

# Stochastic Timed Automata

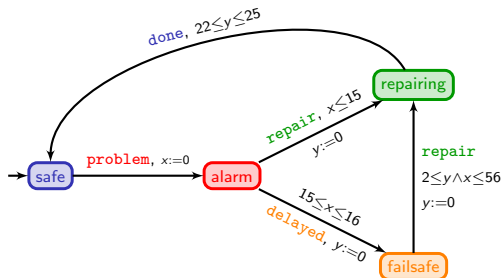
Patricia Bouyer-Decitre

LSV, CNRS & ENS Cachan, France

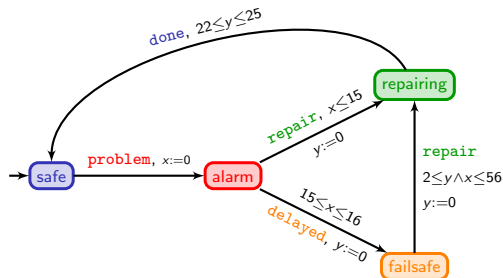
Based on joint works with Nathalie Bertrand, Thomas Brihaye,  
Pierre Carlier, Quentin Menet, Christel Baier, ...



# The model of timed automata



# The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\text{delayed}}$	failsafe	...
x	0		23		0		15.6		15.6	...
y	0		23		23		38.6		0	
	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\text{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\text{done}}$	safe	
...	15.6		17.9		17.9		40		40	
	0		2.3		0		22.1		22.1	

# An example: The task graph scheduling problem

Compute  $D \times (C \times (A+B)) + (A+B) + (C \times D)$  using two processors:

$P_1$  (fast):



time	
+	2 picoseconds
×	3 picoseconds

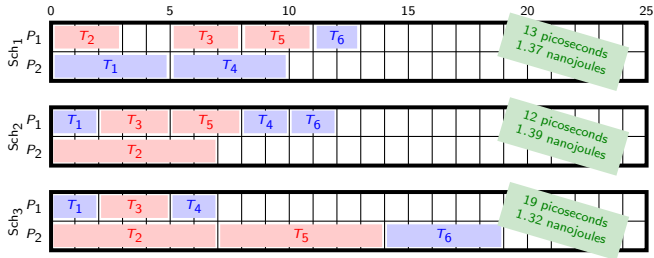
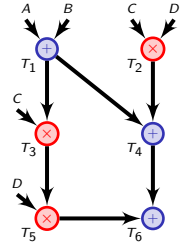
energy	
idle	10 Watt
in use	90 Watts

$P_2$  (slow):



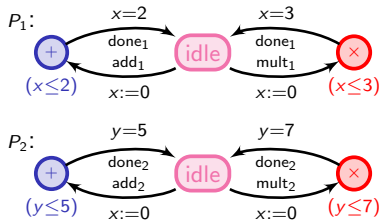
time	
+	5 picoseconds
×	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts

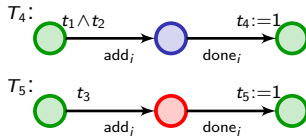


# Modelling the task graph scheduling problem

## Processors

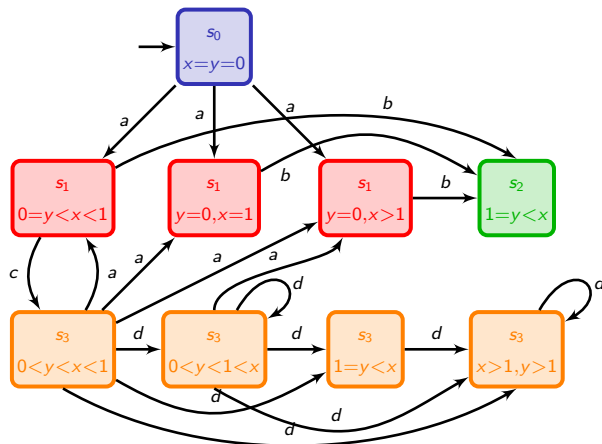
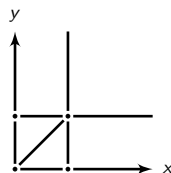
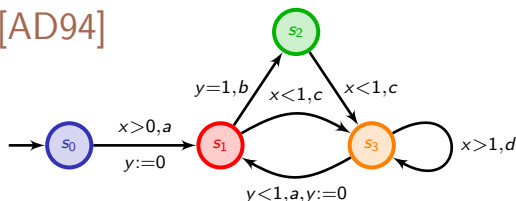


