

Semântica operacional para a linguagem IMP

- \mathbb{Z} - n, m
- $T = \{\text{true}, \text{false}\}$
- $Vars$ - variáveis de programa - X, Y
- $Aexp$ - expressões aritméticas - a
- $Bexp$ - expressões booleanas - b
- Com - comandos - c
- $Aexp$:
 $a ::= n \mid X \mid a_0 + a_1 \mid a_0 - a_1 \mid a_0 \times a_1$
- $Bexp$:
 $b ::= \text{true} \mid \text{false} \mid a_0 = a_1 \mid a_0 \leq a_1 \mid \neg b \mid b_0 \wedge b_1 \mid b_0 \vee b_1$
- Com :
 $c ::= \text{skip} \mid X := a \mid c_0; c_1 \mid \text{if } b \text{ then } c_0 \text{ else } c_1 \mid \text{while } b \text{ do } c$
- Σ - conjunto de estados
- $\sigma : Vars \longrightarrow \mathbb{Z}$

Avaliação de expressões aritméticas

- $\langle n, \sigma \rangle \rightarrow n$
- $\langle X, \sigma \rangle \rightarrow \sigma(X)$
- $\frac{\langle a_0, \sigma \rangle \rightarrow n \quad \langle a_1, \sigma \rangle \rightarrow m}{\langle a_0 + a_1, \sigma \rangle \rightarrow k}$ para $k = n + m$
- $\frac{\langle a_0, \sigma \rangle \rightarrow n \quad \langle a_1, \sigma \rangle \rightarrow m}{\langle a_0 - a_1, \sigma \rangle \rightarrow k}$ para $k = n - m$
- $\frac{\langle a_0, \sigma \rangle \rightarrow n \quad \langle a_1, \sigma \rangle \rightarrow m}{\langle a_0 \times a_1, \sigma \rangle \rightarrow k}$ para $k = n * m$

Avaliação de expressões booleanas

- $\langle \text{true}, \sigma \rangle \rightarrow \text{true}$
- $\langle \text{false}, \sigma \rangle \rightarrow \text{false}$
- $\frac{\langle a_0, \sigma \rangle \rightarrow n \quad \langle a_1, \sigma \rangle \rightarrow m}{\langle a_0 = a_1, \sigma \rangle \rightarrow \text{true}}$ se $n = m$

- $$\frac{\langle a_0, \sigma \rangle \rightarrow n \quad \langle a_1, \sigma \rangle \rightarrow m}{\langle a_0 = a_1, \sigma \rangle \rightarrow \mathbf{false}} \quad \text{se } n \neq m$$
- $$\frac{\langle a_0, \sigma \rangle \rightarrow n \quad \langle a_1, \sigma \rangle \rightarrow m}{\langle a_0 \leq a_1, \sigma \rangle \rightarrow \mathbf{true}} \quad \text{se } n \leq m$$
- $$\frac{\langle a_0, \sigma \rangle \rightarrow n \quad \langle a_1, \sigma \rangle \rightarrow m}{\langle a_0 \leq a_1, \sigma \rangle \rightarrow \mathbf{false}} \quad \text{se } n \not\leq m$$
- $$\frac{\langle b, \sigma \rangle \rightarrow \mathbf{true}}{\langle \neg b, \sigma \rangle \rightarrow \mathbf{false}} \quad \frac{\langle b, \sigma \rangle \rightarrow \mathbf{false}}{\langle \neg b, \sigma \rangle \rightarrow \mathbf{true}}$$
- $$\frac{\frac{\langle b_0, \sigma \rangle \rightarrow \mathbf{false}}{\langle b_0 \wedge b_1, \sigma \rangle \rightarrow \mathbf{false}} \quad \frac{\langle b_0, \sigma \rangle \rightarrow \mathbf{true} \quad \langle b_1, \sigma \rangle \rightarrow \mathbf{false}}{\langle b_0 \wedge b_1, \sigma \rangle \rightarrow \mathbf{false}}}{\frac{\langle b_0, \sigma \rangle \rightarrow \mathbf{true} \quad \langle b_1, \sigma \rangle \rightarrow \mathbf{true}}{\langle b_0 \wedge b_1, \sigma \rangle \rightarrow \mathbf{true}}}$$
- $$\frac{\frac{\langle b_0, \sigma \rangle \rightarrow \mathbf{true}}{\langle b_0 \vee b_1, \sigma \rangle \rightarrow \mathbf{true}} \quad \frac{\langle b_0, \sigma \rangle \rightarrow \mathbf{false} \quad \langle b_1, \sigma \rangle \rightarrow \mathbf{true}}{\langle b_0 \vee b_1, \sigma \rangle \rightarrow \mathbf{true}}}{\frac{\langle b_0, \sigma \rangle \rightarrow \mathbf{false} \quad \langle b_1, \sigma \rangle \rightarrow \mathbf{false}}{\langle b_0 \wedge b_1, \sigma \rangle \rightarrow \mathbf{false}}}$$

Execução de comandos

- $\langle \mathbf{skip}, \sigma \rangle \rightarrow \sigma$
- $$\frac{\langle a, \sigma \rangle \rightarrow m}{\langle X := a, \sigma \rangle \rightarrow \sigma[m/X]}$$
- $$\frac{\langle c_0, \sigma \rangle \rightarrow \sigma' \quad \langle c_0, \sigma' \rangle \rightarrow \sigma''}{\langle c_0; c_1, \sigma \rangle \rightarrow \sigma''}$$
- $$\frac{\langle b, \sigma \rangle \rightarrow \mathbf{true} \quad \langle c_0, \sigma \rangle \rightarrow \sigma'}{\langle \mathbf{if } b \text{ then } c_0 \text{ else } c_1, \sigma \rangle \rightarrow \sigma'} \quad \frac{\langle b, \sigma \rangle \rightarrow \mathbf{false} \quad \langle c_1, \sigma \rangle \rightarrow \sigma'}{\langle \mathbf{if } b \text{ then } c_0 \text{ else } c_1, \sigma \rangle \rightarrow \sigma'}$$
- $$\frac{\langle b, \sigma \rangle \rightarrow \mathbf{false}}{\langle \mathbf{while } b \text{ do } c, \sigma \rangle \rightarrow \sigma} \quad \frac{\langle b, \sigma \rangle \rightarrow \mathbf{true} \quad \langle c, \sigma \rangle \rightarrow \sigma' \quad \langle \mathbf{while } b \text{ do } c, \sigma' \rangle \rightarrow \sigma''}{\langle \mathbf{while } b \text{ do } c, \sigma \rangle \rightarrow \sigma''}$$