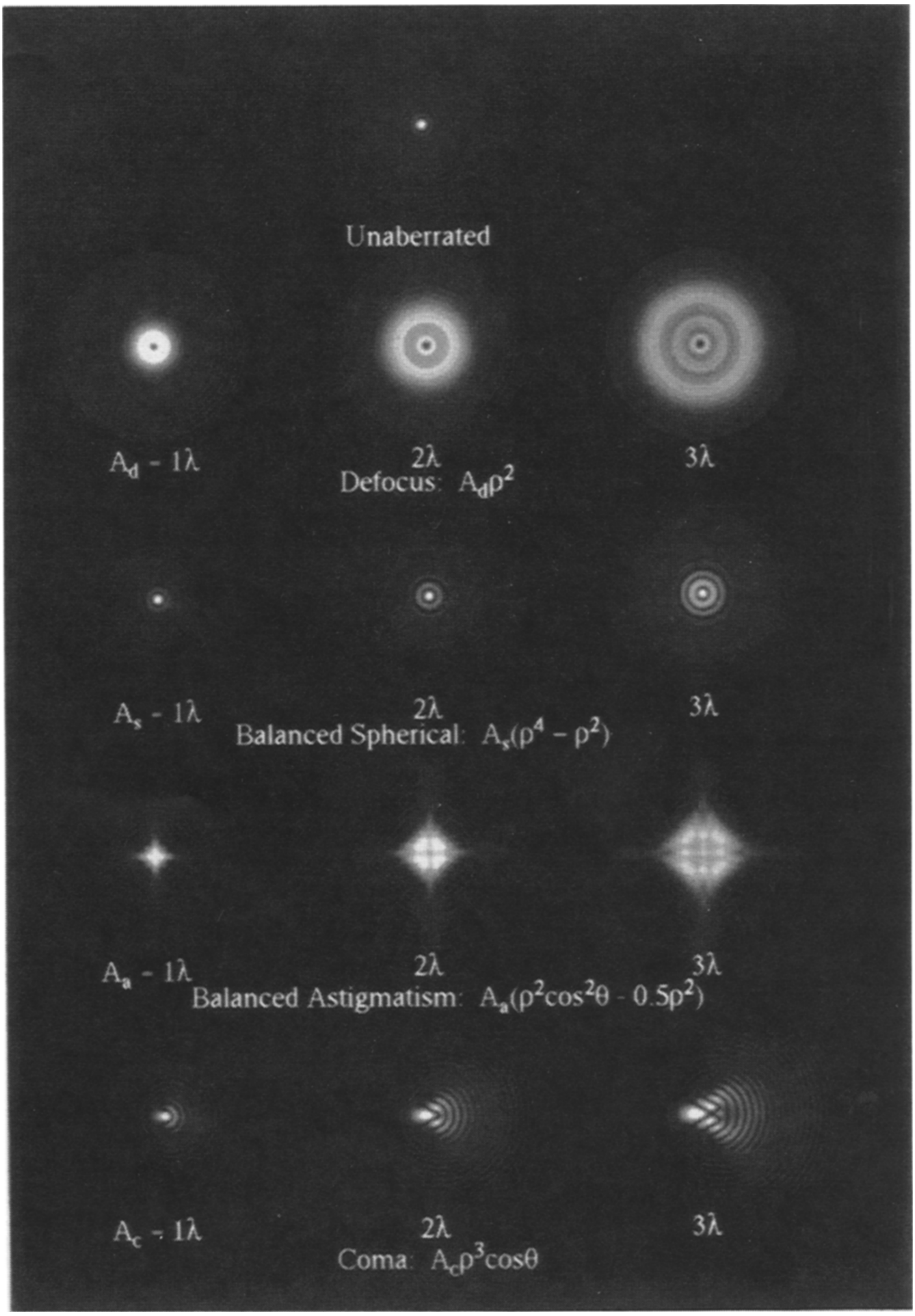
Aberration Theory Made Simple



Unaberrated and aberrated images of a point object as indicated. For details, see Sec. 8.4. Photo (this page and cover) by Dr. Richard Boucher, The Aerospace Corporation.

