Exercise Sheet n°2

Exercise 1:

Give context free grammars that generate the following languages.

1. the language of well-bracketed words (cf. exercise 3.3 Exercise sheet n°1);
2. the language of all strings over the alphabet $\{(,),+,\cdot,a\}$ which represent syntactically correct arithmetic expressions with variable a such as
$$a,\ a+a\cdot a,\ (a+a)\cdot a,\ \ldots;$$
3. the language of all well-formed formulas of first order logic with variables among $x_1,\cdots,x_n$. That is all strings on the alphabet
$$\{\forall,\exists,(,),\wedge,\vee,\neg,=,x_1,\ldots,x_n\}$$
which represent valid formulas of first order logic.

Is the language of first order formulas with variables among $\{x_n \mid n\in\omega\}$ context free?

Exercise 2:

Give a pushdown automaton which recognises the language generated by the grammar $G=(V,\Sigma,R,S)$ where

$$V=\{S\},\quad \Sigma=\{(,),[,]\},\quad R=\{S\to\epsilon, S\to SS, S\to[S], S\to(S)\}.$$

Exercise 3:

A context free grammar $G=(V,\Sigma,R,S)$ is called *regular* if the set of relations is such that

$$R\subseteq(V\times\Sigma^*V)\cup(V\times\Sigma^*),$$

i.e. each rule is either of the form $N\to wM$ for some $N,M\in V$ and $w\in\Sigma^*$ or of the form $O\to w$ for some $O\in V$ and $w\in\Sigma^*$.

1. Define for any regular grammar G a NFA $N(G)$ which recognises the language generated by G.
2. Define for any language L recognised by a DFA a regular grammar $G(L)$ which generates L.
3. Conclude that a language is recognised by a DFA if and only if it is generated by a regular grammar.

Exercise 4:

1. Use the languages $\{a^m b^n c^n \mid m,n\in\omega\}$ and $\{a^n b^n c^m \mid m,n\in\omega\}$ and the fact that $\{a^n b^n c^n \mid n\in\omega\}$ is not context free to show that the context free languages are not closed under intersection.

2. Let C be a context free language and R be a regular language. Show that $C \cap R$ is context free.

3. Assuming that $\{a^n b^n c^n \mid n \in \omega\}$ is not context free, show that

$$\{w \in \{a, b, c\}^* \mid w \text{ contains an equal number of } a,\ b, \text{ and } c.\}$$

is not context free.