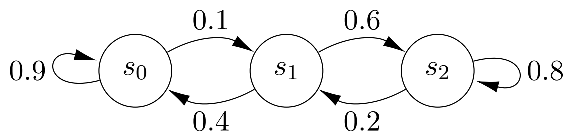
Quantitative Verification 7

Ex 1: Steady State

Compute the steady state distribution of the following Markov Chain by solving the corresponding linear equation system.



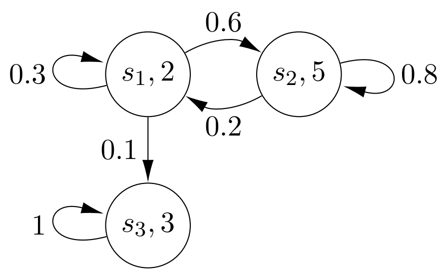
Ex 2: Proof

Prove the following statements:

- Let P be a stochastic matrix, i.e. the matrix representation of some Markov Chain. Then, $\frac{1}{2}P + \frac{1}{2}I$ is aperiodic, where I is the unit matrix.
- There exists a finite state Markov Chain with an unique stationary distribution π^*, but for any $n \in \mathbb{N}$ we have that $\pi_n = P^n \pi_0 \neq \pi^*$.
- In the lecture, we saw that if all states are irreducible, aperiodic and recurrent non-null in a Markov Chain, there is an unique limiting distribution which does not depend on π_0. Show that each of these properties is required by finding a Markov Chain which does (i) not have a unique limiting distribution, and (ii) satisfies all but one of the properties.

Ex 3: Average Reward

For each initial distribution π, compute the average reward obtained in the following Markov Chain.



Given an initial distribution π and step bound n, how can you compute the average (accumulated) n-step reward? What is the 2-step average reward for $\pi = \{s_1 \mapsto 1\}$?