:'
for x,y in np.nditer([a,b]):
    print "%d:%d" % (x,y),

```

Its output would be as follows:

```

First array is:
[[ 0  5 10 15]
 [20 25 30 35]
 [40 45 50 55]]

Second array is:
[1 2 3 4]

Modified array is:
0:1 5:2 10:3 15:4 20:1 25:2 30:3 35:4 40:1 45:2 50:3 55:4

```

13. NumPy – Array Manipulation

Several routines are available in NumPy package for manipulation of elements in ndarray object. They can be classified into the following types:

Changing Shape

reshape	Gives a new shape to an array without changing its data
flat	A 1-D iterator over the array
flatten	Returns a copy of the array collapsed into one dimension
ravel	Returns a contiguous flattened array

Transpose Operations

transpose	Permutates the dimensions of an array
ndarray.T	Same as self.transpose()
rollaxis	Rolls the specified axis backwards
swapaxes	Interchanges the two axes of an array

Changing Dimensions

broadcast	Produces an object that mimics broadcasting
broadcast_to	Broadcasts an array to a new shape
expand_dims	Expands the shape of an array
squeeze	Removes single-dimensional entries from the shape of an array

Joining Arrays

concatenate	Joins a sequence of arrays along an existing axis
stack	Joins a sequence of arrays along a new axis
hstack	Stacks arrays in sequence horizontally (column wise)
vstack	Stacks arrays in sequence vertically (row wise)

Splitting Arrays

split	Splits an array into multiple sub-arrays
hsplit	Splits an array into multiple sub-arrays horizontally (column-wise)
vsplit	Splits an array into multiple sub-arrays vertically (row-wise)

Adding / Removing Elements

resize	Returns a new array with the specified shape
append	Appends the values to the end of an array
insert	Inserts the values along the given axis before the given indices
delete	Returns a new array with sub-arrays along an axis deleted
unique	Finds the unique elements of an array

numpy.reshape

This function gives a new shape to an array without changing the data. It accepts the following parameters:

```
numpy.reshape(arr, newshape, order')
```

Where,

arr	Array to be reshaped
newshape	int or tuple of int. New shape should be compatible to the original shape
order	'C' for C style, 'F' for Fortran style, 'A' means Fortran like order if an array is stored in Fortran-like contiguous memory, C style otherwise

Example 1

```
import numpy as np
a = np.arange(8)
print 'The original array:'
print a
print '\n'

b = a.reshape(4,2)
print 'The modified array:'
print b
```

Its output would be as follows:

```
The original array:
[0 1 2 3 4 5 6 7]

The modified array:
[[0 1]
 [2 3]
 [4 5]
 [6 7]]
```

numpy.ndarray.flat

This function returns a 1-D iterator over the array. It behaves similar to Python's built-in iterator.

Example 2

```
import numpy as np
a = np.arange(8).reshape(2,4)
print 'The original array:'
print a
print '\n'
print 'After applying the flat function:'
# returns element corresponding to index in flattened array
print a.flat[5]
```

Its output is as follows:

```
The original array:
[[0 1 2 3]
 [4 5 6 7]]

After applying the flat function:
5
```

numpy.ndarray.flatten

This function returns a copy of an array collapsed into one dimension. The function takes the following parameters.

```
ndarray.flatten(order)
```

Where,

order	'C': row major (default. 'F': column major 'A': flatten in column-major order, if a is Fortran contiguous in memory, row-major order otherwise 'K': flatten a in the order the elements occur in the memory
--------------	--

Example 3

```
import numpy as np
a = np.arange(8).reshape(2,4)
print 'The original array is:'
print a
print '\n'

# default is column-major
print 'The flattened array is:'
print a.flatten()
print '\n'

print 'The flattened array in F-style ordering:'
print a.flatten(order='F')
```

The output of the above program would be as follows:

```
The original array is:
[[0 1 2 3]
 [4 5 6 7]]

The flattened array is:
[0 1 2 3 4 5 6 7]

The flattened array in F-style ordering:
[0 4 1 5 2 6 3 7]
```

numpy.ravel

This function returns a flattened one-dimensional array. A copy is made only if needed. The returned array will have the same type as that of the input array. The function takes one parameter.

```
numpy.ravel(a, order)
```

The constructor takes the following parameters.

Order	'C': row major (default. 'F': column major 'A': flatten in column-major order if a is Fortran contiguous in memory, row-major order otherwise 'K': flatten a in the order the elements occur in memory.
--------------	--

Example 4

```
import numpy as np
a = np.arange(8).reshape(2,4)
print 'The original array is:'
print a
print '\n'

print 'After applying ravel function:'
print a.ravel()

print '\n'
print 'Applying ravel function in F-style ordering:'
print a.ravel(order='F')
```

Its output would be as follows:

```
The original array is:
[[0 1 2 3]
 [4 5 6 7]]

After applying ravel function:
[0 1 2 3 4 5 6 7]

Applying ravel function in F-style ordering:
[0 4 1 5 2 6 3 7]
```

numpy.transpose

This function permutes the dimension of the given array. It returns a view wherever possible. The function takes the following parameters.

```
numpy.transpose(arr, axes)
```

where,

arr	The array to be transposed
axes	List of ints, corresponding to the dimensions. By default, the dimensions are reversed

Example 5

```
import numpy as np
a = np.arange(12).reshape(3,4)
print 'The original array is:'
print a

print '\n'
print 'The transposed array is:'
print np.transpose(a)
```

Its output would be as follows:

```
The original array is:
[[ 0  1  2  3]
 [ 4  5  6  7]
 [ 8  9 10 11]]

The transposed array is:
[[ 0  4  8]
 [ 1  5  9]
 [ 2  6 10]
 [ 3  7 11]]
```

numpy.ndarray.T

This function belongs to **ndarray** class. It behaves similar to **numpy.transpose**.

Example 6

```
import numpy as np
a = np.arange(12).reshape(3,4)
print 'The original array is:'
print a
print '\n'

print 'Array after applying the function:'
print a.T
```

The output of the above program would be:

The original array is:

```
[[ 0  1  2  3]
 [ 4  5  6  7]
 [ 8  9 10 11]]
```

Array after applying the function:

```
[[ 0  4  8]
 [ 1  5  9]
 [ 2  6 10]
 [ 3  7 11]]
```

numpy.swapaxes

This function interchanges the two axes of an array. For NumPy versions after 1.10, a view of the swapped array is returned. The function takes the following parameters.

```
numpy.swapaxes(arr, axis1, axis2)
```

Where,

arr	Input array whose axes are to be swapped
axis1	An int corresponding to the first axis
axis2	An int corresponding to the second axis

Example 7

```
# It creates a 3 dimensional ndarray
import numpy as np
a = np.arange(8).reshape(2,2,2)
print 'The original array:'
print a
print '\n'

# now swap numbers between axis 0 (along depth) and axis 2 (along width)
print 'The array after applying the swapaxes function:'
print np.swapaxes(a, 2, 0)
```

Its output would be as follows:

The original array:

```
[[[0 1]
   [2 3]]

 [[4 5]
   [6 7]]]
```

The array after applying the swapaxes function:

```
[[[0 4]
   [2 6]]

 [[1 5]
   [3 7]]]
```

numpy.rollaxis

This function rolls the specified axis backwards, until it lies in a specified position. The function takes three parameters.

