Tutorial Solutions

4 Optical Transfer Function

Range of problems associated with the OTF. The first 5 problems are essential to the understanding of this material. In addition the computer examples in problem 7 & 8 are very useful for insight into how the OTF behaves under aberrations.

4.1 Coherent Imaging

For coherently illuminated system at unit magnification the input amplitude transmission is $f_a(x,y)$ and the output is

$$v(x,y) = f_a(x,y) \odot u_2(x,y)$$

where $u_2(x,y)$ is the Amplitude PSF. Show that this can be expressed in Fourier space by

$$V(u,v) = F(u,v)U(u,v)$$

where we have that

$$U(u,v) = p(u\lambda z_1, v\lambda z_2)$$

where $p(x,y)$ is the pupil function of the lens.

Solution

We have that

$$v(x,y) = f_a(x,y) \odot u_2(x,y)$$

where $u_2(x,y)$ is the Amplitude PSF, so straight from the *convolution theorem* we have that,

$$V(u,v) = F(u,v)U(u,v)$$

where

$$U(u,v) = \iint u_2(x,y) \exp(-\imath 2\pi(ux+vy))\, \mathrm{d}x\mathrm{d}y$$

But $u_2(x,y)$ is the Amplitude PSF in the back focal plane of the lens, so if the pupil function of the lens is $p(x,y)$, then from Lecture 4, we have that $u_2(x,y)$ is the scaled FT of $p(x,y)$ plus a few phase terms. If the distance from the lens to its focal plane is z_1 we have that:

$$u_2(x,y) = \hat{B_0} \iint p(s,t) \exp\left(-\imath \frac{\kappa}{z_1}(sx+ty)\right) \mathrm{d}s\mathrm{d}t$$

so ignoring the phase term, we have that $p(x,y)$ is given by the scaled Inverse Fourier Transform of the $u_2(x,y)$, so that:

$$p(s,t) = \iint u_2(x,y) \exp\left(\imath \frac{\kappa}{z_1}(sx+ty)\right) \mathrm{d}x\mathrm{d}y$$

If we now substitute

$$s = -u\lambda z_1 \qquad t = -v\lambda z_1$$

we then get that,

$$p(-u\lambda z_1, -v\lambda z_1) = \frac{1}{\lambda^2 z_1^2} \iint u_2(x,y) \exp\left(-\imath 2\pi(ux+vy)\right) \mathrm{d}u\mathrm{d}v$$

so from above we have that

$$U(u,v) = \lambda^2 z_1^2 \, p(-u\lambda z_1, -v\lambda z_1)$$

By convention we set $U(0,0) = 1$ and we also ignore the $-$signs. (If fact as almost all pupil functions are symmetric so $-$ have no effect), we have that

$$U(u,v) = p(u\lambda z_1, v\lambda z_1)$$

as stated in lectures.

Aside: This analysis has ignored the Phase Term in the amplitude PSF which **is** *important in Coherent Imaging. The effect of this being that the results are qualitively correct, but we have lost the speckle effects which occur in coherent light. Some of the effects of the phase term are considered in the Optical Processing lectures, but a detailed description of Speckle effects are beyond this course.*

4.2 Conditions for Use

State **all** the conditions for the image formed by an imaging system to be fully described by the equation

$$g(x,y) = f(x,y) \odot h(x,y)$$

where $f(x,y)$ is the object and $h(x,y)$ is the intensity PSF.

Describe one optical system that obeys these assumptions, and one that does not stating which assumptions are violated and why.

Solution

For a system to be characterised by the convolution relation we mut assume that the system is

1. **Linear:** Image of any two points is just the sum of the images of each separate point.
2. **Incoherent:** No interference between any points in the object plane.
3. **Space Invariant:** Imageing characteristics do not depend on the location in the object plane.

A good quality 35mm camera is effectively *space invariant*, and if used to photograph naturally lit scenes it will be *incohenent*. In general film is **not** linear, but with careful photographic processing the two stage printing process *can* be *linear* (see lecture 10 for details).

A big telescope with a parabolic mirror is *not space invariant* since the mirror in only parabolic for objects very close to the optical axis, and if used in low light levels you count photons, which is *not linear*. However such a system would be *incoherent*.

4.3 Image of Grating

A 100 mm focal length circular lens with an $F_{No} = 4$ is used to image a 100 line/mm cosine grating with a contrast of 0.5 in a one-to-one imaging system illuminated with incoherent light of approximately 500 nm. Calculate the contrast of the image.

If this same lens is used to image this grating with a magnification of a) 0.25, b) 0.5, c) 2, d) 4. What is the contrast of the image in each case.

Solution

This solution covers all the key aspects OTF and shoule be studied carefully by all.

