

$$P = (x_1, y_1), Q = (x_2, y_2)$$

(lhs)

$$\begin{aligned} & ((x_1, y_1) \oplus_0 (x_2, y_2)) \oplus_0 \tau i(x_2, y_2) \xrightarrow{(x_2, y_2) = \tau \rho^l((x_1, y_1) \oplus_0 (x_2, y_2))} \\ &= (\tau \rho^{-l}(x_2, y_2)) \oplus_0 \tau i(x_2, y_2) \xrightarrow{\text{inversion invariance and } \tau \circ \tau = \text{id}} \\ &= \rho^{-l}(x_2, y_2) \oplus_0 i(x_2, y_2) \xrightarrow{\text{rotation invariance}} \\ &= \rho^{-l}((x_2, y_2) \oplus_0 i(x_2, y_2)) \xrightarrow{\text{inverse generalised (there is no delta condition for this in the projective setting)}} \\ &= \rho^{-l}(1, 0) \end{aligned}$$

(rhs) ~~$((x_1, y_1) \oplus_0 (x_2, y_2)) \oplus_0 \tau i(x_2, y_2)$~~

if we assume that we get to the right hand-side

$$\text{we get } [\tau P, 0] = [\tau(x_1, y_1), 0] = \{(x_1, y_1, 0), (x_1, y_1, 1)\}$$

since $x_1 \neq 0, y_1 \neq 0$ and (x_1, y_1) is an affine point.

$$\text{However } [\rho^{-l}(1, 0), 0] = \{(\rho^{-l}(1, 0), 0)\} \text{ since } \rho^{-l}(1, 0) \text{ has one zero component and is an affine point.}$$

The index 0 of the sum above is fixed by the deduction and cannot be changed as after applying associativity ---

$$((x_1, y_1) \oplus_0 (x_2, y_2)) \oplus_0 \tau i(x_2, y_2) =$$

$$= [(x_1, y_1) \oplus_0 (x_2, y_2)] \oplus_0 [\tau i(x_2, y_2)] =$$

$$= [(x_1, y_1) \oplus_0 (x_2, y_2)] \oplus_0 [(x_2, y_2)] =$$

$$[(x_1, y_1) \oplus_0 (x_2, y_2)] \oplus_0 [(x_2, y_2)] =$$

$$[(x_1, y_1) \oplus_0 (x_2, y_2)] \oplus_0 [(x_2, y_2)] =$$