```
numpy.rollaxis(arr, axis, start)
```

Where,

arr	Input array
axis	Axis to roll backwards. The position of the other axes do not change relative to one another
start	Zero by default leading to the complete roll. Rolls until it reaches the specified position

Example 8

```
# It creates 3 dimensional ndarray
import numpy as np
a = np.arange(8).reshape(2,2,2)
print 'The original array:'
print a
print '\n'
```

```
# to roll axis-2 to axis-0 (along width to along depth)
print 'After applying rollaxis function:'
print np.rollaxis(a,2)

# to roll axis 0 to 1 (along width to height)
print '\n'
print 'After applying rollaxis function:'
print np.rollaxis(a,2,1)
```

Its output is as follows:

```
The original array:
[[[0 1]
   [2 3]
   [4 5]
   [6 7]]]

After applying rollaxis function:
[[[0 2]
   [4 6]]
 [[1 3]
   [5 7]]]

After applying rollaxis function:
[[[0 2]
   [1 3]]
 [[4 6]
   [5 7]]]
```

numpy.broadcast

As seen earlier, NumPy has in-built support for broadcasting. This function mimics the broadcasting mechanism. It returns an object that encapsulates the result of broadcasting one array against the other.

The function takes two arrays as input parameters. Following example illustrates its use.

Example 9

```
import numpy as np
x = np.array([[1], [2], [3]])
y = np.array([4, 5, 6])

# to broadcast x against y
b = np.broadcast(x,y)

# it has an iterator property, a tuple of iterators along self's "components."
print 'Broadcast x against y:'
r,c = b.iters
print r.next(), c.next()
print r.next(), c.next()
print '\n'

# shape attribute returns the shape of broadcast object
print 'The shape of the broadcast object:'
print b.shape
print '\n'

# to add x and y manually using broadcast
b = np.broadcast(x,y)
c = np.empty(b.shape)
print 'Add x and y manually using broadcast:'
print c.shape
print '\n'

c.flat = [u+v for (u,v) in b]
print 'After applying the flat function:'
print c
print '\n'

# same result obtained by NumPy's built-in broadcasting support
print 'The summation of x and y:'
print x+y
```

Its output is as follows:

```
Broadcast x against y:
1 4
1 5

The shape of the broadcast object:
(3, 3)

Add x and y manually using broadcast:
(3, 3)

After applying the flat function:
[[ 5.  6.  7.]
 [ 6.  7.  8.]
 [ 7.  8.  9.]]

The summation of x and y:
[[5 6 7]
 [6 7 8]
 [7 8 9]]
```

numpy.broadcast_to

This function broadcasts an array to a new shape. It returns a read-only view on the original array. It is typically not contiguous. The function may throw `ValueError` if the new shape does not comply with NumPy's broadcasting rules.

Note: This function is available *version 1.10.0* onwards.

The function takes the following parameters.

```
numpy.broadcast_to(array, shape, subok)
```

Example 10

```
import numpy as np
a = np.arange(4).reshape(1,4)
print 'The original array:'
print a
print '\n'

print 'After applying the broadcast_to function:'
print np.broadcast_to(a,(4,4))
```

It should produce the following output:

```
[[0  1  2  3]
 [0  1  2  3]
 [0  1  2  3]
 [0  1  2  3]]
```

numpy.expand_dims

This function expands the array by inserting a new axis at the specified position. Two parameters are required by this function.

```
numpy.expand_dims(arr, axis)
```

Where,

arr	Input array
axis	Position where new axis to be inserted

Example 11

```
import numpy as np
x = np.array([[1,2],[3,4]])
print 'Array x:'
print x
print '\n'

y = np.expand_dims(x, axis=0)
print 'Array y:'
print y
print '\n'
```

```
print 'The shape of X and Y array:'
print x.shape, y.shape
print '\n'

# insert axis at position 1
y = np.expand_dims(x, axis=1)
print 'Array Y after inserting axis at position 1:'
print y
print '\n'

print 'x.ndim and y.ndim:'
print x.ndim,y.ndim
print '\n'

print 'x.shape and y.shape:'
print x.shape, y.shape
```

The output of the above program would be as follows:

```
Array x:
[[1 2]
 [3 4]]

Array y:
[[[1 2]
  [3 4]]]

The shape of X and Y array:
(2, 2) (1, 2, 2)

Array Y after inserting axis at position 1:
[[[1 2]
  [3 4]]]

x.ndim and y.ndim:
2 3
```



```
x.shape and y.shape:
(2, 2) (2, 1, 2)
```

numpy.squeeze

This function removes one-dimensional entry from the shape of the given array. Two parameters are required for this function.

```
numpy.squeeze(arr, axis)
```

Where,

arr	Input array
axis	int or tuple of int. selects a subset of single dimensional entries in the shape

Example 12

```
import numpy as np
x = np.arange(9).reshape(1,3,3)
print 'Array X:'
print x
print '\n'

y = np.squeeze(x)
print 'Array Y:'
print y
print '\n'

print 'The shapes of X and Y array:'
print x.shape, y.shape
```

Its output is as follows:

```
Array X:
[[[0 1 2]
  [3 4 5]
  [6 7 8]]]
```

Array Y:

```
[[0 1 2]
 [3 4 5]
 [6 7 8]]
```

The shapes of X and Y array:

```
(1, 3, 3) (3, 3)
```

numpy.concatenate

Concatenation refers to joining. This function is used to join two or more arrays of the same shape along a specified axis. The function takes the following parameters.

```
numpy.concatenate((a1, a2, ...), axis)
```

Where,

a1,a2..	Sequence of arrays of the same type
axis	Axis along which arrays have to be joined. Default is 0

Example 13

```
import numpy as np
a=np.array([[1,2],[3,4]])
print 'First array:'
print a
print '\n'

b = np.array([[5,6],[7,8]])
print 'Second array:'
print b
print '\n'

# both the arrays are of same dimensions
print 'Joining the two arrays along axis 0:'
print np.concatenate((a,b))
print '\n'

print 'Joining the two arrays along axis 1:'
print np.concatenate((a,b),axis=1)
```

Its output is as follows:

```
First array:
[[1 2]
 [3 4]]
Second array:
[[5 6]
 [7 8]]
Joining the two arrays along axis 0:
[[1 2]
 [3 4]
 [5 6]
 [7 8]]

Joining the two arrays along axis 1:
[[1 2 5 6]
 [3 4 7 8]]
```

numpy.stack

This function joins the sequence of arrays along a new axis. This function has been added since NumPy version 1.10.0. Following parameters need to be provided.

Note: This function is available in *version 1.10.0* onwards.