## Tasks



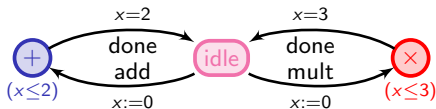
A schedule is a path in the product automaton

# An example [AD94]



# How to model uncertainty over delays?

- Using timed games



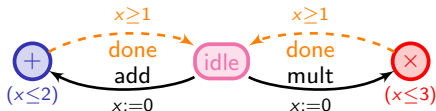
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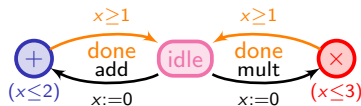


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- Using stochastic delays

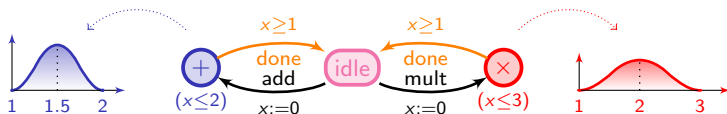


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# Existing models?

## Models based on timed automata

54 s.

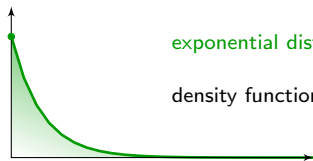
- Probabilistic timed automata [KNSS99]  
     $\leadsto$  only discrete probabilities over edges
- Continuous probabilistic timed automata [KNSS00]  
     $\leadsto$  resets of clocks are randomized, but only few results

[KNSS99] Kwiatkowska, Norman, Segala, Sproston. Automatic verification of real-time systems with discrete probability distributions (*ARTS'99*).

[KNSS00] Kwiatkowska, Norman, Segala, Sproston. Verifying quantitative properties of continuous probabilistic timed automata (*CONCUR'00*).

# How can we attach probabilities to delays?

- The example of continuous-time Markov chains

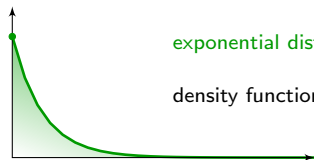


exponential distribution

$$\text{density function } t \mapsto \begin{cases} \lambda \cdot \exp(-\lambda t) & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

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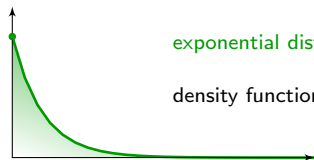
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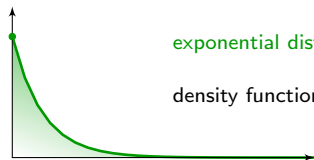
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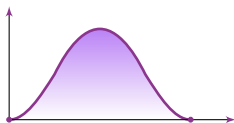


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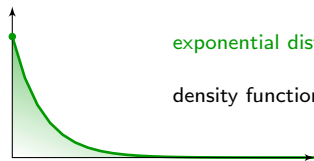
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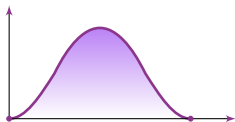


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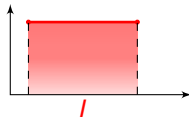
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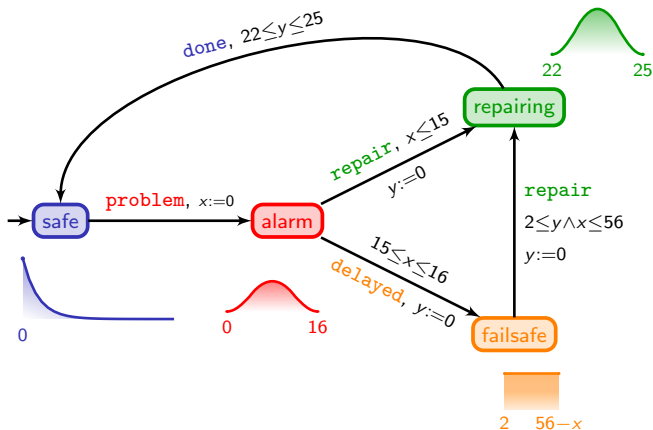
truncated normal distribution



uniform distribution

$$\text{density function } t \mapsto \begin{cases} \frac{1}{|I|} & \text{if } t \in I \\ 0 & \text{otherwise} \end{cases}$$

# How does a STA look like?



## Some remarks

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3:15

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- **Continuous-time Markov chains** = STA with a single “useless” clock which is reset on all transitions. The distributions on delays are exponential distributions with a rate per location
- Finite-state **generalized semi-Markov processes** (residual-lifetime semantics) are STAs (if no fixed-delay events)
- Allows to express richer timing constraints

# Almost-sure model-checking

We are interested in (automatic) model-checking algorithms!

