PHYS 259 '07

We'll only do Fabry-Perot portion

Laboratory 8
Interferometers

Physics ~~281~~
~~November~~ 6 and 13, 2006

Depending on the level of completeness with which the experiments are performed, this laboratory will probably require two weeks, one for each instrument.

Objectives:

1. To gain a basic familiarity with Fabry-Perot and Michelson interferometers.

2. Use a Fabry-Perot etalon to measure the fine-structure splitting of the Balmer α line of atomic deuterium, as well as the H-D isotope shift.

3. Use a Michelson interferometer to determine an unknown laser wavelength in terms of the known wavelength of a He-Ne laser, and to determine the refractive index of a glass plate.

Equipment:

1. He-Ne and diode lasers, and diverging lenses.
2. Michelson interferometers.
4. Hydrogen and deuterium discharge lamps.
5. Solid fused silica étalons (*EXPENSIVE: do not touch the surfaces*!)
6. White screens for viewing interference fringes.
7. Firewire cameras and NI-Vision software for acquiring and analyzing etalon fringes.

Theory

A. Michelson interferometer

The Michelson interferometer is in a sense the simplest two-beam interferometer, and operates by dividing a single input beam into two parts, passing them along separate paths, then recombining them on the same beamsplitter used to separate the incident beam. The basic layout is shown in Fig. 1. instrument is described thoroughly in all of the common optics texts, so a full account is not repeated here (see, for example, Fig. 8-1 of Pedrotti[3] and p. 408 of Hecht, 4th Edition). Each path contains a retroreflecting mirror, one fixed and the other adjustable to vary the length of the corresponding path. Since the exit beam is at a right angle to the incident beam, the interference pattern is easily viewed by eye or on a screen.