```
numpy.stack(arrays, axis)
```

Where,

arrays	Sequence of arrays of the same shape
axis	Axis in the resultant array along which the input arrays are stacked

Example 14

```
import numpy as np
a = np.array([[1,2],[3,4]])
print 'First Array:'
print a
print '\n'
```

```
b = np.array([[5,6],[7,8]])
print 'Second Array:'
print b
print '\n'

print 'Stack the two arrays along axis 0:'
print np.stack((a,b),0)
print '\n'

print 'Stack the two arrays along axis 1:'
print np.stack((a,b),1)
```

It should produce the following output:

```
First array:
[[1 2]
 [3 4]]

Second array:
[[5 6]
 [7 8]]

Stack the two arrays along axis 0:
[[[1 2]
   [3 4]]
 [[5 6]
   [7 8]]]

Stack the two arrays along axis 1:
[[[1 2]
   [5 6]]
 [[3 4]
   [7 8]]]
```

numpy.hstack and numpy.vstack

Variants of numpy.stack function to stack so as to make a single array horizontally or vertically.

Example 15

```
import numpy as np
a = np.array([[1,2],[3,4]])
print 'First array:'
print a
print '\n'

b = np.array([[5,6],[7,8]])
print 'Second array:'
print b
print '\n'

print 'Horizontal stacking:'
c = np.hstack((a,b))
print c
print '\n'

print 'Vertical stacking:'
c = np.vstack((a,b))
print c
```

It would produce the following output:

```
First array:
[[1 2]
 [3 4]]

Second array:
[[5 6]
 [7 8]]

Horizontal stacking:
[[1 2 5 6]
 [3 4 7 8]]
```

Vertical stacking:

```
[[1 2]
 [3 4]
 [5 6]
 [7 8]]
```

numpy.split

This function divides the array into subarrays along a specified axis. The function takes three parameters.

```
numpy.split(ary, indices_or_sections, axis)
```

Where,

ary	Input array to be split
indices_or_sections	Can be an integer, indicating the number of equal sized subarrays to be created from the input array. If this parameter is a 1-D array, the entries indicate the points at which a new subarray is to be created
axis	Default is 0

Example 16

```
import numpy as np
a = np.arange(9)
print 'First array:'
print a
print '\n'

print 'Split the array in 3 equal-sized subarrays:'
b = np.split(a,3)
print b
print '\n'

print 'Split the array at positions indicated in 1-D array:'
b = np.split(a,[4,7])
```

```
print b
```

Its output is as follows:

First array:

```
[0 1 2 3 4 5 6 7 8]
```

Split the array in 3 equal-sized subarrays:

```
[array([0, 1, 2]), array([3, 4, 5]), array([6, 7, 8])]
```

Split the array at positions indicated in 1-D array:

```
[array([0, 1, 2, 3]), array([4, 5, 6]), array([7, 8])]
```

numpy.hsplit and numpy.vsplit

The `numpy.hsplit` is a special case of `split()` function where axis is 1 indicating a horizontal split regardless of the dimension of the input array.

Similarly, `numpy.vsplit` is a special case of `split()` function where axis is 0 indicating a vertical split regardless of the dimension of the input array. The following example makes this clear.

Example 17

```
import numpy as np
a = np.arange(16).reshape(4,4)
print 'First array:'
print a
print '\n'

print 'Horizontal splitting:'
b = np.hsplit(a,2)
print b
print '\n'
print 'Vertical splitting:'
b = np.vsplit(a,2)
print b
```

Its output would be as follows:

First array:

```
[[ 0  1  2  3]
```

```
[ 4  5  6  7]
[ 8  9 10 11]
[12 13 14 15]]
```

Horizontal splitting:

```
[array([[ 0,  1],
        [ 4,  5],
        [ 8,  9],
        [12, 13]]), array([[ 2,  3],
        [ 6,  7],
        [10, 11],
        [14, 15]])]
```

Vertical splitting:

```
[array([[0, 1, 2, 3],
        [4, 5, 6, 7]]), array([[ 8,  9, 10, 11],
        [12, 13, 14, 15]])]
```

numpy.resize

This function returns a new array with the specified size. If the new size is greater than the original, the repeated copies of entries in the original are contained. The function takes the following parameters.

```
numpy.resize(arr, shape)
```

Where,

arr	Input array to be resized
shape	New shape of the resulting array

Example 18

```
import numpy as np
a = np.array([[1,2,3],[4,5,6]])
print 'First array:'
print a
print '\n'
```



```
print 'The shape of first array:'
print a.shape
print '\n'

b = np.resize(a, (3,2))
print 'Second array:'
print b
print '\n'

print 'The shape of second array:'
print b.shape
print '\n'

# Observe that first row of a is repeated in b since size is bigger
print 'Resize the second array:'
b = np.resize(a,(3,3))
print b
```

The above program will produce the following output:

```
First array:
[[1 2 3]
 [4 5 6]]

The shape of first array:
(2, 3)

Second array:
[[1 2]
 [3 4]
 [5 6]]

The shape of second array:
(3, 2)

Resize the second array:
```

```
[[1 2 3]
 [4 5 6]
 [1 2 3]]
```

numpy.append

This function adds values at the end of an input array. The append operation is not in-place, a new array is allocated. Also the dimensions of the input arrays must match otherwise ValueError will be generated.

The function takes the following parameters.

```
numpy.append(arr, values, axis)
```

Where,

arr	Input array
values	To be appended to arr. It must be of the same shape as of arr (excluding axis of appending)
axis	The axis along which append operation is to be done. If not given, both parameters are flattened

Example 19

```
import numpy as np
a = np.array([[1,2,3],[4,5,6]])
print 'First array:'
print a
print '\n'

print 'Append elements to array:'
print np.append(a, [7,8,9])
print '\n'

print 'Append elements along axis 0:'
print np.append(a, [[7,8,9]],axis=0)
print '\n'

print 'Append elements along axis 1:'
print np.append(a, [[5,5,5],[7,8,9]],axis=1)
```

Its output would be as follows:

First array:

```
[[1 2 3]
 [4 5 6]]
```

Append elements to array:

```
[1 2 3 4 5 6 7 8 9]
```

Append elements along axis 0:

```
[[1 2 3]
 [4 5 6]
 [7 8 9]]
```

Append elements along axis 1:

```
[[1 2 3 5 5 5]
 [4 5 6 7 8 9]]
```

numpy.insert

This function inserts values in the input array along the given axis and before the given index. If the type of values is converted to be inserted, it is different from the input array. Insertion is not done in place and the function returns a new array. Also, if the axis is not mentioned, the input array is flattened.

The insert() function takes the following parameters:

```
numpy.insert(arr, obj, values, axis)
```

Where,

arr	Input array
obj	The index before which insertion is to be made
values	The array of values to be inserted
axis	The axis along which to insert. If not given, the input array is flattened

Example 20

```

import numpy as np
a = np.array([[1,2],[3,4],[5,6]])
print 'First array:'
print a
print '\n'

print 'Axis parameter not passed. The input array is flattened before
insertion.'
print np.insert(a,3,[11,12])
print '\n'

print 'Axis parameter passed. The values array is broadcast to match input
array.'
print 'Broadcast along axis 0:'
print np.insert(a,1,[11],axis=0)
print '\n'

print 'Broadcast along axis 1:'
print np.insert(a,1,11,axis=1)

```

Its output would be as follows:

