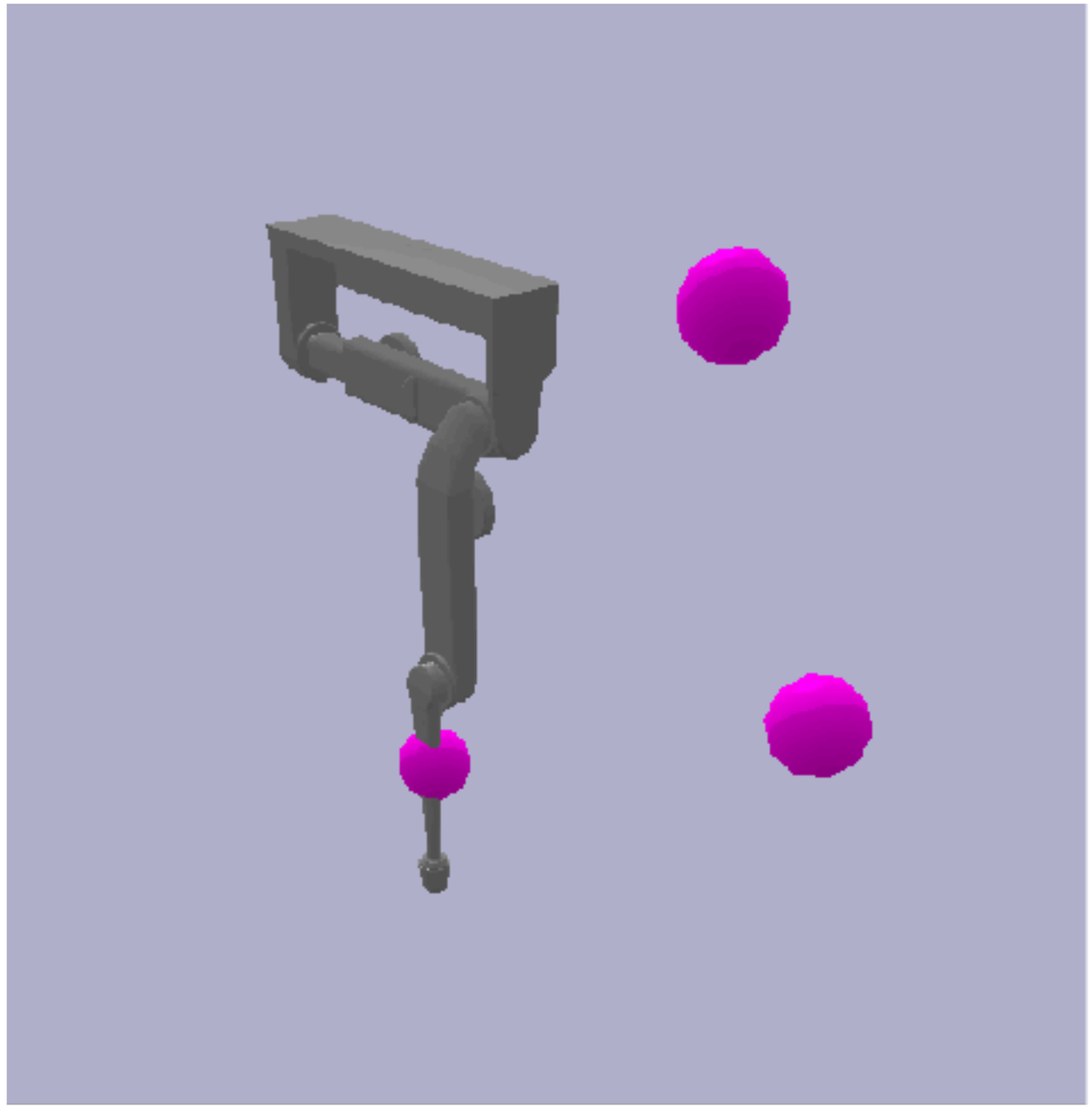
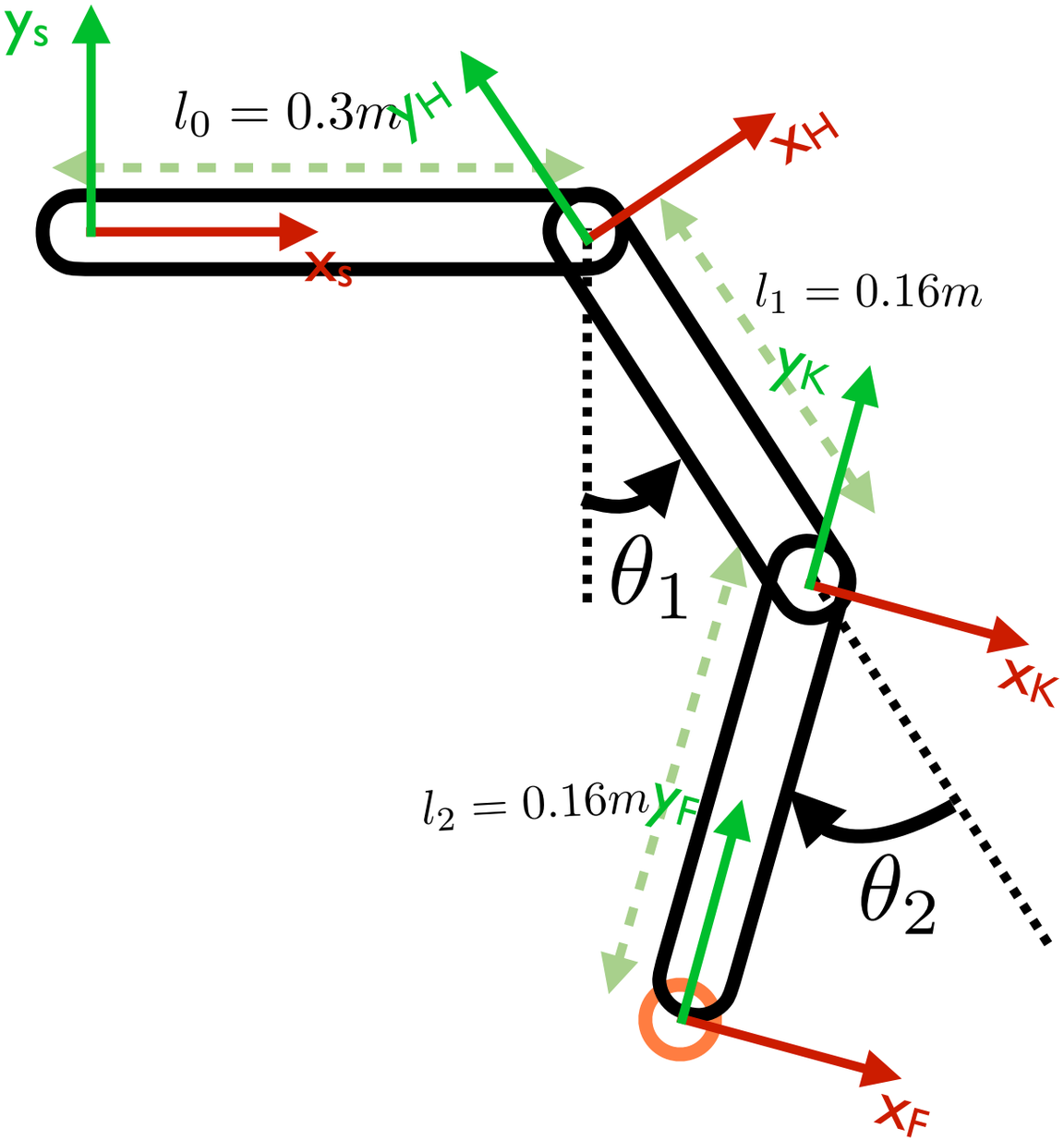
ROB2004
Robotic Manipulation and Locomotion

