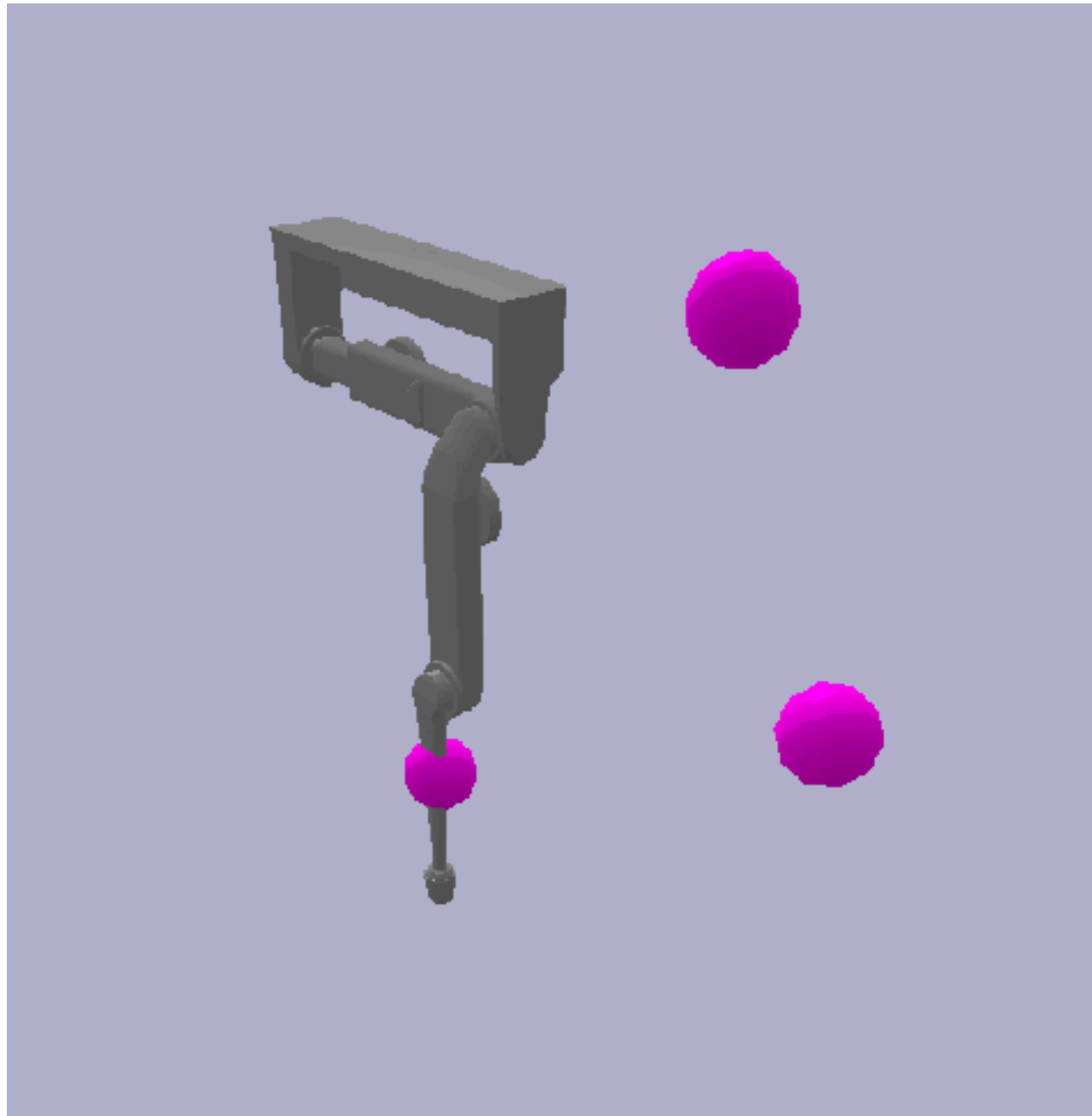


ROB2004

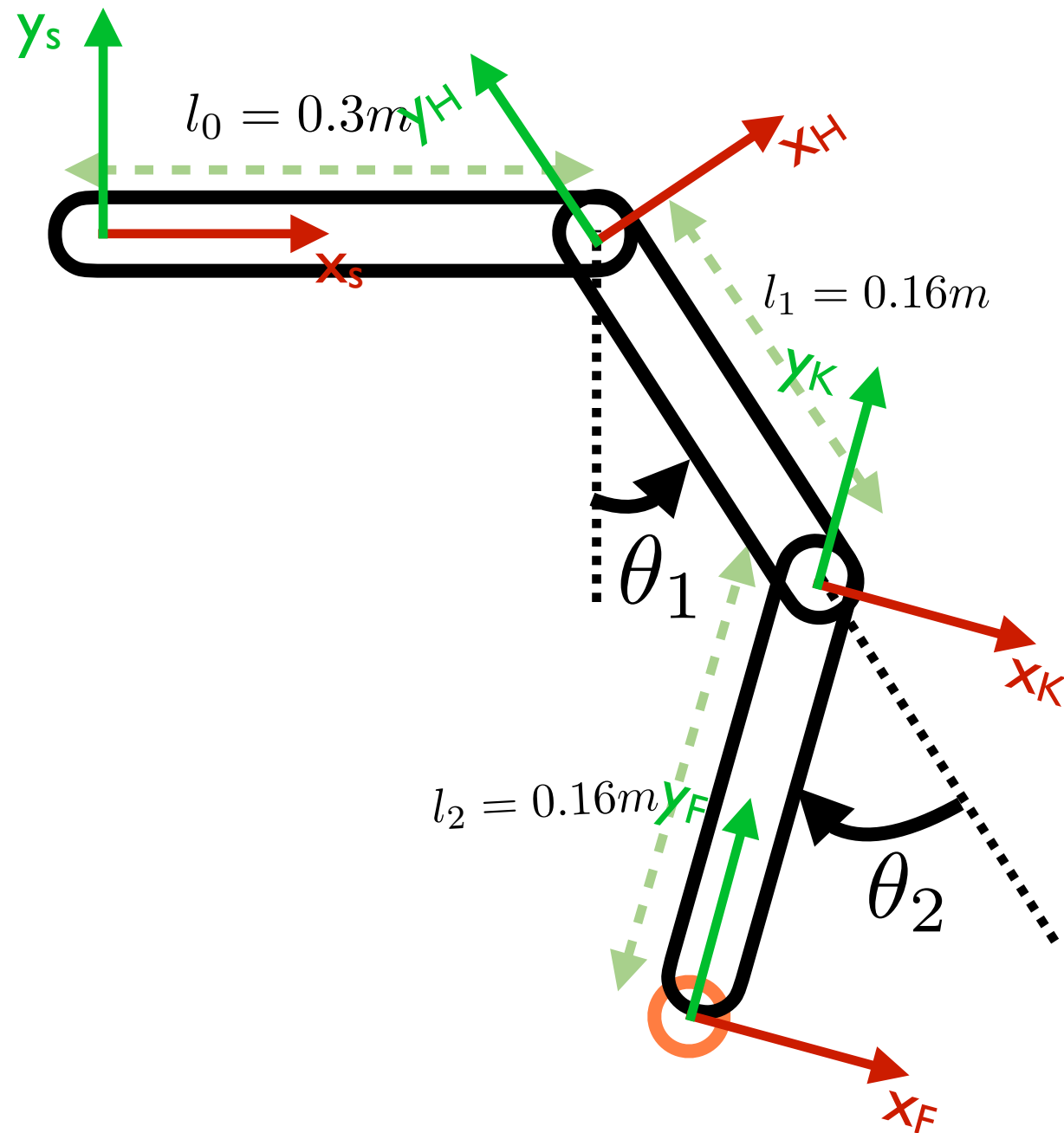
Robotic Manipulation and Locomotion

Laboratory III: Analytic Inverse Kinematics

The goal of the laboratory is to build a complete controller capable of reaching target objects in the environment



# The robot and its kinematics



The foot orientation  $R_{SF} = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) \end{bmatrix}$