```

First array:
[[1 2]
 [3 4]
 [5 6]]

Axis parameter not passed. The input array is flattened before insertion.
[ 1  2  3 11 12  4  5  6]

Axis parameter passed. The values array is broadcast to match input array.
Broadcast along axis 0:
[[ 1  2]
 [11 11]
 [ 3  4]
 [ 5  6]]

```

Broadcast along axis 1:

```
[[ 1 11  2]
 [ 3 11  4]
 [ 5 11  6]]
```

numpy.delete

This function returns a new array with the specified subarray deleted from the input array. As in case of insert() function, if the axis parameter is not used, the input array is flattened. The function takes the following parameters:

```
Numpy.delete(arr, obj, axis)
```

Where,

arr	Input array
obj	Can be a slice, an integer or array of integers, indicating the subarray to be deleted from the input array
axis	The axis along which to delete the given subarray. If not given, arr is flattened

Example 21

```
import numpy as np
a = np.arange(12).reshape(3,4)
print 'First array:'
print a
print '\n'

print 'Array flattened before delete operation as axis not used:'
print np.delete(a,5)
print '\n'

print 'Column 2 deleted:'
print np.delete(a,1,axis=1)
print '\n'

print 'A slice containing alternate values from array deleted:'
```

```
a = np.array([1,2,3,4,5,6,7,8,9,10])
print np.delete(a, np.s_[:2])
```

Its output would be as follows:

```
First array:
[[ 0  1  2  3]
 [ 4  5  6  7]
 [ 8  9 10 11]]

Array flattened before delete operation as axis not used:
[ 0  1  2  3  4  6  7  8  9 10 11]

Column 2 deleted:
[[ 0  2  3]
 [ 4  6  7]
 [ 8 10 11]]

A slice containing alternate values from array deleted:
[ 2  4  6  8 10]
```

numpy.unique

This function returns an array of unique elements in the input array. The function can be able to return a tuple of array of unique vales and an array of associated indices. Nature of the indices depend upon the type of return parameter in the function call.

```
numpy.unique(arr, return_index, return_inverse, return_counts)
```

Where,

arr	The input array. Will be flattened if not 1-D array
return_index	If True, returns the indices of elements in the input array
return_inverse	If True, returns the indices of unique array, which can be used to reconstruct the input array
return_counts	If True, returns the number of times the element in unique array appears in the original array

Example 22

```
import numpy as np
a = np.array([5,2,6,2,7,5,6,8,2,9])
print 'First array:'
print a
print '\n'

print 'Unique values of first array:'
u = np.unique(a)
print u
print '\n'

print 'Unique array and Indices array:'
u,indices=np.unique(a, return_index=True)
print indices
print '\n'

print 'We can see each number corresponds to index in original array:'
print a
print '\n'

print 'Indices of unique array:'
u,indices=np.unique(a,return_inverse=True)
print u
print '\n'
print 'Indices are:'
print indices
print '\n'

print 'Reconstruct the original array using indices:'
print u[indices]
print '\n'

print 'Return the count of repetitions of unique elements:'
u,indices=np.unique(a,return_counts=True)
print u
```

```
print indices
```

Its output is as follows:

First array:

```
[5 2 6 2 7 5 6 8 2 9]
```

Unique values of first array:

```
[2 5 6 7 8 9]
```

Unique array and Indices array:

```
[1 0 2 4 7 9]
```

We can see each number corresponds to index in original array:

```
[5 2 6 2 7 5 6 8 2 9]
```

Indices of unique array:

```
[2 5 6 7 8 9]
```

Indices are:

```
[1 0 2 0 3 1 2 4 0 5]
```

Reconstruct the original array using indices:

```
[5 2 6 2 7 5 6 8 2 9]
```

Return the count of repetitions of unique elements:

```
[2 5 6 7 8 9]
```

```
[3 2 2 1 1 1]
```


16. NumPy – Mathematical Functions

Quite understandably, NumPy contains a large number of various mathematical operations. NumPy provides standard trigonometric functions, functions for arithmetic operations, handling complex numbers, etc.

Trigonometric Functions

NumPy has standard trigonometric functions which return trigonometric ratios for a given angle in radians.

Example

```
import numpy as np
a = np.array([0,30,45,60,90])
print 'Sine of different angles:'
# Convert to radians by multiplying with pi/180
print np.sin(a*np.pi/180)
print '\n'

print 'Cosine values for angles in array:'
print np.cos(a*np.pi/180)
print '\n'

print 'Tangent values for given angles:'
print np.tan(a*np.pi/180)
```

Here is its output:

```
Sine of different angles:
[ 0.          0.5          0.70710678  0.8660254   1.          ]

Cosine values for angles in array:
[ 1.00000000e+00  8.66025404e-01  7.07106781e-01  5.00000000e-01
 6.12323400e-17]

Tangent values for given angles:
[ 0.00000000e+00  5.77350269e-01  1.00000000e+00  1.73205081e+00
 1.63312394e+16]
```

arcsin, **arccos**, and **arctan** functions return the trigonometric inverse of sin, cos, and tan of the given angle. The result of these functions can be verified by **numpy.degrees()** function by converting radians to degrees.

Example

```
import numpy as np
a = np.array([0,30,45,60,90])
print 'Array containing sine values:'
sin=np.sin(a*np.pi/180)
print sin
print '\n'

print 'Compute sine inverse of angles. Returned values are in radians.'
inv=np.arcsin(sin)
print inv
print '\n'

print 'Check result by converting to degrees:'
print np.degrees(inv)
print '\n'

print 'arccos and arctan functions behave similarly:'
cos=np.cos(a*np.pi/180)
print cos
print '\n'

print 'Inverse of cos:'
inv=np.arccos(cos)
print inv
print '\n'

print 'In degrees:'
print np.degrees(inv)
print '\n'

print 'Tan function:'
tan=np.tan(a*np.pi/180)
print tan
```

```

print '\n'

print 'Inverse of tan:'
inv = np.arctan(tan)
print inv
print '\n'

print 'In degrees:'
print np.degrees(inv)

```

Its output is as follows:

```

Array containing sine values:
[ 0.          0.5          0.70710678  0.8660254   1.          ]

Compute sine inverse of angles. Returned values are in radians.
[ 0.          0.52359878  0.78539816  1.04719755  1.57079633]

Check result by converting to degrees:
[ 0.  30.  45.  60.  90.]

arccos and arctan functions behave similarly:
[ 1.00000000e+00  8.66025404e-01  7.07106781e-01  5.00000000e-01
 6.12323400e-17]

Inverse of cos:
[ 0.          0.52359878  0.78539816  1.04719755  1.57079633]

In degrees:
[ 0.  30.  45.  60.  90.]

Tan function:
[ 0.00000000e+00  5.77350269e-01  1.00000000e+00  1.73205081e+00

Inverse of tan:
[ 0.          0.52359878  0.78539816  1.04719755  1.57079633]

In degrees:

```

```
[ 0.  30.  45.  60.  90.]
```

Functions for Rounding

numpy.around()

This is a function that returns the value rounded to the desired precision. The function takes the following parameters.