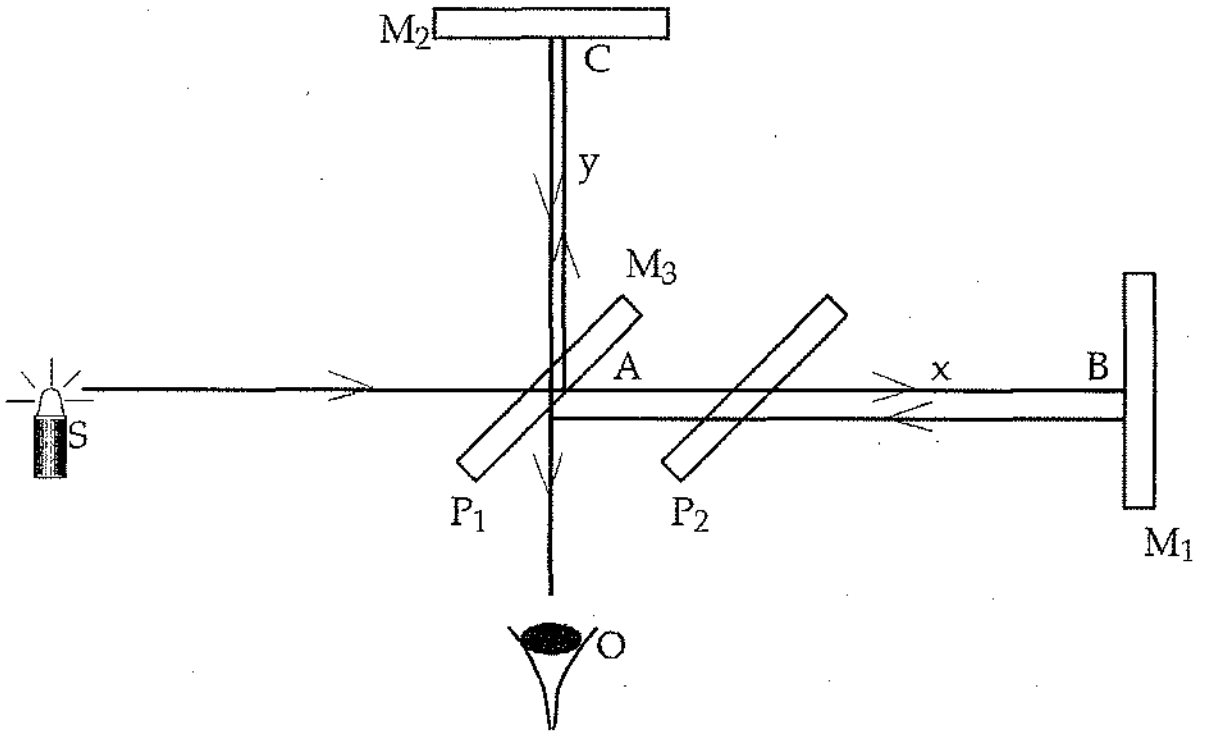

Fig. 1. Michelson interferometer layout, from a Physics 233 lab handout at www.sfu.edu.

If the source is divergent, either because it is spatially extended or because a lens has been used to spread it, the resulting interference pattern is a series of concentric circles. If the difference between path lengths 1 and 2 is d, the dark fringes occur when

$$2d\cos\theta = m\lambda, \text{ where } m=0,1,2,\cdots. \quad (1)$$

Note that the central fringe with d=0 is dark because one beam encounters two external reflections but the other only one, so there's a net phase shift of π. Also, when d is very small the angular separation between fringes becomes large, so the circles become extremely large when the path lengths are nearly equal. For conventional light sources like mercury or sodium lamps it's necessary to operate fairly close to d=0 because the *coherence length* is limited. The source has a finite spectral width, and if the path length difference includes too many wavelengths, the interference pattern washes out. With laser sources this ordinarily is not a significant constraint—the frequency is well-defined to about 1 part in 10^6, so the path length difference has to approach 10^6 wavelengths before you're in trouble.

B. Fabry-Perot interferometer, or étalon

The Fabry-Perot interferometer differs from the Michelson design in making use of multiple-beam interference, giving the possibility of far higher spectral resolution. The instrument consists of nothing more than two parallel, highly reflective planes. An easy way to make one is to coat both sides of a high-quality optical flat with a reflective multi-layer intererence coating. This version is called a solid étalon (often spelled without the accent mark), and will be used for today's laboratory. For a collimated incident beam, all of the back-and-forth reflections interfere constructively when an integral number of half-wavelengths fit in the cavity, giving a transmitted

beam. For any wavelength not satisfying this condition, destructive interference cancels the transmission partially or completely.

When viewing an extended source with a range of incident angles, the interference pattern consists of concentric circles just as for the Michelson interferometer. This time there is no relative phase shift of π, so the same formula that gives the dark fringes for the Michelson interferometer *now describes the bright* fringes,

$$2nd\cos\phi = m\lambda, \text{ where } m \text{ is the integer } \textit{order number}. \tag{2}$$

Here λ is the wavelength in vacuum and ϕ is the angle inside the etalon. The corresponding laboratory angle θ is the incidence angle when viewing a point source, or the angle subtended at the detector when viewing an extended source. It is related to ϕ by Snell's law,

$$\sin\theta = n\sin\phi. \tag{3}$$

For an ideal etalon with uniform illumination, the fringes are described quantitatively by an Airy function (see pp. 420 and 423 of Hecht),

$$\frac{I_t}{I_i} = \frac{1}{1+F\sin^2(\delta/2)}, \text{ where } F \equiv \frac{4R}{(1-R)^2} \text{ and } \delta \equiv \frac{4\pi nd}{\lambda}\cos\phi. \tag{4}$$

The etalons available to us are made of fused silica, with reflectance R=0.95 and refractive index n=1.4570 at 632 nm and 1.4564 at 656 nm (obviously it changes little with wavelength in this range). The thickness d is 3 mm. The sharpness of the interference pattern and thus the wavelength resolution of the instrument is quantified by the *finesse*, the ratio of the spacing between fringes to the width of a fringe. Because the Fabry-Perot uses multiple reflections its finesse can be quite large. In the best instruments it can exceed 50,000. In ours it's determined by the R=95% reflectance of the coatings and by imperfections in the surface flatness. A simple calculation shows that if the only limit is the reflectance, the finesse is given by

$$F = \frac{\pi\sqrt{R}}{1-R}. \tag{5}$$

In part II of this laboratory you will use an etalon to make a remarkably precise determination of the very small fine structure of atomic deuterium, as well as the slight isotope shift between the Balmer-α lines of atomic hydrogen and deuterium. While Eqs. 2 and 4 contain all of the basic physics, they are not directly usable because we don't know the absolute order number m for any of the fringes. Noting that the value of m decreases when moving away from the center of the pattern, it makes sense to label the innermost fringe as m_{max}, as shown in Fig. 2.