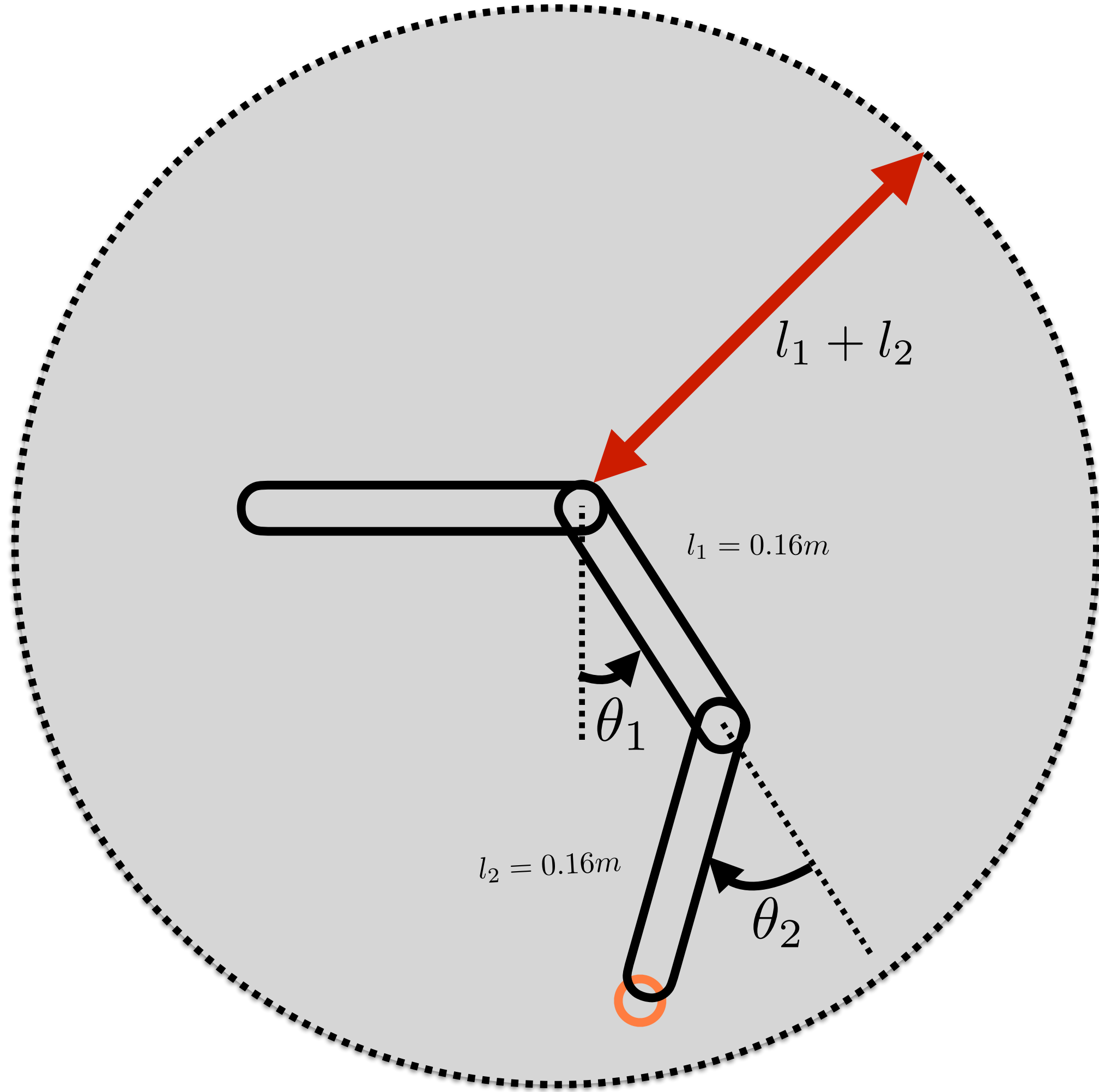
The foot position is  $p_{SF} = \begin{pmatrix} l_2 \sin(\theta_1 + \theta_2) + l_1 \sin \theta_1 + l_0 \\ -l_2 \cos \theta_1 + \theta_2) - l_1 \cos \theta_1 \end{pmatrix}$

