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About Lock-In Amplifiers

Application Note #3

Lock-in amplifiers are used to detect and measure very small AC signals—all the way down to a few nanovolts. Accurate measurements may be made even when the small signal is obscured by noise sources many thousands of times larger. Lock-in amplifiers use a technique known as phase-sensitive detection to single out the component of the signal at a specific reference frequency and phase. Noise signals, at frequencies other than the reference frequency, are rejected and do not affect the measurement.

Why Use a Lock-In?

Let's consider an example. Suppose the signal is a 10 nV sine wave at 10 kHz. Clearly some amplification is required to bring the signal above the noise. A good low-noise amplifier will have about 5 nV/√Hz of input noise. If the amplifier bandwidth is 100 kHz and the gain is 1000, we can expect our output to be 10 µV of signal (10 nV × 1000) and 1.6 mV of broadband noise (5 nV/√Hz × √100 kHz × 1000). We won't have much luck measuring the output signal unless we single out the frequency of interest.

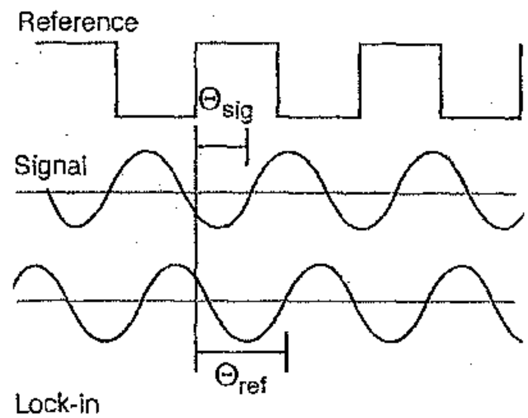
If we follow the amplifier with a band pass filter with a Q=100 (a VERY good filter) centered at 10 kHz, any signal in a 100 Hz bandwidth will be detected (10 kHz/Q). The noise in the filter pass band will be 50 µV (5 nV/√Hz × √100 Hz × 1000), and the signal will still be 10 µV. The output noise is much greater than the signal, and an accurate measurement can not be made. Further gain will not help the signal-to-noise problem.

Now try following the amplifier with a phase-sensitive detector (PSD). The PSD can detect the signal at 10 kHz with a bandwidth as narrow as 0.01 Hz! In this case, the noise in the detection bandwidth will be 0.5 µV (5 nV/√Hz × √.01 Hz × 1000), while the signal is still 10 µV. The signal-to-noise ratio is now 20, and an accurate measurement of the signal is possible.

What is Phase-Sensitive Detection?

Lock-in measurements require a frequency reference. Typically, an experiment is excited at a fixed frequency (from an oscillator or function generator), and the lock-in detects the response from the experiment at the reference frequency. In the following diagram, the reference signal is a square wave at frequency ω_r. This might be the sync output from a function generator. If the sine output from the function generator is used to excite the experiment, the response might be the signal waveform shown below. The signal is $V_{sig}\sin(\omega_r t + \theta_{sig})$ where V_{sig} is the signal amplitude, ω_r is the signal frequency, and θ_{sig} is the signal's phase.

Lock-in amplifiers generate their own internal reference signal usually by a phase-locked-loop locked to the external reference. In the diagram, the external reference, the lock-in's reference, and the signal are all shown. The internal reference is $V_L\sin(\omega_L t + \theta_{ref})$.



The lock-in amplifies the signal and then multiplies it by the lock-in reference using a phase-sensitive detector or multiplier. The output of the PSD is simply the product of two sine waves.

$$V_{psd} = V_{sig}V_L\sin(\omega_r t + \theta_{sig})\sin(\omega_L t + \theta_{ref})$$

$$= \tfrac{1}{2}V_{sig}V_L\cos([\omega_r - \omega_L]t + \theta_{sig} - \theta_{ref}) - \tfrac{1}{2}V_{sig}V_L\cos([\omega_r + \omega_L]t + \theta_{sig} + \theta_{ref})$$

The PSD output is two AC signals, one at the difference frequency $(\omega_r - \omega_L)$ and the other at the sum frequency $(\omega_r + \omega_L)$.