Laboratory III: Analytic Inverse Kinematics

The goal of the laboratory is to build a complete controller capable of reaching target objects in the environment



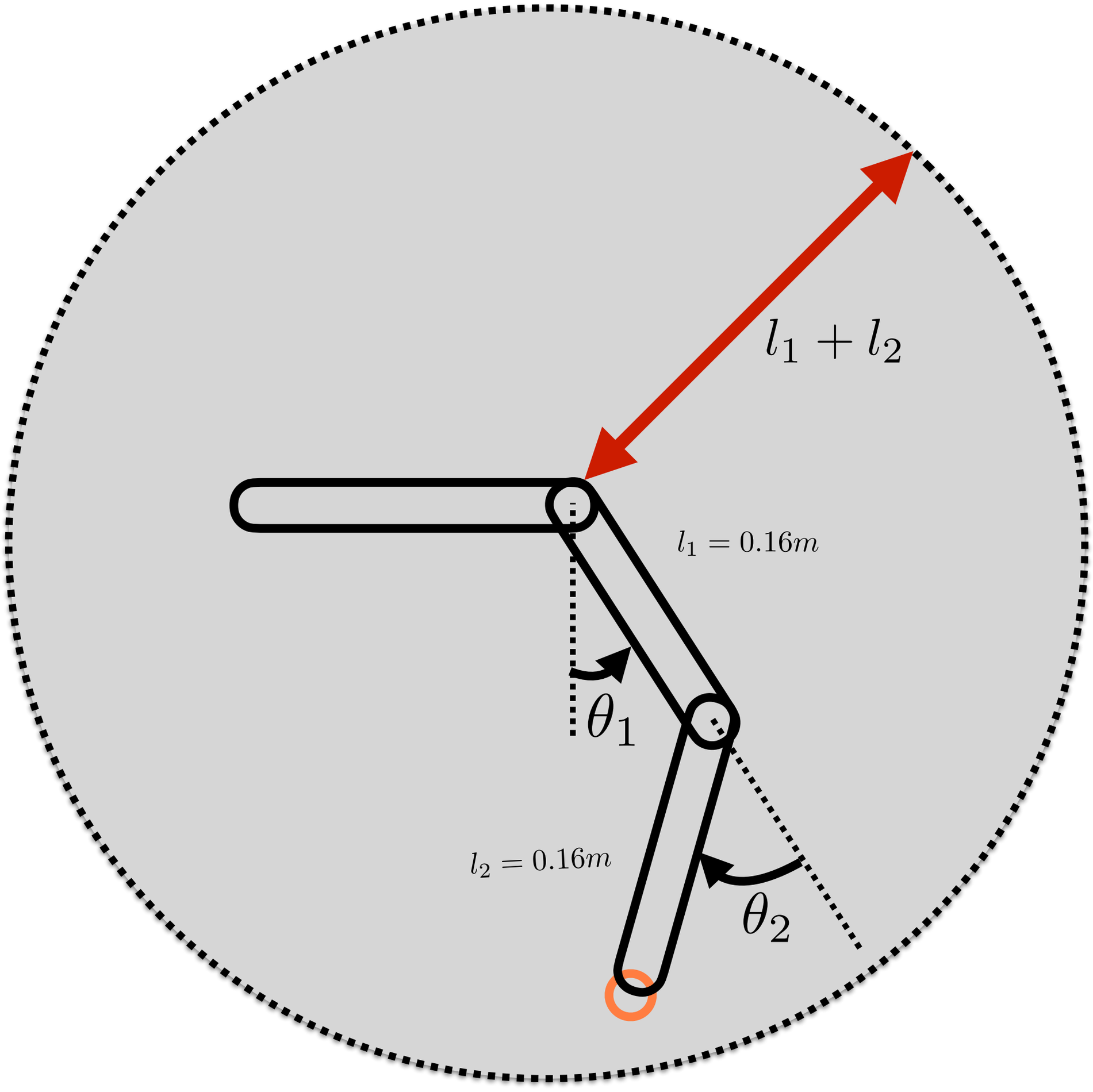
The robot and its kinematics



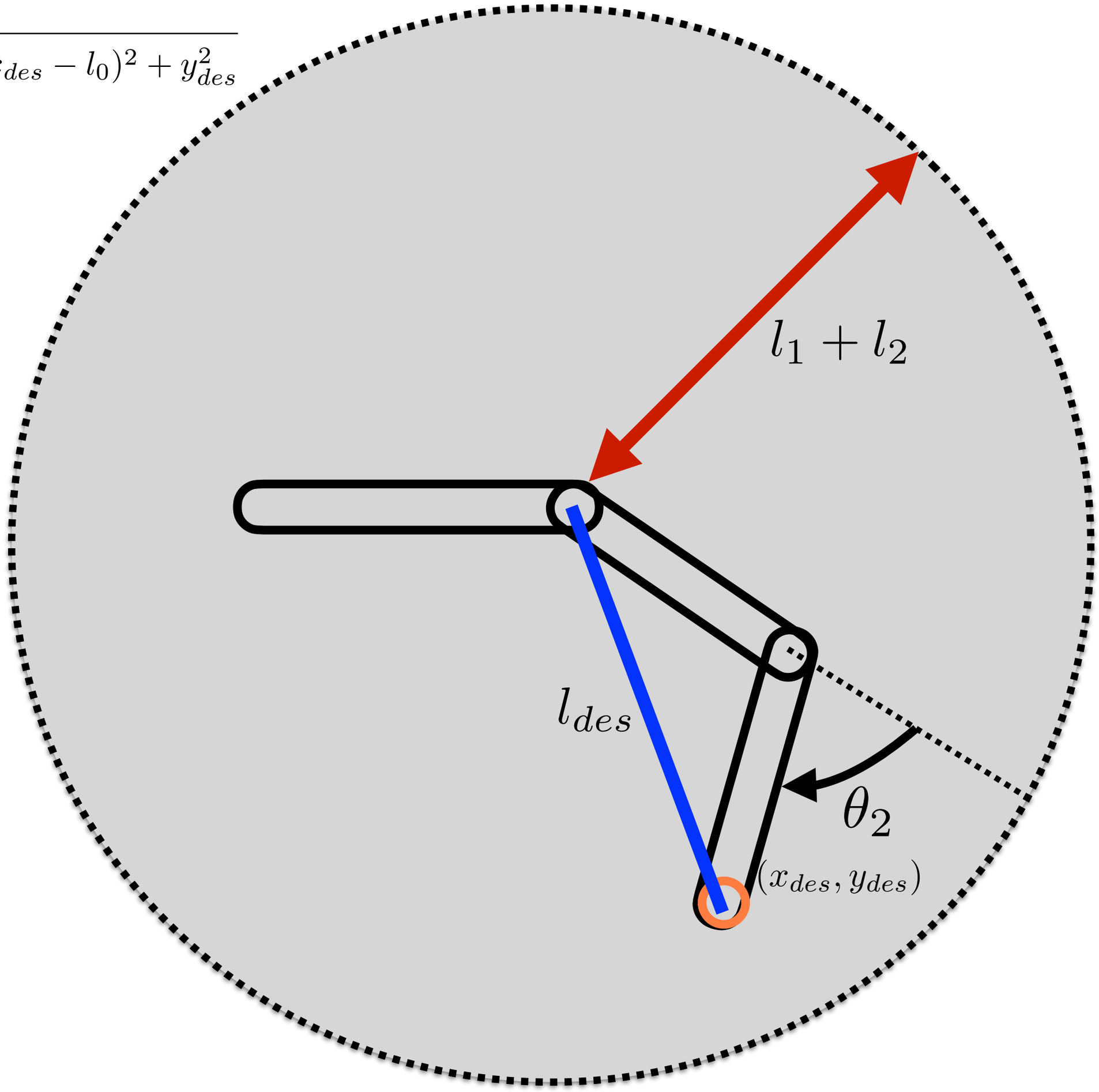
The foot orientation $R_{SF} = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) \end{bmatrix}$

The foot position is $p_{SF} = \begin{pmatrix} l_2 \sin(\theta_1 + \theta_2) + l_1 \sin\theta_1 + l_0 \\ -l_2 \cos\theta_1 + \theta_2) - l_1 \cos\theta 1 \end{pmatrix}$