Frame  $\{s\}$  is our fixed frame, i.e. the spatial frame

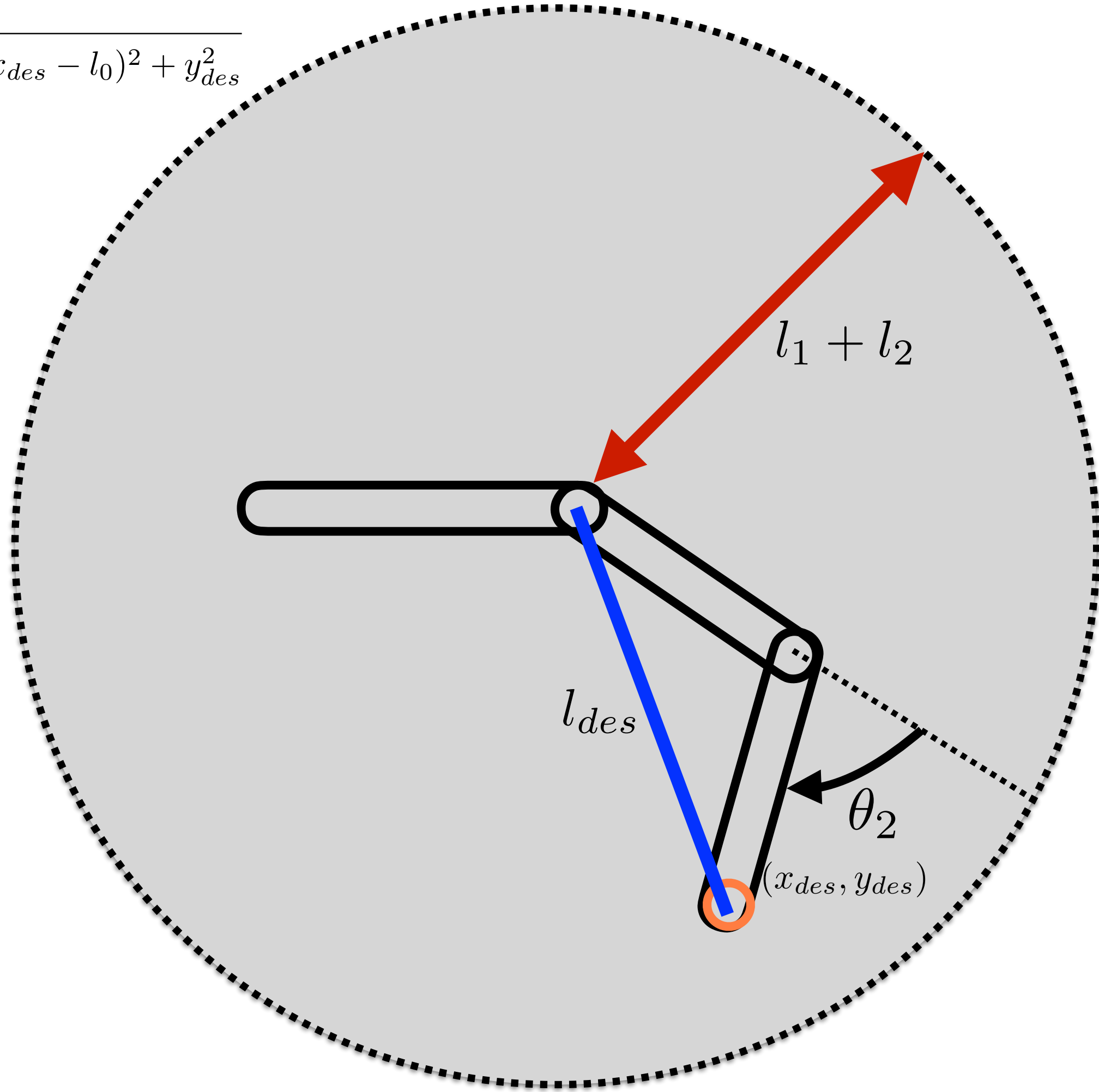
The hip frame  $\{H\}$  is translated by  $l_0$  and rotated by  $\theta_0$  with respect to frame  $\{s\}$

The knee frame  $\{K\}$  is translated by  $l_1$  and rotated by  $\theta_1$  with respect to frame  $\{H\}$