Aberration Theory Made Simple

Virendra N. Mahajan

The Aerospace Corporation and
University of Southern California

Donald C. O'Shea, Series Editor
Georgia Institute of Technology

TUTORIAL TEXTS IN OPTICAL ENGINEERING

Volume TT 6

SPIE OPTICAL ENGINEERING PRESS

A Publication of SPIE—The International Society for Optical Engineering
Bellingham, Washington USA

Library of Congress Cataloging-in-Publication Data

Mahajan, Virendra N.,
Aberration theory made simple / Virendra N. Mahajan
p. cm. — (Tutorial texts in optical engineering : v. TT 6)
Includes bibliographical references and index.
ISBN 0-8194-0536-1
1. Aberration. 2. Imaging Systems. 3. Optics, Geometrical.
I. Title. II. Series.
QC671.M34 1991
621.36—dc20 90-48633
CIP

Published by
SPIE—The International Society for Optical Engineering
P.O. Box 10
Bellingham, Washington 98227-0010

Printed in the United States of America

Fourth Printing

To My

Wife	**Shashi Prabha**
Son	**Vinit Bharati**
Daughter	**Sangita Bharati**

Books in the SPIE Tutorial Texts Series

Digital Image Compression Techniques
Majid Rabbani and Paul W. Jones (1991)

Aberration Theory Made Simple
Virendra N. Mahajan (1991)

Single Frequency Semiconductor Lasers
Jens Buus (1991)

An Introduction to Biological and Artificial Neural Networks for Pattern Recognition
Steven K. Rogers and Matthew Kabrisky (1991)

Laser Beam Propagation in the Atmosphere
Hugo Weichel (1990)

Infrared Fiber Optics
Paul Klocek and George H. Sigel, Jr. (1989)

Spectrally Selective Surfaces for Heating and Cooling Applications
C. G. Granqvist (1989)

Introduction to the Series

These Tutorial Texts provide an introduction to specific optical technologies for both professionals and students. Based on selected SPIE short courses, they are intended to be accessible to readers with a basic physics or engineering background. Each text presents the fundamental theory to build a basic understanding as well as the information necessary to give the reader practical working knowledge. The included references form an essential part of each text for the reader requiring a more in-depth study.

Many of the books in the series will be aimed to readers looking for a concise tutorial introduction to new technical fields, such as CCDs, fiber optic amplifiers, sensor fusion, computer vision, or neural networks, where there may be only limited introductory material. Still others will present topics in classical optics tailored to the interests of a specific audience such as mechanical or electrical engineers. In this respect the Tutorial Text serves the function of a textbook. With its focus on a specialized or advanced topic, the Tutorial Text may also serve as a monograph, although with a marked emphasis on fundamentals.

As the series develops, a broad spectrum of technical fields will be represented. One advantage of this series and a major factor in the planning of future titles is our ability to cover new fields as they are developing, giving people the basic knowledge necessary to understand and apply new technologies.

Donald C. O'Shea
Georgia Institute of Technology

June 1991

Foreword

It is a distinct pleasure for me to write this short foreword to Dr. Virendra Mahajan's tutorial text, *Aberration Theory Made Simple*. I write it not because I am particularly knowledgeable about aberration theory—in fact, it may be because I am not particularly knowledgeable that I was invited! This is a tutorial text, and as a lifelong educator I am also a lifelong learner and I should be able to learn from this text; and I did.

This text is prepared in the ideal way for a tutorial. It comes as a direct result of teaching this material to a wide range of audiences in a wide range of locations; so it has been tried and tested. The "student guinea pigs" have performed their invaluable service so that those of us who come along later have the benefit of their and the author's labors.

Dr. Mahajan has lived up to his title and made aberration theory simple. Of course, I should caution the reader that simple is relative. Some topics do not yield easily to simple yet accurate descriptions. Those readers who insist that "rays" are the most important components of any analysis of optical systems, whether aberrant or not, will be very satisfied with the first half of the book, but may wish to ignore the second half. They should not. Those who are enamored with the wave approach (like me) will immediately read the second half of this book and applaud, but not go back and read the first half. They should! I did!

