A Clockwork RNN

Jan Koutník HKOU@IDSIA.CH
Klaus Greff KLAUS@IDSIA.CH
Faustino Gomez TINO@IDSIA.CH
Jürgen Schmidhuber JUERGEN@IDSIA.CH
IDSIA, USI&SUPSI, Manno-Lugano, CH-6928, Switzerland

Abstract

Sequence prediction and classification are ubiquitous and challenging problems in machine learning that can require identifying complex dependencies between temporally distant inputs. Recurrent Neural Networks (RNNs) have the ability, in theory, to cope with these temporal dependencies by virtue of the short-term memory implemented by their recurrent (feedback) connections. However, in practice they are difficult to train successfully when long-term memory is required. This paper introduces a simple, yet powerful modification to the simple RNN (SRN) architecture, the *Clockwork RNN* (CW-RNN), in which the hidden layer is partitioned into separate modules, each processing inputs at its own temporal granularity, making computations only at its prescribed clock rate. Rather than making the standard RNN models more complex, CW-RNN reduces the number of SRN parameters, improves the performance significantly in the tasks tested, and speeds up the network evaluation. The network is demonstrated in preliminary experiments involving three tasks: audio signal generation, TIMIT spoken word classification, where it outperforms both SRN and LSTM networks, and online handwriting recognition, where it outperforms SRNs.

1. Introduction

Recurrent Neural Networks (RNNs; Robinson & Fallside, 1987; Werbos, 1988; Williams, 1989) are a class of connectionist models that possess internal state or *short-term memory* due to recurrent feed-back connections, that make them suitable for dealing with sequential problems, such as speech classification, prediction and generation.

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Simple RNNs trained with stochastic gradient-descent have difficulty learning long-term dependencies (i.e. spanning more that 10 time-steps) encoded in the input sequences due to *vanishing gradient* (Hochreiter, 1991; Hochreiter et al., 2001). This problem has been addressed for example by using a specialized neuron structure, or cell, in Long Short-Term Memory (LSTM) networks (Hochreiter & Schmidhuber, 1997) that maintains constant backward flow in the error signal; second-order optimization methods (Martens & Sutskever, 2011) preserve the gradient by estimating its curvature; or using informed random initialization (Sutskever et al., 2013) which allows for training the networks with momentum and stochastic gradient-descent only.

This paper presents a novel modification to the simple RNN (SRN; Elman, 1988) architecture and, *mutatis mutandis*, an associated error back-propagation through time (Rumelhart et al., 1986; Werbos, 1988; Williams, 1989) training algorithm, that show superior performance in the generation and classification of sequences that contain long-term dependencies. Here, the long-term dependency problem is addressed by having different parts (modules) of the RNN hidden layer run at different clock speeds, timing their computation with different, discrete clock periods, hence the name Clockwork Recurrent Neural Network (CW-RNN). CW-RNNs train and evaluate faster since not all modules are executed at every time step, and have a smaller number of weights compared to SRNs, because slower modules do not receive inputs from faster ones.

The next section provides an overview of the related work, section 3 describes the CW-RNN architecture in detail. Experiments with audio signal generation, spoken word classification using the TIMIT dataset and online handwriting recognition are presented in section 4. Sections 5 and 6 discuss the results of experiments and future potential of CW-RNNs, respectively.