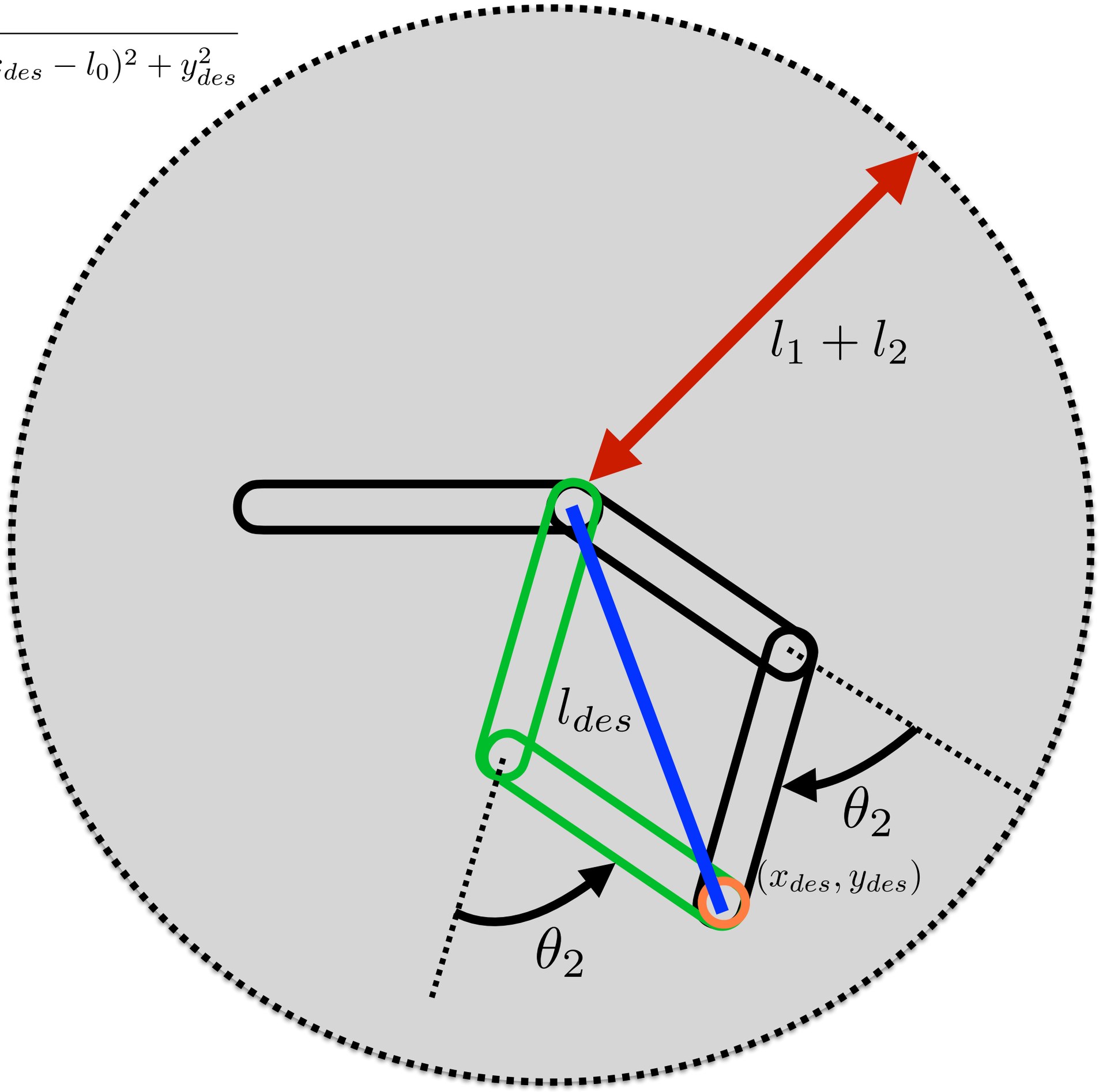
Frame {s} is our fixed frame, i.e. the spatial frame
The hip frame {H} is translated by l_0 and rotated by θ_0 with respect to frame $\{s\}$
The knee frame {K} is translated by l_1 and rotated by θ_1 with respect to frame $\{H\}$
The foot frame {F} is translated by l_2 with respect to frame $\{K\}$


