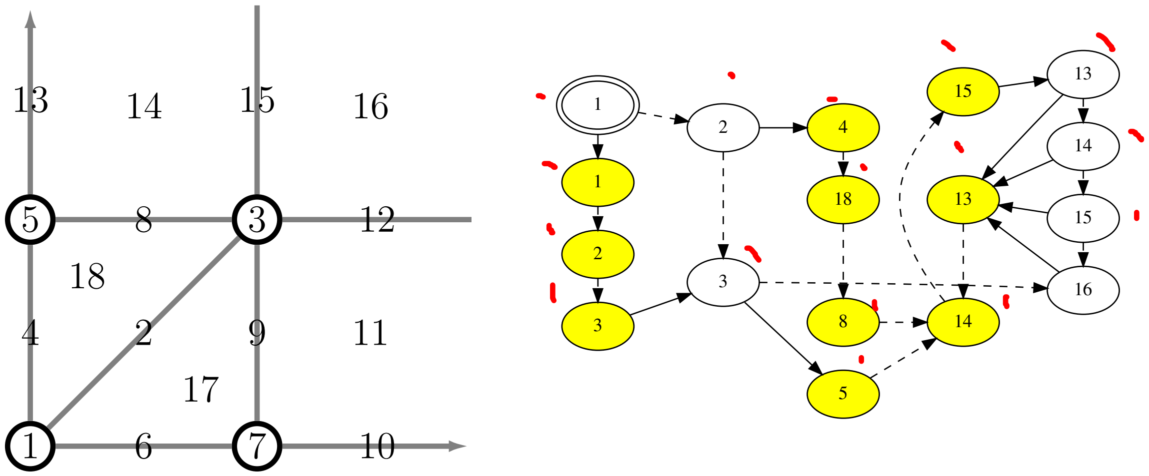
Technische Universität München
Institut für Informatik I07
Theoretical Computer Science WS 2018/19

Quantitative Verification 5 - Solutions

Ex 1: TCTL



Region Graph and corresponding Transition System. Yellow states are `on`, white are `off`.

Recall that the clock z, i.e. the y-axis, represents total elapsed time. As we can reach, e.g., the state $(\texttt{on}, 1)$ in the Region Transition System, $\exists\Diamond^{\leq 1}\texttt{on}$ holds. For $\forall\Diamond^{\leq 1}\texttt{on}$, observe that we have the trace $(\texttt{off}, 1)$, $(\texttt{off}, 2)$, $(\texttt{off}, 3)$, $(\texttt{off}, 16)$, which violates the property.

Ex 2: Reachability

The probability of reaching s_1 in at most n steps is $1 - 0.999^n \to 1$.

Ex 3: Matrix Representation

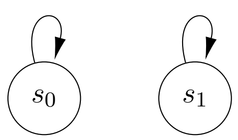
Transition matrix:

$$P = \begin{bmatrix} 0.9 & 0.1 & 0.0 \\ 0.4 & 0.0 & 0.6 \\ 0.0 & 0.2 & 0.8 \end{bmatrix}$$

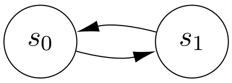
Transient distribution: $\pi_0 \cdot P^3 = (0.9, 0.1, 0) \cdot P^2 = [0.85, 0.09, 0.06] \cdot P = [0.801, 0.097, 0.102]$.

0.1 Ex 4: Proof

- For any state, we have $p_{ii}^1 \geq \frac{1}{2}$, hence $d_i = 1$.
- See the Markov Chain from Ex 2 with initial distribution $\pi_0 = [1, 0]$. Its stationary distribution equals $\pi^* = [0, 1]$, but we have $\pi_n = [0.999^n, 1 - 0.999^n]$.
- **Irreducible:** Each π_0 gives a distinct stationary distribution, namely itself.



Aperiodic: Take $\pi_0 = [1, 0]$, then $\pi_1 = [0, 1]$, $\pi_2 = [1, 0]$, ...



Recurrent non-null: No state is recurrent non-null, all recurrent null. No limiting distribution exists, since π_n converges to 0 point-wise.