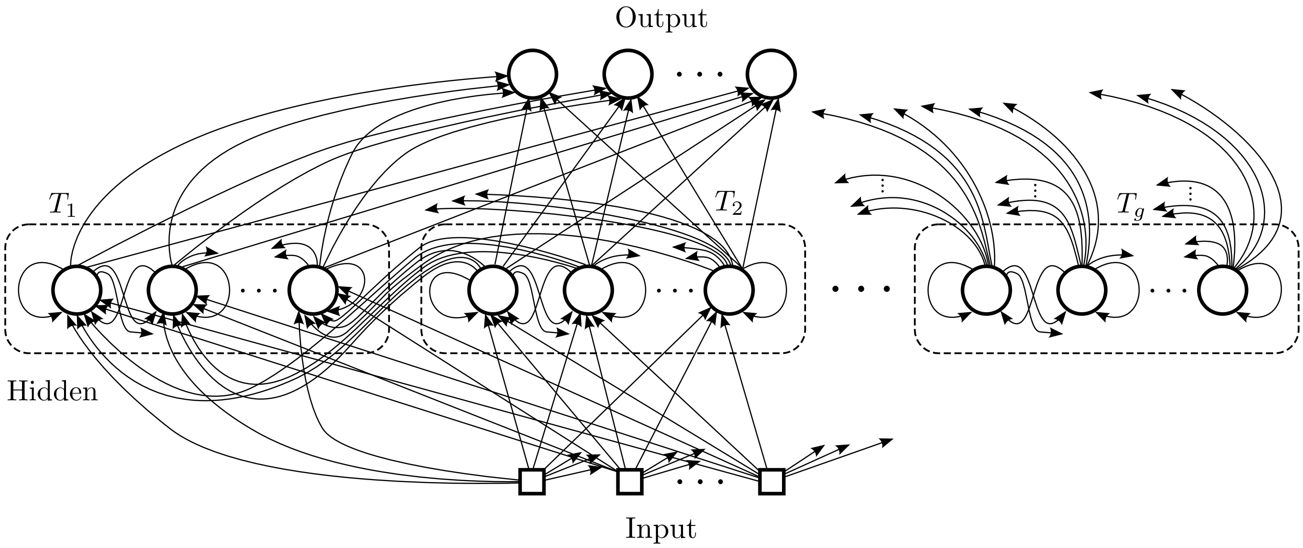

Figure 1. CW-RNN architecture is similar to a simple RNN with an input, output and hidden layer. The hidden layer is partitioned into g modules each with its own clock rate. Within each module the neurons are fully interconnected. Neurons in faster module i are connected to neurons in a slower module j only if a clock period $T_i < T_j$.

2. Related Work

Contributions to sequence modeling and recognition that are relevant to CW-RNNs are presented in this section. The primary focus is on RNN extensions that deal with the problem of bridging long time lags.

Hierarchical Recurrent Neural Networks (Hihi & Bengio, 1996) are the most similar to CW-RNNs work, having both delayed connections and units operating at different time-scales. Five hand-designed architectures were tested on simple long time-lag toy-problems finding that multi-time-scale units significantly help in learning those tasks.

Another similar model is the NARX RNN[1] (Lin et al., 1996). But, instead of simplifying the network, it introduces additional sets of recurrent connections with time lags of 2,3..k time steps. These additional connections help to bridge long time lags, but introduce many additional parameters that make NARX RNN training more difficult and run k times slower.

Long Short-Term Memory (LSTM; Hochreiter & Schmidhuber, 1997) uses a specialized architecture that allows information to be stored in linear units called *constant error carousels* (CECs) indefinitely. CECs are contained in *cells* that have a set of multiplicative units (gates) connected to other cells that regulate when new information enters the CEC (input gate), when the activation of the CEC is output to the rest of the network (output gate), and when the activation decays or is ”forgotten” (forget gate). These networks have been very successful recently in speech and handwriting recognition (Graves et al., 2005; 2009; Sak et al., 2014).

Stacking LSTMs (Fernandez et al., 2007; Graves & Schmidhuber, 2009) into a hierarchical sequence processor, equipped with Connectionist Temporal Classification (CTC; Graves et al., 2006), performs simultaneous segmentation and recognition of sequences. Its deep variant currently holds the state-of-the-art result in phoneme recognition on the TIMIT database (Graves et al., 2013).

Temporal Transition Hierarchies (TTHs; Ring, 1993) incrementally add high-order neurons in order to build a memory that is used to disambiguate an input at the current time step. This approach can, in principle, bridge time intervals of any length, but at a cost of a proportional growth in network size. The model was recently improved by adding recurrent connections (Ring, 2011) that reduces the number of high-order neurons needed to encode the temporal dependencies.

One of the earliest attempts to enable RNNs to handle long-term dependencies is the Reduced Description Network (Mozer, 1992; 1994). It uses leaky neurons whose activation changes only by a small amount in response to its inputs. This technique was recently picked up by Echo State Networks (ESN; Jaeger, 2002).

A similar technique has been used by Sutskever & Hinton (2010) to solve some serial recall tasks. These Temporal-Kernel RNNs add a weighted connection from each neuron to itself that decays exponentially in time. Its performance is still slightly inferior to LSTM.

Evolino (Schmidhuber et al., 2005; 2007) feeds the input

[1]NARX stands for Non-linear Auto-Regressive model with eXogeneous inputs

to an RNN (which can be e.g. LSTM to cope with long time lags) and then transforms the RNN outputs to the target sequences via an optimal linear mapping, that is computed analytically by pseudo-inverse. The RNN is trained by an evolutionary algorithm, and therefore does not suffer from the vanishing gradient problem. Evolino outperformed LSTM on a set of synthetic problems and was used to perform complex robotic manipulation (Mayer et al., 2006).

