Gabarito A2 - PE - Computação - 21/05/15

2,0 ① média: 2 mortes por 50.000 hab.

(a) $2 - 50.000 \Rightarrow \lambda = \dfrac{2}{50.000}$; $N_{200.000}$: nº mortes em $\dfrac{200.000}{t}$

$\lambda - 1$

média p/ hab

$\lambda . t = 200.000 \cdot \dfrac{2}{50.000} = \dfrac{400.000}{50.000} = \dfrac{40}{5} = 8 \Rightarrow N_{200000} \sim P(8)$

$P(N_{200000} = 5) = \dfrac{e^{-8} . 8^5}{5!} \approx 0,091603662 \approx \boxed{0,091604}$ 0,5

(b) $N_{112.500} \sim P\left(\dfrac{2}{50.000} . 112.500\right) = P(4,5)$

$P(N_{112500} \geq 2) = 1 - \left[P(N_{112500} = 0) - P(N_{112500} = 1)\right] = 1 - e^{-4,5}\left[\dfrac{4,5^0}{0!} + \dfrac{4,5^1}{1!}\right] =$

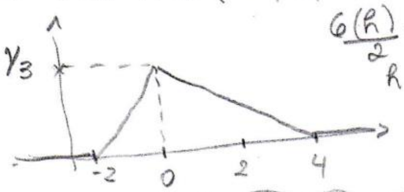
$= 1 - e^{-4,5}\left[1 + 4,5\right] = 1 - 5,5\, e^{-4,5} \approx 0,938900519 \approx \boxed{0,938901}$ 1,0

(c) Como $\lambda t = 4,5$ (média de mortes em 112.500), temos:

$P(N_{112500} = 4) = \dfrac{e^{-4,5}\, 4,5^4}{4!} \approx 0,189808$ logo, o nº mais provável é $\boxed{4 \text{ mortes}}$ 0,5

$P(N_{112500} = 5) = \dfrac{e^{-4,5}\, 4,5^5}{5!} \approx 0,170827$

3,0 ② $X \sim TRI(-2, 0, 4)$



$\dfrac{6(h)}{2} = 1$

$h = \dfrac{1}{3}$

$y = ax + b$

$\begin{cases} 0 = -2a + b \\ \frac{1}{3} = 0.a + b \end{cases}$

$b = \frac{1}{3}$

$a = \dfrac{-1/3}{-2} = \dfrac{1}{6}$

$y = \dfrac{x}{6} + \dfrac{1}{3}$

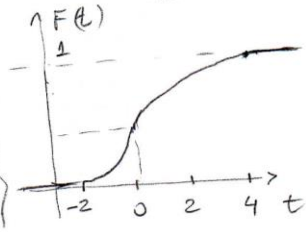
$y = ax + b$

$\begin{cases} \frac{1}{3} = 0.a + b \\ 0 = 4a + b \end{cases}$

$b = \frac{1}{3}$

$a = \dfrac{-1/3}{4} = -\dfrac{1}{12}$

$y = -\dfrac{x}{12} + \dfrac{1}{3}$



(a) 1,0 $f(x) = \begin{cases} 0, & x < -2 \\ \frac{x}{6} + \frac{1}{3}, & -2 \leq x \leq 0 \\ -\frac{x}{12} + \frac{1}{3}, & 0 \leq x \leq 4 \\ 0, & x > 4 \end{cases}$

(b) $F(t) = \begin{cases} 0, & t < -2 \\ \int_{-2}^{t} \left(\frac{x}{6} + \frac{1}{3}\right) dx, & -2 \leq t \leq 0 \\ \frac{1}{3} + \int_{0}^{t} \left(-\frac{x}{12} + \frac{1}{3}\right) dx, & 0 \leq t \leq 4 \\ 1, & t > 4 \end{cases}$

$\int_{-2}^{t} \left(\dfrac{x}{6} + \dfrac{1}{3}\right) dx = \dfrac{1}{6(2)} x^2 \Big|_{-2}^{t} + \dfrac{1}{3} x \Big|_{-2}^{t} =$