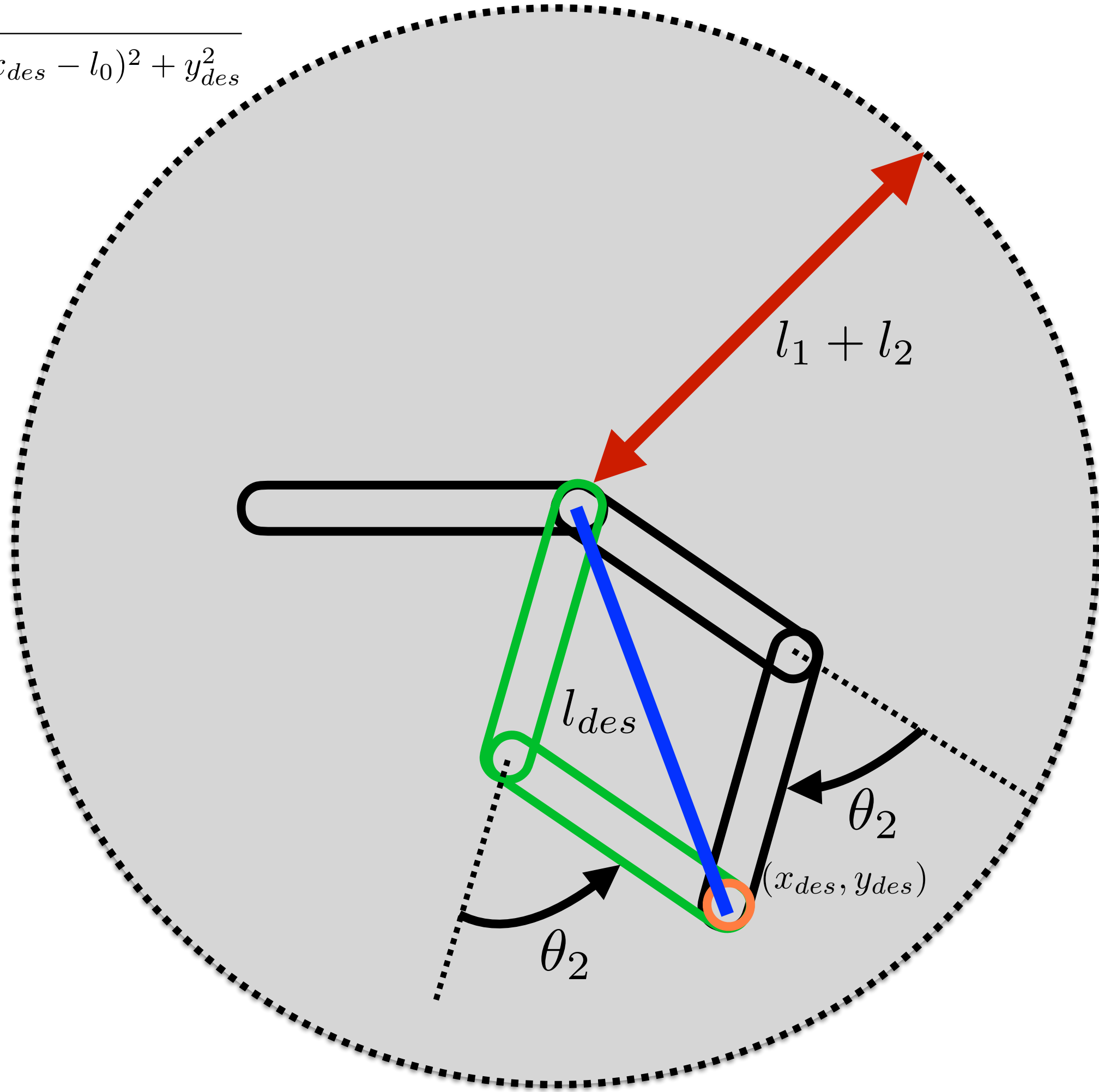
The foot frame  $\{F\}$  is translated by  $l_2$  with respect to frame  $\{K\}$