```
numpy.around(a, decimals)
```

Where,

a	Input data
decimals	The number of decimals to round to. Default is 0. If negative, the integer is rounded to position to the left of the decimal point

Example

```
import numpy as np
a = np.array([1.0, 5.55, 123, 0.567, 25.532])
print 'Original array:'
print a
print '\n'

print 'After rounding:'
print np.around(a)
print np.around(a, decimals=1)
print np.around(a, decimals=-1)
```

It produces the following output:

```
Original array:
[ 1.    5.55 123.    0.567 25.532]

After rounding:
[ 1.  6. 123.  1.  26.]
[ 1.  5.6 123.  0.6 25.5]
[ 0. 10. 120.  0. 30.]
```

numpy.floor()

This function returns the largest integer not greater than the input parameter. The floor of the **scalar x** is the largest **integer i**, such that $i \leq x$. Note that in Python, flooring always is rounded away from 0.

Example

```
import numpy as np
a = np.array([-1.7, 1.5, -0.2, 0.6, 10])
print 'The given array:'
print a
print '\n'

print 'The modified array:'
print np.floor(a)
```

It produces the following output:

```
The given array:
[ -1.7   1.5  -0.2   0.6  10. ]

The modified array:
[ -2.   1.  -1.   0.  10.]
```

numpy.ceil()

The ceil() function returns the ceiling of an input value, i.e. the ceil of the **scalar x** is the smallest **integer i**, such that $i \geq x$.

Example

```
import numpy as np
a = np.array([-1.7, 1.5, -0.2, 0.6, 10])
print 'The given array:'
print a
print '\n'

print 'The modified array:'
print np.ceil(a)
```

It will produce the following output:

The given array:

```
[ -1.7   1.5  -0.2   0.6  10. ]
```

The modified array:

```
[ -1.   2.  -0.   1.  10.]
```

17. NumPy – Arithmetic Operations

Input arrays for performing arithmetic operations such as `add()`, `subtract()`, `multiply()`, and `divide()` must be either of the same shape or should conform to array broadcasting rules.

Example

```
import numpy as np
a = np.arange(9, dtype=np.float_).reshape(3,3)
print 'First array:'
print a
print '\n'

print 'Second array:'
b = np.array([10,10,10])
print b
print '\n'

print 'Add the two arrays:'
print np.add(a,b)
print '\n'

print 'Subtract the two arrays:'
print np.subtract(a,b)
print '\n'

print 'Multiply the two arrays:'
print np.multiply(a,b)
print '\n'

print 'Divide the two arrays:'
print np.divide(a,b)
```

It will produce the following output:

First array:

```
[[ 0.  1.  2.]  
 [ 3.  4.  5.]  
 [ 6.  7.  8.]]
```

Second array:

```
[10 10 10]
```

Add the two arrays:

```
[[ 10.  11.  12.]  
 [ 13.  14.  15.]  
 [ 16.  17.  18.]]
```

Subtract the two arrays:

```
[[ -10.  -9.  -8.]  
 [  -7.  -6.  -5.]  
 [  -4.  -3.  -2.]]
```

Multiply the two arrays:

```
[[  0.  10.  20.]  
 [ 30.  40.  50.]  
 [ 60.  70.  80.]]
```

Divide the two arrays:

```
[[ 0.   0.1  0.2]  
 [ 0.3  0.4  0.5]  
 [ 0.6  0.7  0.8]]
```


Let us now discuss some of the other important arithmetic functions available in NumPy.

numpy.reciprocal()

This function returns the reciprocal of argument, element-wise. For elements with absolute values larger than 1, the result is always 0 because of the way in which Python handles integer division. For integer 0, an overflow warning is issued.

Example

```
import numpy as np
a = np.array([0.25, 1.33, 1, 0, 100])
print 'Our array is:'
print a
print '\n'

print 'After applying reciprocal function:'
print np.reciprocal(a)
print '\n'

b = np.array([100], dtype=int)
print 'The second array is:'
print b
print '\n'

print 'After applying reciprocal function:'
print np.reciprocal(b)
```

It will produce the following output:

```
Our array is:
[  0.25   1.33   1.    0.  100. ]

After applying reciprocal function:
main.py:8: RuntimeWarning: divide by zero encountered in reciprocal
  print np.reciprocal(a)
[  4.          0.7518797  1.          inf  0.01    ]

The second array is:
[100]
```

After applying reciprocal function:
[0]

numpy.power()

This function treats elements in the first input array as base and returns it raised to the power of the corresponding element in the second input array.

```
import numpy as np
a = np.array([10,100,1000])
print 'Our array is:'
print a
print '\n'

print 'Applying power function:'
print np.power(a,2)
print '\n'

print 'Second array:'
b = np.array([1,2,3])
print b
print '\n'

print 'Applying power function again:'
print np.power(a,b)
```

It will produce the following output:

```
Our array is:
[10  100 1000]

Applying power function:
[100  10000 1000000]

Second array:
[1 2 3]
Applying power function again:
[10  10000 1000000000]
```

numpy.mod()

This function returns the remainder of division of the corresponding elements in the input array. The function **numpy.remainder()** also produces the same result.

```
import numpy as np
a = np.array([10,20,30])
b = np.array([3,5,7])
print 'First array:'
print a
print '\n'

print 'Second array:'
print b
print '\n'

print 'Applying mod() function:'
print np.mod(a,b)
print '\n'

print 'Applying remainder() function:'
print np.remainder(a,b)
```

It will produce the following output:

```
First array:
[10 20 30]

Second array:
[3 5 7]

Applying mod() function:
[1 0 2]

Applying remainder() function:
[1 0 2]
```

The following functions are used to perform operations on array with complex numbers.

- **numpy.real()** returns the real part of the complex data type argument.
- **numpy.imag()** returns the imaginary part of the complex data type argument.
- **numpy.conj()** returns the complex conjugate, which is obtained by changing the sign of the imaginary part.
- **numpy.angle()** returns the angle of the complex argument. The function has degree parameter. If true, the angle in the degree is returned, otherwise the angle is in radians.