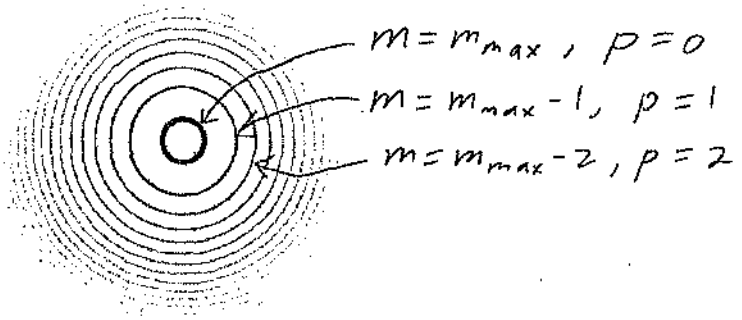

Fig. 2. Fabry-Perot fringes from a He-Ne laser (actual photograph from the Physics 281 lab), labeled with relative index p.

The figure also defines a new index p, used to label the fringes in the obvious way from the center out. It is related to the order m by

$$p = m_{\max} - m. \tag{6}$$

The angles ϕ and θ are always small in practice, so it is quite accurate to use small-angle approximations in Eqs. 2 and 3. Doing so, we get the dependence of the order m on the laboratory angle θ,

$$2nd\left[1 - \frac{\theta^2}{2n^2}\right] = m\lambda. \tag{7}$$

Since m is related to p by a constant and a minus sign, it has the opposite slope. Thus a plot of the observed fringe index p against θ^2 will look something like this:

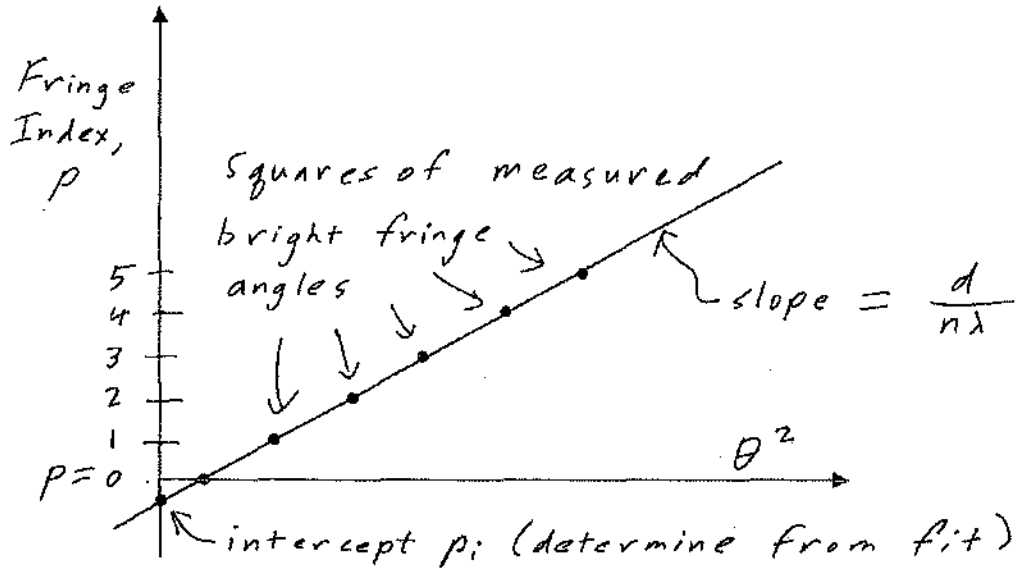

Fig. 3. Plot of fringe index p against θ^2.