A modern theory of why RNNs fail to learn long-term dependencies is that simple gradient-descent fails to optimize them correctly. One attempt to mitigate this problem is Hessian Free (HF) optimization (Martens & Sutskever, 2011), an adapted second-order training method that has been demonstrated to work well with RNNs. HF allows RNNs to solve some long-term lag problems that are considered challenging for stochastic gradient-descent. Their performance on rather synthetic, long-term memory benchmarks is approaching that of LSTM, though the number of optimization steps required in HF-RNN is usually greater. Training networks by HF optimization is an orthogonal approach to the network architecture, so both LSTM and CW-RNN can still benefit from it.

HF optimization allowed for training of Multiplicative RNN (MRNN; Sutskever et al., 2011) that port the concept of multiplicative gating units to SRNs. The gating units are represented by a factored 3-way tensor in order to reduce the number of parameters. Extensive training of an MRNN on a GPU provided impressive results in text generation.

Training RNNs with Kalman filters (Pérez-Ortiz et al., 2003; Williams, 1992) has shown advantages in bridging long time lags as well, although this approach is computationally unfeasible for larger networks.

The methods mentioned above are strictly synchronous–elements of the network clock at the same speed. The *Sequence Chunker*, Neural History Compressor or Hierarchical Temporal Memory (Schmidhuber, 1991; 1992) consists of a hierarchy or stack of RNN that may run at different time scales, but, unlike the simpler CW-RNN, it requires unsupervised event predictors: a higher-level RNN receives an input only when the lower-level RNN below is unable to predict it. Hence the clock of the higher level may speed up or slow down, depending on the current predictability of the input stream. This contrasts with the CW-RNN, in which the clocks always run at the same speed, some slower, some faster.

3. A Clockwork Recurrent Neural Network

Clockwork Recurrent Neural Networks (CW-RNN) like SRNs, consist of input, hidden and output layers. There are forward connections from the input to hidden layer, and from the hidden to output layer, but, unlike the SRN,

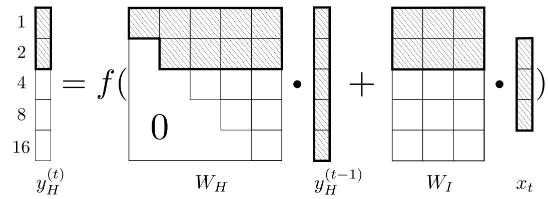

Figure 2. Calculation of the hidden unit activations at time step $t = 6$ in CW-RNN according to equation (1). Input and recurrent weight matrices are partitioned into blocks. Each block-row in $\mathbf{W}_H$ and $\mathbf{W}_I$ corresponds to the weights of a particular module. At time step $t = 6$, the first two modules with periods $T_1 = 1$ and $T_2 = 2$ get evaluated (highlighted parts of $\mathbf{W}_H$ and $\mathbf{W}_I$ are used) and the highlighted outputs are updated. Note that, while using exponential series of periods, the active parts of $\mathbf{W}_H$ and $\mathbf{W}_I$ are always contiguous.

the neurons in the hidden layer are partitioned into g modules of size k. Each of the modules is assigned a clock period $T_n \in \{T_1, \ldots, T_g\}$. Each module is internally fully-interconnected, but the recurrent connections from module j to module i exists only if the period T_i is smaller than period T_j. Sorting the modules by increasing period, the connections between modules propagate the hidden state *right-to-left*, from slower modules to faster modules, see Figure 1.

The SRN output, $y_O^{(t)}$, at a time step t is calculated using the following equations:

$$\mathbf{y}_H^{(t)} = f_H(\mathbf{W}_H \cdot \mathbf{y}^{(t-1)} + \mathbf{W}_I \cdot \mathbf{x}^{(t)}), \tag{1}$$

$$\mathbf{y}_O^{(t)} = f_O(\mathbf{W}_O \cdot \mathbf{y}_H^{(t)}), \tag{2}$$

where $\mathbf{W}_H$, $\mathbf{W}_I$ and $\mathbf{W}_O$ are the hidden, input and output weight matrices, $\mathbf{x}_t$ is the input vector, and $\mathbf{y}_H^{(t)}$ is a vector representing the activation of the hidden units at time step t. Functions $f_H(.)$ and $f_O(.)$ are non-linear activation functions. For simplicity, neuron biases are omitted from the equations.

The main difference between CW-RNN and a SRN is that at each CW-RNN time step t, only the output of modules i that satisfy $(t \text{ MOD } T_i) = 0$ are active. The choice of the set of periods $\{T_1, \ldots, T_g\}$ is arbitrary. In this paper, we use the exponential series of periods: module i has clock period of $T_i = 2^{i-1}$.