```
import numpy as np
a = np.array([-5.6j, 0.2j, 11. , 1+1j])
print 'Our array is:'
print a
print '\n'

print 'Applying real() function:'
print np.real(a)
print '\n'

print 'Applying imag() function:'
print np.imag(a)
print '\n'

print 'Applying conj() function:'
print np.conj(a)
print '\n'

print 'Applying angle() function:'
print np.angle(a)
print '\n'

print 'Applying angle() function again (result in degrees)'
print np.angle(a, deg=True)
```

It will produce the following output:

Our array is:

```
[ 0.-5.6j  0.+0.2j 11.+0.j   1.+1.j ]
```

Applying `real()` function:

```
[ 0.  0. 11.  1.]
```

Applying `imag()` function:

```
[-5.6  0.2  0.  1.]
```

Applying `conj()` function:

```
[ 0.+5.6j  0.-0.2j 11.-0.j   1.-1.j ]
```

Applying `angle()` function:

```
[-1.57079633  1.57079633  0.      0.78539816]
```

Applying `angle()` function again (result in degrees)

```
[-90.  90.  0.  45.]
```

18. NumPy – Statistical Functions

NumPy has quite a few useful statistical functions for finding minimum, maximum, percentile standard deviation and variance, etc. from the given elements in the array. The functions are explained as follows:

numpy.amin() and numpy.amax()

These functions return the minimum and the maximum from the elements in the given array along the specified axis.

Example

```
import numpy as np
a = np.array([[3,7,5],[8,4,3],[2,4,9]])
print 'Our array is:'
print a
print '\n'

print 'Applying amin() function:'
print np.amin(a,1)
print '\n'

print 'Applying amin() function again:'
print np.amin(a,0)
print '\n'

print 'Applying amax() function:'
print np.amax(a)
print '\n'

print 'Applying amax() function again:'
print np.amax(a, axis=0)
```

It will produce the following output:

```
Our array is:
[[3 7 5]
 [8 4 3]
 [2 4 9]]
Applying amin() function:
[3 3 2]

Applying amin() function again:
[2 4 3]

Applying amax() function:
9

Applying amax() function again:
[8 7 9]
```

numpy.ptp()

The **numpy.ptp()** function returns the range (maximum-minimum) of values along an axis.

```
import numpy as np
a = np.array([[3,7,5],[8,4,3],[2,4,9]])
print 'Our array is:'
print a
print '\n'

print 'Applying ptp() function:'
print np.ptp(a)
print '\n'

print 'Applying ptp() function along axis 1:'
print np.ptp(a, axis=1)
print '\n'

print 'Applying ptp() function along axis 0:'
```

```
print np.ptp(a, axis=0)
```

It will produce the following output:

Our array is:

```
[[3 7 5]
 [8 4 3]
 [2 4 9]]
```

Applying ptp() function:

```
7
```

Applying ptp() function along axis 1:

```
[4 5 7]
```

Applying ptp() function along axis 0:

```
[6 3 6]
```

numpy.percentile()

Percentile (or a centile) is a measure used in statistics indicating the value below which a given percentage of observations in a group of observations fall. The function **numpy.percentile()** takes the following arguments.

```
numpy.percentile(a, q, axis)
```

Where,

a	Input array
q	The percentile to compute must be between 0-100
axis	The axis along which the percentile is to be calculated

Example

```
import numpy as np
a = np.array([[30,40,70],[80,20,10],[50,90,60]])
print 'Our array is:'
print a
print '\n'

print 'Applying percentile() function:'
print np.percentile(a,50)
print '\n'

print 'Applying percentile() function along axis 1:'
print np.percentile(a,50, axis=1)
print '\n'

print 'Applying percentile() function along axis 0:'
print np.percentile(a,50, axis=0)
```

It will produce the following output:

```
Our array is:
[[30 40 70]
 [80 20 10]
 [50 90 60]]

Applying percentile() function:
50.0

Applying percentile() function along axis 1:
[ 40.  20.  60.]

Applying percentile() function along axis 0:
[ 50.  40.  60.]
```

numpy.median()

Median is defined as the value separating the higher half of a data sample from the lower half. The **numpy.median()** function is used as shown in the following program.

Example

```
import numpy as np
a = np.array([[30,65,70],[80,95,10],[50,90,60]])
print 'Our array is:'
print a
print '\n'

print 'Applying median() function:'
print np.median(a)
print '\n'

print 'Applying median() function along axis 0:'
print np.median(a, axis=0)
print '\n'

print 'Applying median() function along axis 1:'
print np.median(a, axis=1)
```

It will produce the following output:

```
Our array is:
[[30 65 70]
 [80 95 10]
 [50 90 60]]

Applying median() function:
65.0

Applying median() function along axis 0:
[ 50.  90.  60.]

Applying median() function along axis 1:
[ 65.  80.  60.]
```

numpy.mean()

Arithmetic mean is the sum of elements along an axis divided by the number of elements. The **numpy.mean()** function returns the arithmetic mean of elements in the array. If the axis is mentioned, it is calculated along it.

Example

```
import numpy as np
a = np.array([[1,2,3],[3,4,5],[4,5,6]])
print 'Our array is:'
print a
print '\n'

print 'Applying mean() function:'
print np.mean(a)
print '\n'

print 'Applying mean() function along axis 0:'
print np.mean(a, axis=0)
print '\n'

print 'Applying mean() function along axis 1:'
print np.mean(a, axis=1)
```

It will produce the following output:

```
Our array is:
[[1 2 3]
 [3 4 5]
 [4 5 6]]

Applying mean() function:
3.66666666667

Applying mean() function along axis 0:
[ 2.66666667  3.66666667  4.66666667]

Applying mean() function along axis 1:
[ 2.  4.  5.]
```

numpy.average()

Weighted average is an average resulting from the multiplication of each component by a factor reflecting its importance. The **numpy.average()** function computes the weighted average of elements in an array according to their respective weight given in another array. The function can have an axis parameter. If the axis is not specified, the array is flattened.

Considering an array [1,2,3,4] and corresponding weights [4,3,2,1], the weighted average is calculated by adding the product of the corresponding elements and dividing the sum by the sum of weights.

Weighted average = $(1*4+2*3+3*2+4*1)/(4+3+2+1)$

Example

```
import numpy as np
a = np.array([1,2,3,4])
print 'Our array is:'
print a
print '\n'

print 'Applying average() function:'
print np.average(a)
print '\n'

# this is same as mean when weight is not specified
wts=np.array([4,3,2,1])
print 'Applying average() function again:'
print np.average(a,weights=wts)
print '\n'

# Returns the sum of weights, if the returned parameter is set to True.
print 'Sum of weights'
print np.average([1,2,3, 4],weights=[4,3,2,1], returned=True)
```

It will produce the following output:

```
Our array is:
[1 2 3 4]

Applying average() function:
2.5
```

Applying average() function again:

2.0

Sum of weights

(2.0, 10.0)

In a multi-dimensional array, the axis for computation can be specified.

Example

```
import numpy as np
a = np.arange(6).reshape(3,2)
print 'Our array is:'
print a
print '\n'

print 'Modified array:'
wt=np.array([3,5])
print np.average(a, axis=1, weights=wt)
print '\n'

print 'Modified array:'
print np.average(a, axis=1, weights=wt, returned=True)
```

It will produce the following output:

```
Our array is:
[[0 1]
 [2 3]
 [4 5]]

Modified array:
[ 0.625  2.625  4.625]

Modified array:
(array([ 0.625,  2.625,  4.625]), array([ 8.,  8.,  8.]))
```

Standard Deviation

Standard deviation is the square root of the average of squared deviations from mean. The formula for standard deviation is as follows:

```
std = sqrt(mean(abs(x - x.mean())**2))
```

If the array is [1, 2, 3, 4], then its mean is 2.5. Hence the squared deviations are [2.25, 0.25, 0.25, 2.25] and the square root of its mean divided by 4, i.e., $\sqrt{5/4}$ is 1.1180339887498949.

Example