For a round lens we have that the OTF is given by

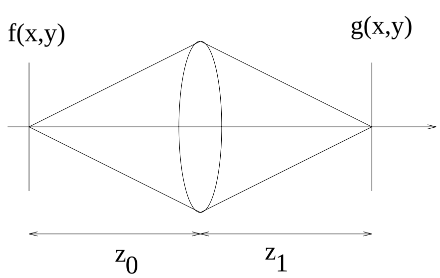
$$H(w) = \frac{2}{\pi}\left[\cos^{-1}\left(\frac{w}{v_0}\right) - \frac{w}{v_0}\left(1-\left(\frac{w}{v_0}\right)^2\right)^{\frac{1}{2}}\right]$$

where the spatial frequency limit v_0 is given by:

$$v_0 = \frac{2a}{\lambda z_1}$$

where z_1 is the image distance and a is the radius of the lens.

For one-to-one imaging systems, we have



with

$$z_0 = z_1 = 2f$$

So if $f = 100$mm, $F_{No} = 4$ so diameter of lens is 25 mm, so $a = 12.5$mm. So frequency limit

$$v_0 = 250 \text{ lines/mm}$$

Input is a vertical grating of contrast 0.5, so

$$f(x,y) = 1 + 0.5\cos(2\pi bx)$$

Aside:

$$\text{Constrast} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{1}{2}$$

so with one-to-one imaging the output is

$$g(x,y) = 1 + 0.5H(b)\cos(2\pi bx)$$

so the contrast is

$$0.5H(b)$$

We have that $b = 100\text{mm}^{-1}$ and $v_0 = 250\text{mm}^{-1}$, so

$$H(b) = 0.504 \rightarrow 0.5H(b) = 0.252 = \text{Contrast}$$

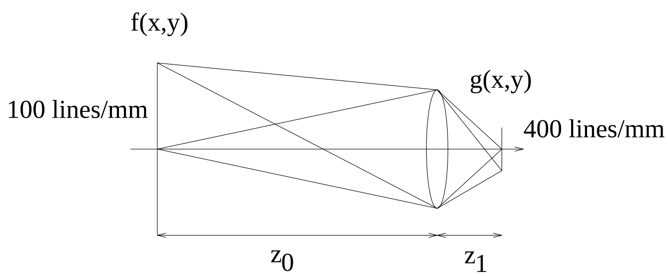
a:) Magnification of 0.25: We have that

$$z_0 = 4z_1 \rightarrow z_1 = \frac{5}{4}f$$

so with the same lens, in this imaging configuration,

$$v_0 = 400 \text{ lines/mm}$$

If the input image has spatial frequency of 100mm^{-1}, then with magnification of 0.25 the spatial frequency in the output will be 400mm^{-1}



So contrast of the output with $b = 100\text{mm}^{-1}$,

$$0.5H(4b) = 0$$

so no image of the grating is formed.

b:) Magnification of 0.5: We have that

$$z_0 = 2z_1 \rightarrow z_1 = \frac{3}{2}f$$

so with the same lens, in this imaging configuration,

$$v_0 = 333 \text{ lines/mm}$$

Spatial frequency in the image plane is $2b = 200\text{mm}^{-1}$, so Contrast is:

$$0.5H(2b) = 0.142$$

c:) Magnification of 2: We have that

$$z_0 = \frac{1}{2}z_1 \rightarrow z_1 = 3f$$

so with the same lens, in this imaging configuration,

$$v_0 = 166 \text{ lines/mm}$$

Spatial frequency in the image plane is $0.5b = 50\text{mm}^{-1}$, so Contrast is:

$$0.5H\left(\frac{b}{2}\right) = 0.311$$

d:) Magnification of 4: We have that

$$z_0 = \frac{1}{4}z_1 \rightarrow z_1 = 5f$$

so with the same lens, in this imaging configuration,

$$v_0 = 100 \text{ lines/mm}$$

Spatial frequency in the image plane is $0.25b = 25\text{mm}^{-1}$, so Contrast is:

$$0.5H\left(\frac{b}{4}\right) = 0.342$$

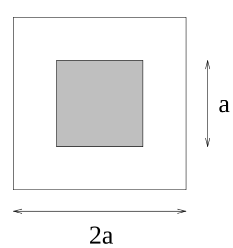
So contrast improves with magnification, but in a complicated way since v_0 also varies.

4.4 Square Annular Aperture

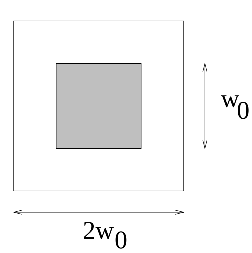
Calculate the OTF for a square annular aperture, where the square central obstruction is half the size of the aperture. Plot a graph of this OTF and, by comparison with the result for a square aperture without the central obstruction, show that the central obstruction gives rise to increased imaging quality at high spatial frequencies.