$$l_{des} = \sqrt{(x_{des} - l_0)^2 + y_{des}^2}$$

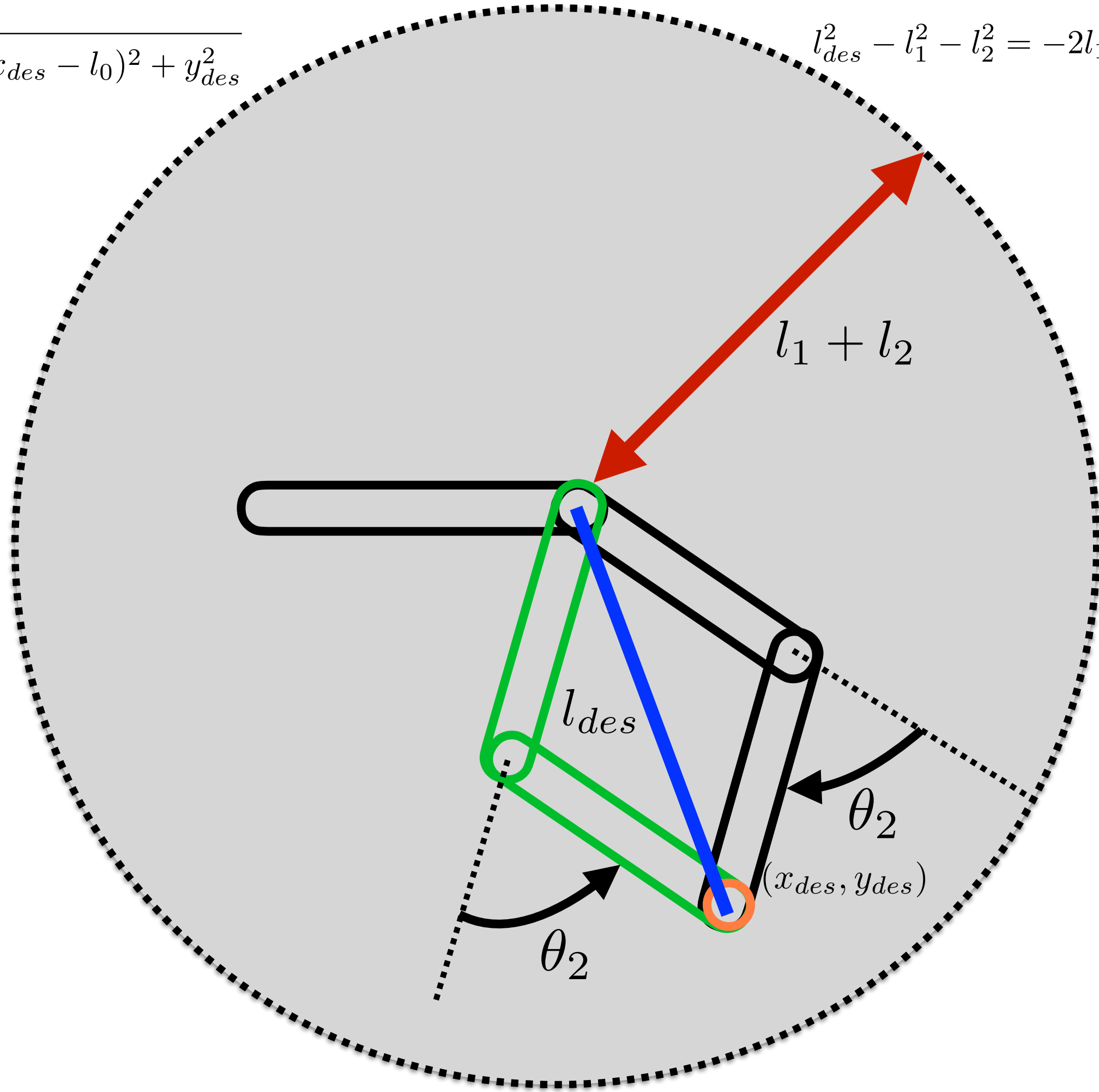