```
import numpy as np
print np.std([1,2,3,4])
```

It will produce the following output:

```
1.1180339887498949
```

Variance

Variance is the average of squared deviations, i.e., **`mean(abs(x - x.mean())**2)`**. In other words, the standard deviation is the square root of variance.

Example

```
import numpy as np
print np.var([1,2,3,4])
```

It will produce the following output:

```
1.25
```

19. NumPy – Sort, Search & Counting Functions

A variety of sorting related functions are available in NumPy. These sorting functions implement different sorting algorithms, each of them characterized by the speed of execution, worst case performance, the workspace required and the stability of algorithms. Following table shows the comparison of three sorting algorithms.

kind	speed	worst case	work space	stable
'quicksort'	1	$O(n^2)$	0	no
'mergesort'	2	$O(n \log(n))$	$\sim n/2$	yes
'heapsort'	3	$O(n \log(n))$	0	no

numpy.sort()

The sort() function returns a sorted copy of the input array. It has the following parameters:

```
numpy.sort(a, axis, kind, order)
```

Where,

a	Array to be sorted
axis	The axis along which the array is to be sorted. If none, the array is flattened, sorting on the last axis
kind	Default is quicksort
order	If the array contains fields, the order of fields to be sorted

Example

```
import numpy as np
a = np.array([[3,7],[9,1]])
print 'Our array is:'
print a
print '\n'
```

```

print 'Applying sort() function:'
print np.sort(a)
print '\n'

print 'Sort along axis 0:'
print np.sort(a, axis=0)
print '\n'

# Order parameter in sort function
dt = np.dtype([('name', 'S10'),('age', int)])
a = np.array([("raju",21),("anil",25),("ravi", 17), ("amar",27)], dtype=dt)
print 'Our array is:'
print a
print '\n'

print 'Order by name:'
print np.sort(a, order='name')

```

It will produce the following output:

```

Our array is:
[[3 7]
 [9 1]]

Applying sort() function:
[[3 7]
 [1 9]]

Sort along axis 0:
[[3 1]
 [9 7]]

Our array is:
[('raju', 21) ('anil', 25) ('ravi', 17) ('amar', 27)]

Order by name:
[('amar', 27) ('anil', 25) ('raju', 21) ('ravi', 17)]

```


numpy.argsort()

The **numpy.argsort()** function performs an indirect sort on input array, along the given axis and using a specified kind of sort to return the array of indices of data. This indices array is used to construct the sorted array.

Example

```
import numpy as np
x = np.array([3, 1, 2])
print 'Our array is:'
print x
print '\n'

print 'Applying argsort() to x:'
y = np.argsort(x)
print y
print '\n'

print 'Reconstruct original array in sorted order:'
print x[y]
print '\n'

print 'Reconstruct the original array using loop:'
for i in y:
    print x[i],
```

It will produce the following output:

```
Our array is:
[3 1 2]

Applying argsort() to x:
[1 2 0]

Reconstruct original array in sorted order:
[1 2 3]

Reconstruct the original array using loop:
1 2 3
```

numpy.lexsort()

function performs an indirect sort using a sequence of keys. The keys can be seen as a column in a spreadsheet. The function returns an array of indices, using which the sorted data can be obtained. Note, that the last key happens to be the primary key of sort.

Example

```
import numpy as np
nm = ('raju','anil','ravi','amar')
dv = ('f.y.', 's.y.', 's.y.', 'f.y.')

ind = np.lexsort((dv,nm))
print 'Applying lexsort() function:'
print ind
print '\n'

print 'Use this index to get sorted data:'
print [nm[i] + ", " + dv[i] for i in ind]
```

It will produce the following output:

```
Applying lexsort() function:
[3 1 0 2]

Use this index to get sorted data:
['amar, f.y.', 'anil, s.y.', 'raju, f.y.', 'ravi, s.y.']
```

NumPy module has a number of functions for searching inside an array. Functions for finding the maximum, the minimum as well as the elements satisfying a given condition are available.

numpy.argmax() and numpy.argmin()

These two functions return the indices of maximum and minimum elements respectively along the given axis.

Example

```
import numpy as np
a = np.array([[30,40,70],[80,20,10],[50,90,60]])
print 'Our array is:'
print a
print '\n'
```

```
print 'Applying argmax() function:'
print np.argmax(a)
print '\n'

print 'Index of maximum number in flattened array'
print a.flatten()
print '\n'

print 'Array containing indices of maximum along axis 0:'
maxindex = np.argmax(a, axis=0)
print maxindex
print '\n'

print 'Array containing indices of maximum along axis 1:'
maxindex=np.argmax(a, axis=1)
print maxindex
print '\n'

print 'Applying argmin() function:'
minindex=np.argmin(a)
print minindex
print '\n'

print 'Flattened array:'
print a.flatten()[minindex]
print '\n'

print 'Flattened array along axis 0:'
minindex = np.argmin(a, axis=0)
print minindex
print '\n'

print 'Flattened array along axis 1:'
minindex = np.argmin(a, axis=1)
print minindex
```

It will produce the following output:

```
Our array is:
[[30 40 70]
 [80 20 10]
 [50 90 60]]

Applying argmax() function:
7

Index of maximum number in flattened array
[30 40 70 80 20 10 50 90 60]

Array containing indices of maximum along axis 0:
[1 2 0]

Array containing indices of maximum along axis 1:
[2 0 1]

Applying argmin() function:
5

Flattened array:
10

Flattened array along axis 0:
[0 1 1]

Flattened array along axis 1:
[0 2 0]
```

numpy.nonzero()

The **numpy.nonzero()** function returns the indices of non-zero elements in the input array.

Example

```
import numpy as np
a = np.array([[30,40,0],[0,20,10],[50,0,60]])
print 'Our array is:'
print a
print '\n'

print 'Applying nonzero() function:'
print np.nonzero (a)
```

It will produce the following output:

```
Our array is:
[[30 40  0]
 [ 0 20 10]
 [50  0 60]]

Applying nonzero() function:
(array([0, 0, 1, 1, 2, 2]), array([0, 1, 1, 2, 0, 2]))
```

numpy.where()

The where() function returns the indices of elements in an input array where the given condition is satisfied.

Example

```
import numpy as np
x = np.arange(9.).reshape(3, 3)
print 'Our array is:'
print x

print 'Indices of elements > 3'
y=np.where(x>3)
print y

print 'Use these indices to get elements satisfying the condition'
print x[y]
```

It will produce the following output:

Our array is:

```
[[ 0.  1.  2.]
 [ 3.  4.  5.]
 [ 6.  7.  8.]]
```

Indices of elements > 3

```
(array([1, 1, 2, 2, 2]), array([1, 2, 0, 1, 2]))
```

Use these indices to get elements satisfying the condition

```
[ 4.  5.  6.  7.  8.]
```

numpy.extract()

The **extract()** function returns the elements satisfying any condition.

