Legend

c : Country

r : Subcontinent

$\boldsymbol{\psi_{c,m,t,s}}$: Latent variable used to infer the proportion $\phi_{c,t,m,s}$

$\boldsymbol{\phi_{c,m,t,s}}$: Proportion of modern contraceptive m, supplied by sector s, at time t in country c.

$\boldsymbol{\alpha^{coun.}_{c,m,s}}$: Hierarchically estimated intercept term, informed by the most recently observed level (k*) supplied by sector s of method m in country c.

$\boldsymbol{\beta_{c,m,s,k}}$: Spline coefficient, informed by estimated cross-method correlations.

$\boldsymbol{B_{c,k}(t)}$: Basis functions for country c at knot k.

$\boldsymbol{\delta_{c,m,s,k}}$: Difference between spline coefficients K and K+1 for sector s of method m in country c.

$\boldsymbol{\sigma^2_{\alpha^x_s}}$: Sector-specific variance for x = Country / Subcontinent

$\boldsymbol{\hat{\Sigma}^{global}_{\delta,s}}$: Variance-covariance matrix estimated using national-level data from all countries available for sector s.

Spline model:

$$\psi_{c,m,t,s} = \sum_{k=1}^{K} \beta_{c,m,s,k} B_{c,k}(t)$$

At knot k,

$$\beta_{c,m,s,k} = \begin{cases} \alpha_{c,m,s} & k = k^*, \\ \beta_{c,m,s,k+1} - \delta_{c,m,s,k} & k < k^*, \\ \beta_{c,m,s,k-1} + \delta_{c,m,s,k-1} & k > k^*. \end{cases}$$

Proportions:

$$\phi_{c,t,m,1} = logit^{-1}(\psi_{c,t,m,1})$$

$$\phi_{c,t,m,2} = (1 - \phi_{c,t,m,1}) logit^{-1}(\psi_{c,t,m,2})$$

$$\phi_{c,t,m,3} = 1 - (\phi_{c,t,m,1} + \phi_{c,t,m,2})$$

Multi-country national estimation

Single-country national estimation

Multi-country national estimates inform single-country national estimates

$$\boldsymbol{\delta}_{c,1:M,s,k} | \Sigma_{\delta,s} \sim MVN(\mathbf{0}, \Sigma_{\delta,s})$$

$$\Sigma_{\delta,s} = \hat{\Sigma}^{global}_{\delta,s}$$

The variance-covariance matrix of multi-country national model informs the Wishart prior of single-country national model.

$$\delta_{c,1:M,s,k} | \Sigma_{\delta,s} \sim MVN\left(0, \hat{\Sigma}^{global}_{\delta,s}\right)$$

$$\alpha^{coun.}_{c,m,s} | \alpha^{subcont.}_{r[c],m,s}, \sigma^2_{\alpha^{coun.}_s} \sim N\left(\alpha^{subcont.}_{r[c],m,s}, \sigma^2_{\alpha^{coun.}_s}\right)$$

$$\alpha^{subcont.}_{r,m,s} | \alpha^{world}_{m,s}, \sigma^2_{\alpha^{subcont.}_s} \sim N\left(\alpha^{world}_{m,s}, \sigma^2_{\alpha^{subcont.}_s}\right),$$

$$\alpha^{world}_{m,s} \sim N(0, 10^2)$$

$$\alpha^{subcont.}_{r[c],m,s} = \hat{\alpha}^{subcont.}_{r[c],m,s}$$

$$\sigma^2_{\alpha^{coun.}_s} = \hat{\sigma}^2_{\alpha^{coun.}_s}$$

Hierarchically estimated intercept of multi-country national model informs the prior of intercept in single-country national model.

$$\alpha^{coun.}_{c,m,s} | \hat{\alpha}^{subcont.}_{r[c],m,s}, \hat{\sigma}^2_{\alpha_{c,s}} \sim N(\hat{\alpha}^{subcont.}_{r[c],m,s}, \hat{\sigma}^2_{\alpha^{coun.}_s})$$