Solution

We have a square aperture with a central stop of half the size, so the Pupil Function is



The Coherent Transfer Function (CTF) is just the scaled pupil function, so it is just



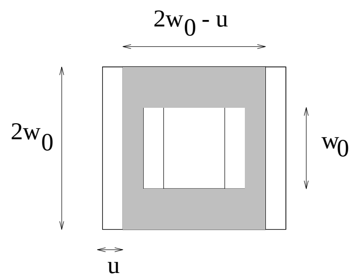
where, if the lens is of size $2a \times 2a$ and the image distance is z_1,

$$w_0 = \frac{a}{\lambda z_1}$$

To to get the OTF we need to take the normalsied Autocorrelation of this.

There are **4** ranges of shift:

1): $u \leq \frac{w_0}{2}$,



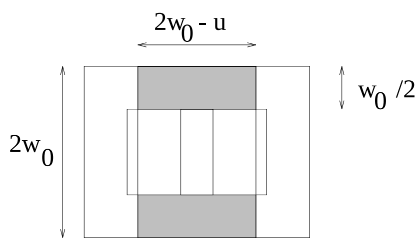
The shaded area is

$$2w_0(2w_0 - u) - w_0(w_0 + u) = 3w_0^2 - 3w_0 u$$

So the normalsied OTF

$$H(u,0) = 1 - \frac{u}{w_0} \quad \text{for } u \leq w_0/2$$

2): $\frac{w_0}{2} \leq u \leq w_0$,



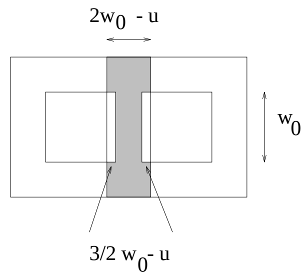
The shaded area is

$$2(2w_0 - u)\frac{w_0}{2} = 2w_0^2 - uw_0$$

So the normalised OTF is

$$H(u,0) = \frac{2}{3} - \frac{u}{3w_0} \quad \text{for } w_0/2 \leq u \leq w_0$$

3): $w_0 \leq u \leq \frac{3w_0}{2}$



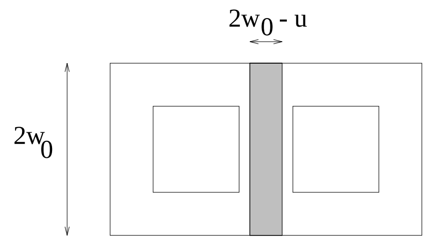
The shaded area is now

$$2w_0(2w_0-u)-2\left[w_0\left[\frac{3}{2}w_0-u\right)\right]=w_0^2$$

So the normalised OTF is

$$H(u,0)=\frac{1}{3}=\text{Constant}\quad\text{for } w_0\leq u\leq 3/2w_0$$

4): $\frac{3w_0}{2}\leq u2w_0$:



The shaded area is

$$(2w_0-u)2w_0=4w_0^2-2w_0u$$

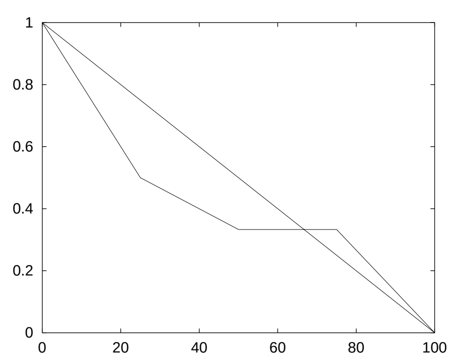
so the normalsied OTF is

$$H(u,0)=\frac{4}{3}-\frac{2u}{3w_0}\quad\text{for } 3/2w_0\leq u\leq 2w_0$$

For a plane square of size $2a\times 2a$ we get, (as shown in lectures), that the OTF is given by

$$H_0(u,0)=1-\frac{u}{2w_0}$$

The two plots with $2w_0=100$ are shown below,



The OTF of the apetrure with the central obstruction in reduced at low spatial frequencies, but is **increased** at high spatial frequencies. Note this is consistent with question 5.1, which shows that the resolution is increased in when a central stop is added. (Not what you would expect).

4.5 Resolution Limit

Use the Rayleigh criteria for two point resolution to estimate the maximum spatial frequency resolved by a lens (hint: consider an array of equally spaced point sources). Compare this value with the maximum spatial frequency obtained from the OTF measure and compare results.

Repeat the above question for the Sparrow resolution limit and comment on your results.