```
import numpy as np
x = np.arange(9.).reshape(3, 3)
print 'Our array is:'
print x

# define a condition
condition = np.mod(x,2)==0
print 'Element-wise value of condition'
print condition

print 'Extract elements using condition'
print np.extract(condition, x)
```

It will produce the following output:

Our array is:

```
[[ 0.  1.  2.]
 [ 3.  4.  5.]
 [ 6.  7.  8.]]
```

Element-wise value of condition

```
[[ True False  True]
 [False  True  False]
 [ True False  True]]
```

Extract elements using condition

```
[ 0.  2.  4.  6.  8.]
```

23. NumPy – Linear Algebra

NumPy package contains **numpy.linalg** module that provides all the functionality required for linear algebra. Some of the important functions in this module are described in the following table.

dot	Dot product of the two arrays
vdot	Dot product of the two vectors
inner	Inner product of the two arrays
matmul	Matrix product of the two arrays
det	Computes the determinant of the array
solve	Solves the linear matrix equation
inv	Finds the multiplicative inverse of the matrix

numpy.dot()

This function returns the dot product of two arrays. For 2-D vectors, it is the equivalent to matrix multiplication. For 1-D arrays, it is the inner product of the vectors. For N-dimensional arrays, it is a sum product over the **last axis of a** and the **second-last axis of b**.

```
import numpy.matlib
import numpy as np
a = np.array([[1,2],[3,4]])
b = np.array([[11,12],[13,14]])
np.dot(a,b)
```

It will produce the following output:

```
[[37 40]
 [85 92]]
```

Note that the dot product is calculated as:

```
[[1*11+2*13, 1*12+2*14],[3*11+4*13, 3*12+4*14]]
```

numpy.vdot()

This function returns the dot product of the two vectors. If the first argument is complex, then its conjugate is used for calculation. If the argument is multi-dimensional array, it is flattened.

Example

```
import numpy as np
a = np.array([[1,2],[3,4]])
b = np.array([[11,12],[13,14]])
print np.vdot(a,b)
```

It will produce the following output:

```
130
```

Note: $1*11 + 2*12 + 3*13 + 4*14 = 130$

numpy.inner()

This function returns the inner product of vectors for 1-D arrays. For higher dimensions, it returns the sum product over the last axes.

Example

```
import numpy as np
print np.inner(np.array([1,2,3]),np.array([0,1,0]))
# Equates to 1*0+2*1+3*0
```

It will produce the following output:

```
2
```

Example

```
# Multi-dimensional array example
import numpy as np
a = np.array([[1,2], [3,4]])
print 'Array a:'
print a
b = np.array([[11, 12], [13, 14]])
print 'Array b:'
print b
print 'Inner product:'
print np.inner(a,b)
```

It will produce the following output:

Array a:

```
[[1 2]
 [3 4]]
```

Array b:

```
[[11 12]
 [13 14]]
```

Inner product:

```
[[35 41]
 [81 95]]
```

In the above case, the inner product is calculated as:

```
1*11+2*12, 1*13+2*14
3*11+4*12, 3*13+4*14
```

numpy.matmul()

The **numpy.matmul()** function returns the matrix product of two arrays. While it returns a normal product for 2-D arrays, if dimensions of either argument is >2 , it is treated as a stack of matrices residing in the last two indexes and is broadcast accordingly.

On the other hand, if either argument is 1-D array, it is promoted to a matrix by appending a 1 to its dimension, which is removed after multiplication.

Example

```
# For 2-D array, it is matrix multiplication
import numpy.matlib
import numpy as np
a = [[1,0],[0,1]]
b = [[4,1],[2,2]]
print np.matmul(a,b)
```

It will produce the following output:

```
[[4 1]
 [2 2]]
```

Example

```
# 2-D mixed with 1-D
import numpy.matlib
import numpy as np
a=[[1,0],[0,1]]
b=[1,2]
print np.matmul(a,b)
print np.matmul(b,a)
```

It will produce the following output:

```
[1  2]
[1  2]
```

Example

```
# one array having dimensions > 2
import numpy.matlib
import numpy as np
a=np.arange(8).reshape(2,2,2)
b=np.arange(4).reshape(2,2)
print np.matmul(a,b)
```

It will produce the following output:

```
[[[2  3]
  [6 11]]
 [[10 19]
  [14 27]]]
```

Determinant

Determinant is a very useful value in linear algebra. It is calculated from the diagonal elements of a square matrix. For a 2x2 matrix, it is simply the subtraction of the product of the top left and bottom right element from the product of other two.

In other words, for a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is computed as $'ad-bc'$. The larger square matrices are considered to be a combination of 2x2 matrices.

The **numpy.linalg.det()** function calculates the determinant of the input matrix.

```
import numpy as np
a = np.array([[1,2], [3,4]])
```

```
print np.linalg.det(a)
```

It will produce the following output:

```
-2.0
```

Example

```
import numpy as np
b = np.array([[6,1,1], [4, -2, 5], [2,8,7]])
print b
print np.linalg.det(b)
print 6*(-2*7 - 5*8) - 1*(4*7 - 5*2) + 1*(4*8 - -2*2)
```

It will produce the following output:

```
[[ 6  1  1]
 [ 4 -2  5]
 [ 2  8  7]]

-306.0

-306
```

numpy.linalg.solve()

The **numpy.linalg.solve()** function gives the solution of linear equations in the matrix form.

Considering the following linear equations:

$$x + y + z = 6$$

$$2y + 5z = -4$$

$$2x + 5y - z = 27$$

They can be represented in the matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix}$$

If these three matrices are called **A**, **X** and **B**, the equation becomes:

$$A \cdot X = B$$

Or

$$X = A^{-1}B$$

We use **numpy.linalg.inv()** function to calculate the inverse of a matrix. The inverse of a matrix is such that if it is multiplied by the original matrix, it results in identity matrix.

Example

```
import numpy as np
x = np.array([[1,2],[3,4]])
y = np.linalg.inv(x)
print x
print y
print np.dot(x,y)
```

It should produce the following output:

```
[[ 1  0]
 [ 0  1]]
```

Example

Let us now create an inverse of matrix A in our example.

```
import numpy as np
a = np.array([[1,1,1],[0,2,5],[2,5,-1]])
print 'Array a:'
print a
ainv=np.linalg.inv(a)
print 'Inverse of a:'
print ainv

print 'Matrix B is:'
b = np.array([[6],[-4],[27]])
print b

print 'Compute A-1B:'
```

```
x = np.linalg.solve(a,b)
print x

# this is the solution to linear equations x=5, y=3, z=-2
```

It will produce the following output:

```
Array a:
[[ 1  1  1]
 [ 0  2  5]
 [ 2  5 -1]]

Inverse of a:
[[ 1.28571429 -0.28571429 -0.14285714]
 [-0.47619048  0.14285714  0.23809524]
 [ 0.19047619  0.14285714 -0.0952381  ]]

Matrix B is:
[[ 6]
 [-4]
 [27]]

Compute A-1B:
[[ 5.]
 [ 3.]
 [-2.]]
```

The same result can be obtained by using the function:

```
x = np.dot(ainv,b)
```