Matrices $\mathbf{W}_H$ and $\mathbf{W}_I$ are partitioned into g blocks-rows:

$$\mathbf{W}_H = \begin{pmatrix} \mathbf{W}_{H_1} \\ \vdots \\ \mathbf{W}_{H_g} \end{pmatrix} \qquad \mathbf{W}_I = \begin{pmatrix} \mathbf{W}_{I_1} \\ \vdots \\ \mathbf{W}_{I_g} \end{pmatrix}, \tag{3}$$

and $\mathbf{W}_H$ is a block-upper triangular matrix, where each block-row, $\mathbf{W}_{H_i}$, is partitioned into block-columns

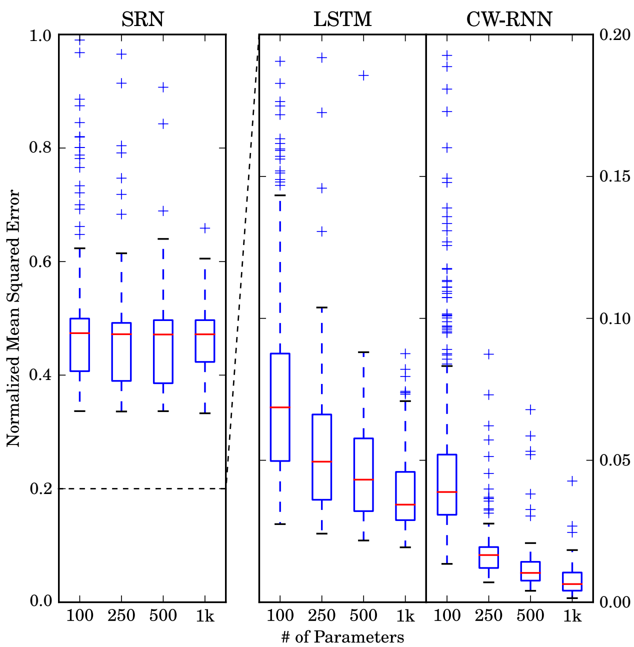

Figure 3. Normalized mean squared error for the **sequence generation task**, with one box-whisker (showing mean value, 25% and 75% quantiles, minimum, maximum and outliers) for each tested network size and type. Note that the plot for the SRN has a different scale than the other two.

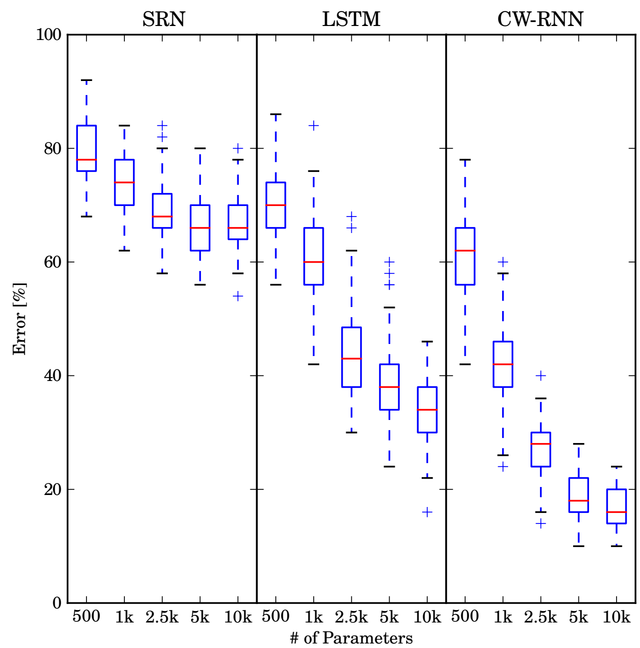

Figure 4. Classification error for the **word classification task**, with one box-whisker for each tested network size and type.

$\{\mathbf{0}_1, \ldots, \mathbf{0}_{i-1}, \mathbf{W}_{\mathbf{H_{i,i}}}, \ldots, \mathbf{W}_{\mathbf{H_{i,g}}}\}$. At each time step of a forward pass, only the block-rows of $\mathbf{W}_H$ and $\mathbf{W}_I$ that correspond to the active modules are used for evaluation in Equation (1):

$$\mathbf{W}_{H_i} = \begin{cases} \mathbf{W}_{H_i} & \text{if } (t \text{ MOD } T_i) = 0 \\ \mathbf{0} & \text{otherwise,} \end{cases} \tag{4}$$

and the corresponding parts of the output vector, $\mathbf{y}_H$, are updated. The other modules retain their output values from the previous time-step. Calculation of the hidden activation at time step $t = 6$ is illustrated in Figure 2.

