

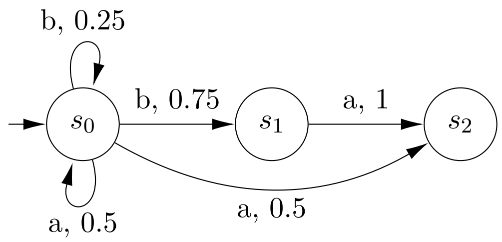
Quantitative Verification 11 - Solutions

Ex 1: Reachability LP

First, we compute $S_0^{\max} = \{s_5, s_4, s_3, s_1, s_0\}$, hence $S_0^{\min} = \{s_2\}$. We have that $S \setminus (B \cup S_0^{\max}) = \{s_0, s_1, s_3, s_4\}$. Then, we can derive the LP as follows:

$$\begin{aligned}
\text{minimize} \quad & \sum_{s\in S} x(s) \\
\text{where} \quad & x(s_5) = 1 \\
& x(s_2) = 0 \\
& x(s_0) \geq 0.5 \cdot x(s_1) + 0.5 \cdot x(s_2) \\
& x(s_1) \geq 0.5 \cdot x(s_0) + 0.5 \cdot x(s_3) \\
& x(s_3) \geq 0.5 \cdot x(s_0) + 0.5 \cdot x(s_4) \\
& x(s_4) \geq 0.25 \cdot x(s_4) + 0.75 \cdot x(s_5) \\
& x(s_4) \geq x(s_5)
\end{aligned}$$

Ex 2: Bounded Reachability

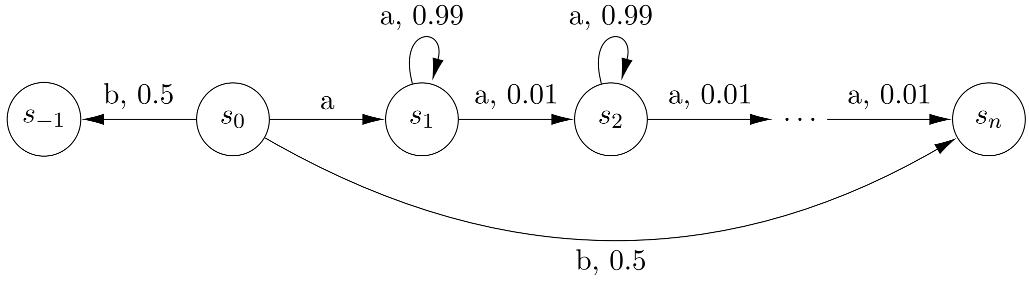


Target: $B = \{s_2\}$, Bound: $n = 2$.

The optimal reachability probability of can only be achieved by playing b and then a if remaining in s_0. This yields a total probability of $0.5 + 0.5 \cdot 0.75 = 0.875$, compared to the 0.75 achieved when playing only either a or b.

Solving bounded reachability: Value Iteration ($x_n(s) = \sup_\Theta \mathcal{P}_s^\Theta [\mathsf{F}^{\leq n} B]$) or encoding the step bound into an MDP and solving the unbounded reachability query.

Ex 3: Sound Value Iteration



To reach $\{s_n\}$, the optimal strategy is to play a everywhere, guaranteeing a probability of 1. Yet, value iteration will eventually stop with a value of 0.5 by playing b in s_0.

For more information, see [HM14; Bai+17; Brá+14].

References

[Bai+17] Christel Baier et al. "Ensuring the reliability of your model checker: Interval iteration for markov decision processes". In: *International Conference on Computer Aided Verification*. Springer. 2017, pp. 160–180.

[Brá+14] Tomáš Brázdil et al. "Verification of Markov decision processes using learning algorithms". In: *International Symposium on Automated Technology for Verification and Analysis*. Springer. 2014, pp. 98–114.

[HM14] Serge Haddad and Benjamin Monmege. "Reachability in MDPs: Refining convergence of value iteration". In: *International Workshop on Reachability Problems*. Springer. 2014, pp. 125–137.