$l_1 + l_2$
$l_1 = 0.16m$
θ_1
$l_2 = 0.16m$
θ_2

$$l_{des} = \sqrt{(x_{des} - l_0)^2 + y_{des}^2}$$

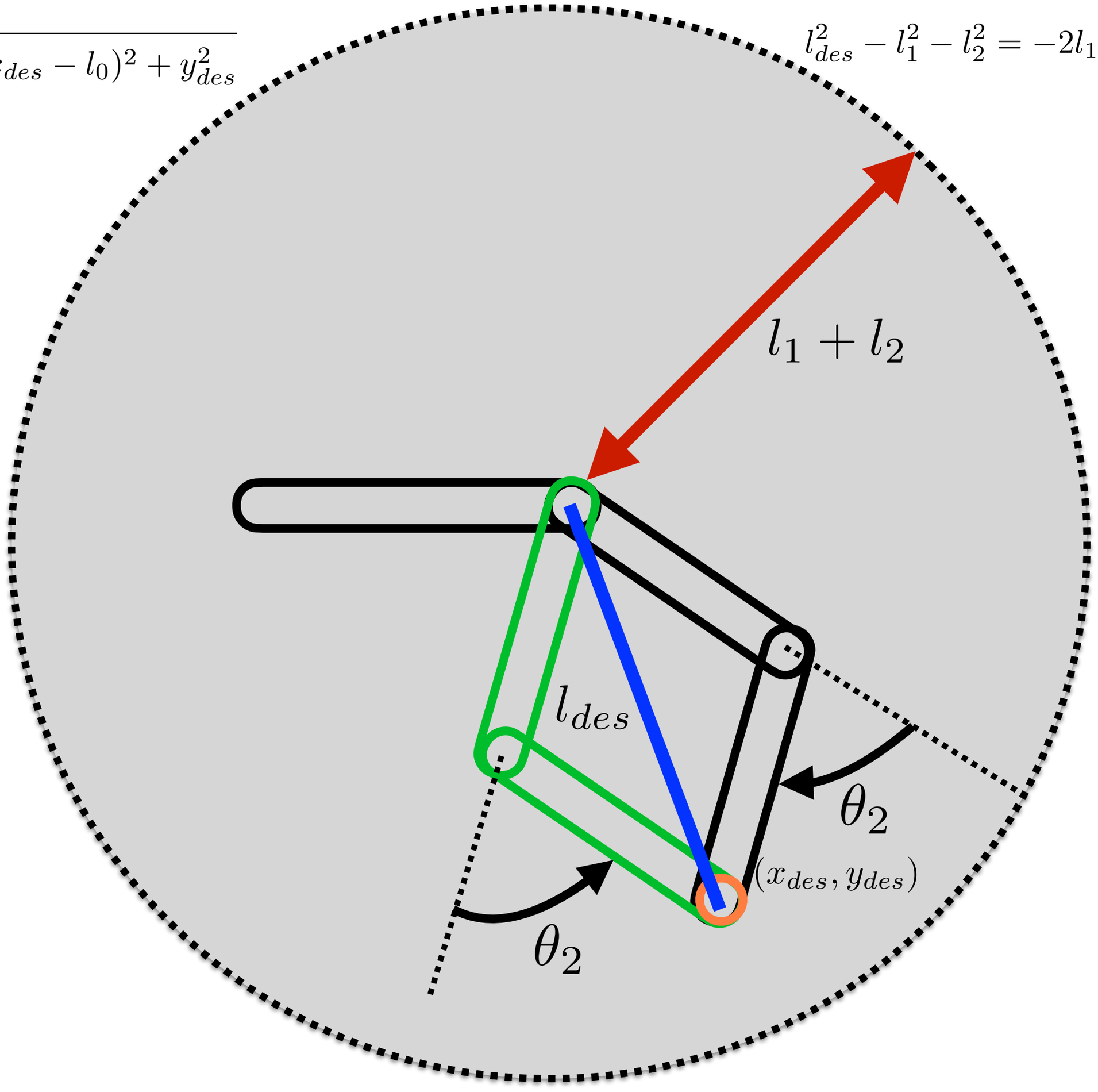


$$l_{des} = \sqrt{(x_{des} - l_0)^2 + y_{des}^2}$$