As a result, the low-clock-rate modules process, retain and output the long-term information obtained from the input sequences (not being distracted by the high speed modules), whereas the high-speed modules focus on the local, high-frequency information (having the context provided by the low speed modules available).

The backward pass of the error propagation is similar to SRN as well. The only difference is that the error propagates only from modules that were executed at time step t. The error of non-activated modules gets copied back in time (similarly to copying the activations of nodes not activated at the time step t during the corresponding forward pass), where it is added to the back-propagated error.

A CW-RNN with the same number of nodes runs much faster than a SRN since not all modules are evaluated at every time step. The lower bound for the CW-RNN speedup compared to a SRN with the same number of neurons is $g/4$ in the case of this exponential clock setup, see Appendix for a detailed derivation.

4. Experiments

CW-RNNs were compared to the SRN and LSTM networks. All networks have one hidden layer with the *tanh* activation function, and the number of nodes in the hidden layer was chosen to obtain (approximately) the same number of parameters for all three methods (in the case of CW-RNN, the clock periods were included in the parameter count). activation

Initial values for all the weights were drawn from a Gaussian distribution with zero mean and standard deviation of 0.1. Initial values of all internal state variables for all hidden activations were set to 0. Each setup was run 100 times with different random initialization of parameters. All networks were trained using Stochastic Gradient Descent (SGD) with Nesterov-style momentum (Sutskever et al., 2013). initialization optimizer

4.1. Sequence Generation

The goal of this task is to train a recurrent neural network, that receives no input, to generate a target sequence as accurately as possible. The weights of the network can be seen

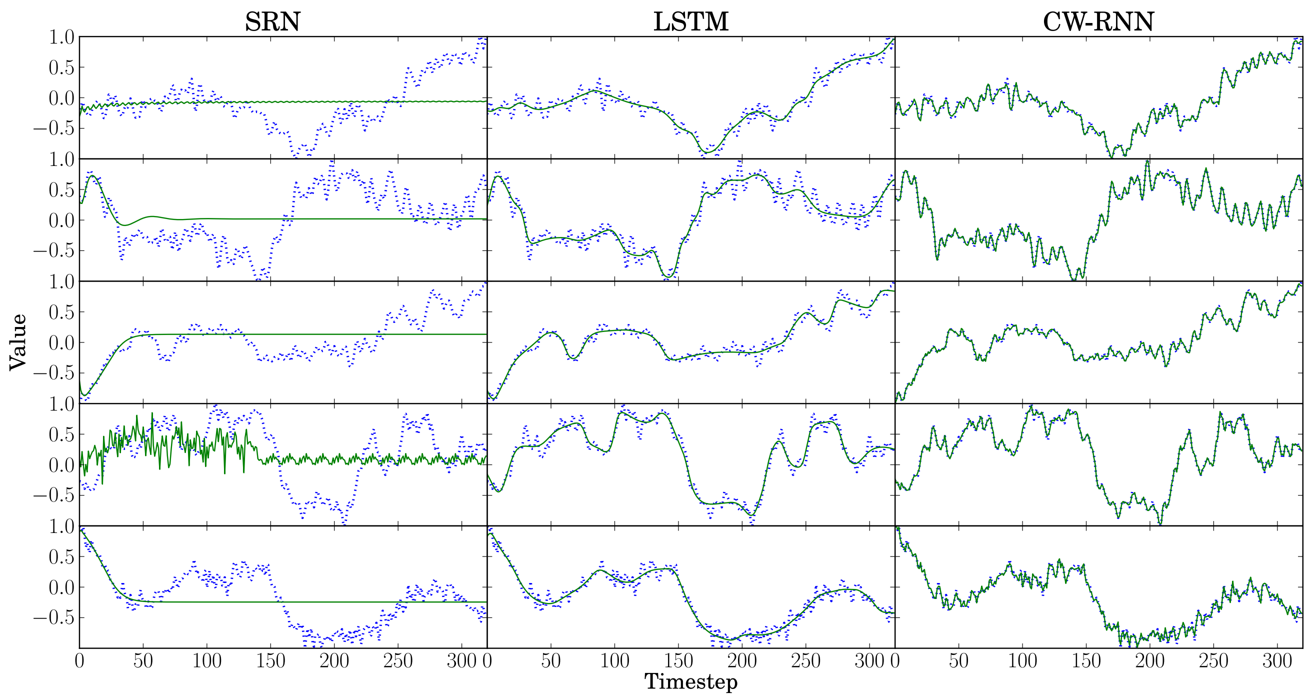

Figure 5. Sequence generation task. Output of the best performing network (solid, green) compared to the target signal (dotted, blue) for each method (column) and each training sequence (row). SRN tends to learn the first few steps of the sequence and then generates the mean of the remaining portion, while the output of LSTM resembles a sliding average, and CW-RNN approximates the sequence much more accurately.