The intercept p_i of this plot is particularly useful. It will be a negative number between 0 and -1. Using Eqs. 2 and 6, it is related to the wavelength by

$$\frac{1}{\lambda} = \frac{1}{2nd}\left(m_{\max} - p_i\right). \tag{8}$$

If the difference between two wavelengths λ_1 and λ_2, is needed, as in part II of this laboratory, it can be found from the intercepts p_{i1} and p_{i2} for the two interference patterns. In practice it's usually easier to work with the reciprocal of the wavelength, since the mathematical expressions are simpler. For example, for the transition wavelengths λ_D and λ_H of the Balmer α line in hydrogen and deuterium,

$$\frac{1}{\lambda_D} - \frac{1}{\lambda_H} = \frac{1}{2nd}\left[p_{iH} - p_{iD} + q\right]. \tag{9}$$

The integer q is needed because we don't know whether the value of m_{max} is the same for both wavelengths. The fringe pattern repeats itself whenever the optical frequency changes by a multiple of the *free spectral range*,

$$FSR = \frac{c}{2nd}. \tag{10}$$

To say it another way, there is no direct way to tell whether the difference between two interference rings is 0.3 fringes or 12.3 fringes; the appearance is the same. In principle the slope of p vs. θ^2 can be used to find the thickness exactly, and thus to find the exact order numbers, but in practice this is an exacting exercise because the order number m is nearly 10,000. In part II of this laboratory we will take advantage of the fact that we already know the approximate answer to eliminate all possible values of q in Eq. 9 except for the one that gives a physically reasonable answer.

C. The Balmer lines of the atomic hydrogen spectrum

According to the Bohr model, the energy levels of atomic hydrogen should be given by the simple formula,

$$E_n = \frac{Ry}{n^2}. \tag{10}$$

Here Ry is the Rydberg constant. For this lab, and for most spectroscopic work, it is easiest to measure spectral intervals in terms of the reciprocal of the vacuum wavelength; this is related to the energy levels by factors of h and c. In these unusual units of "energy" the Rydberg constant is given by Ry = 109737.3157 cm^{-1} . The Balmer α transition is the red line of the hydrogen spectrum corresponding to the transition from n=3 to 2, so it has a reciprocal wavelength of $1/\lambda$ = 5/36 Ry. Several corrections affect this simple picture. The biggest, the reduced mass correction, exists even in semiclassical physics. If you take into account the finite mass of the proton, the Rydberg constant must be corrected by the reduced mass of the system according to the formula,

$$Ry = R_\infty \left[\frac{1}{1 + m_e / m_{nucleus}} \right]. \tag{11}$$

All of the energy levels are therefore shifted by amounts that depend on the nuclear mass and the value of n. For hydrogen the corrected Rydberg constant is 109677.583 cm^{-1} , while for deuterium, with an extra neutron in the nucleus, it is 109707.427 cm^{-1} . There is thus a small difference in the wavelength of the Balmer line that depends on the isotope. For D the reciprocal wavelength is 15237.1426 cm^{-1} , and for H it is 15232.9977 cm^{-1} , so the isotope shift is predicted semiclassically to be 4.145 cm^{-1} . There are small quantum mechanical corrections to this result, but they only enter at the 4th decimal place in the shift.

In addition, the n=2 state has a small *fine structure* splitting, both in hydrogen and deuterium, due to the magnetic interaction of the electron spin with the orbital magnetic moment. There are two of these fine-structure levels, since quantum mechanically the electron spin can be oriented either up or down. Thus the Balmer α line is actually a closely spaced *doublet*, with a splitting of 0.366 cm^{-1}, both for hydrogen and for deuterium.

(A few parenthetical details: Actually, there are more than just two components, because transitions can occur both to the 2S state, with orbital angular momentum L=0, and the 2P state, with L=1. However, this is inconsequential because the 2S state has almost exactly the same energy as the lower component of the 2P state. Further, the n=3 state has analogous internal splittings for both fine structure and L, but they are too small to affect our experiments.)