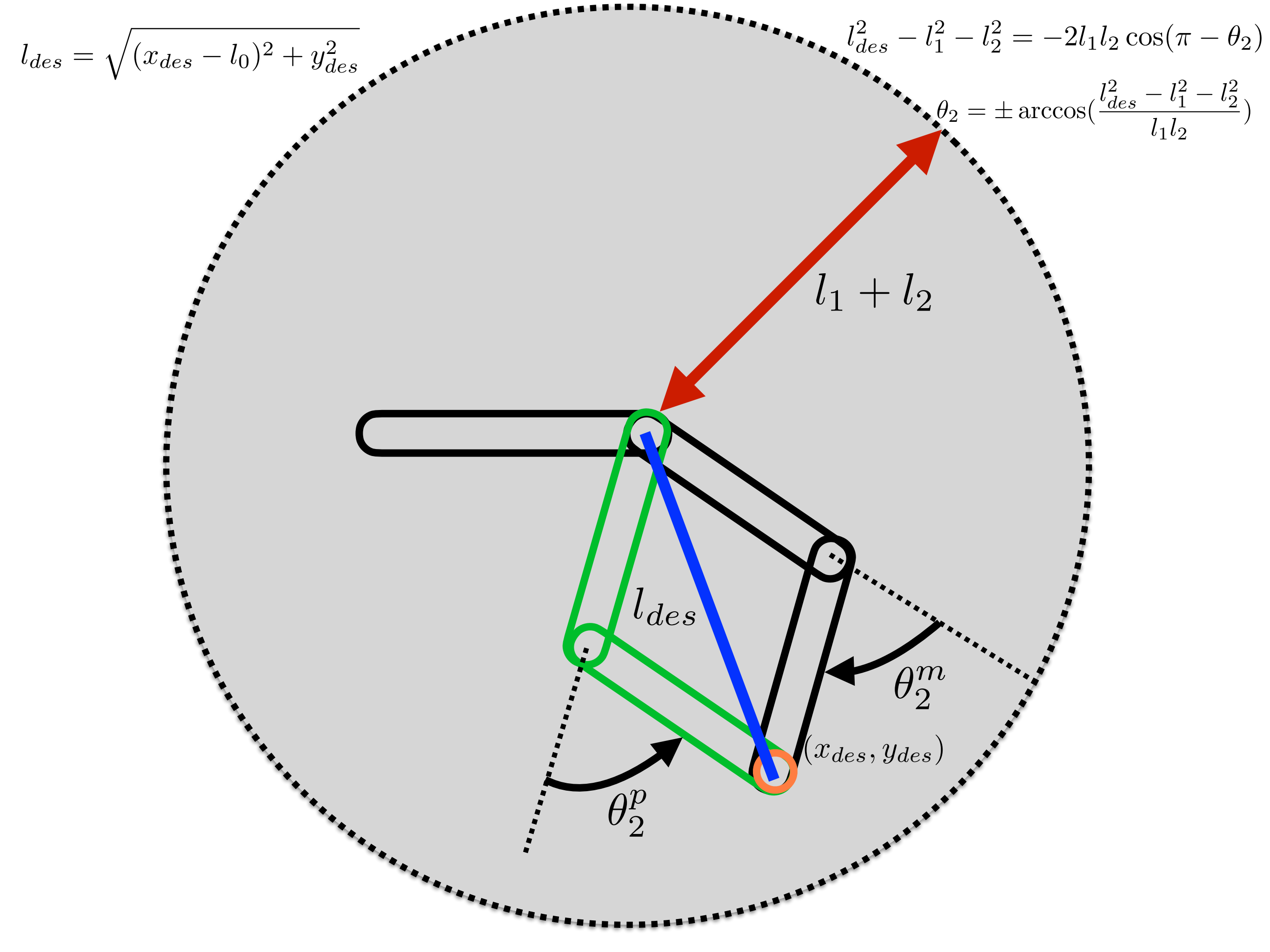
$$l_{des} = \sqrt{(x_{des} - l_0)^2 + y_{des}^2}$$