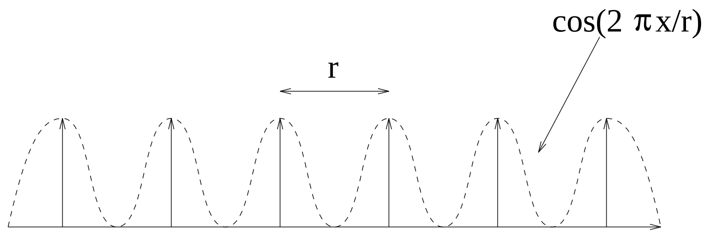
Solution

Rayleigh resolution limit for two point is that their separation in the image plane is at least

$$r_0 = 0.61\frac{\lambda f}{a}$$

where a is the radius of the lens.

Consider a 1-D array of δ-functions, separated,



The Rayleigh criteria is that this object will just be resolved if $r = r_0$.

The fundamental spatial frequency for such an array is $1/r$, so at the Rayleigh limit the maximum spatial frequency is

$$v_R = \frac{1}{r_0} = 1.64\frac{a}{\lambda f}$$

The maximum spatial frequency if we consider the OTF limit is

$$v_0 = \frac{2a}{\lambda f}$$

which is greater than the Rayleigh limit. Why the difference?

The OTF of a round lens of radius a is

$$H(w) = \frac{2}{\pi}\left[\cos^{-1}\left(\frac{w}{v_0}\right)\frac{w}{v_0}\left(1-\left(\frac{w}{v_0}\right)^2\right)^{\frac{1}{2}}\right]$$

so at the Rayleigh limit of v_R we have that

$$H(v_R) = 0.089$$

so the lens will pass a grating of this spatial frequency with a contrast of 0.09.

This is a reasonable practical limit since v_R gives the highest spatial frequency that can be easily detected, and is perhaps a better limit that the absolute spatial frequency limit given by considering the OTF.

The OTF at the 0.1 (10%) contrast is frequently quoted as a practical limit for camera lenses.

From question 3.5 the Sparrow Limit for separation of two points is given by,

$$r_s = 0.476\frac{\lambda f}{a}$$

which, if we apply the same arguments as above, gives a Sparrow frequency limit of

$$v_S = \frac{2.1a}{\lambda f}$$

which is "greater" than the absolute limit given by the OTF scheme which look inconsistent! The problem is that the Sparrow limit is an absolute limit for two isolated points but is not valid for more than two points since the seconary maximas have been ignored. This again shows that the Sparrow limit not actually a useful measure since it cannot be obtained in practice.

4.6 OTF under defocus

Derive the OFT for a square aperture under defocus. Plot the OTF under defocus at the Strehl limit and compare with the result for the case at ideal focus.

What is the criteria for the OTF to have a zero. Calculate the minumum defocus when a zero occurs and the spatial frequency at which it occurs.

Plot the OTF for a range of defocus up to $5\lambda/4$, and show that for medium defocus, the location of the maximum contrast reversal is approximately given by

$$\frac{u_{\mathrm{Max}}}{v_0} \approx \frac{1}{2}\left(1-\sqrt{1-\frac{3\lambda}{4\Delta W}}\right)$$

You wish to demonstrate this constrast reversal effect, using a "fan" image, (as in lectures and Goodman page 150), using the slide projector on Lecture Theatre A, which has a 230 mm 5.6 F_{No} lens. Suggest a suitable system to show this effect and estimate how far you would have to defocus the lens.

Solution

For a square aperture at focus, the pupil function is

$$\begin{aligned} p(x,y) &= 1 \quad \text{for } |x| \ \& \ |y| \le a \\ &= 0 \quad \text{else} \end{aligned}$$

so the CTF is the scaled PSF, so is given by,

$$\begin{aligned} U(u,v) &= 1 \quad \text{for } |u| \ \& \ |u| \le w_0 \\ &= 0 \quad \text{else} \end{aligned}$$

where $w_0 = \frac{a}{\lambda z_1}$ and z_1 is the image distance.
The OTF of this square in one dimension, given in lectures , is

$$H_0(u,0) = \left(1-\frac{|u|}{2w_0}\right)$$

With a defocus of ΔW, the pupil function becomes,

$$\begin{aligned} p(x,y) &= \exp\left(\imath\kappa\Delta W\frac{(x^2+y^2)}{a^2}\right) \quad \text{for } |x| \ \& \ |y| \le a \\ &= 0 \quad \text{else} \end{aligned}$$

The CTF is still the scaled PSF, and is the same "shape" as the pupil function, so that

$$\begin{aligned} U(u,v) &= \exp\left(\imath\kappa\Delta W\frac{(u^2+v^2)}{w_0^2}\right) \quad \text{for } |u| \ \& \ |v| \le w_0 \\ &= 0 \quad \text{else} \end{aligned}$$

Make the substitution that

$$\alpha = \frac{\kappa\Delta W}{w_0^2}$$

so we can write

$$U(u,v) = \exp\left(\imath\alpha(u^2+v^2)\right) \quad \text{for } |u| \,\&\, |v| \leq w_0$$