Table 1. Number of hidden neurons (cells in the case of LSTM) for SRN, LSTM and CW-RNN for each network size specified in terms of the number of parameters (weights).

Task	# of Params.	SRN	LSTM	CW-RNN
Sequence generation	100	9	4	11
	250	15	7	19
	500	22	10	27
	1000	31	15	40
Spoken word classif.	500	10	5	10
	1000	18	8	19
	2500	34	17	40
	5000	54	26	65
	10000	84	41	102
Online handwriting recognition	10 k	40	25	45
	50 k	121	67	148
	100 k	185	100	231

as a (lossy) encoding of the whole sequence, which could be used for compression.

Five different target sequences were created by sampling a piece of music[2] at 44.1 Hz for 7 ms. The resulting sequences of 320 data points were scaled to the interval $[-1, 1]$.

All networks used the same architecture: no inputs, one hidden layer and a single linear output neuron. Each network type was run with 4 different sizes: 100, 250, 500, and 1000 parameters, see Table 1 for the summary of number of hidden nodes. The networks were trained for 2000 epochs to minimize the mean squared error. Momentum was set to 0.95, with a learning that was optimized separately for each method, but kept the same for all network sizes: 3.0×10^{-4} for SRN and CW-RNN, and 3.0×10^{-5} for LSTM. For LSTM it was also crucial to initialize the bias of the forget gates to a high value (5.0 in this case) to encourage the long-term memory. The hidden units of CW-RNN were divided into nine aproximately equally sized groups with exponential clock-timings $\{1, 2, 4, \ldots, 256\}$. Whenever the number of neurons was not divisible by 9, the remaining neurons were distributed among the faster modules.

[2]taken from the beginning of the first track *Manýrista* of album *Musica Deposita* by *Cuprum*

The results of the experiments are shown in Figure 3. It is obvious that SRNs fail to generate the target sequence, and they do not seem to improve with network size. LSTM does much better, especially as the networks get bigger. CW-RNNs give by far the best results, with the smallest one being roughly on par with the second-biggest LSTM network. Also, all but the smallest CW-RNN have significantly less variance than all the other methods. To get an intuitive understanding of what is happening, Figure 5 shows the output of the best network of each type on each of the five audio samples. The average error of the best networks is shown in Table 2 (row 1).

Table 2. Normalized mean square error and standard deviation (averaged over 100 runs) for the largest (best) LSTM, SRN and CW-RNN on the **sequence generation task**, where the CW-RNN was $5.7\times$ better than LSTM, and the percentage of the wrongly classified words in the **spoken word classification**, where the CW-RNN was more than $2\times$ better than LSTM.

Task	SRN	LSTM	CW-RNN
Seq. gen.	$0.46{\pm}0.08$	$0.04{\pm}0.01$	$0.007{\pm}0.004$
Word classif.	$66.8{\pm}4.7$	$34.2{\pm}5.6$	$16.8{\pm}3.5$

4.2. Spoken Word Classification

The second task is sequence classification instead of generation. Each sequence contains an audio signal of one spoken word from the TIMIT Speech Recognition Benchmark (Garofolo et al., 1993). The dataset contains 25 different words (classes) arranged in 5 groups based on their phonetic suffix. Because of the suffix-similarity, the network needs to learn long-term dependencies in order to disambiguate the words. The words are:

Group 1: `making, walking, cooking, looking, working`

Group 2: `biblical, cyclical, technical, classical, critical`

Group 3: `tradition, addition, audition, recognition, competition`

Group 4: `musicians, discussions, regulations, accusations, conditions`

Group 5: `subway, leeway, freeway, highway, hallway`

For every word there are 7 examples from different speakers, which were partitioned into 5 for training and 2 for testing, for a total of 175 sequences (125 train, 50 test). Each sequence element consists of 12-dimensional MFCC vector (Mermelstein, 1976) plus energy, sampled every 10 ms over a 25 ms window with a pre-emphasis coefficient of 0.97. Each of the 13 channels was normalized to have zero mean and unit variance over the whole training set.

All network types used the same architecture: 13 inputs, a single hidden and a softmax output layer with 25 units. Five hidden layer sizes were chosen such that the total number of parameters for the whole network was roughly 0.5 k, 1 k, 2.5 k, 5 k, and 10 k, see Table 1.