If the PSD output is passed through a low pass filter, the AC signals are removed. What will be left? In the general case, nothing. However, if ω_r equals ω_L, the difference frequency component will be a DC signal. In this case, the filtered PSD output will be:

$$V_{psd} = \tfrac{1}{2}V_{sig}V_L\cos(\theta_{sig} - \theta_{ref})$$

This is a very nice signal—it is a DC signal proportional to the signal amplitude.

It's important to consider the physical nature of this multiplication and filtering process in different types of lock-ins. In traditional analog lock-ins, the signal and reference are analog voltage signals. The signal and reference are multiplied in an analog multiplier, and the result is filtered with one or more stages of RC filters. In a digital lock-in, such as the SR830 or SR850, the signal and reference are represented by sequences of numbers. Multiplication and filtering are performed mathematically by a digital signal processing (DSP) chip. We'll discuss this in more detail later.

Narrow Band Detection

Let's return to our generic lock-in example. Suppose that instead of being a pure sine wave, the input is made up of signal plus noise. The PSD and low pass filter only detect signals whose frequencies are very close to the lock-in reference frequency. Noise signals, at frequencies far from the reference, are attenuated at the PSD output by the low pass filter (neither $\omega_{noise} - \omega_{ref}$ nor $\omega_{noise} + \omega_{ref}$ are close to DC). Noise at frequencies very close to the reference frequency will result in very low frequency AC outputs from the PSD ($|\omega_{noise} - \omega_{ref}|$ is small). Their attenuation depends upon the low pass filter bandwidth and rolloff. A narrower bandwidth will remove noise sources very close to the reference frequency; a wider bandwidth allows these signals to pass. The low pass filter bandwidth determines the bandwidth of detection. Only the signal at the reference frequency will result in a true DC output and be unaffected by the low pass filter. This is the signal we want to measure.

Where Does the Lock-In Reference Come From?

We need to make the lock-in reference the same as the signal frequency, i.e. $\omega_r = \omega_L$. Not only do the frequencies have to be the same, the phase between the signals can not change with time. Otherwise, $\cos(\theta_{sig} - \theta_{ref})$ will change and V_{psd} will not be a DC signal. In other words, the lock-in reference needs to be phase-locked to the signal reference.

Lock-in amplifiers use a phase-locked loop (PLL) to generate the reference signal. An external reference signal (in this case, the reference square wave) is provided to the lock-in. The PLL in the lock-in amplifier locks the internal reference oscillator to this external reference, resulting in a reference sine wave at ω_r with a fixed phase shift of θ_{ref}. Since the PLL actively tracks the external reference, changes in the external reference frequency do not affect the measurement.

Internal Reference Sources

In the case just discussed, the reference is provided by the excitation source (the function generator). This is called an external reference source. In many situations the lock-in's internal oscillator may be used instead. The internal oscillator is just like a function generator (with variable sine output and a TTL sync) which is always phase-locked to the reference oscillator.

Magnitude and Phase

Remember that the PSD output is proportional to $V_{sig}\cos\theta$, where $\theta = (\theta_{sig} - \theta_{ref})$. θ is the phase difference between the signal and the lock-in reference oscillator. By adjusting θ_{ref} we can make θ equal to zero. In which case we can measure V_{sig} ($\cos\theta = 1$). Conversely, if θ is 90°, there will be no output at all. A lock-in with a single PSD is called a single-phase lock-in and its output is $V_{sig}\cos\theta$.

This phase dependency can be eliminated by adding a second PSD. If the second PSD multiplies the signal with the reference oscillator shifted by 90°, i.e. $V_L\sin(\omega_L t + \theta_{ref} + 90°)$, its low pass filtered output will be:

$$V_{psd2} = \tfrac{1}{2} V_{sig} V_L \sin(\theta_{sig} - \theta_{ref})$$

$$V_{psd2} \sim V_{sig}\sin\theta$$

Now we have two outputs: one proportional to $\cos\theta$ and the other proportional to $\sin\theta$. If we call the first output X and the second Y,

$$X = V_{sig}\cos\theta \qquad Y = V_{sig}\sin\theta$$

these two quantities represent the signal as a vector relative to the lock-in reference oscillator. X is called the 'in-phase' component and Y the 'quadrature' component. This is because when $\theta = 0$, X measures the signal while Y is zero.