In a discharge lamp, the fine-structure doublet is partially obscured by the *Doppler broadening* that occurs because in a gas cell the atoms are moving randomly, so there is a considerable range of different Doppler shifts for different atoms. This limits the spectral resolution to a level well below the intrinsic resolution of the Fabry-Perot etalon. At room temperature the Doppler-broadened lineshape for hydrogen is a gaussian distribution with a full width at half maximum of 0.19 cm^{-1}, and for deuterium it is 0.096 cm^{-1}. In the discharge these widths are increased further due to heating and collisional effects, so the fine structure can be clearly observed only for the case of deuterium.

I. Interferometric length, wavelength, and refractive index determinations with a Michelson interferometer

A. Setup and measurement of angles for real fringes from a diverging laser beam

To start, set up a Michelson interferometer with a helium-neon laser, diverged through a microscope objective or lens with a focal length of about $f = 1$ to 2 cm. Align the instrument until a clear pattern of interference rings can be seen on a screen about 1 m away. Set the instrument for a path length difference of d=0 by observing the size of the fringes. Now adjust it to $d = 5$ mm, and measure the angles θ_i of the central three or four fringes. Compare these observed spacings with the predicted functional form in Eq. 1. You can use the same procedure as for the Fabry-Perot etalon (see Fig. 3), if you keep in mind that Eq. 1 describes the nulls, not the bright fringes—the bright rings occur at m=1/2, 3/2, 5/2, etc.

Because the fringes observed on the screen are real fringes resulting from a point source, to determine the angles θ_i you will need to measure the distance of the viewing screen from this source, which is located one focal length beyond the lens (assuming a positive lens is used). The down-and-back path length of the interferometer should be included in this distance.

B. Comparing an unknown wavelength with a He-Ne laser

The idea here is simply to count the large number of interference fringes that one sweeps through in moving between two points on the distance scale of the instrument. This can be used to calibrate the distance scale in terms of the He-Ne laser wavelength λ=632.8164 nm, and the calibration can subsequently be used to make additional measurements.

Start counting fringes with the path length difference d adjusted far enough from d=0 that the fringes are easily viewed. All mechanical instruments exhibit *hysteresis*, or backlash, in the threaded motions, so you should decide to consistently take all of your measurements moving either clockwise or counterclockwise. If you move in the other direction even briefly, you must move a considerable distance to overshoot the desired position, then come back to it in the chosen direction to remove the backlash. Read the starting position accurately, and note the state of the central fringe so you can end the measurement at the same state. Carefully scan the interferometer while counting fringes, for at least 100 fringes and preferably more, then record the stopping position. A photodiode and frequency counter can be put to good use to make this task eas-

ier, allowing the measurement of many hundreds of fringes in just a few minutes. You can now calibrate the micrometer scale by determining the distance traveled from your fringe count, since from Eq. 1 above, if Δm is the number of fringes counted,

$$\lambda \Delta m = 2\Delta d \,. \tag{12}$$

Be cautioned that some instruments have a periodic error associated with pitch irregularities in the drive screw; you may want to try to go through a whole number of revolutions to minimize this problem.

Now that you have calibrated the distance scale in terms of the known He-Ne laser wavelength, you can use the instrument measure an unknown wavelength, in this case a diode laser. Proceed by replacing the He-Ne laser with a diode laser. Then measure the fringes just as before, but reverse the analysis: use the known calibration of the micrometer dial to determine the unknown wavelength of the laser. Be sure to make an estimate of the uncertainty of your determination.

C. Index of refraction of a thin glass plate

Place a thin glass plate in one arm of the Michelson interferometer, mounting it on a good rotation stage so that its angle can accurately be measured. Set it perpendicular to the laser path by observing how the interference fringes move as the plate is rotated. Now rotate the plate through a known angle while counting fringes. Finally, measure its thickness with a micrometer, and use your results to calculate the index of refraction n.

D. *Optional*: other stuff

The experiments described above all use real fringes from a point source. Although this works very well with a laser source, it is actually not the classic configuration of the Michelson interferometer. Try placing a ground-glass screen at the entrance to the instrument. If the laser is sufficiently spread out on the screen, it will act as a classical extended source. You should now be able to view virtual fringes by looking into the exit of the interferometer, imaged at $-\infty$. Some realignment of the mirrors may be required, depending on how the laser was originally aimed.