All networks used a learning rate of 3×10^{-4}, a momentum of 0.9, and were trained to minimize the Multinomial Cross Entropy Error. Every experiment was repeated 100 times with different random initializations.

Because the dataset is so small, Gaussian noise with a standard deviation of 0.6 was added to the inputs during training to guard against overfitting. Training was stopped once the error on the *noise-free* training set did not decrease for 5 epochs. To obtain good results with LSTM, it was again important to initialize the forget gate bias to 5. For the CW-RNNs the neurons were divided evenly into 7 groups with exponentially increasing periods: $\{1, 2, 4, 8, 16, 32, 64\}$.

Figure 4 shows the classification error of the different networks on the word classification task. Here again, SRNs perform the worst, followed by LSTMs, which give substantially better results, especially with more parameters. CW-RNNs beat both SRN and LSTM networks by a considerable margin of 8-20% on average irrespective of the number of parameters. The error of the largest networks is summarized in Table 2 (row 2).

4.3. Online Handwriting Recognition

In the third task, online handwriting classification, timed sequences of pen coordinates must be translated into character strings by the network. The dataset (Liwicki & Bunke, 2005) consists of 5364 hand-written lines of text in the training set and 3859 lines in the test set, and two validation sets that were combined to form one validation set of size 2956. Each line consists of various numbers of stroke sequences, where each element is a 3-dimensional vector of (x, y)-coordinates and a time stamp t. All vectors were transformed into 4-dimensions where the first two elements are the x and y coordinates, relative to the top left corner of the line, t is the time relative to the beginning of the line and the fourth dimension contains 1 when the pen was lifted from the writing surface (end of the stroke), zero otherwise. This input format is referred to as *raw* in Graves (2008). All stroke sequences were sub-sampled by a factor of 3 in order to reduce the amount of data. The target strings contain 81 different characters (lower and upper case letters, numbers and special characters). All strokes within a line were merged into a single input sequence and the first 3 dimensions were standardized.

Three different network sizes were tested for each architec-

Table 3. Character error of the three network architectures in the **online handwriting recognition task**. The largest CW-RNN is roughly 20% better than the corresponding SRN. LSTM is still $2\times$ better than CW-RNN.

# of Params.	SRN	LSTM	CW-RNN
10 k	52.8	34.2	67.2
50 k	43.4	21.4	46.5
100 k	47.5	19.2	39.5

ture, such that the number of parameters were roughly 10 k, 50 k and 100 k (see Table 1). The CW-RNN used 9 modules with exponentially increasing periods (up to 256). All the networks have 4 inputs and 82 outputs (all characters + an *empty label*). Each network was bidirectional (Liwicki et al., 2007; Schuster & Paliwal, 1997), i.e. with an additional hidden layer that processes the temporal context backwards.

The networks were trained with the *connectionist temporal classification* (CTC; Graves et al., 2006) error function which calculates a probabilistic mapping error of the network outputs to the desired label sequence. The SRN and CW-RNN learning rate was set to 3.0×10^{-4}, the LSTM learning rate was 3.0×10^{-3}, and momentum was 0.9 in all cases. All the weights were initialized from a Gaussian distribution with zero mean and standard deviation of 0.1 (no forget gate bias for LSTM). The training was stopped after there was no improvement of the error on the validation set for 50 epochs.

At test time we report the Character Error Rate as obtained by the best path decoding heuristic (Graves, 2008). Neither a word dictionary nor a language model was used.

Every experiment was repeated 3 times. For each of the methods we observed that, often one of the experimental runs converged prematurely to a bad local optimum. These runs were not considered. The average character error rates of the remaining runs are summarized in Table 3.

5. Discussion

The experimental results show that the simple mechanism of running subsets of neurons at different speeds allows an RNN to efficiently learn the different dynamic time-scales inherent in complex signals.

CW-RNN showed superior performance on the speech data classification among all three models tested. Note that, unlike in the standard approach, in which the speech signal frequency coefficients are first translated to phonemes which are modeled with a standard approach like Hidden Markov Modes for complete words, the CW-RNN attempts to model and recognize the complete words directly, where it benefits from the modules running at multiple speeds.

For the task of handwriting recognition we observe that for small networks (10 k parameters), the performance of SRN is better that the one of CW-RNN, but this trend reverses for large networks (100 k parameters). This indicates, that there is a lot of complexity in the short-term structure of the task. Because a CW-RNN has fewer fast changing units it seems to suffer more from the size constraint. As we add more parameters, SRNs have trouble using the long-term structure and thus fall behind CW-RNN. In the end, the large CW-RNN are roughly 20% better than SRN. LSTM networks are better than both CW-RNN and SRN regardless to the network size.