We now need to form the Autocorrelation of U, which is

$$H(u,v) = \iint U(\zeta,\eta)\, U^*(\zeta-u,\eta-v)\mathrm{d}\zeta\mathrm{d}\eta$$

which can be considered at taking the function and shifting it by (u,v), multiplying and integrating. Now take the substitution that

$$s = \zeta - \frac{u}{2} \quad \& \quad t = \eta - \frac{v}{2}$$

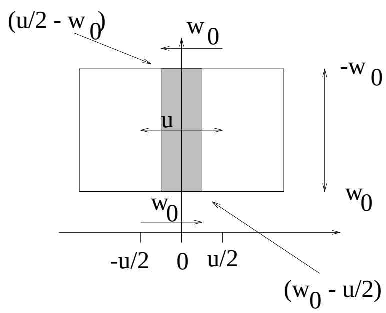
which is equivalent of shifting one function by $(u/2, v/2)$, and the second by $(-u/2,-v/2)$, so we can write

$$H(u,v) = \iint U\left(s+\frac{u}{2},t+\frac{v}{2}\right)\, U^*\left(s-\frac{u}{2},t-\frac{v}{2}\right)\mathrm{d}s\mathrm{d}t$$

Due the the symmetry of the system, consider a shift in the u direction only, so that

$$H(u,0) = \iint U\left(s+\frac{u}{2},t\right)\, U^*\left(s-\frac{u}{2},t\right)\mathrm{d}s\mathrm{d}t$$

Look at the limits of integration,



The limits are the area of overlap of the shifted apetures being the shaded area of,

$$-\left(w_0 - \frac{u}{2}\right) \to \left(w_0 - \frac{u}{2}\right)$$

So now substituting for U we get that

$$H(u,0) = \int_{-w_0}^{w_0}\int_{-\left(w_0-\frac{u}{2}\right)}^{\left(w_0-\frac{u}{2}\right)} \exp\left[\imath\alpha\left(\left(s+\frac{u}{2}\right)^2+t^2\right)\right]\exp\left[-\imath\alpha\left(\left(s-\frac{u}{2}\right)^2+t^2\right)\right]\mathrm{d}s\mathrm{d}t$$

Note the complex conjugate. Now if we expand the $()^2$ we find that most of the terms vanish, and we are left with,

$$H(u,o) = \int_{-w_0}^{w_0}\mathrm{d}t\int_{-b}^{b}\exp(\imath 2\alpha us)\,\mathrm{d}s$$

where we have let

$$b = w_0 - \frac{u}{2}$$

which we can easily integrate to get

$$H(u,0) = 2w_0\left[\frac{\exp(\imath 2\alpha us)}{\imath 2\alpha u}\right]_{-b}^{b} = 2w_0\frac{sin(2\alpha ub)}{\alpha u}$$

which we can write as

$$H(u,0) = 4w_0 b \frac{\sin(2\alpha ub)}{2\alpha ub} = 4w_0 b \,\mathrm{sinc}(2\alpha ub)$$

Finally we normalise the OTF, so noting that $H(0,0) = 4w_0^2$, then we get that the normalised OTF is

$$H(u,0) = \frac{b}{w_0}\mathrm{sinc}(2\alpha bu)$$

Note now that

$$\frac{b}{w_0} = 1 - \frac{u}{2w_0}$$

which is just the OFT of the square lens without any defocus, so under defocus we get that

$$H(u,0) = H_0(u,0)\mathrm{sinc}(2\alpha bu)$$

where $H_0(u,v)$ is the OTF without defocus.
Therefore in two dimensions, the full expression is given by

$$H(u,v) = H_0(u,v)\mathrm{sinc}(2\alpha bu)\mathrm{sinc}(2\alpha bv)$$

Now look at the term within the $\mathrm{sinc}()$. Noting that

$$\alpha = \frac{\kappa\Delta W}{w_0^2} \quad \& \quad b = w_0 - \frac{u}{2}$$

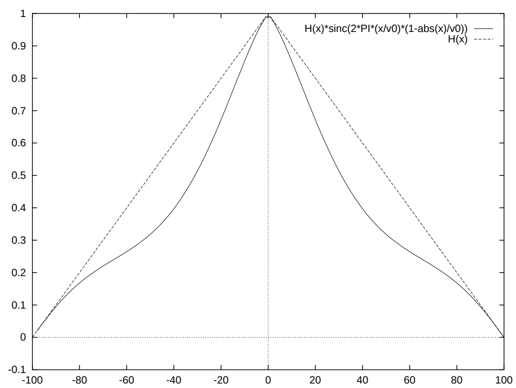
then of we let $v_0 = 2w_0$, we get that

$$2\alpha bu = 4\kappa\Delta W \left(\frac{u}{v_0}\right)\left(1 - \frac{u}{v_0}\right)$$

Strehl Limit: we have that $\Delta W = \lambda/4$, so that

$$2\alpha bu = 2\pi \left(\frac{u}{v_0}\right)\left(1 - \frac{u}{v_0}\right)$$

so the shape of the OTF for $v_0 = 100$ is given by,



Not that there is no major changes in the shape, and the biggest "drop" occurs at $u = v_0/2$ where the OTF drops from $0.5 \to 0.32$.