I am pleased that Dr. Mahajan has provided a significant list of references in addition to the bibliography at the end of the book. This will be of considerable value to the reader. Not incidentally, SPIE Optical Engineering Press will also publish a Milestone volume on Aberrations in Imaging Systems with Dr. Mahajan as the co-editor. Thus, each of us will be able to have an authoritative companion volume that contains reprints from the world's literature that will no doubt verify that this current tutorial text is indeed *Aberration Theory Made Simple.*

Brian J. Thompson
Rochester, New York

June 1991

Table of Contents

PREFACE

Aberration theory is a subject that is as old and fascinating as the field of optics. It is, however, a cumbersome subject that many students of optics do not appreciate fully. The purpose of this tutorial book is to provide a clear, concise, and consistent exposition of what aberrations are, how they arise in optical imaging systems, and how they affect the quality of images formed by them. Its emphasis is on physical insight, problem solving, and numerical results. It is intended for engineers and scientists who have a need and/or a desire for a deeper and better understanding of aberrations and their role in optical imaging and wave propagation. Although some knowledge of Gaussian optics and an appreciation for aberrations would be useful, they are not prerequisites. What is needed is dedication and perseverance. A novice trying to learn this subject without investing much time will probably be disappointed in spite of the title of the book. The book is not intended for teaching lens design or optical testing. However, it is hoped that those working in these fields will benefit from it. It should be useful to students who may want to learn aberration theory without having to go through any lengthy derivations. These derivations are omitted out of necessity for brevity and in keeping with the spirit of these tutorials.

These tutorials have been adapted from my lectures for a graduate course entitled "Advanced Geometrical Optics," which I have been teaching in the Electrical Engineering–Electrophysics Department of the University of Southern California since 1984. They were originally developed for a short course on optical imaging and aberrations, which I taught at The Aerospace Corporation to Aerospace and Air Force personnel. They were then expanded for a short course I have been teaching at the Optical Society of America and SPIE meetings. Generally speaking, only the primary aberrations of optical systems are discussed here; they provide the first and a significant step beyond Gaussian imaging. Although a knowledge of these aberrations is very useful, they may not sufficiently describe the imaging properties of a high-quality optical system. Higher-order aberrations in such systems are often determined by ray tracing them.

This book is organized in two parts: Part I is on ray geometrical optics and Part II is on wave diffraction optics. The first chapter introduces the concepts of aperture stop and entrance and exit pupils of an optical imaging system. The wave and ray aberrations are defined and wavefront defocus and tilt aberrations are discussed. Various forms of the primary aberration function of a rotationally symmetric system are given, and how this function changes as the aperture stop of the system is moved from one position to another is discussed. The aberration function for the simplest imaging system, namely, a single spherical refracting surface, is given. Finally, a procedure by which the aberration function of a multielement system may be calculated is described. This chapter provides a foundation for the next six chapters.

Chapters 2–6 give the primary aberrations of simple systems, such as a thin lens, plane-parallel plate, spherical mirror, Schmidt camera, and a conic mirror. Numerical problems are discussed here and there to illustrate how to apply the formulas given in these chapters. Part I of the book ends with chapter 7, where the aberrated images of a point object based on geometrical optics are discussed. Thus the ray spot diagrams and, in particular, the spot sizes for primary aberrations are discussed. The concept of aberration balancing, based on geometrical optics to reduce the size of an image spot, is introduced.

In Part II, chapters 8–11 discuss the effects of aberrations on the image of a point object based on wave diffraction optics. Chapter 8 considers systems with circular exit pupils. The aberration-free characteristics of such systems are discussed in terms of their point-spread and optical transfer functions. How the aberrations affect these functions is discussed and aberration tolerances are obtained for a given Strehl or a Hopkins ratio. The concept of aberration balancing, based on wave diffraction optics, to maximize Strehl or Hopkins ratios is discussed. Systems with annular and Gaussian pupils are considered in chapter 9. The effect of obscuration on the point-spread function and on aberration tolerance is discussed. Similarly, the effect of Gaussian amplitude at the exit pupil is discussed. The content of this chapter provides a basis for assessing the effects of aberrations on the optical performance of reflecting telescopes, such as Cassegrain and Ritchey-Chrétien, and on the propagation of laser beams.

The line of sight of an aberrated system is discussed in chapter 10 in terms of the centroid of its point-spread function. It is pointed out that only coma type aberrations change the centroid. Random aberrations are considered in chapter 11, where the average point-spread and optical transfer functions for random image motion and aberrations introduced by atmospheric turbulence are discussed. Part II of the book ends with chapter 12, where a brief discussion is given on how the aberrations of a system may be observed and recognized interferometrically.