A possible reason for LSTM being the best architecture so far for handwriting recognition could be that both short-term (spanning one or two characters) and long-term (a dot placed above the letter *i* to finish writing a word) information and its precise timing is important. An SRN is not capable of learning the long-term context and CW-RNNs exponential distribution of module timing is not flexible enough for identifying and utilizing the information that occurs far in the history.

6. Future Work

Future work will start by conducting a detailed analysis of the internal dynamics taking place in the CW-RNN to understand how the network is allocating resources for a given type of input sequence. Further testing on other classes of problems, such as reinforcement learning, and comparison to the larger set of connectionist models for sequential data processing are also planned.

Other functions could be used to set the module periods: linear, Fibonacci, logarithmic series, or even fixed random periods. These were not considered in this paper because the intuitive setup of using an exponential series, for the most part, worked well in these preliminary experiments. Another option would be to learn the periods as well, which, to use error back-propagation, would require a differentiable modulo function for triggering the clocks. Alternatively, one could train the clocks (together with the weights) using evolutionary algorithms which do not require a closed form for the gradient. Note that the lowest period in the network can be greater than 1. Such a network would not be able to change its output at every time step, which may be useful as a low-pass filter when the data contains noise.

Also, the modules do not have to be all of the same size. One could adjust them according to the expected information in the input sequences, by e.g. using frequency analysis of the data and setting up module sizes and clock rates proportional to the spectrum.

The low-speed modules read only a single input at a time and the part of the sequence that passes by while they are not active may not affect their output. One could use multiple low-speed modules that run at the same clock periods but interleave with a different offset, employ a buffer for the sequence and pass the low-speed modules a function of this buffer (e.g. mean) or transform the data first into some other domain (e.g. wavelet) and let the CW-RNN process the transformed information.

Appendix

CW-RNN has fewer total parameters and even fewer operations per time step than a SRN with the same number of neurons. Assume CW-RNN consists of g modules of size k for a total of $n = kg$ neurons. Because a neuron is only connected to other neurons with the same or larger period, the number of parameters N_H for the recurrent matrix is:

$$N_H = \sum_{i=1}^{g} \sum_{j=1}^{k} k(g-i+1) = k^2 \sum_{i=0}^{g-1} (g-i) = \frac{n^2}{2} + \frac{nk}{2}.$$

Compared to the n^2 parameters in the recurrent matrix $\mathbf{W}_H$ of SRN this results in roughly half as many parameters:

$$\frac{N_H}{n^2} = \frac{\frac{n^2}{2} + \frac{nk}{2}}{n^2} = \frac{n^2 + nk}{2n^2} = \frac{n+k}{2n} = \frac{g+1}{2g} \approx \frac{1}{2}.$$

Each module i is evaluated only every T_i-th time step, therefore the number of operations at a time step is:

$$O_H = k^2 \sum_{i=0}^{g-1} \frac{g-i}{T_i}.$$

For exponentially scaled periods, $T_i = 2^i$, the upper bound for number of operations, O_H, needed for $\mathbf{W}_H$ per time step is:

$$O_h = k^2 \sum_{i=0}^{g-1} \frac{g-i}{2^i} = k^2 \left(g \underbrace{\sum_{i=0}^{g-1} \frac{1}{2^i}}_{\leq 2} + \underbrace{\sum_{i=0}^{g-1} \frac{i}{2^i}}_{\leq 2} \right) \leq$$
$$\leq k^2(2g-2) \leq 2nk,$$

because $g \geq 2$ this is less than or equal to n^2. Recurrent operations in CW-RNN are faster than in a SRN with the same number of neurons by a factor of at least $g/2$, which, for typical CW-RNN sizes ends up being between 2 and 5. Similarly, upper bound for the number of input weight evaluations, E_I, is:

$$O_I = \sum_{i=0}^{g-1} \frac{km}{T_i} = km \sum_{i=0}^{g-1} \frac{1}{T_i} \leq 2km$$

Therefore, the overall CW-RNN speed-up w.r.t SRN is:

$$\frac{n^2 + nm + n}{O_R + O_I + 2n} = \frac{k^2 g^2 + kgm + kg}{k^2(2g-2) + 2km + 2kg} =$$
$$= \frac{g(kg+m+1)}{2(k(g-1)+m+g)} = \frac{g}{2} \underbrace{\frac{(kg+m+1)}{k(g-1)+m+g}}_{\geq \frac{1}{2}} \geq \frac{g}{4}$$

Note that this is a conservative lower bound.

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