Now that the interferometer is set up for an extended source, try placing a sodium lamp or a mercury lamp with a uv-blocking filter in front of the ground-glass screen. You should still be able to see fringes, and if time permits you can measure the wavelength by counting fringes, just as in Part B above.

If time permits and the equipment is available, consider adding a gas cell to one arm of your interferometer, and using it to measure the *index of refraction of air*, by pumping out the air and then slowly re-admitting it while counting the fringes. Be sure to measure the length of the cell, since the optical phase delay caused by the air-filled cell is $\delta = n_{\text{air}}L$. Use this information to find the index of refraction, n_{air}.

II. Measuring deuterium fine structure and the H-D isotope shift with an etalon.

A. Setting up the Fabry-Perot etalon with a He-Ne laser

Because it's easiest to work with an extended light source, set up a pair of ground-glass screens to diffuse the light from a He-Ne laser. This will produce a virtual interference pattern, in the form of well-defined rings at $-\infty$. As shown in the figure below, place the etalon in the beam path, followed by a 630 nm long-pass filter and a firewire camera with a lens. You can also view the interference rings by eye, but you will want the camera for the actual measurements. If available, one of the ground-glass filters should be a wheel on a slowly turning motor—this will average out the effects of *laser speckle* due to diffraction of the laser light on the rough surfaces.

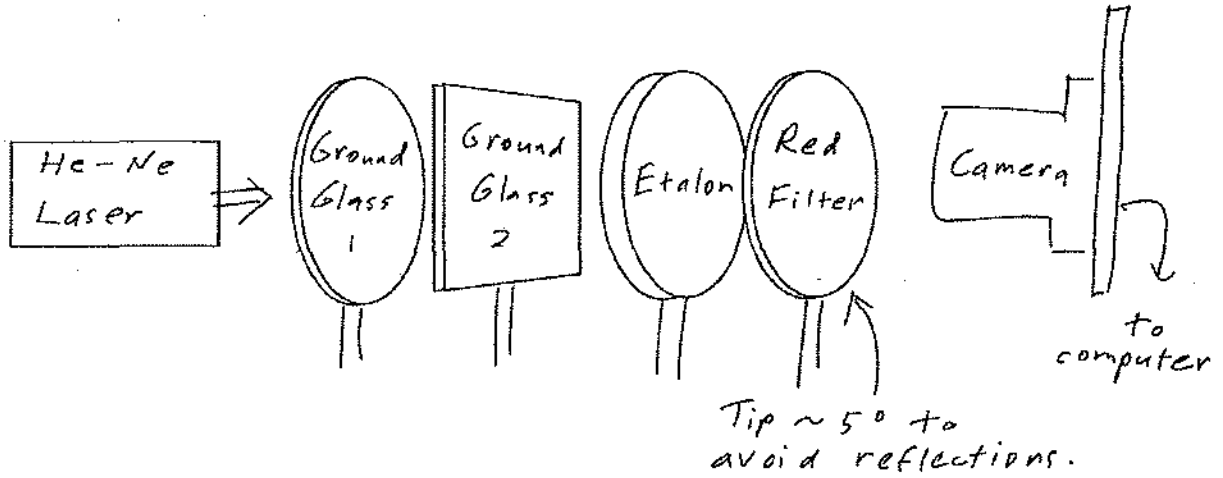

Fig. 4. Experimental layout for Fabry-Perot interferometer and He-Ne laser.

View your camera image in real time using the National Instruments Vision Assistant software. Focus your camera carefully on the interference rings, and take care to adjust the aperture and the shutter speed so as not to saturate the camera in the brightest regions. A false-color image is helpful—set the exposure so there are only a few narrow regions of red, which denotes the brightest range of pixel values just below saturation. Try moving the camera closer to the etalon, and further away from it. You will find that the size of the rings does not vary at all, and that they are always in focus—this verifies that they are characterized only by their angular extent, and that they are viewed at $-\infty$.

Now store a few images of the rings, and use the line-scan tool in Vision Assistant to take profiles through the centers of the interference patterns and store them in an Excel spreadsheet. Be sure that the line you draw is either exactly vertical or exactly horizontal, so that you only have to deal with one coordinate in your data file. You will use this intensity profile for all of the subsequent analysis.

