Quantitative Verification 8 - Solutions

Ex 1: Reachability

We have $S_{=0} = \{s_3\}$ and $S_? = \{s_0, s_2, s_5\}$. The resulting equation system is
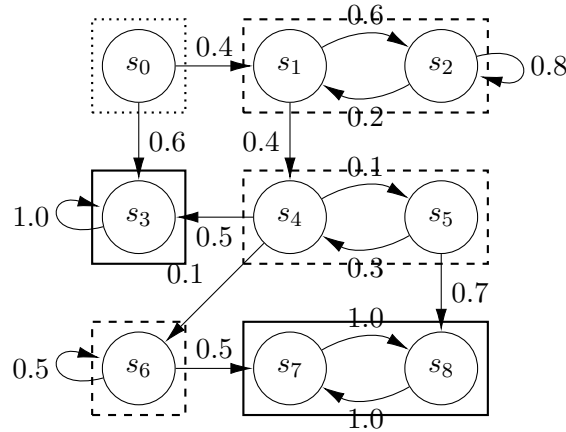
$$x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0.7 & 0 \end{bmatrix} x + \begin{bmatrix} 0.4 \\ 0.2 \\ 0.3 \end{bmatrix}$$

Reordering gives

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & -0.7 & 1 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.2 \\ 0.3 \end{bmatrix}$$

and we obtain $p_0 = 0.4$, $p_2 = 1$, and $s_5 = 1$.

Ex 2: Connected Components

Ex 3: Proof

There is a path between two states in the induced graph iff there is a sequence of transitions $p_i > 0$ between those states. This path has a probability of $\prod p_i > 0$.

Ex 4: Cost-Bounded Reachability

We can encode the currently accumulated costs in the state space ($S' = S \times \{0, \ldots, T\}$) and apply the usual methods.