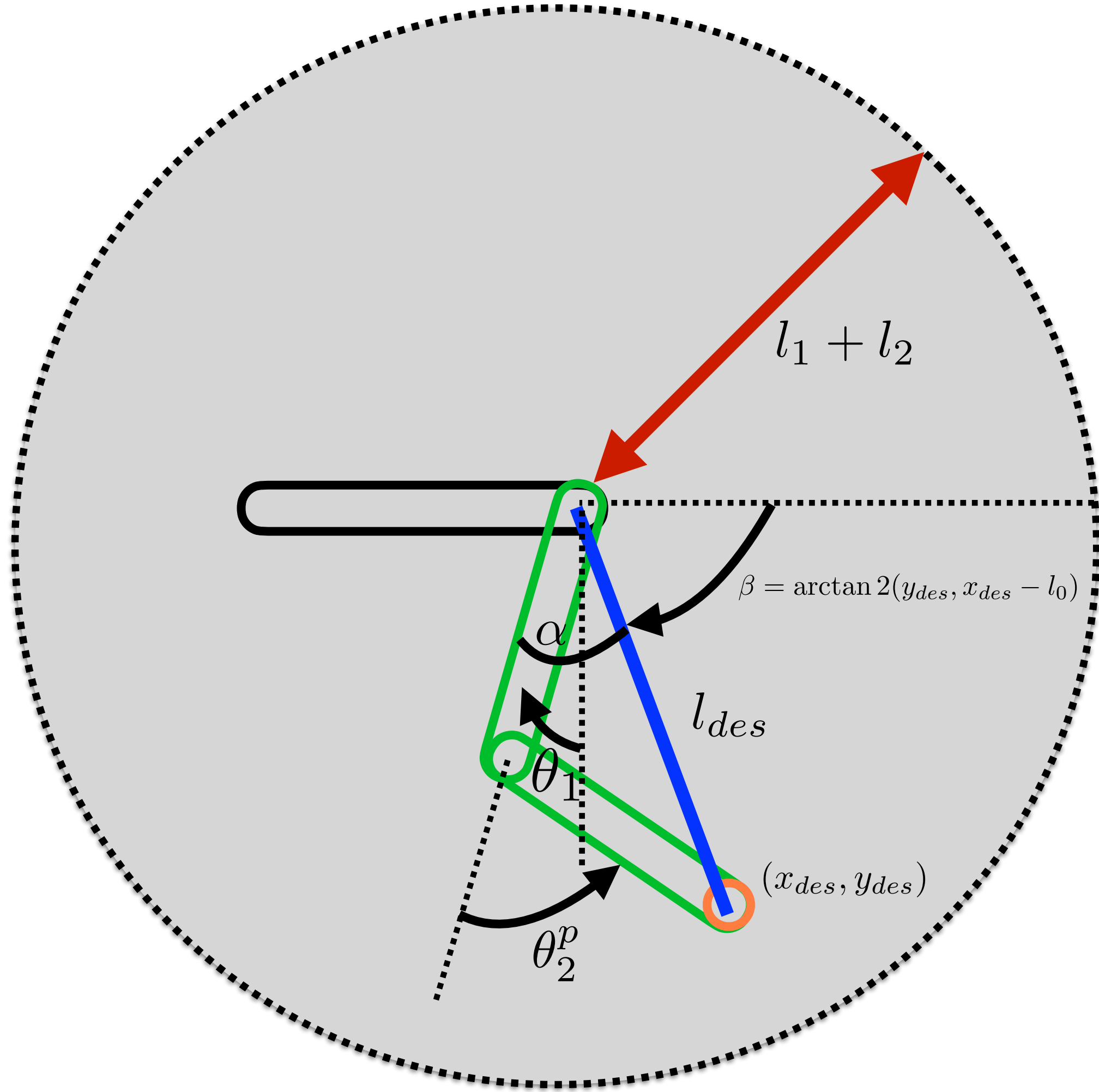
$$l_{des} = \sqrt{(x_{des} - l_0)^2 + y_{des}^2}$$

$$l_{des}^2 - l_1^2 - l_2^2 = -2l_1l_2 \cos(\pi - \theta_2)$$









$$l_{des} = \sqrt{(x_{des} - l_0)^2 + y_{des}^2}$$

$$l_{des}^2 - l_1^2 - l_2^2 = -2l_1l_2 \cos(\pi - \theta_2)$$

$$\theta_2 = \pm \arccos\left(\frac{l_{des}^2 - l_1^2 - l_2^2}{2l_1l_2}\right)$$