This is consistent with good imaging at the Strehl limit, where we get only slight reduction in the contrast, but no reduction in the spatial frequency limit.

Large Defocus:

The $\mathrm{sinc}(x)$ will go negative if $x > \pi$, so there will be negative regions if

$$2\alpha bu > \pi \quad \text{for } 0 \leq u \leq v_0$$

or in more useful term, if

$$4\kappa\Delta W \left(\frac{u}{v_0}\right)\left(1 - \frac{u}{v_0}\right) > \pi \quad \text{for } 0 \leq u \leq v_0$$

Firstly find the location of the maximum. This occurs at

$$\frac{\mathrm{d}}{\mathrm{d}u}\left(\frac{u}{v_0}\right)\left(1 - \frac{u}{v_0}\right) = 0$$

so occurs at

$$\frac{1}{v_0} - \frac{2u}{v_0^2} = 0 \quad \Rightarrow \quad u = \frac{1}{2}v_0$$

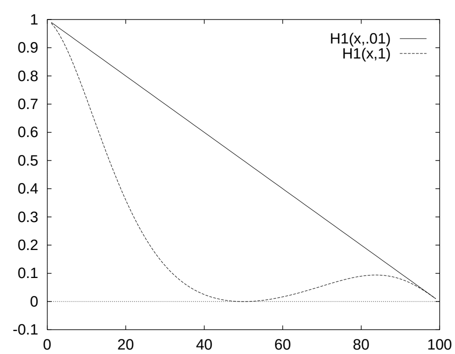
so the maximum value is,

$$2\alpha ub = 4\kappa\Delta W\left(\frac{1}{4}\right) = \frac{2\pi}{\lambda}\Delta W$$

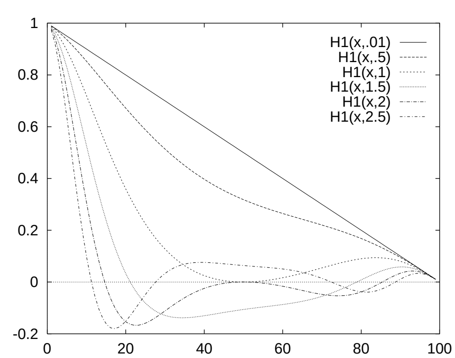
We get zero if

$$\frac{2\pi}{\lambda}\Delta W \geq \pi \quad \Rightarrow \quad \Delta W > \frac{\lambda}{2}$$

so we get zero(s) if the defocus is more than **twice** the Strehl limit. Plot of OTF with $\Delta W = \lambda/2$ shows the expected zero at $u = v_0/2$,



while larger defocus gives rise to negative regions, with plot showing otfs with $\Delta W = 0, \lambda/4, \lambda/2, 3\lambda/4, \lambda, 5\lambda/4$



To a first approximation, the location of the point of maximum contrast reversal is given by the location of the first minimum of the sinc() function, which occurs when

$$2\alpha bu = \frac{3\pi}{2}$$

so if we write $\hat{u} = u/v_0$, we get the

$$4\kappa\Delta W\hat{u}(1-\hat{u}) = \frac{3\pi}{2}$$

which we can rearrange to get

$$\hat{u}^2 - \hat{u} + \frac{3\lambda}{16\Delta W} = 0$$

this is now a quadratic with roots of

$$\hat{u} = \frac{1 \pm \sqrt{1-\frac{3\lambda}{4\Delta W}}}{2}$$

but, as noted from the graph above, for $\Delta W > \lambda$ we have that $u_{\mathrm{Max}} < v_0/2$, so we get,

$$\frac{u_{\mathrm{Max}}}{v_0} \approx \frac{1}{2}\left(1-\sqrt{1-\frac{3\lambda}{4\Delta W}}\right)$$

If $\Delta W = \lambda$ then we get that

$$\frac{u_{\mathrm{Max}}}{v_0} = \frac{1}{4}$$

which corresponds well to the graph.

The full expression for the location of the maximum contrast would be found by taking the full expression for $H(u)$ and finding the location of the first differential. Maple experts may wish to try this. I would expect the difference to be small.