$= \dfrac{1}{12}(t^2 - 4) + \dfrac{1}{3}(t + 2) = \dfrac{t^2}{12} + \dfrac{t}{3} + \dfrac{1}{3}$

$\int_{0}^{t} \left(-\dfrac{x}{12} + \dfrac{1}{3}\right) dx = -\dfrac{1}{12(2)} x^2 \Big|_{0}^{t} + \dfrac{1}{3} x \Big|_{0}^{t} =$

$= -\dfrac{1}{24}(t^2) + \dfrac{1}{3}(t) = -\dfrac{t^2}{24} + \dfrac{t}{3}$

(b) 1,0

$F(t) = \begin{cases} 0, & t < -2 \\ \frac{t^2}{12} + \frac{t}{3} + \frac{1}{3}, & -2 \leq t \leq 0 \\ -\frac{t^2}{24} + \frac{t}{3} + \frac{1}{3}, & 0 \leq t \leq 4 \\ 1, & t > 4 \end{cases}$

cont. Gabarito A2 - PE - Computação - 21/05/15

(c) (i) $P(X>1) = 1 - P(X \leq 1) = 1 - F(1) = 1 - \left[-\frac{1}{24} + \frac{1}{3} + \frac{1}{3}\right] = 1 - \left[\frac{-1+16}{24}\right] = \frac{9}{24}$ $0,375$

(ii) $P(-1 < X < 3) = F(3) - F(-1) = \left[-\frac{9}{24} + \frac{3}{3} + \frac{1}{3}\right] - \left[+\frac{1}{12} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}}\right] =$ 0,5

$= -\frac{3}{8} + \frac{4}{3} - \frac{1}{12} = \frac{-9+32-2}{24} = \frac{21}{24} = \boxed{\frac{7}{8} = 0,875}$ 0,5

2,5 (3) $X \sim UD_{[1,25]}$

(a) $\boxed{P(X=k) = \frac{1}{25} : k = 1,2,\ldots,25}$ 0,5

(b) $F(t) = \begin{cases} 0, & t < 1 \\ \frac{k}{25}, & k \leq t < k+1,\ k = 1,2,\ldots,24 \\ 1, & t \geq 25 \end{cases}$ 1,0

(c) (i) $P(X \geq 20) = 1 - P(X < 20) = 1 - [F(20) - P(X=20)] =$

$= 1 - \left[\frac{20}{25} - \frac{1}{25}\right] = 1 - \frac{19}{25} = \frac{6}{25}$ 0,5

(ii) $P(8 \leq X < 15) = F(15) - F(8) + P(X=8) - P(X=15) =$

$= \frac{15}{25} - \frac{8}{25} + \frac{1}{25} - \frac{1}{25} = \frac{7}{25}$ 0,5

2,5 (4) Y: nº pacientes curados entre 7 doentes $Y \sim B(7;\ 0,8)$

(a) $P(Y=7) = \binom{7}{7} 0,8^7 0,2^0 = 0,8^7 \approx \boxed{0,209715}$ 0,5

(b) ñ curados 2, 3, ... 7
curados 5, 4, ... 0

$P(0 \leq Y \leq 5) = 1 - [P(Y=6) + P(Y=7)] =$

$= 1 - \left[\binom{7}{6} 0,8^6 0,2^1 + \binom{7}{7} 0,8^7 0,2^0\right] =$

$= 1 - [7 \cdot 0,8^6 \cdot 0,2 + 0,8^7] = 1 - 0,8^6 [7(0,2) + 0,8] =$

$= 1 - 0,8^6 (2,2) \approx \boxed{0,423283}$ 1,0

(c) $P(Y \geq 5) = P(Y=5) + P(Y=6) + P(Y=7) =$

$= \binom{7}{5} 0,8^5 0,2^2 + \binom{7}{6} 0,8^6 0,2^1 + \binom{7}{7} 0,8^7 0,2^0 = 21 \cdot 0,8^5 \cdot 0,04 + 7 \cdot 0,8^6 \cdot 0,2 + 0,8^7 =$

$= 0,8^5 [21 \cdot 0,04 + 1,4 \cdot 0,8 + 0,64] = 2,6 \cdot 0,8^5 \approx \boxed{0,851968}$ 1,0