Each chapter is written to be as independent of the others as possible, although some are more so than others. For example, chapter 7 may be followed by chapter 1. Except for the first few sections of chapter 1, it is not necessary to understand Part I in order to understand Part II. However, reading Part II without Part I would be like knowing half of a story. Chapter 12 may be read at any time; however, the reason for using certain specific values of defocus, for example, in the case of spherical aberration, may not be understood unless the concepts of aberration balancing discussed in chapters 7 and 8 are understood.

On the matter of references to the literature on aberration theory, I have listed under bibliography those books that treat this subject to some or a large extent. These are the ones I have had the opportunity to read and benefit from. On the wave diffraction optics, I have given references in the text either for historical reasons (such as the papers by Airy and Lord Rayleigh) or because the work is relatively recent and has not appeared in books. Additional references are given after the bibliography for further study on part of the reader.

Finally, I would like to thank those who have helped me with the preparation of this book. I have had many discussions with Dr. Bill Swantner on geometrical optics

and Dr. Richard Boucher on diffraction optics. Dr. Boucher also did computer simulations of the point-spread functions and interferograms and prepared the photographs for this book. Prof. Don O'Shea provided critical and valuable comments when he reviewed this book. Helpful comments were also provided by Prof. R. Shannon. The Sanskrit verse and its translation on p. xx were provided by Dr. S. Sutherland, University of California at Berkeley. The manuscript and its many revisions were typed by Iva Moore. The final version was produced by Betty Wenker and Candy Worshum. I thank The Aerospace Corporation for providing help and facilities to prepare this book. I also thank Dr. Roy Potter and Eric Pepper of the SPIE staff for suggesting and facilitating the preparation of this book, which was carefully edited by Rick Hermann. I cannot thank my wife and children enough for their patience during the course of this work and so I dedicate this book to them.

Los Angeles
1991

Virendra N. Mahajan

SYMBOLS AND NOTATION

a	radius of exit pupil
a_i	aberration coefficient
A_i	peak aberration coefficient
AS	aperture stop
CR	chief ray
e	eccentricity
EnP	entrance pupil
ExP	exit pupil
f	focal length
F	focal ratio or f-number, focal point
GR	general ray
h	object height
h'	image height
H	Hopkins ratio
I	irradiance
J_n	nth-order Bessel function of the first kind
L	image distance from exit pupil
m	pupil-image magnification
M	object-image magnification
MR	marginal ray
N	Fresnel number
n	refractive index
OA	optical axis
OTF	optical transfer function
p	position factor
P	object point, exit pupil power
P'	Gaussian image point
PSF	point spread function
q	shape factor
Q	aberration difference function
r_c	radius of a circle

r_0	atmospheric coherence length or diameter
R	radius of curvature of a surface or reference sphere
s	entrance pupil distance
s'	exit pupil distance
S_p	exit pupil area
S	object distance, Strehl ratio
S'	image distance
t	thickness
W	wave aberration
x,y	rectangular coordinates of a point
z	sag, observation distance
β	field angle
γ	Gaussian beam truncation parameter
δ	phase
δF	figure error
ΔR	longitudinal defocus
ϵ	obscuration ratio
r,θ	polar coordinates of a point
λ	optical wavelength
$(\xi,\eta) = \frac{1}{a}(x,y)$	normalized rectangular coordinates
$\rho = r/a$	normalized radial coordinate in the pupil plane
ν,ϕ	spatial frequency coordinates
σ_Φ	standard deviation of phase aberration
σ_W	standard deviation of wave aberration
τ	optical transfer function
Φ	phase aberration
ψ	angular deviation of a ray
Ψ	phase transfer function
ω	Gaussian beam radius
$R_n^m(\rho)$	radial Zernike polynomial
$< >$	average

अनन्तरत्नप्रभवस्य यस्य हिमं न सौभाग्यविलोपि जातम् ।
एको हि दोषो गुणसन्निपाते निमज्जतीन्दोः किरणेष्विवाङ्कः ॥

anantaratnaprabhavasya yasya himaṃ na saubhāgyavilopi jātam |
eko hi doso gunasannipāte nimajjatīndoḥ kiraṇeṣv ivāṅkaḥ ||

The snow does not diminish the beauty of the Himālayan mountains which are the source of countless gems. Indeed, one flaw is lost among a host of virtues, as the moon's dark spot is lost among its rays.

Kālidāsa *Kumārasambhava* 1.3