By computing the magnitude (R) of the signal vector, the phase dependency is removed.

$$R = (X^2 + Y^2)^{1/2} = V_{sig}$$

R measures the signal amplitude and does not depend upon the phase between the signal and lock-in reference.

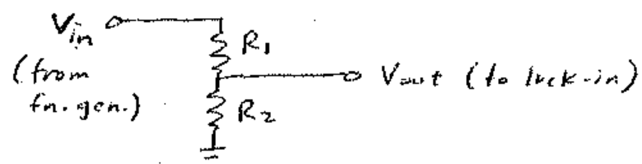
A dual-phase lock-in has two PSDs with reference oscillators 90° apart, and can measure X, Y and R directly. In addition, the phase (θ) between the signal and lock-in is defined as:

$$\theta = \tan^{-1}(Y/X)$$

The operating principle of the lock-in amplifier is discussed in the accompanying technical note. Here we describe what you will actually do in the lab. There are two basic experiments. The first is completely electronic while the second involves measuring an optical signal.

Electronic measurement

Here we demonstrate the ability of a lock-in amplifier to detect a very small electronic signal. You will use a function generator as the reference oscillator – it should be plugged into the "external reference" of the lock-in. Using the oscilloscope, set the frequency to approximately 1 kHz and the amplitude to 1 V. A highly attenuated version of this oscillating voltage will serve as the signal (recall that the signal must be synchronized with the reference). The attenuation will be achieved using a voltage divider, as shown below. This should be set up on one of the white "breadboards".



For $R_1 >> R_2$, the output voltage is related to the input voltage by:

$$V_{out} = \left(\frac{R_2}{R_1+R_2}\right) V_{in} \simeq \left(\frac{R_2}{R_1}\right) V_{in}$$

Choose a value for R_1 between 1 MΩ and 10 MΩ (use the color code, then verify your selection with the digital multimeter). For R_2, start with a value that will give an attenuation factor of roughly 1000. Plug the attenuated voltage into the signal input of the lock-in amplifier. Start with the least sensitive gain setting (all the way CW) and the phase set to zero. Look for a meter reading (which indicates the signal level) at phase settings of both 0° and 90°. If no signal is seen, increase the sensitivity and look again, using phase settings of both 0° and 90°. Repeat until a signal is seen. Now adjust the phase to optimize the signal. It is often easier to maximize the signal by finding the phase at which it goes to zero and then adding or subtracting 90°.

Now vary the phase over 360° in 10° steps (this won't be very precise with the old lock-in) and record the signal (including the sign) at each point. Compare your data to a sinusoidal variation.

Now increase the attenuation (in steps of 10) by using smaller values of R_2. You will need to increase the sensitivity as the signals become smaller. Optimize each signal using the phase adjustment. See how small a signal you can detect (a reasonable criterion for detecting a signal is if the meter moves noticeably when you change the phase by 90°). Record the signal levels and make a log-log plot of signal level vs. R_2. Are the results what you would expect?

Optical Measurement

Here you will use the lock-in amplifier to measure an optical signal which is synchronized to the reference voltage. Connect the function generator to the "external reference" and to the oscilloscope (for monitoring the frequency and amplitude). Also connect this oscillating voltage to a light-emitting diode (LED) in series with a 100 Ω current-limiting resistor. Set the frequency to approximately 1 kHz and the voltage amplitude to approximately 1 V. Now slowly increase the voltage amplitude up to 2 V. You should see a weak glow from the LED. Because the current limit of the LED is 20 mA, DO NOT EXCEED 2 V. Now place the photodiode about 1 cm away from the LED and connect to the input of the other channel of the oscilloscope. Since the photodiode detects light from the LED, you should see a small AC signal at the reference frequency. Note that it will not be sinusoidal due to the nonlinearity of the LED (recall the Planck's constant lab from PHYS 258). Now connect the photodiode to the signal input of the lock-in amplifier. Adjust the sensitivity and phase to optimize the signal. Now increase the distance d between the photodiode and LED. How small a signal can you detect? Record the signal level at 10 different distances and make a plot of signal vs. d^{-2}. Do your results make sense?