For a projector in Lecture Theatre A with a lens of focal length 230 mm, we have a magnification is $\approx \times 70$, (see Tutorial question from Physics 3 optics, or work it out for yourself!), so $z_1 \approx f$, so that

$$v_0 = \frac{2a}{\lambda f} = \frac{1}{\mathrm{F_{No}}\lambda} \approx 325\mathrm{mm}^{-1}$$

We want to have the contrast easily visible on the screen, so we want this to occur at about 5mm grating on the screen, so with a magnification of $\times 70$, at the object plane we need $u_{\mathrm{Max}} \approx 15\mathrm{mm}^{-1}$. So we have that

$$\frac{u_{\mathrm{Max}}}{v_0} = \frac{1}{2}\left(1-\sqrt{1-\frac{3\lambda}{4\Delta W}}\right) \approx \frac{15}{325}$$

We can now solve for ΔW, which gives

$$\Delta W \approx 4.25\lambda$$

From lectures, we have, that if we move a lens (or object) a distance Δz, then

$$\Delta W = \frac{\Delta z}{2}\left(\frac{a}{z_1}\right)^2$$

in this case, $z_1 = f = 230\,\mathrm{mm}$ and $a = 20.5\,\mathrm{mm}$, so that

$$\Delta z \approx 0.6\mathrm{mm}$$

So a "fan" slide with spatial frequencies centered on about 15 line/mm would show this effect if the projector was defocused by about 0.6 mm.

There are a **lot** of assumptions here, the biggest of which is that the same formula applies to square and round lens. This introduces an error of about 10%, which is reasonable. The biggest problem in this demonstration is that this theory applied to "diffraction limited" optics, and most projectors lenses have significant other aberrations that swamp this contrast reversal effect. The normal 80 mm $F_{No} = 2.8$ projector lens on the portable projectors are not good enough to show this effect, but the long focus 230 mm lenses in the lecture theatres are.

4.7 Computer Example I: Calculation of OTF

Experiment with the computer program `otf` which will form the OTF of a round lens under various aberrations. When you run the program you will be prompted for the aberrations, and the OTF is then displayed via `xv`. In addition the horizontal line scan of the OTF will be stored to file in a format that can be input into `xgraph` or `xmgr`.

The programme is located in:

`~wjh/mo4/examples/otf`

Tasks to try with this programme:

1. Show that the OTF of a annular aperture is slightly larger at high spatial freqnecies than that for the full aperture.
2. Show that at the Strehl limit of defocus that the OTF does not vary significantly in shape to the ideal OTF. (Note at the Strehl limit $S_4 = 0.5$.)
3. Show that with large defocus parts of the OTF become negative. (Try $S_4 = 2.$).
4. Find the mininum defocus to obtain a zero in the OTF and compare this with the analytical result for the square aperture.
5. Experiment with the off-axis aberrations an observe what happens to the shape of the psf.

Technical note: The OTF is always normalised so that

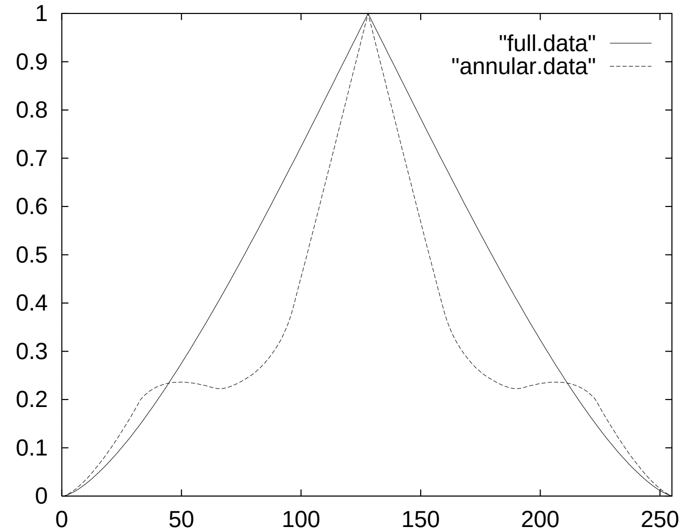
$$H(0,0) = 1.0$$

Neither the line scan data or the image send to `xv` *are processed in any way. However* `xv` *display intensity, so negative regions of the displayed OTF are lost.*