- **Qualitative model-checking:** decide whether

$$\mathbb{P}(\{\varrho \in \text{Runs}(s) \mid \varrho \models \varphi\}) = 1$$

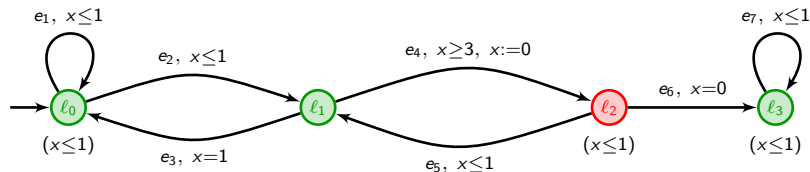
We write  $s \models \varphi$  whenever it is the case.

This is the almost-sure model-checking problem.

- **Quantitative model-checking:** compute (or approximate) the value

$$\mathbb{P}(\{\varrho \in \text{Runs}(s) \mid \varrho \models \varphi\})$$

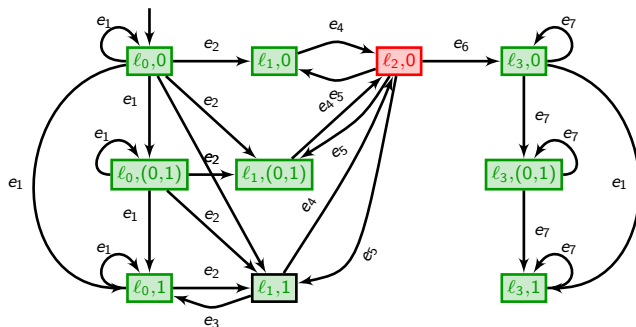
# An example



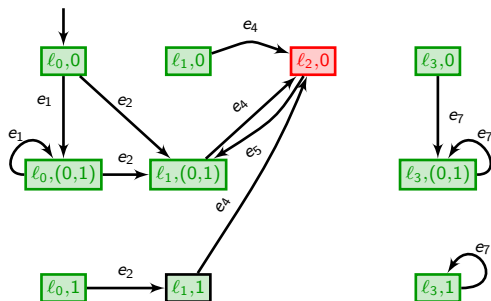
$$\mathcal{A} \not\models \mathbf{G}(\text{green} \Rightarrow \mathbf{F} \text{red}) \quad \text{but} \quad \mathbb{P}(\mathcal{A} \models \mathbf{G}(\text{green} \Rightarrow \mathbf{F} \text{red})) = 1$$

Indeed, almost surely, paths are of the form  $e_1^* e_2 (e_4 e_5)^\omega$

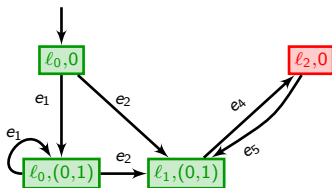
# The classical region automaton



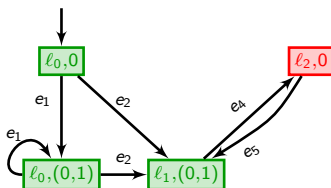
# The pruned region automaton



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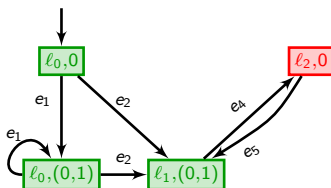


... viewed as a finite Markov chain  $MC(\mathcal{A})$

It holds as well that:

$$\mathbb{P}(MC(\mathcal{A}) \models \mathbf{G}(\text{green} \Rightarrow \mathbf{F} \text{red})) = 1$$

# The pruned region automaton



... viewed as a finite Markov chain  $MC(\mathcal{A})$

15:40

It holds as well that:

$$\mathbb{P}(MC(\mathcal{A}) \models \mathbf{G}(\text{green} \Rightarrow \mathbf{F} \text{red})) = 1$$

When is that the case that

$$\mathbb{P}(\mathcal{A} \models \varphi) = 1 \quad \text{iff} \quad \mathbb{P}(MC(\mathcal{A}) \models \varphi) = 1 ?$$