$$l_{des} = \sqrt{(x_{des} - l_0)^2 + y_{des}^2}$$

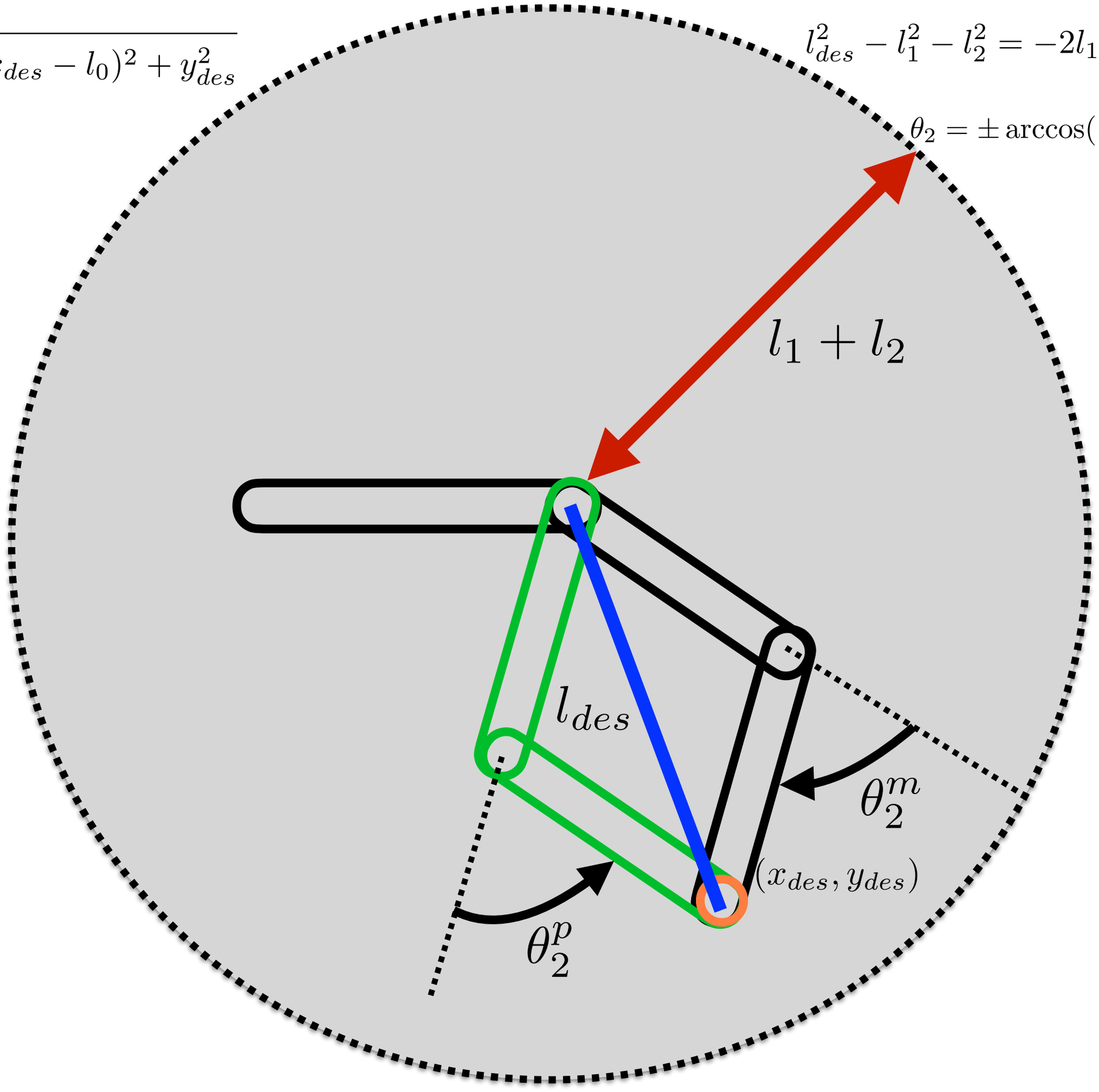
$$l_{des}^2 - l_1^2 - l_2^2 = -2l_1 l_2 \cos(\pi - \theta_2)$$