Solution

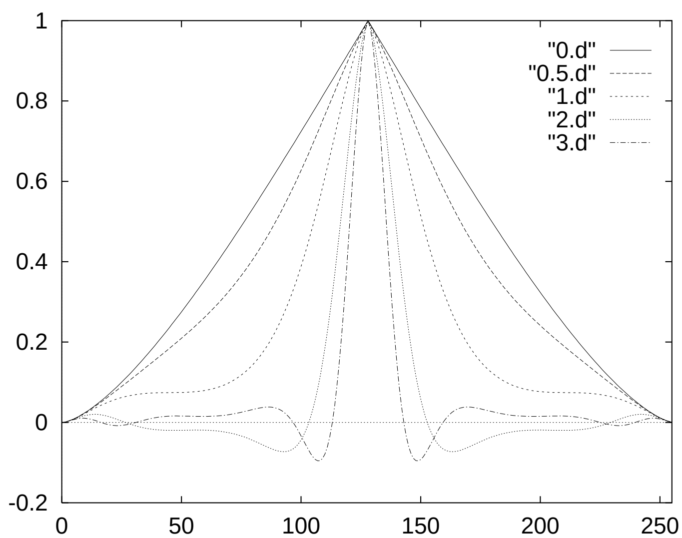
You can try most of this yourself, but here are a few results.

Annular Aperture: Comparison of the OTF of the full and annular aperture, where the annular obstruction is half the radius. The cross-section plot is:



This shows the *expected* slight increase at high spatial frequencies but a significant reduction and the mid spatial frequencies. This is consistent with the exact calculation for the square aperture above, but the graph is smoother due to the apertures being circular.

Range of Defocus: Series of cross-section plots for varying amounts of defocus. The plots for $S_4 = 0, 0.5, 1, 2, 3$ is shown below,

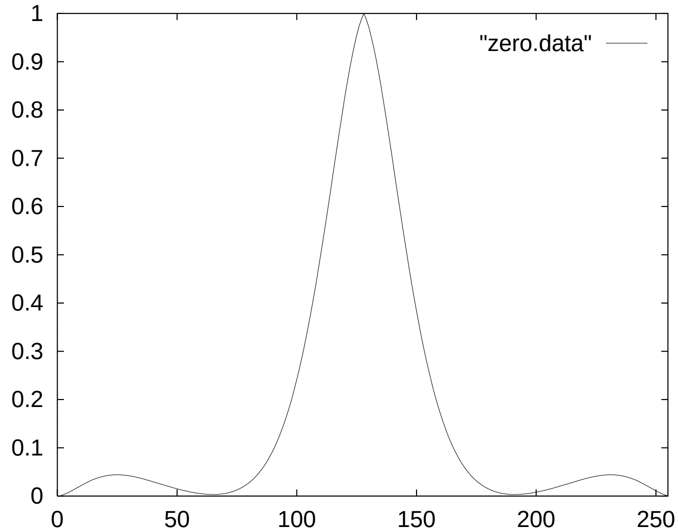


note that

$$S_4 = \frac{2\Delta}{\lambda}$$

so that the Strehl limit occurs at $S_4 = 0.5$. As expected the OTF at the Strehl limit is only slightly reduced from the ideal, and as expected at significant defocus, $S_4 > 2$, (so $\Delta W > \lambda$), we get negative regions, so the expected contrast reversal. However for the round lens the first zero does **not** occur at $\Delta W = \lambda/2$ ($S_4 = 1$), but at a slightly large defocus.

Location of first zero: From the above plots, the defocus that corresponds to a zero occurring in the OTF is between $S_4 = 1 \to 2$, possibly closer to 1. There is no simple analysis of this condition, but simple iterative search using the OTF program shows that the first condition for a zero to occur at $S_4 \approx 1.27$ as shown below:



which gives $\Delta W \approx 0.63\lambda$. This is about 27% greater defocus that a square aperture of size $2a \times 2a$.

These plots show that the OTF for a square and circular aperture have the same basic shape, but with the circular aperture the effects effects of a central obstruction or defocus are reduced.

4.8 Computer Example II: Digital Defocus

Experiment with the computer `defocus` which implements digitally the defocus of an image. You will be promted of an image file (in `pgm` format) and the amount of defocus you want to apply. The value of defocus is set by the S_4, so the Strehl limit occurs when $S_4 = 0.5$. The resultant defocused image will ne calculated and displayed with `xv`. As with the previous programs you may have to modify the image contrast with the *Colo(u)r Edit* facility to make the images features visible.

The programme is located in:

```
~wjh/mo4/examples/defocus
```

and supplied images are,

`toucan.pgm`	Image of Toucan
`grating.pgm`	Horizontal grating
`fringe.pgm`	Fringe pattern
`fan.pgm`	Fan image used.

These are located in the directory:

```
~wjh/mo4/examples/
```

Solution

Try the program for yourself and see what you get! Here are some typical defocused “toucan” images showing defocus of 1,2 and 3 wavelengths respectively.

This shown that up to about twice the Strehl limit we still get “reasonable” images which appear to be somewhat blurred but still useful, however images with defocus of $S_4 \geq 2$ images are severely blurred. This is consistent with the above solutions which show that with $S_4 > 2$ we get negative regions of the OTF which severely degrade the imaging.