

Require: Samples from the previous iteration $(\mathcal{T}_t, \mathcal{T}'_t, \mathcal{M}_t, \mathcal{M}'_t)$ for $t = 1, \dots, T$ and $(\sigma^2, \mathbf{s}_{13})$, and data (y_i, A_i, \mathbf{X}_i) for $i = 1, \dots, N$

1: **for** $r = 1, \dots, M$ iteration **do**
2: **for** $i = 1, \dots, N$ **do**
3: $Z_i \sim \begin{cases} N\left(\sum_{t=1}^T g_1(\mathbf{X}_i; \mathcal{T}_t, \mathcal{M}_t), 1\right) I_{(Z_i > 0)} & \text{for } A_i = 1; \\ N\left(\sum_{t=1}^T g_1(\mathbf{X}_i; \mathcal{T}_t, \mathcal{M}_t), 1\right) I_{(Z_i \leq 0)} & \text{for } A_i = 0 \end{cases} \quad \triangleright \text{latent exposure variable}$
4: **end for**
5: **for** $j = 1, \dots, T$ **do**
6: **for** $i = 1, \dots, N$ **do**
7: $R_{i,-j}^{(r)} = Z_i - \sum_{t \neq j} g_1(\mathbf{X}_i; \mathcal{T}_t, \mathcal{M}_t) \quad \triangleright \text{residual of the exposure model}$
8: $H_{i,-j}^{(r)} = y_i - \sum_{t \neq j} g_3(\mathbf{X}_i; \mathcal{T}'_t, \mathcal{M}'_t) \quad \triangleright \text{residual of the outcome model}$
9: **end for**
10: $\mathcal{T}_j^{(r)} \sim [\mathcal{T}_j | R_{1,-j}^{(r)}, \dots, R_{N,-j}^{(r)}, 1] \quad \triangleright \text{based on one of the three acceptance ratios}$
11: $\mathcal{T}'_j{}^{(r)} \sim [\mathcal{T}'_j | H_{1,-j}^{(r)}, \dots, H_{N,-j}^{(r)}, \sigma^2] \quad \triangleright \text{based on one of the three acceptance ratios}$
12: $\mathcal{M}_j^{(r)} \sim [\mathcal{M}_j | \mathcal{T}_j^{(r)}, R_{1,-j}^{(r)}, \dots, R_{N,-j}^{(r)}, 1]$
13: $\mathcal{M}'_j{}^{(r)} \sim [\mathcal{M}'_j | \mathcal{T}'_j{}^{(r)}, H_{1,-j}^{(r)}, \dots, H_{N,-j}^{(r)}, \sigma^2]$
14: **end for**
15:

$(\sigma^2)^{(r)} \sim \text{Inv.Gamma}\left(a_\sigma + \frac{N}{2}, b_\sigma + \frac{1}{2} \left\{ \sum_{i=1}^N \left(y_i - \sum_{t=1}^T g_3(\mathbf{X}_i; \mathcal{T}'_t{}^{(r)}, \mathcal{M}'_t{}^{(r)}) \right)^2 \right\}\right)$

16: Update $\mathbf{s}_{13}^{(r)}$ based on the M-H algorithm:

17:

Proposal: $\mathbf{s}_{13}^{(r)} \sim \mathcal{D}(n_{0,3} + c + \alpha/P, n_{1,1} + n_{1,3} + \alpha/P, \dots, n_{P,1} + n_{P,3} + \alpha/P)$

18:

Acceptance Ratio: $P_{\text{AR}}(\mathbf{s}_{13} \rightarrow \mathbf{s}_{13}^{(r)}) = \min \left\{ 1, \left(\frac{1 - \sum_{j=1}^P s_j}{1 - \sum_{j=1}^P s_j^{(r)}} \right)^{\sum_{j=1}^J n_{j,3}} \right\}$

19: **end for**