To calibrate the angular magnification of the camera, aim it at a calibrated wall chart or ruler about 1-2 m away without making any changes to the focal length or focus of the lens. Save an image of the wall chart, again using the line profiling tool. Use a ruler to measure the spacing of the wall chart marks and the distance to the chart.

In your analysis, start with the calibration data. Calculate the angular separation of the chart marks and then use Excel or Mathcad to analyze the corresponding image. From the measured number of camera pixels between chart marks you can calibrate the angular magnification of the camera, in units of milliradians per pixel.

Using this calibration, analyze the best of your He-Ne laser fringe profiles. Determine the radii of each of the first five or six fringes from the line scan, and convert the results to angular measure. Index the fringes by labeling them p=0, 1, 2, and so forth as described in the theory section. If there's a bright fringe right in the center you won't be able to measure it accurately, so just ignore it and if the value of p_i in your fits is greater than 0, subtract 1 from it.

Make a plot of the fringe index p vs. θ^2, just as in Fig. 3 above. This plot should be a straight line. Calculate its slope and compare with the predicted value of $d/n\lambda$ for d=3 mm. Finally, use the intercept p_i from the plot to calculate a theoretical interference pattern that you can compare directly with your line scan, by using the Airy function of Eq. 4. You will probably find that the actual finesse is not as high as the calculated value, due to optical imperfections, camera limitations, and wavefront distortion. Try adjusting the reflectance R to obtain the best empirical fit to the shape of the first few interference fringes in your data.

B. Fine structure of the 2*p* state of deuterium

Replace the He-Ne laser and the pair of diffusers with a deuterium discharge lamp and a single piece of ground glass. Adjust the camera aperture and shutter speed to obtain a clear image of the interference rings, which are much dimmer than from the laser. Be sure to use a red filter to isolate the Balmer α line at 656 nm from the rest of the spectrum. Note that the finesse is degraded greatly by the Doppler width of the atomic spectrum, and that each ring is doubled because of the atomic fine structure.

Take an image and a line scan of the spectrum, just as in Part A above. Your camera calibration will be unchanged as long as you have not altered the lens focal length and focus. In your analysis, find the fringe intercept p_i for each of the two sets of rings, and use their difference, with Eq. 9, to measure the atomic fine-structure splitting in cm^{-1}. Because its expected size is only 0.366 cm^{-1}, less than the free spectral range $1/2nd$ = 1.144 cm^{-1} of the interferometer, the integer q will typically be zero, although it might be q=1 if the measurement happens to "wrap around" through zero: for example, $p_1 = -0.9$ and $p_2 = -0.21$ would yield a difference, after including q, of 0.31 fringes or 0.36 cm^{-1}.

C. The H-D isotope shift

To measure the shift between the two stable isotopes of hydrogen, start by retaking the deuterium data if more than a few minutes have elapsed, in case the etalon thickness has drifted slightly due to temperature changes. Now replace the deuterium discharge lamp with a hydrogen lamp and take a corresponding set of images and line scans for hydrogen. Note that the Doppler width is larger for hydrogen due to its lighter mass, so the fine structure is not as well resolved even though the splitting is the same.

Analyze each data set by plotting p vs. θ^2 and extrapolating to find the intercept p_i. Since the fine structure is not of interest for present purposes, just take an approximate average of the two components by measuring to the darkest area in between them.

To determine the isotope shift using Eq. 9, you need to know the number of integer order numbers q between the H and D results, since a shift of, say, 3.3 fringes looks the same as a shift of 0.3 fringes. Since you already know the approximate size of the isotope shift, the easiest course is to "bootstrap" by working backwards. Since the fringes occur every $1/2nd = 1.144$ cm^{-1}, the number of skipped orders q must be either 3 or 4. Try both values in Eq. 9 together with your values of p_{iH} and p_{iD} ; only one will give a reasonable result. You can thus use your knowledge of the approximate result to give a much more refined measurement, a standard technique in interferometry. If you have done a careful job, the isotope shift can be measured to significantly better than 1%; this means that using only a simple chunk of reflective glass, you have made a measurement accurate to 2 parts in 10^6 of the total transition wavelength!