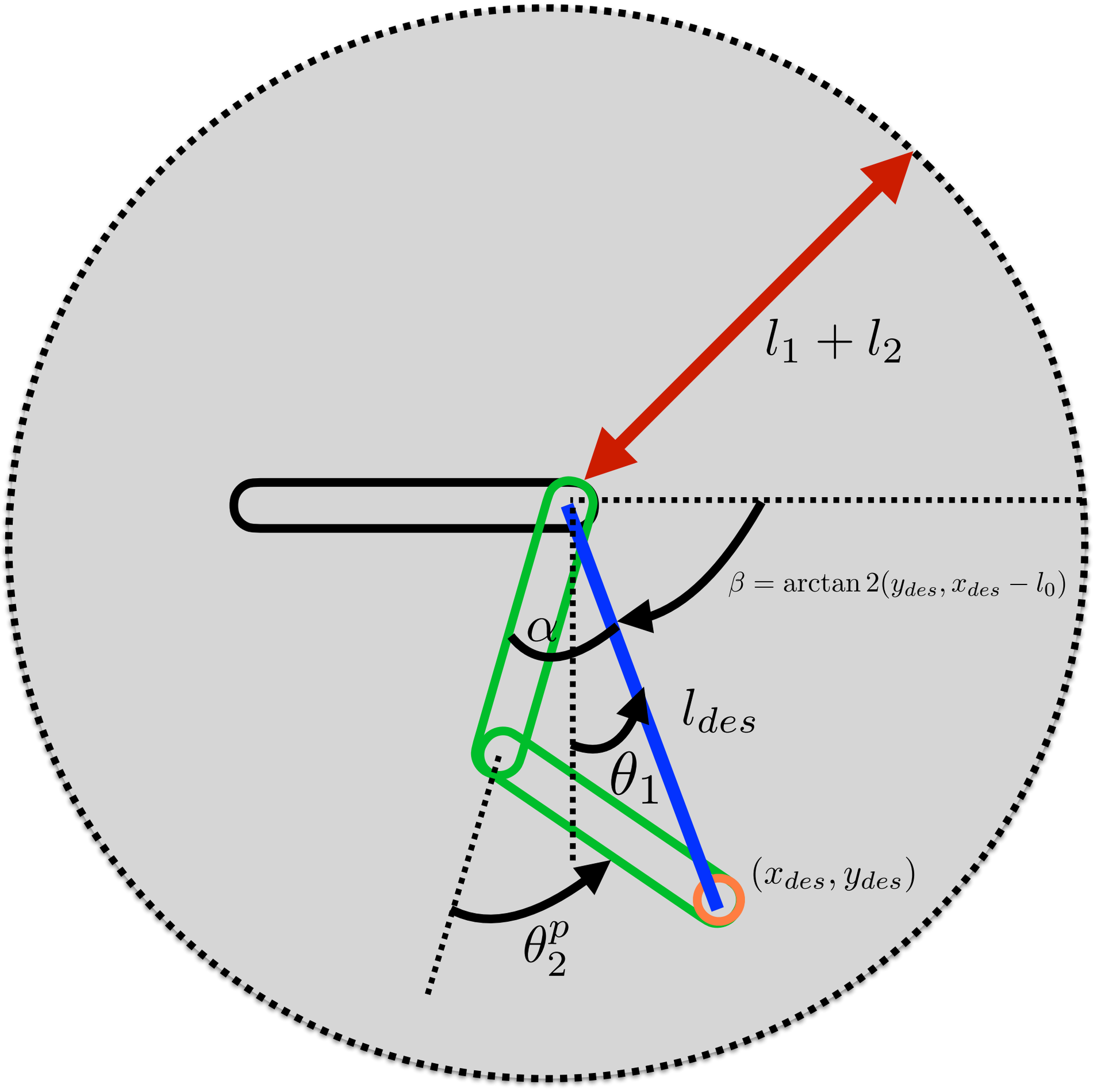


$$l_{des} = \sqrt{(x_{des} - l_0)^2 + y_{des}^2}$$

$$l_{des}^2 - l_1^2 - l_2^2 = -2 l_1 l_2 \cos(\pi - \theta_2)$$

$$\theta_2 = \pm \arccos\left(\frac{l_{des}^2 - l_1^2 - l_2^2}{l_1 l_2}\right)$$




$l_1 + l_2$
$\beta = \arctan 2(y_{des}, x_{des} - l_0)$
α
l_{des}
θ_1
(x_{des}, y_{des})
θ_2^p

$$l_{des} = \sqrt{(x_{des} - l_0)^2 + y_{des}^2}$$

$$l_{des}^2 - l_1^2 - l_2^2 = -2 l_1 l_2 \cos(\pi - \theta_2)$$

$$\theta_2 = \pm \arccos\left(\frac{l_{des}^2 - l_1^2 - l_2^2}{l_1 l_2}\right)$$